

Tindeliumte Feliultet Feeulty	Occate Institution I	Don artimont		
iedekunta – Fakultet – Faculty Osasto – Institution – Department		Department		
Faculty of Science	Department of Geosciences and Geography			
Tekijä – Författare – Author				
Susanna Kukkavuori				
Tutkielman otsikko – Avhandlings titel – Title of thesis				
A methodological comparison of bathymetric modelling in large sub-arctic rivers – case study in Tana River, Finland Koulutusohjelma ja opintosuunta – Utbildningsprogram och studieinriktning – Programme and study track				
Master's programme in geography, Geoinformatics				
	Aika – Datum – Date	Sivumäärä – Sidoantal – Number of pages		
the thesis				
Master's thesis, 30 credits	Month Year	xx (+ x appendixes)		
Tiivistelmä – Referat – Abstract		, , ,		
Master's thesis, 30 credits Month Year xx (+ x appendixes)				
Avainsanat – Nyckelord – Keywords				
Multibeam sonar, laser scanning, satellite imagery, bathymetric mapping, sub-arctic river Säilytyspaikka – Förvaringställe – Where deposited				
University of Helsinki electronic theses library E-thesis/HELDA				
Muita tietoja – Övriga uppgifter – Additional information				

MSc Thesis advancements and results

Susanna Kukkavuori 29/11/2023

Working title: A methodological comparison of bathymetric modelling in large sub-arctic rivers – case study in Tana River, Finland

1. Introduction

Modelling the topography of river bottom with remote sensing (RS) methods plays an important part when studying environmental hydrological processes (Kinzel et al., 2013). For example, forecasting flood catchment areas and urban stormwater (e.g. Beven & Kirkby (1979) and Djokic & Maidment (1991)), predicting flow depth, velocity, spatial flow paths and networks (e.g. Quinn et al. (1991)), as well as modelling water quality and soil erosion (e.g. Vieux (1991) and Wright & Webster (1991)), can be challenging without proper equipment and methods. However, the techniques to model bathymetric data have not been developed as rapidly as general, dry-area topography measurement methods. Especially shallow waters in remote locations are yet to have a proven method suitable for them, as most of the bathymetric data gathering methods are directed for deeper (>10m) waters (Kasvi et al., 2019).

Environmentally, the sub-arctic is facing crucial times. The region has perhaps already crossed a critical threshold of change, as global warming due to climate change causes shifts in ecosystems as in landscapes in Northern Hemisphere now more than ever (Huang et al., 2017; Magnuson et al., 2000; Weyhenmeyer et al., 2011). To predict the consequences that climate change causes in sub-arctic river ecosystems, it is vital to have accurate, multitemporal and continuous monitoring and data acquisition. To achieve this, further studies for implementing adequate methods is necessary. New methods, approaches, applications, models, and testing them in different sub-arctic and artic river environments are much needed to understand the impacts warming weather might cause to river processes, but much of it are still lacking (Lotsari, 2023).

This study conducts a methodological comparison of RS methods for producing bathymetric models of sub-arctic river bottom. The study compares three RS methods on two study sites along the northern border of Finland and Norway, Tana River. The three RS methods are: 1) multibeam sonar, obtained using autonomous surface vehicle (ASV), 2) aerial laser scanning (ALS), using scanner attached to a helicopter, and 3) satellite imagery (SI), utilizing multispectral sensors from satellite Sentinel-2. The study aims to answer these following research questions:

- 1) Are there differences between RS methods, how do they perform in providing information of river depth?
- 2) Are there significant strengths and/or weaknesses in the methods based on modelled results?
- 3) Does the modelling accurately reflect the truthful depth information?
- 4) Do geomorphological and hydrological factors impact the accuracy of the models?

The output of this study contains reflection and discussion on models' qualitative and quantitative performance and what factors affected them. It also evaluates the importance of data quality and temporal and spatial resolution. This study aims to help sub-arctic river monitoring work to become more efficient and to highlight its importance in rapidly changing environmental conditions.

2. Background

In sub-arctic regions there are usually variation in geology, topography and in the processes of earth surface, for example post-glacial rebound (Nylén et al., 2019). Snow and ice covers the land and waters of sub-arctic regions for more than half of the year (Bennett et al., 2022) that impacts the region's environment and ecosystem. River valleys are geodiversity hotspots providing habitats and microhabitats for species above and under water (Gould & Walker, 1999). They also provide livelihoods such as fishery for local communities (Alioravainen et al., 2023) and tourism.

