Two-dimensional MTF for pure tilt modulation

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Abstract

Abstract. This paper introduces a two-dimensional tilt modulation transfer function (MTF). It is an extension to the known one-dimensional tilt MTF for the cases when shadowing can be disregarded. The limiting conditions for the MTF are analyzed numerically for different antenna heights and sea states. The method is(HOPEFULLY) validated by the usage of a coherent radar where the Doppler image serves as ground truth for the wave spectrum.

1 Introduction

The translation of radar images of the ocean into surface maps have long been a challenge. BLA BLA BLA

This work is motivated from the need to correlate radar measurements to numerous ADCP measurements in location of the radar footprint that are not ideal for analysis according to standard methods. Established analysis methods of radar images focus on patches of the radar footprint that are oriented into the main wave direction. The main downside of this approach is the limitation to a small area and not flexible ...

When observing coastal areas it is of great interest to observe a larger area parallel with the shore line. ...

2 Theory

The MTF used herein is an extension of the one-dimensional case described in **?**. The ... assumes that data is used that allow the assumption of no shadows in the image. It is further assumed that some pre-processig of the radar image has taken place, i.e. range-dependent imaging effects related to decay in the electromagnetic energy have been removed. The pure tilt based MTF can then be estimated from geometrical principles. It is assumed that the amplitude of the backscatter signal is proportional to the local incidence angle θ_l that is defined as angle between the surface normal $\mathbf{n}$ and the backscatter vector $\mathbf{b}$ pointing form the surface to the radar antenna.

$$\mathbf{n} \cdot \mathbf{b} = |\mathbf{n}|\,|\mathbf{b}|\cos(\theta_l), \tag{1}$$

$$\mathbf{n} = (-\partial\eta/\partial x, -\partial\eta/\partial y, 1) \tag{2}$$
$$\mathbf{b} = (-x, -y, H-\eta) \tag{3}$$

The lengths of the surface normals are defined by

$$|\mathbf{n}| = \sqrt{(\partial\eta/\,\partial x)^2 + (\partial\eta/\,\partial y)^2. + 1} \tag{4}$$
$$|\mathbf{b}| = \sqrt{r^2 + (H-\eta)^2} \tag{5}$$

and can be approximated by

$$|\mathbf{n}| = 1, \tag{6}$$
$$|\mathbf{b}| = r, \tag{7}$$

assuming $(\partial\eta/\,\partial x)^2 \ll 1$, $(\partial\eta/\,\partial y)^2 \ll 1$ and $(H-\eta)^2 \ll r^2$.

After replacing $x = r\cos(\phi)$ and $y = r\sin(\phi)$, Equation 1 can then be approximated by

$$\cos(\phi)\partial\eta/\partial x + \sin(\phi)\partial\eta/\partial y + \frac{(H-\eta)}{r} = \cos(\theta_l). \tag{8}$$

To find the MTF, Fourier transform $\mathcal{F}$ is applied. The term $\frac{H-\eta}{r}$ contributes energy at low wavenumbers that are typically filtered. The term is therefor neglected (compare **?**). Based on the grazing incidence assumption that θ_l is close to $\pi/2$ we also approximate $\cos(\theta_l) \approx \theta_l$ NOTE: we could also apply the cosine term to the measurement!!!. The two-dimensional tilt forward MTF is hence given by

$$\mathcal{F}\left[\cos(\phi)\right] * \left[\mathrm{i}k_x\mathcal{F}(\eta)\right] + \mathcal{F}\left[\sin(\phi)\right] * \left[\mathrm{i}k_y\mathcal{F}(\eta)\right] \approx \mathcal{F}(\theta_l). \tag{9}$$

Inverting the latter equation is not possible, further assumptions are needed. For limited extensions of the patch to be analyzed the azimuth-angle dependent functions can be approximated by a single value. The dominant values for $\mathcal{F}\left[\cos(\phi)\right]$ and $\mathcal{F}\left[\sin(\phi)\right]$ are the zero-frequency components, $\mathcal{F}\left[\cos(\phi)\right]_{0,0}$ and $\mathcal{F}\left[\sin(\phi)\right]_{0,0}$. Hence, the convolution operator is not longer required and the two-dimensional inverse MTF is given by the following equation

$$\mathcal{F}(\eta) \approx \frac{\mathcal{F}(\theta_l)}{\mathrm{i}\left(k_x\mathcal{F}\left[\cos(\phi)\right]_{0,0} + k_y\mathcal{F}\left[\sin(\phi)\right]_{0,0}\right)}. \tag{10}$$

The validity of the assumptions used herein is explored in the next section. It should however be noted that except for the assumption of a shadow-free analysis patch, the MTF is independent of the sea state. Furthermore, the water depth does not affect the MTF as long as the area that is analyzed can be ... FFT...

