Sparse Unmixing of Hyperspectral Images

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Outline

1. Hyperspectral Images and Spectral Unmixing
   - Introduction
   - Spectral mixing/unmixing process
   - Related works
2. Similarity-guided and ℓ_p-regularized Sparse Unmixing (SLpSU)
   - Similarity-guided regularization
   - ℓ_p-norm regularization
   - Proposed model and numerical algorithm
   - Experiments
3. Framelet based Sparse Unmixing (FSU)
   - Framelets and image representation
   - Framelet based sparse unmixing model
   - Numerical algorithm
   - Experiments and conclusions
4. Conclusion and future work

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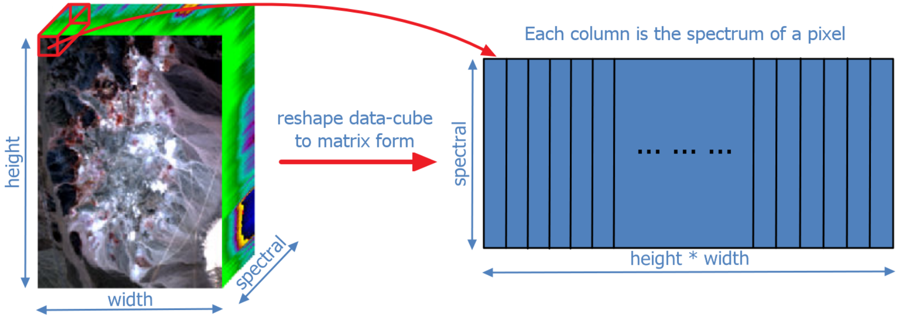
Hyperspectral image

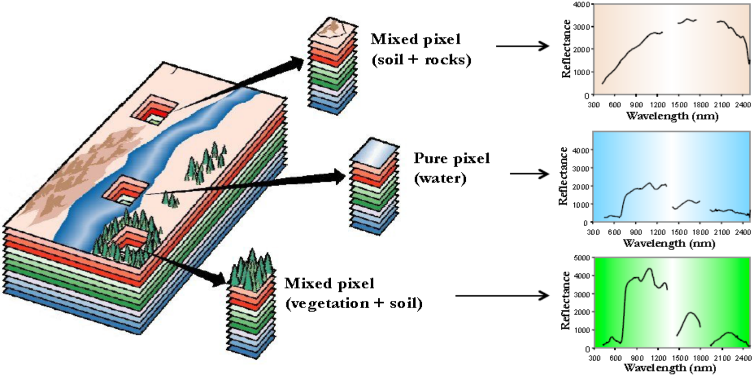
In hyperspectral imaging, the recorded spectra have fine wavelength resolution and cover a wide range of wavelengths.



Hyperspectral image

A single pixel contains several materials (endmembers), due to:

- low spatial resolution
- microscopic material mixing
- multiple scattering



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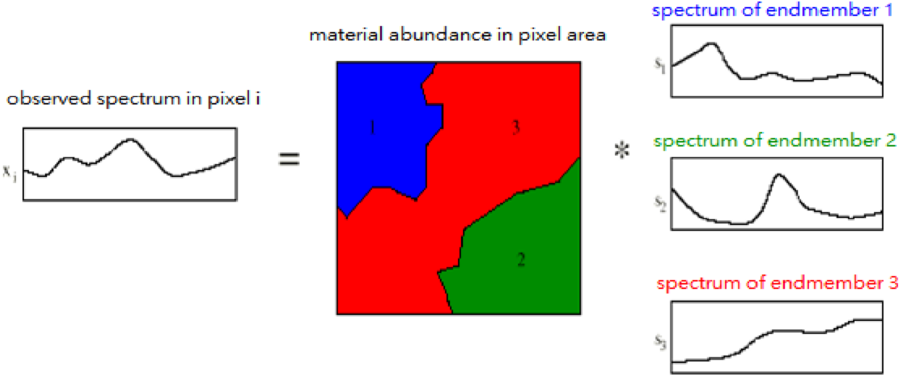
Spectral unmixing

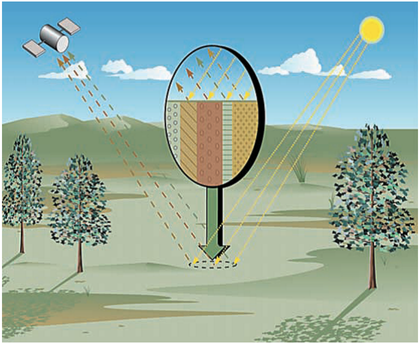
Spectral unmixing aims at extracting pure endmember spectra and corresponding abundance.