Tana River (*Tenojoki*) is circa 361 km long frontier river outlet at the border of Northern Finland and Norway, that enters the Barents Sea at Tanafjord (Collinson, 1970; Eilertsen & Corner, 2011). Its drainage area is approximately 16 380 km² with high spring period (spring floods) and low winter period (frozen) discharge, $> 500 \text{ m}^3 \text{ s}^{-1}$, and $< 70 \text{ m}^3 \text{ s}^{-1}$, respectively (Eilertsen & Corner, 2011). It is the largest and most productive natural salmon river in northern Europe (Fossøy et al., 2022). The river has been divided into three parts by Lax et al. (1993) based on river's geomorphology and flow rate. Slower flow-rate parts of the river bottom consist of sand and faster pace flow parts of bigger rock shores. The annual precipitation is approximately 700-800 mm and is mostly received as falling snow during the winter months. Mean temperature is -0.7 °C. The river freezes usually in November or late October and stays frozen approximately seven months. Discharge takes place a month after the breakup of ice, usually in early June, but sometimes already in May (Collinson, 1970).

A characteristic phenomenon to sub-arctic region, and important event for hydrologic cycle is floods in spring as snow cover starts to melt (Ford & Bedford, 1987). Floods export sediment loads in various size depending on flood magnitude and cause changes in river topography. Climate change has various dimensions influencing the balance of sub-arctic river systems, but are at the moment monitored relatively little, considering the rapidity climate change affects sub-arctic latitudes already (Lotsari et al., 2015, 2020; Magnuson et al., 2000; Rantanen et al., 2022; Weyhenmeyer et al., 2011). With RS methods far more accurate data with adequate resolution, from wider

spatial range and remote locations that can be hard-to-reach could be acquired less time consumingly and more often. Multitemporal data could reveal knowledge of how climate change would impact river dynamics in a long run and provide possible tools for prevention work.

Environmental factors influence the remote sensed radiation (Curran & Novo, 1988). For convenience they are grouped into those that relate to atmosphere, the boundary between atmosphere and water, water column and depth, and sediment. Atmosphere can affect the amount and quality of electromagnetic radiation from target to sensor. Natural aquatic systems are seldom pure waters, as in free from all organic, e.g. phytoplankton, and inorganic, e.g. suspended minerals material. The state of the water quality can be assessed with subsurface volumetric radiance information. In inland and nearshore waters, the impact of suspended material can play a significant role as investigating the spectral reflectance of the water body (Miller & McKee, 2004). Energy transfers knowingly by three modes: conduction, convection, and radiation (Kairu, 1982). In RS, the energy transmission through radiation by electromagnetic waves is the essential part of producing information. RS instruments gather information of the sediment from spectral reflectance, grain size and texture, mineral composition, and moisture content, that all response to electromagnetic radiation either by scattering, absorbing, fluorescing, polarizing, transmitting and/or penetrating (Campbell et al., 2022).

Conventional bathymetric measurement methods for estimating bottom topography have been single beam echo sound (SBES), multibeam echo sound (or sonar) (MBES), and LiDAR (Evagorou et al., 2022). The measurements are achieved by installing the measuring device into for example boats, aerial platforms, remotely controlled or autonomous underwater vehicles (Janowski et al., 2021). The limitations in these are rather small spatial coverage, although the spatial resolution and accuracy is particularly high. Recent advantages in satellite-derived bathymetry (SDB) techniques have proved itself to be rather cost-effective and able to monitor large spatial coverage and is an advantage especially in remote and hard-to-get areas. Satellites can also provide continuous data frequently (Evagorou et al., 2022; Jawak et al., 2015).

Comparison of the RS methods' ability to map river bathymetry have been conducted by, for example, Kasvi et al. (2019) studying efficiency of echo sounding and photogrammetry in clear and shallow river in subarctic Finland, Shintani & Fonstad (2017) in their research studying the abilities of photogrammetry and spectral depth approaches in a gravel-bed river in Oregon, United States, and Woodget et al. (2015) quantifying the performance of photogrammetry and LiDAR.