3 Validation of Assumptions

3.1 Shadow free parts of the image

3.2 High pass filtering

The high pass filter introduces an error (the original wave has always some low frequency contributions)... apply some windowing upfront? if used in wavenumber frequency, can we than distiguish between static patterns and wave patterns at low frequencies? For the analysis of the other contributions low frequencies are filtered...

3.3 Approximated incidence angle

Test a range of conditions; how well does Equation 8 approximate Equation 1.

Based on different wave fields we investigate how well the simplified local incidence angle matches the exact incidence angle. The smallest wavenumber ($k < 0.03$) contributions have been eliminated by a high pass filter. To distinguish the two incidence angles, a tilde is added to the approximation in the current secion ($\widetilde{\theta_l}$).

3.4 Simplified azimuth convolution

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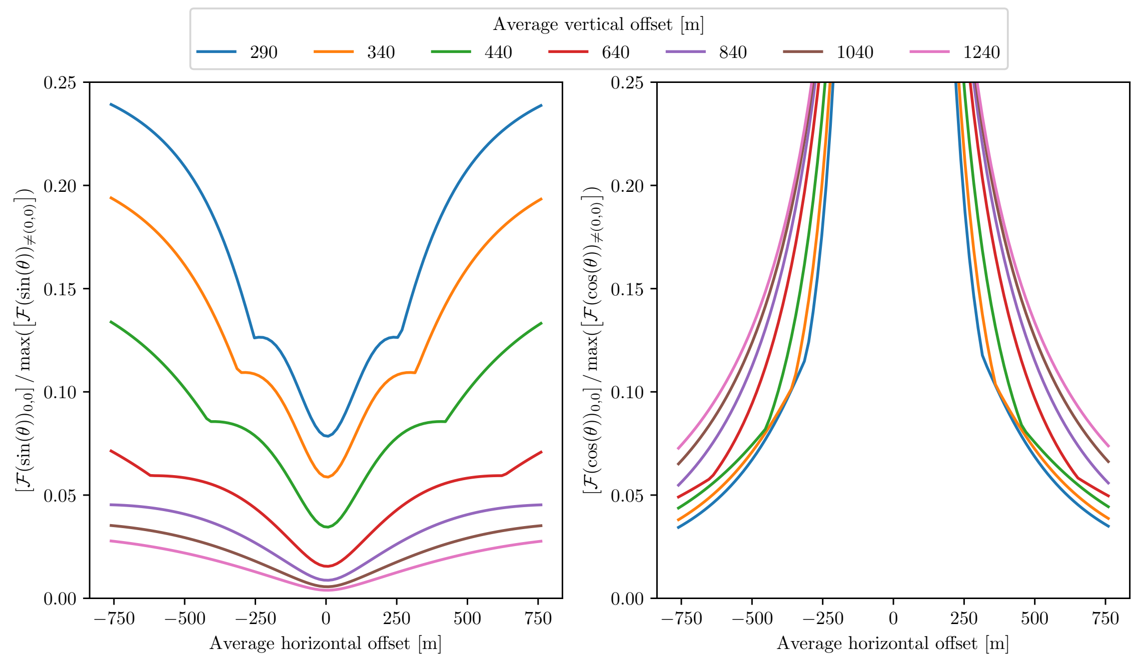

Figure 1: The figure illustrates that the validity of replacing the azimuth convolution by a single value depending of the position of the analysis window. The patch size is 64 by 64 points with a resolution of 7.5 m in both directions. The subplots show the respective ratios of $\mathcal{F}\left[\cos(\phi)\right]_{0,0}$ and $\mathcal{F}\left[\sin(\phi)\right]_{0,0}$ and the next biggest value of the $\mathcal{F}\left[\cos(\phi)\right]$ and $\mathcal{F}\left[\sin(\phi)\right]$.