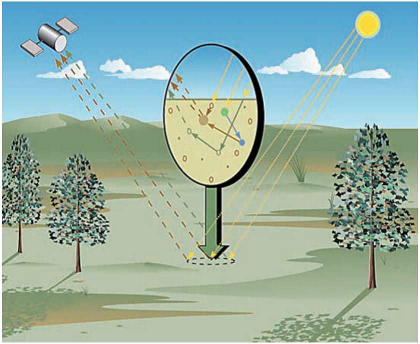


Mixing model

The mixture models can be divided into two categories:

- linear mixture model and nonlinear mixture model.





Hyperspectral unmixing process

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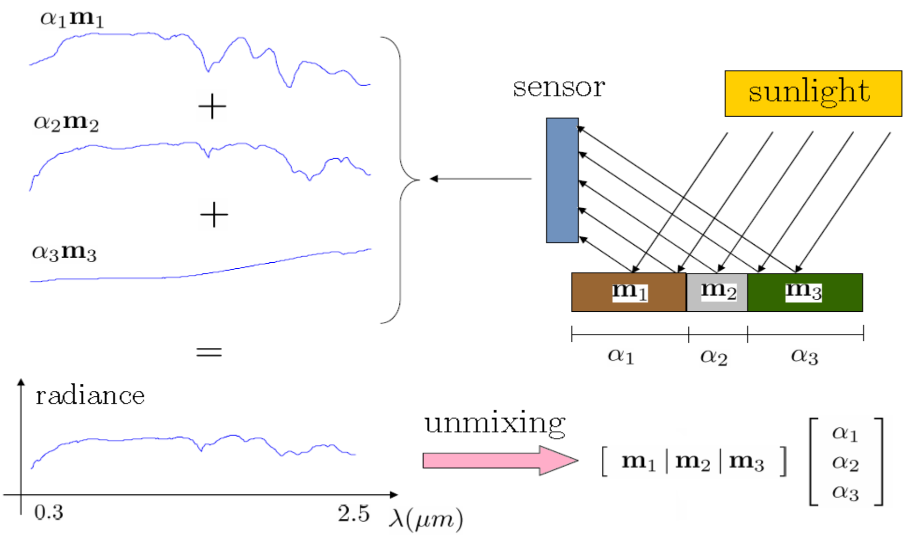
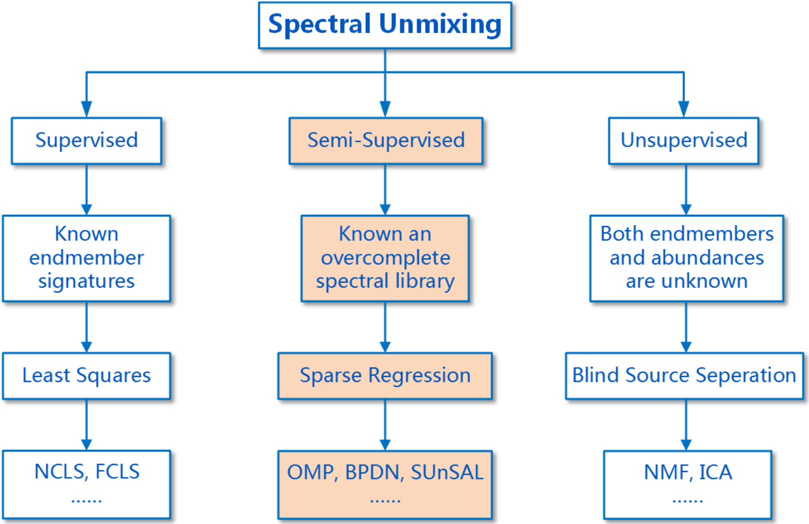


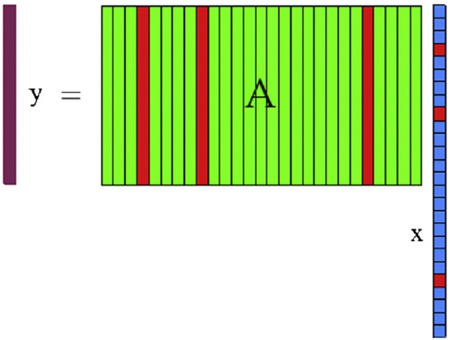
Hyperspectral unmixing process



Concept of the sparse hyperspectral unmixing

The pixel observation y can be expressed as *a linear combination* of spectral signatures in a spectral library A as:

$$y = Ax + n$$



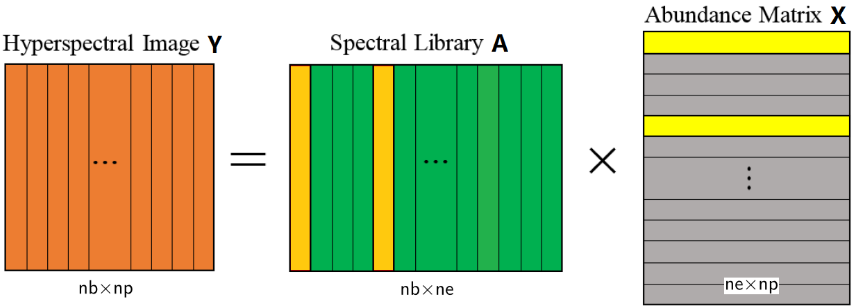
Noise-tolerant sparse regression optimization problem is:

$$\min_{x} ||x||_0 \quad \text{subject to} \quad ||y - Ax||_2 \leq \delta, \quad x \geq 0, \quad \mathbf{1}^T x = 1$$