3. Study area

The study area is located along the border of northern Finland and Norway, between $69.90^{\circ} - 70.10^{\circ}$ N and $27.00^{\circ} - 28.00^{\circ}$ E. I selected two study sites: Utsjoki, near Holiday village Valle, and Nuorgam, near Nuorgam elementary school (fig. 1). They represent two different parts of the large sub-arctic river by physical and fluvial characteristics, upstream and downstream, respectively.

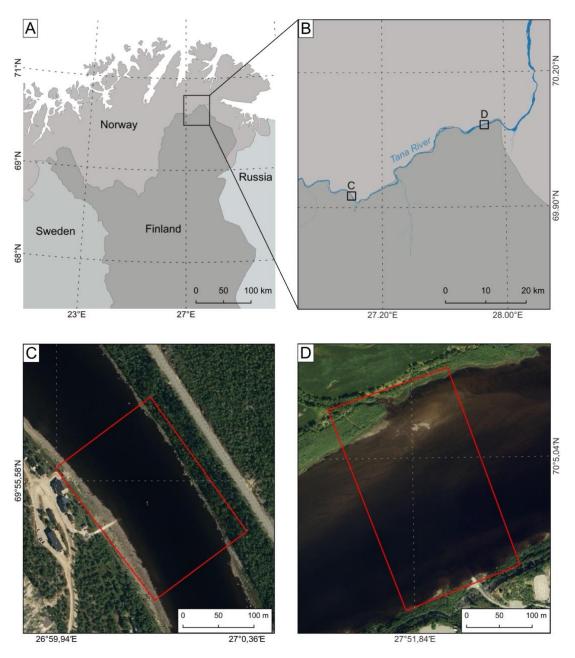


Fig. 1. (A) The study area is located along the border of northern Finland and Norway, near Tana River's lower reaches. (B) The locations of the two study sites along the river. (C) Utsjoki study site, representing the upstream part, and (D) Nuorgam study site, representing the downstream part of the river. Aerial imagery in C and D is obtained from Norwegian Mapping Authority ©Kartverket, www.norgeskart.no.

4. Materials and methods

This study compares three RS methods for producing bathymetric models: multibeam sonar, acquired using ASV, aerial laser scanning (ALS) and satellite imagery (SI) from Sentinel-2 satellite. I evaluate the methods by quantitative and qualitative performance, how the models perform under different hydrological conditions caused by weather, location within the river (downstream and upstream), and season. Along with detecting the differences and errors, stating the RS methods' strengths and weaknesses is also in the central focus of this study. The data are high in quality but have their limitations. The data used in this study went through pre-processing before I conducted bathymetric modelling. Both ASV and ALS data are 3D point cloud data, which were interpolated with Inverse Distance Weighting (IDW). SI was obtained using pre-processing in Google Earth Engine and calculating estimated water depth with Lyzenga's algorithm (Lyzenga, 1978).

4.1 ASV

Utsjoki site has 160 556 XYZ-points and Nuorgam site 198 489 points that were measured in September 2022 with unmanned surface vehicle (USV) Otter. Tana River is approximately three times wider at the site of Nuorgam than in Utsjoki, but the Utsjoki site was scanned more densely, meaning most survey lines were scanned more than once, while Nuorgam site had more parts which were scanned only once, leading for less dense measurements. Both point clouds were interpolated with IDW to resolution of 0.25m bathymetric raster.

4.2 ALS

Helicopter collected laser scanning data on sites in September 2022. The data was delivered in LAZ format by National Land Survey of Finland (NLS). Utsjoki site's point cloud consists of 1 771 687 points and Nuorgam site of 2 883 542 points. Utsjoki points has been classified into three categories: ground (44.5%), medium vegetation (18.3%) and unclassified (37.2%). Nuorgam points have two classifications: reserved (32.2%) and unclassified (67.8%). LAZ files were first filtered from outliers and possible noise. Filtering and rasterizing were implemented with Point Data Abstraction Library's (PDAL) pipelines (PDAL Contributors, 2022). Generating DSM happens with GDAL writer, that uses IDW interpolation algorithm. Data type was defined to GeoTIFF. Resolution was set to 0.25m, the same as ASV.

