Übungsblatt 3

Exercice 1. Let M be a smooth manifold and let $f\colon M \longrightarrow \mathbb{R}^k$ be a smooth function. Show that $f \circ \varphi^{-1}\colon \varphi(U) \longrightarrow \mathbb{R}^k$ is smooth for *any* chart (U, φ) for M.

Exercice 2. Let M, N, P be smooth manifolds. Show that the following hold.

(1) Any constant map $c\colon M \longrightarrow N$ is smooth.
(2) The identity map $M \longrightarrow M$, $p \mapsto p$, is smooth.
(3) Let $U \subset M$ be an open subset and consider it as manifold (with the induced topology). Then the inclusion $U \hookrightarrow M$ is smooth.
(4) If $F\colon M \longrightarrow N$ und $G\colon N \longrightarrow P$ are smooth, then $G \circ F\colon M \longrightarrow P$ is smooth.

Exercice 3. Let $\mathbb{B}$ be the unit sphere in $\mathbb{R}^n$. We consider the maps $F\colon \mathbb{B} \longrightarrow \mathbb{R}^n$ und $G\colon \mathbb{R}^n \longrightarrow \mathbb{B}$

$$F(x) = \frac{x}{\sqrt{1-|x|^2}}, \quad G(y) = \frac{y}{\sqrt{1+|y|^2}}$$

Show that F und G are smooth and inverse to each other.

Exercice 4. Let $f\colon M \longrightarrow N$ be a smooth map between manifolds. Show that f is continuous. Conclude that if f is a diffeomorphism, then f is open.

Exercice 5. Let $\mathbb{S}^1$ be the unit circle in $\mathbb{R}^2$. Show that for any integer $n \geqslant 1$, the map $\mathbb{S}^1 \longrightarrow \mathbb{S}^1$, $z \mapsto z^n$, is smooth.

Exercice 6. Show that $\mathbb{RP}^n$ is a smooth manifold and that the quotient map $\pi : \mathbb{R}^{n+1}\backslash\{0\} \longrightarrow \mathbb{RP}^n$ is smooth.