Sparsity-based simultaneous linear unmixing

Linear mixing model written in a matrix form is:

$$Y = AX + N$$



Noise-tolerant sparse regression optimization problem is:

$$\min_{X} ||X||_0 \quad \text{subject to} \quad ||Y - AX||_F \leq \delta, \quad X \geq 0, \quad \sum_{i=1}^{ne} X_{ij} = 1 \quad \forall j$$

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Related works

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Table: Some popular sparse unmixing methods

Methods	Data fidelity term	Regularization term
SUnSAL	$\|Ax - y\|_2^2$	$\|x\|_1$
SUnSAL-TV	$\|AX - Y\|_F^2$	$\|X\|_{1,1} + \lambda_{TV}\mathrm{TV}(X)$
DSUnADM	$\|HXA - Y\|_F^2$	$\|X\|_{1,1} + \lambda_{TV}\mathrm{TV}(X)$
CLSUnSAL	$\|AX - Y\|_F^2$	$\|X\|_{2,1}$
[F. Chen, 13]	$\|Ax - y\|_2^2$	$\|x\|_p^p,\ 0 < p < 1$
[Giampouras P, 15]	$\|AX - Y\|_F^2$	$\|R \odot X\|_1 + \lambda\|X\|_{b,*}$

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Similarity-guided regularization

- Linear mixing model:

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \boldsymbol{\Theta} \tag{1}$$

- Assumption: pixels with similar spectra have similar abundances.
- Weighting matrix:

$$\mathbf{W}_{ij} = \begin{cases} e^{-\frac{||\mathbf{y_i}-\mathbf{y_j}||_2}{\sigma}} & j \in \eta_i \\ 0 & j \notin \eta_i \end{cases}$$

  where σ denotes the kernel's bandwidth, and η_i represents a small window center on the ith pixel.

- $\mathbf{W}_{ij}$ measures the similarity of the two spectrums $\mathbf{y_i}$ and $\mathbf{y_j}$.
- The more similar the two pixels, the larger the value of $\mathbf{W}_{ij}$.

Similarity-guided regularization

- Our similarity-weighted abundance constraint:

$$R_1(\mathbf{X}) = \frac{1}{2}\sum_{i,j=1}^{k} ||\mathbf{x}_i - \mathbf{x_j}||_2^2 \mathbf{W}_{i,j} \tag{2}$$

where $\mathbf{x_i}$ is the ith column of $\mathbf{X}$.

- $R_1(\mathbf{X})$ makes good use of the inherent structural information of the data as a priori, leading the abundances estimation to become data-guided.

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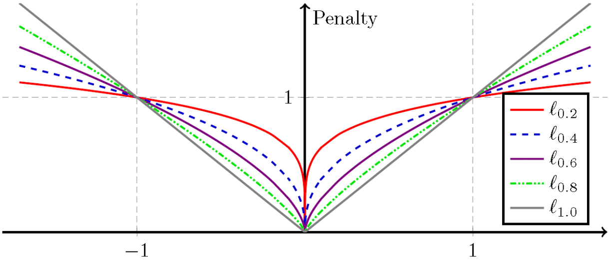
ℓ_p-norm regularization

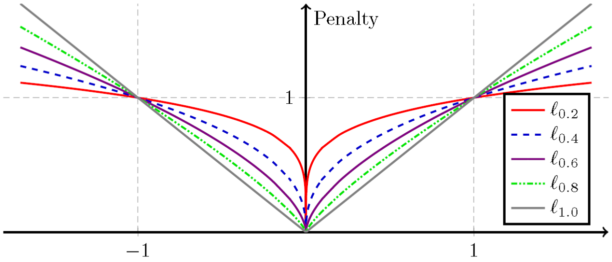
- We use the ℓ_p $(0 < p < 1)$ norm as the sparsity regularization with respect to $\mathbf{X}$, i.e.

$$R_2(\mathbf{X}) = ||\mathbf{X}||_p^p = \sum_{i=1}^{m} \sum_{j=1}^{k} |\mathbf{X}_{ij}|^p. \tag{3}$$



Superior of ℓ_p-norm

- Nonconvex but better approximates the ℓ_0-norm.
- Similar solution but more sparse.
- Confirmed by ℓ_p-norm regularized unmixing method.
- Able to enforce the sum-to-one constraint.