The SI data used in this analysis was Sentinel-2 level 2A, provided by the European Space Agency (ESA). It is equipped with large range of technologies supporting the Copernicus Land Monitoring studies, such as multi-spectral imaging instruments for vegetation, soil, water, and atmospheric monitoring. For this study Sentinel-2 imagery from July 2022 was, since it was the closest date to the closer-to-ground studies, when there was imagery available without clouds above the study sites. The spatial resolution is 10 meters.

4.4 Ground point measurements

With Trimble R10 GNSS receiver device, 31 measurement points were taken from the shorelines of the study sites for the validation analysis and accuracy assessment in June 2023.

5. Results

5.1 Quantitative assessment

The RS method validation results show that laser scanning (ALS) performed better than multibeam sonar (ASV), resulting R²=~0.9 and R²=~0.4, and RMSE=~0.17 and RMSE=~1.6, respectively (all detailed error assessment results seen in tables 1 and 2). However, the validation results of the models' performances are to be questioned, as the validation data acquisition faced contingency of reaching multiple separate depths of the river, and same scanning spots as ASV. Most of ASV depth points values in validation analysis are not based on measured values but interpolated. That results in highly unreliable and predicted values. Thus, depending only on quantitative comparison and assessment of the models' performance is not advisable. The conclusion is that assessing the qualitative performance of the models should be done along with quantitative assessment and give it more weight in this study.

Table 1. The results of root mean squared, absolute and maximum errors and R-squared on IDW modelled river bottom DSMs derived from ASV data, compared to validation point data. N=31.

	Utsjoki ASV	Nuorgam ASV
RMSE	1.7838706318406874	1.1568474726087066
MAE	1.4331780161290262	1.0201863400000006
MAXE	4.261111999999997	2.3705827
R^2	0.44577947248012084	0.3421801604585336

Table 2. The results of root mean squared, absolute and maximum errors and R-squared on IDW modelled river bottom DSMs derived from ALS data, compared to validation point data. N=31.

	Utsjoki ALS	Nuorgam ALS
RMSE	0.16526257931156874	0.17681145003216284
MAE	0.12614583495443576	0.11840849070220001
MAXE	0.427248066521301	0.7277976715493999
\mathbb{R}^2	0.981994064920988	0.9689414156757721

Table 3. The correlation between study sites' ASV and ALS models. N=300.

	Correlation r _{xy}
Utsjoki	-0.1520828076299014
Nuorgam	0.7647202734203874

5.2 Qualitative assessment

Qualitative assessment result show that ALS models have a lot more outlier values even after proper filtering and testing than ASV models (fig. 4 and 5). Outlier values lie mostly on shallower positions, apart from shorelines. ALS models show the river bottom deeper than ASV models (fig. 5), but distinguishably little, approximately 0.01%. Models also show that Nuorgam study site (fig. 3) performed higher correlation (table 3) between ASV and ALS than Utsjoki study site (fig. 2).

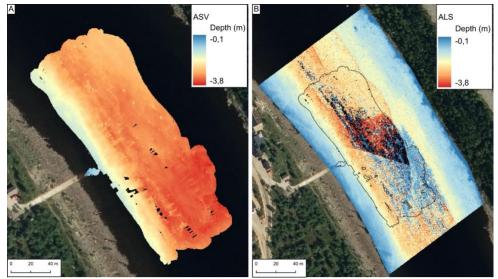


Fig. 2. (A) ASV bathymetric model of Utsjoki study site. (B) ALS bathymetric model of Utsjoki study site. In B image there is the outline of ASV set in proportion in area of DSM from ALS.

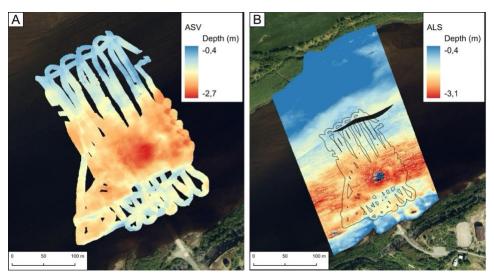


Fig. 3. (A) ASV bathymetric model of Nuorgam study site. (B) ALS bathymetric model of Nuorgam study site. In B image there is the outline of ASV set in proportion in area of DSM from ALS.

