Bachelor Mathematik, Universität Basel, FS2024

Übungsblatt 2

Exercice 1. (1) Show that any finite dimensional vector space is a *smooth* manifold.
(2) Show that $M_n(\mathbb{R})$ und $GL_n(\mathbb{R})$ are *smooth* manifolds for any $n \geqslant 1$.
(3) Show that open subsets of *smooth* manifolds are again *smooth* manifolds.
(4) Show that $\mathbb{RP}^n$ is a *smooth* manifold for any $n \geqslant 1$.

Exercice 2. Let $U \subset \mathbb{R}^n$ be an open set and $f\colon U \longrightarrow \mathbb{R}$ be a smooth function. Let $c \in \mathbb{R}$ and consider the set $f^{-1}(c)$ (called level set of f). Suppose that $Df_a \neq 0$ for any $a \in f^{-1}(c)$. Show that $f^{-1}(c)$ has the structure of a smooth manifold. (Hint: use the implicit function theorem from Analysis II.)

Exercice 3. Let $U \subset \mathbb{R}^n$ be an open set with its standard smooth structure and let $f\colon U \longrightarrow \mathbb{R}^k$ be a map. Show that f is smooth in the sense of the course if and only if it is smooth in the sense of Analysis I&II.

Exercice 4. Let M be a smooth manifold and let $f\colon M \longrightarrow \mathbb{R}^k$ be a smooth function. Show that $f \circ \varphi^{-1}\colon \varphi(U) \longrightarrow \mathbb{R}^k$ is smooth for *any* chart (U, φ) for M.