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Proposed model

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- The proposed unmixing model:

$$\min_{\mathbf{X}} \frac{1}{2}||\mathbf{Y}-\mathbf{A}\mathbf{X}||_F^2 + \frac{\lambda}{2}\sum_{i,j=1}^{k} ||\mathbf{x_i}-\mathbf{x_j}||_2^2 \mathbf{W}_{ij} + \mu||\mathbf{X}||_p^p$$
$$\text{subject to} \quad \mathbf{X} \geq 0, \quad \sum_{i=1}^{m} \mathbf{X}_{ij} = 1 \ \ \forall j. \tag{4}$$

Combining the abundance nonnegative constraint and sum-to-one constraint in (13) into the function:

$$\min_{\mathbf{X}} \frac{1}{2}||\mathbf{Y}-\mathbf{A}\mathbf{X}||_F^2 + \frac{\lambda}{2}\sum_{i,j=1}^{k} ||\mathbf{x_i}-\mathbf{x_j}||_2^2 \mathbf{W}_{i,j} + \mu||\mathbf{X}||_p^p$$
$$+ \delta(\mathbf{X}) + \frac{1}{2}||\mathbf{1}^T\mathbf{X}-\mathbf{1}^T||_F^2 \tag{5}$$

where each element of $\delta(\mathbf{X})$ is zero if $\mathbf{X}_{i,j} \geq 0$ and $+\infty$ otherwise, and $\mathbf{1}^T$ denotes the vector of 1s.

Numerical algorithm

- Variable splitting:

$$\min_{\mathbf{X},\mathbf{V},\mathbf{Z},\mathbf{N}} \frac{1}{2}||\mathbf{Y}-\mathbf{AX}||_F^2 + \frac{\lambda}{2}\sum_{i,j=1}^{k}||\mathbf{v_i}-\mathbf{v_j}||_2^2\mathbf{W}_{i,j} + \mu||\mathbf{Z}||_p^p$$
$$+\delta(\mathbf{N}) + \frac{1}{2}||\mathbf{1}^T\mathbf{X}-\mathbf{1}^T||_F^2 \tag{6}$$
$$\text{subject to}\quad \mathbf{V}=\mathbf{X},\ \mathbf{Z}=\mathbf{X},\ \mathbf{N}=\mathbf{X}.$$

The augmented Lagrangian for problem (6):

$$\mathcal{L}_\rho(\mathbf{X},\mathbf{V},\mathbf{Z},\mathbf{N},\mathbf{D}_1,\mathbf{D}_2,\mathbf{D}_3) =$$
$$\frac{1}{2}||\mathbf{Y}-\mathbf{AX}||_F^2 + \frac{\lambda}{2}\sum_{i,j=1}^{k}||\mathbf{v_i}-\mathbf{v_j}||_2^2\mathbf{W}_{i,j} + \mu||\mathbf{Z}||_p^p$$
$$+\delta(\mathbf{N}) + \frac{1}{2}||\mathbf{1}^T\mathbf{X}-\mathbf{1}^T||_F^2 + \langle\mathbf{D}_1,\mathbf{X}-\mathbf{V}\rangle \tag{7}$$
$$+\frac{\rho}{2}||\mathbf{X}-\mathbf{V}||_F^2 + \langle\mathbf{D}_2,\mathbf{X}-\mathbf{Z}\rangle + \frac{\rho}{2}||\mathbf{X}-\mathbf{Z}||_F^2$$
$$+\langle\mathbf{D}_3,\mathbf{X}-\mathbf{N}\rangle + \frac{\rho}{2}||\mathbf{X}-\mathbf{N}||_F^2$$

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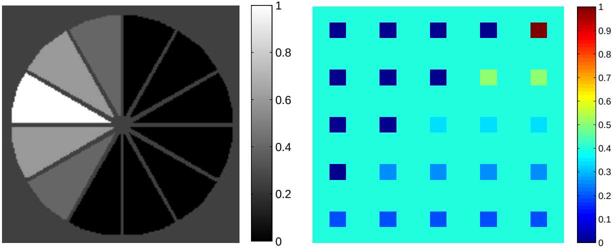
Experiments with simulated data



(a) (b)

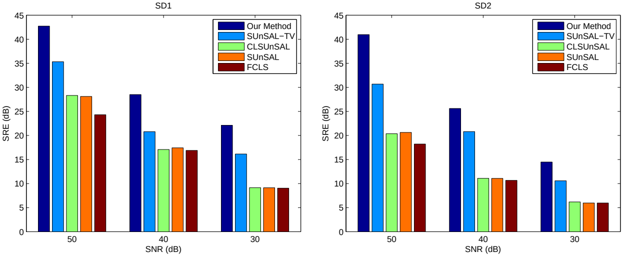
Figure: Two simulated data sets.