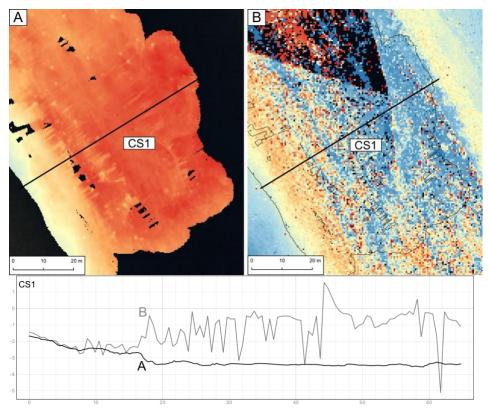


Fig. 4. Utsjoki cross-section 1 (CS1) elevation profile. Closer to Finland's side shore (left side of the map) profiles correlates (-1m to -3m). Closer to -4m depth ALS faces a lot more disturbance than ASV.

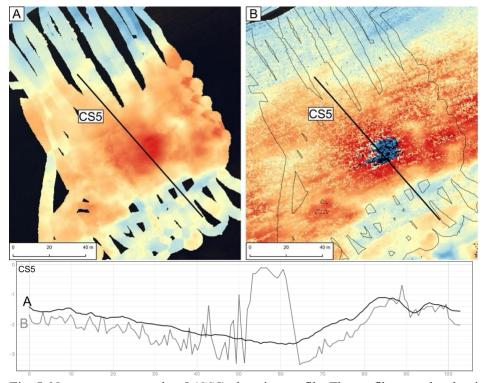


Fig. 5. Nuorgam cross-section 5 (CS5) elevation profile. The profiles correlate but in that approximately 20m wide indentation in the middle of cross-section.



Fig. 6. Vertical differences between ASV and ALS. Unit is meters.

Vertical differences (fig. 6) show that in Utsjoki the difference between models is generally between 0 - 5 meter and in Nuorgam between 0 - 1 meter. That suggests that Nuorgam models are by values closer to each other than Utsjoki. The correlation mentioned earlier in chapter 5.1 (table 3) was performed using 300 random point samples and calculated the Pearson correlation coefficient (PCC) (fig. 7).

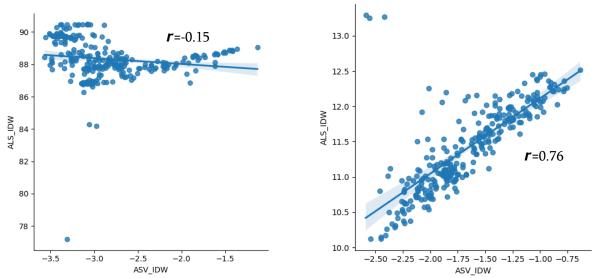


Fig. 7. Correlations between ASV and ALS. Left Utsjoki, right Nuorgam.

- 5.3 SI analysis (unfinished)
- 5.4 Reflecting the results to physical conditions: relations, significant?

6. Discussion and conclusion

- validation results to be questioned
- data quality assessment limitations and possibilities
- did this work produce anything noteworthy for further research in the study field and recommendations
- reflection to main research questions, did they receive answers

References

Alioravainen, N., Orell, P., & Erkinaro, J. (2023). Long-Term Trends in Freshwater and Marine Growth Patterns in Three Sub-Arctic Atlantic Salmon Populations. *Fishes*, 8(9), Article 9. https://doi.org/10.3390/fishes8090441

Beven, K. J., & Kirkby, M. J. (1979). A physically based, variable contributing area model of basin hydrology. *Hydrological Sciences Bulletin*, *24*(1), 43–69. https://doi.org/10.1080/02626667909491834
Bennett, K. E., Miller, G., Busey, R., Chen, M., Lathrop, E. R., Dann, J. B., Nutt, M., Crumley, R., Dillard, S. L., Dafflon, B., Kumar, J., Bolton, W. R., Wilson, C. J., Iversen, C. M., & Wullschleger, S. D. (2022). Spatial patterns of snow distribution in the sub-Arctic. *The Cryosphere*, *16*(8), 3269–3293. https://doi.org/10.5194/tc-16-3269-2022

Campbell, J. B., Thomas, V. A., & Wynne, R. H. (2022). *Introduction to remote sensing* (Sixth edition.). Guilford Publications.