- Evaluation metric: signal-to-reconstruction error (SRE)

$$\mathrm{SRE(dB)} = 10 \log_{10}(\mathbb{E}[\|\mathbf{X}\|_F^2]/\mathbb{E}[\|\mathbf{X} - \hat{\mathbf{X}}\|_F^2]),$$

- Higher SRE(dB) value represents a better unmixing performance.

Experiments with simulated data



(a) (b)

Figure: SRE(dB) values on the simulated data contaminated by noise.

- We obtain on average **5 SRE(dB) values greater** than the second best SUnSAL-TV.
- The per-pixel execution time of our method (0.0052 s) is nearly **one-fifth** of the SUnSAL-TV (0.0259 s)

Experiments with real data

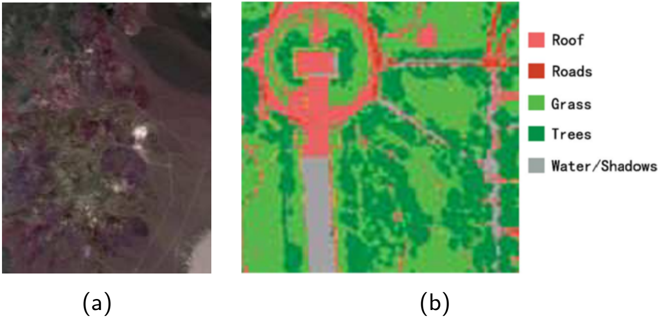
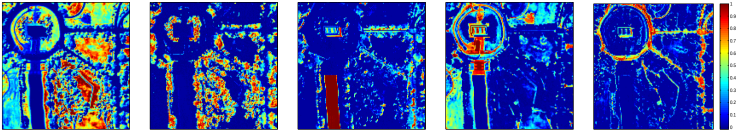


Figure: Two real data sets.

- Reconstruction error (RE): RE $= \sqrt{\frac{1}{size(\mathbf{Y})} \times \|\mathbf{Y} - \mathbf{A}\hat{\mathbf{X}}\|_F^2}$.
- Sparsity: $\frac{1}{size(\hat{\mathbf{X}})} \times \#(\hat{\mathbf{X}} = 0) \approx \frac{1}{size(\hat{\mathbf{X}})} \times \#(\hat{\mathbf{X}} \leq 0.001)$.

Experiments with real data



(a) (b) (c) (d) (e)

Figure: Fractional abundance maps: (a) grass; (b) tree; (c) water/shadows; (d) roof; (e) road.

Table: the reconstruction errors and the sparsity of abundances on the second real data set

	Ours	SUnSAL-TV	CLSUnSAL	SUnSAL	FCLS
RE	**0.0084**	0.0121	0.0152	0.0283	0.0352
Sparsity	**0.4870**	0.4256	0.4657	0.4829	0.4822

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Image representation using framelets

- Let $\chi \subset L^2(\mathbb{R})$ be a countable function subset, if

$$f = \sum_{g \in \chi} \langle f, g \rangle g, \quad \forall f \in L^2(\mathbb{R}),$$

  where $\langle \cdot, \cdot \rangle$ is the inner product in $L^2(\mathbb{R})$. Then χ is called a tight frame of $L^2(\mathbb{R})$.

- The tight frame is a generation of the orthogonal basis which relaxes the requirements of the orthogonality and linear independence.

Image representation using framelets

- Important property:

$$\mathcal{W} = \begin{bmatrix} \mathcal{W}_0 \\ \mathcal{W}_1 \end{bmatrix}, \tag{8}$$

where $\mathcal{W}_0$ denotes the low-pass filter operator, and $\mathcal{W}_1$ consists of remaining band-pass and high-pass filter operators.

- $$\mathcal{W}^T\mathcal{W} = \mathcal{W}_0^T\mathcal{W}_0 + \mathcal{W}_1^T\mathcal{W}_1 = \mathcal{I}, \tag{9}$$

where $\mathcal{W}^T$ and $\mathcal{I}$ are the inverse framelet transform and equivalent transform, respectively.

- $$\mathcal{W}\mathcal{W}^T \neq \mathcal{I}$$

Image representation using framelets

- Given an hyperspatial image Y, the framelet coefficient vector C after one level **framelet decomposition** is given by

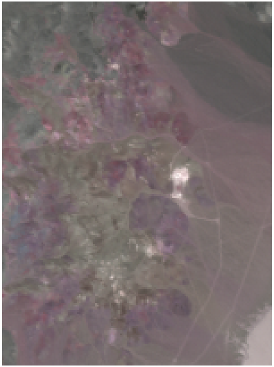
$$C = \mathcal{W}Y = \begin{bmatrix} \mathcal{W}_0 Y \\ \mathcal{W}_1 Y \end{bmatrix}, \tag{10}$$

where $\mathcal{W}_0 Y$ and $\mathcal{W}_1 Y$ are the approximation and detail coefficients, respectively.

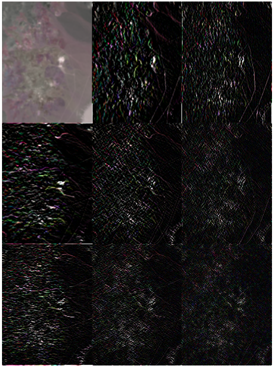
- **framelet reconstruction**:

$$Y = \mathcal{W}^T C = \mathcal{W}_0^T C + \mathcal{W}_1^T C$$

Image representation using framelets



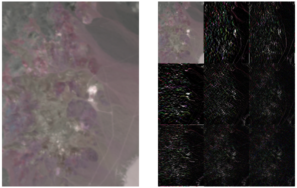
(a) Y



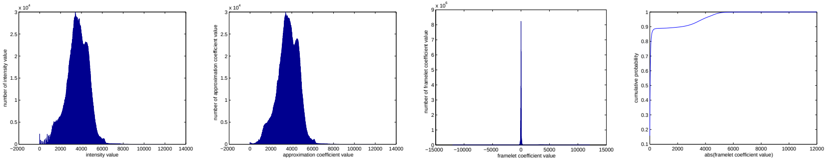
(b) C

Figure: An example of framelet decomposition. (a) the original image (RGB composite of the AVIRIS Cuprite subimage using bands 183, 193, and 203); (b) the result of one level framelet decomposition.

Image representation using framelets



(a) Y (b) C



(c) (d) (e) (f)

Figure: An example of framelet decomposition. (a) the original image; (b) the result of one level framelet decomposition; (c) the histogram of the original image; (d) the histogram of the approximation coefficient value; (e) the histogram of the framelet coefficient value; (f) cumulative probability distribution corresponding to (e).

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Framelet based sparse unmixing model (FSU)

The construction of FSU model utilizes the sparse property of framelet representation.

- Framelet-based sparsity (regularization term)
  - To preserve the sparse property of framelet coefficient value of abundance
- Robust reconstruction (fidelity term)
  - Split framelet coefficient into two components to reduce the interference of the additive noise.

- Framelet-based sparsity (regularization term)
- *Nonnegativity:*

$$\min_X \|\iota(X)\|_{1,1} := \sum_{i,j} |\iota(X_{i,j})|,$$

where

$$\iota(X_{i,j}) = \begin{cases} 0, & X_{i,j} \geq 0 \\ +\infty, & \text{otherwise} \end{cases}.$$

- *Sparsity:*

$$\begin{aligned} \min_X \|\mathcal{W}X\|_{1,1} &= \|\mathcal{W}_0 X\|_{1,1} + \|\mathcal{W}_1 X\|_{1,1} \\ &\approx \|X\|_{1,1} + \|\mathcal{W}_1 X\|_{1,1}. \end{aligned} \tag{11}$$

Framelet based sparse unmixing model (FSU)

- Robust reconstruction (fidelity term)

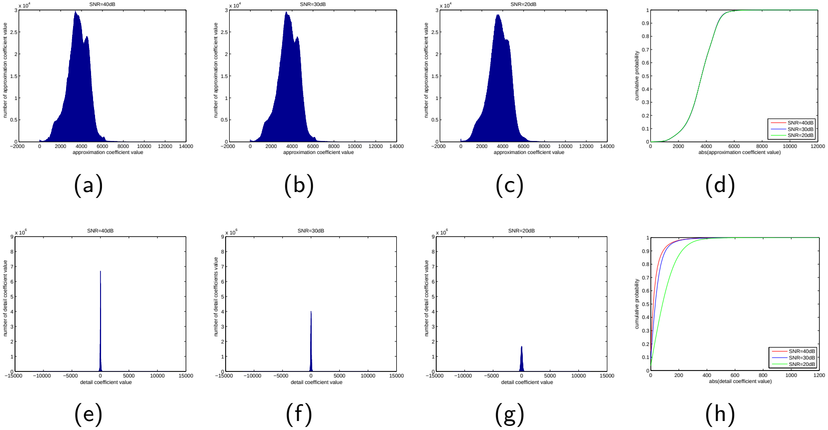

Figure:The histograms of the framelet coefficient value of Cuprite data.

- Robust reconstruction (fidelity term)
  Observation: At different noise levels,
  - the histograms of approximation coefficient value are similar;
  - the histograms of detail coefficient value are significantly distinct with each other.

  Conclusion:
  - The noise has little influence on approximation parts;
  - we can treat the approximation and detail parts separately.