Collinson, J. D. (1970). Bedforms of the Tana River, Norway. *Geografiska Annaler: Series A, Physical Geography*, 52(1), 31–56. https://doi.org/10.1080/04353676.1970.11879807

Curran, P. J., & Novo, E. M. M. (1988). The Relationship Between Suspended Sediment Concentration and Remotely Sensed Spectral Radiance: A Review. *Journal of Coastal Research*, 4(3), 351–368.

Djokic, D., & Maidment, D. R. (1991). Terrain analysis for urban stormwater modelling. *Hydrological Processes*, 5(1), 115–124. https://doi.org/10.1002/hyp.3360050109

Eilertsen, R., & Corner, G. (2011). Role of Scouring and Base-Level Change in Producing Anomalously Thick Fluvial Successions: An Example from the Tana River, Northern Norway (pp. 265–279). https://doi.org/10.2110/sepmsp.097.265

Evagorou, E., Argyriou, A., Papadopoulos, N., Mettas, C., Alexandrakis, G., & Hadjimitsis, D. (2022). Evaluation of Satellite-Derived Bathymetry from High and Medium-Resolution Sensors Using Empirical Methods. *Remote Sensing*, *14*(3), 772. https://doi.org/10.3390/rs14030772

Ford, J., & Bedford, B. L. (1987). The Hydrology of Alaskan Wetlands, U.S.A.: A Review. *Arctic and Alpine Research*, 19(3), 209–229. https://doi.org/10.1080/00040851.1987.12002596

Fossøy, F., Erkinaro, J., Orell, P., Pohjola, J.-P., Brandsegg, H., Andersskog, I. P. Ø., & Sivertsgård, R. (2022). Monitoring the pink salmon invasion in Tana using eDNA. Assessment of pink salmon, Atlantic salmon and European bullhead. In 23. Norwegian Institute for Nature Research (NINA). https://brage.nina.no/nina-xmlui/handle/11250/3036089

Gould, W. A., & Walker, M. D. (1999). Plant communities and landscape diversity along a Canadian Arctic river. *Journal of Vegetation Science*, 10(4), 537–548. https://doi.org/10.2307/3237188

Huang, J., Zhang, X., Zhang, Q., Lin, Y., Hao, M., Luo, Y., Zhao, Z., Yao, Y., Chen, X., Wang, L., Nie, S., Yin, Y., Xu, Y., & Zhang, J. (2017). Recently amplified arctic warming has contributed to a continual global warming trend. *Nature Climate Change*, 7(12), Article 12. https://doi.org/10.1038/s41558-017-0009-5

Janowski, L., Wroblewski, R., Dworniczak, J., Kolakowski, M., Rogowska, K., Wojcik, M., & Gajewski, J. (2021). Offshore benthic habitat mapping based on object-based image analysis and geomorphometric approach. A case study from the Slupsk Bank, Southern Baltic Sea. *Science of The Total Environment*, 801, 149712. https://doi.org/10.1016/j.scitotenv.2021.149712

Jawak, S. D., Vadlamani, S. S., & Luis, A. J. (2015). A Synoptic Review on Deriving Bathymetry Information Using Remote Sensing Technologies: Models, Methods and Comparisons. *Advances in Remote Sensing*, 04(02), Article 02. https://doi.org/10.4236/ars.2015.42013

Kairu, E. N. (1982). An Introduction to Remote Sensing. *GeoJournal*, 6(3), 251–260.