  Leading to:

$$\frac{1}{2}\left\| \begin{bmatrix} \mathcal{W}_0(AX-Y) \\ \sqrt{\alpha}\mathcal{W}_1(AX-Y) \end{bmatrix} \right\|_F^2,$$

  or equivalently:

$$\frac{1}{2}\|\mathcal{W}_0 AX - \mathcal{W}_0 Y\|_F^2 + \frac{\alpha}{2}\|\mathcal{W}_1 AX - \mathcal{W}_1 Y\|_F^2. \tag{12}$$

Total energy:

$$\min_X E(X) = \frac{1}{2}\|\mathcal{W}_0 AX - \mathcal{W}_0 Y\|_F^2 + \frac{\alpha}{2}\|\mathcal{W}_1 AX - \mathcal{W}_1 Y\|_F^2 + \beta\|\mathcal{W}X\|_{1,1} + \|\iota(X)\|_{1,1}, \quad (13)$$

Theorem

Let $Y, A, X \in \mathbb{R}^2$, and $\alpha, \beta > 0$, then the minimization problem (13) admits a unique solution in $\mathbb{R}^2$.

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Numerical algorithm

Total energy:

$$\min_X E(X) = \frac{1}{2}\|\mathcal{W}_0 AX - \mathcal{W}_0 Y\|_F^2 + \frac{\alpha}{2}\|\mathcal{W}_1 AX - \mathcal{W}_1 Y\|_F^2 + \beta\|\mathcal{W}X\|_{1,1} + \|\iota(X)\|_{1,1},$$

rewritten to the following equivalent constrained problem:

$$\min_{X,V_1,V_2,V_3} \frac{1}{2}\|\mathcal{W}_0 V_1 - \mathcal{W}_0 Y\|_F^2 + \frac{\alpha}{2}\|\mathcal{W}_1 V_1 - \mathcal{W}_1 Y\|_F^2 + \beta\|V_2\|_{1,1} + \|\iota(V_3)\|_{1,1},$$
$$\text{subject to} \begin{cases} V_1 = AX \\ V_2 = \mathcal{W}X \\ V_3 = X \end{cases}. \tag{14}$$

Numerical algorithm

Overall algorithm:

- **Input:** spectral library A, observed data Y.
- **Initialize:**
  set $t=0$, $X^{(0)}=(A^TA)^{-1}A^TY$, $V_1^{(0)}=AX^{(0)}$, $V_2^{(0)}=\mathcal{W}X^{(0)}$, $V_3^{(0)}=X^{(0)}$, $D_1^{(0)}=D_2^{(0)}=D_3^{(0)}=0$;
  fix $\mu>0$, $\alpha>0$ and $\beta>0$.
- **Repeat:**
  $$X^{(t+1)}=(A^TA+2I)^{-1}[A^T(V_1^{(t)}+D_1^{(t)})$$
  $$+\mathcal{W}^T(V_2^{(t)}+D_2^{(t)})+(V_3^{(t)}+D_3^{(t)})],$$
  $$V_1^{(t+1)}=\mathcal{F}^{-1}\left(\frac{\mathcal{F}(K)\odot\mathcal{F}(Y)+\mu\mathcal{F}\left(AX^{(t+1)}-D_1^{(t)}\right)}{\mathcal{F}(K)+\mu}\right),$$
  $$V_2^{(t+1)}=\max\{|\mathcal{W}X^{(t+1)}-D_2^{(t)}|-\frac{\beta}{\mu},0\}\frac{\mathcal{W}X^{(t+1)}-D_2^{(t)}}{|\mathcal{W}X^{(t+1)}-D_2^{(t)}|},$$
  $$V_3^{(t+1)}=\max(X^{(t+1)}-D_3^{(t)},0),$$
  $$D_1^{(t+1)}=D_1^{(t)}+V_1^{(t+1)}-AX^{(t+1)},$$
  $$D_2^{(t+1)}=D_2^{(t)}+V_2^{(t+1)}-\mathcal{W}X^{(t+1)},$$
  $$D_3^{(t+1)}=D_3^{(t)}+V_3^{(t+1)}-X^{(t+1)},$$
  update iteration: $t=t+1$,
- **Until** the stopping criterion is satisfied.
- **Output:** abundances X.

Outline

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   - Spectral mixing/unmixing process
   - Related works
2. Similarity-guided and ℓ_p-regularized Sparse Unmixing (SLpSU)
   - Similarity-guided regularization
   - ℓ_p-norm regularization
   - Proposed model and numerical algorithm
   - Experiments
3. Framelet based Sparse Unmixing (FSU)
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   - Experiments and conclusions
4. Conclusion and future work

Experiments with simulated data

Two metrics:

- Signal to reconstruction error (SRE)

$$\text{SRE} \equiv \frac{\mathbb{E}[\|X_{true}\|_F^2]}{\mathbb{E}[\|X_{true} - X\|_F^2]},$$

  Generally, SRE is measured in dB: $\text{SRE(dB)} \equiv 10\log_{10}(\text{SRE})$.

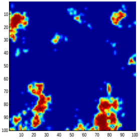
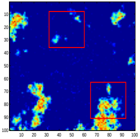
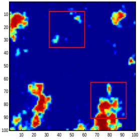
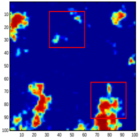
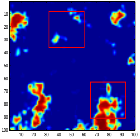
- Root mean square error (RMSE)

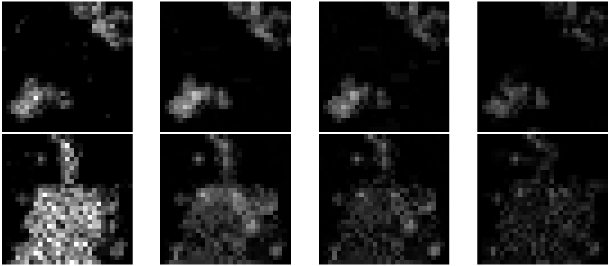
$$\text{RMSE} = \frac{1}{\sqrt{size(X)}}\mathbb{E}[\|X_{true} - X\|_F^2],$$

- The higher SRE and lower RMSE value represent the better unmixing performance.

Experiments with simulated data

華東師範大學 EAST CHINA NORMAL UNIVERSITY

	Groundtruth	CLSUnSAL	DSUnADM	SUnSAL-TV	FSU
SNR=20 dB					



Experiments with simulated data

Table: SRE(dB) and RMSE values of different unmixing methods to the simulated data cube on different noise levels

SNR		CLSUnSAL	DSUnADM	SUnSAL-TV	FSU
40 dB	SRE	14.6597	18.1897	18.2274	**19.4870**
	RMSE	0.0134	0.0089	0.0089	**0.0077**
30 dB	SRE	8.1536	12.9399	12.9881	**14.6790**
	RMSE	0.0283	0.0163	0.0162	**0.0134**
20 dB	SRE	4.1745	7.2897	7.3858	**9.9667**
	RMSE	0.0448	0.0313	0.0310	**0.0230**

Experiments with real data

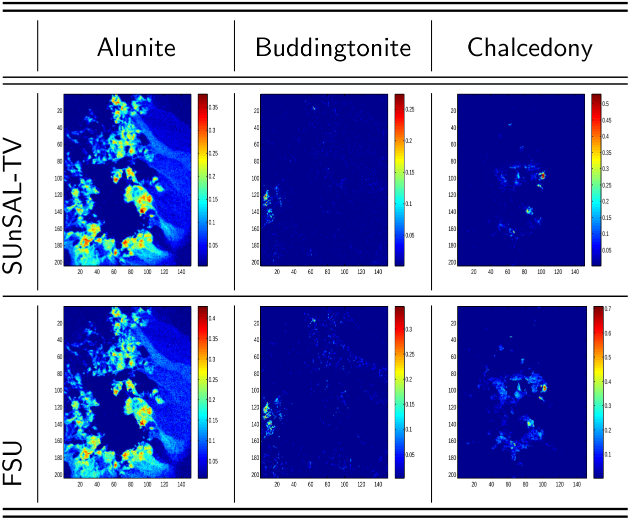


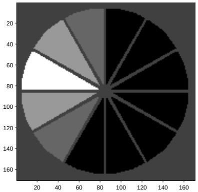
Figure: The abundance maps estimated by SUnSAL-TV, and FSU for the subset of the AVIRIS Cuprite scene at SNR=40 dB noise level.

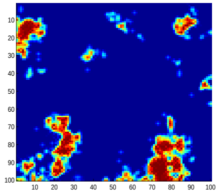
Experiments with real data

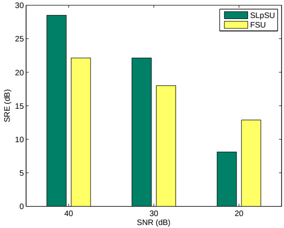
Table: the reconstruction errors (RE) and the sparsity of abundances on real data experiment

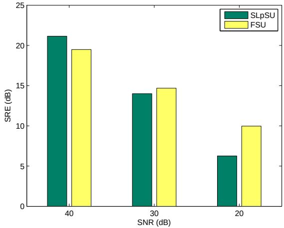
Algorithms	CLSUnSAL	DSUnADM	SUnSAL-TV	FSU
RE	0.0338	0.0088	0.0080	**0.0078**
Sparsity	0.6499	0.4293	0.6356	**0.6988**

Conclusion









- SLpSU better than FSU in piecewise smooth data with small noise.
- FSU better than SLpSU in complex data with large noise.

Future work

- Construct a proper low-rank unmixing model.
- Unmixing the data in tensor form.
- Unmixing the data with incomplete spectral library.

Thanks!