- Kasvi, E., Salmela, J., Lotsari, E., Kumpula, T., & Lane, S. N. (2019). Comparison of remote sensing based approaches for mapping bathymetry of shallow, clear water rivers. *Geomorphology*, *333*, 180–197. https://doi.org/10.1016/j.geomorph.2019.02.017
- Kinzel, P. J., Legleiter, C. J., & Nelson, J. M. (2013). Mapping River Bathymetry With a Small Footprint Green LiDAR: Applications and Challenges1. *JAWRA Journal of the American Water Resources Association*, 49(1), 183–204. https://doi.org/10.1111/jawr.12008
- Lax, H. G., Koskenniemi, E., Sevola, P., & Bagge, P. (1993). Tenojoen pohjaeläimistö ympärstön laadun kuvaajana. *Vesi- ja ympäristöhallitus, Vesi-ja ympäristöhallinnon julkaisuja, sarja A*(131), 124.
- Lotsari, E. (2023, October 27). *Changing cold climate river environments*. The Geographical Society of Finland Palmén colloquium, Zoom.
- Lotsari, E., Hackney, C., Salmela, J., Kasvi, E., Kemp, J., Alho, P., & Darby, S. e. (2020). Sub-arctic river bank dynamics and driving processes during the open-channel flow period. *Earth Surface Processes and Landforms*, 45(5), 1198–1216. https://doi.org/10.1002/esp.4796
- Lotsari, E., Wang, Y., Kaartinen, H., Jaakkola, A., Kukko, A., Vaaja, M., Hyyppä, H., Hyyppä, J., & Alho, P. (2015). Gravel transport by ice in a subarctic river from accurate laser scanning. *Geomorphology*, 246, 113–122. https://doi.org/10.1016/j.geomorph.2015.06.009
- Lyzenga, D. R. (1978). Passive remote sensing techniques for mapping water depth and bottom features. *Applied Optics*, 17(3), 379. https://doi.org/10.1364/AO.17.000379
- Magnuson, J. J., Robertson, D. M., Benson, B. J., Wynne, Randolph H., Livingstone, D. M., Arai, T., Assel, R. A., Barry, R. G., Card, V., Kuusisto, E., Granin, N. G., Prowse, T. D., Stewart, K. M., & Vuglinski, V. S. (2000). Historical Trends in Lake and River Ice Cover in the Northern Hemisphere. *Science*, 289(5485), 1743–1746. https://doi.org/10.1126/SCIENCE.289.5485.1743
- Miller, R. L., & McKee, B. A. (2004). Using MODIS Terra 250 m imagery to map concentrations of total suspended matter in coastal waters. *Remote Sensing of Environment*, 93(1), 259–266. https://doi.org/10.1016/j.rse.2004.07.012
- Nylén, T., Kasvi, E., Salmela, J., Kaartinen, H., Kukko, A., Jaakkola, A., Hyyppä, J., & Alho, P. (2019). Improving distribution models of riparian vegetation with mobile laser scanning and hydraulic modelling. *PLOS ONE*, *14*(12), e0225936. https://doi.org/10.1371/journal.pone.0225936
- PDAL Contributors (2022). PDAL Point Data Abstraction Library. https://doi.org/10.5281/zenodo.2616780
- Quinn, P., Beven, K., Chevallier, P., & Planchon, O. (1991). The prediction of hillslope flow paths for distributed hydrological modelling using digital terrain models. *Hydrological Processes*, *5*(1), 59–79. https://doi.org/10.1002/hyp.3360050106
- Rantanen, M., Karpechko, A. Y., Lipponen, A., Nordling, K., Hyvärinen, O., Ruosteenoja, K., Vihma, T., & Laaksonen, A. (2022). The Arctic has warmed nearly four times faster than the globe since 1979. *Communications Earth & Environment*, 3(1), Article 1. https://doi.org/10.1038/s43247-022-00498-3
- Shintani, C., & Fonstad, M. A. (2017). Comparing remote-sensing techniques collecting bathymetric data from a gravel-bed river. *International Journal of Remote Sensing*, 38(8–10), 2883–2902. https://doi.org/10.1080/01431161.2017.1280636
- Vieux, B. E. (1991). Geographic information systems and non-point source water quality and quantity modelling. *Hydrological Processes*, 5(1), 101–113. https://doi.org/10.1002/hyp.3360050108
- Weyhenmeyer, G. A., Livingstone, D. M., Meili, M., Jensen, O., Benson, B., & Magnuson, J. J. (2011). Large geographical differences in the sensitivity of ice-covered lakes and rivers in the Northern Hemisphere to

temperature changes. *Global Change Biology*, *17*(1), 268–275. https://doi.org/10.1111/j.1365-2486.2010.02249.x

Woodget, A. S., Carbonneau, P. E., Visser, F., & Maddock, I. P. (2015). Quantifying submerged fluvial topography using hyperspatial resolution UAS imagery and structure from motion photogrammetry. *Earth Surface Processes and Landforms*, 40(1), 47–64. https://doi.org/10.1002/esp.3613

Wright, A. C., & Webster, R. (1991). A stochastic distributed model of soil erosion by overland flow. *Earth Surface Processes and Landforms*, 16(3), 207–226. https://doi.org/10.1002/esp.3290160303