First Order Logic

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Limitations

- Propositional logic is not sufficiently powerful for general inference systems.
  - Cannot represent “quantifiers”
  - Cannot represent variables
  - Cannot represent functional information
  - Cannot represent uncertainty
- Next, we will consider a more expressive logic formalism—first-order logic.

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First-Order Logic

- Also called Predicate Calculus
- Extend language beyond propositions
  - Constants (individuals in the world)—*KingJohn*, 2, *MSU,* …
  - Functions (map individuals to individuals)—*Sqrt*, *LeftLegOf*, …
  - Predicates (map individuals to truth values)—*Brother*, >, ==, …
  - Variables (e.g., *x*, *y*)
  - Connectives ($\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\Leftrightarrow$)
  - Quantifiers (universal "$\forall$" and existential "$\exists$")
  - Equality (i.e., =)

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Atomic Sentences

- $AtomicSentence \Leftrightarrow predicate(term_1, \ldots, term_n) | term_1 = term_2$
- $term \Leftrightarrow function(term_1, \ldots, term_n) | constant | variable$
- Examples:
  - *Brother*(*KingJohn*, *RichardTheLionheart*)
  - *GT*(*Length*(*LeftLegOf*(*Richard*)),
    Length(*LeftLegOf*(*KingJohn*)))

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Complex Sentences

- Made up from atomic sentences using connectives.
  - $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$
- Examples:
  - *Sibling*(*KingJohn*,*Richard*) $\Rightarrow$ *Sibling*(*Richard*,*KingJohn*)
  - GT(1,2) $\vee$ LE(1,2)
  - GT(1,2) $\wedge$ $\neg$GT(1,2)

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Truth in First-Order Logic

- Sentences are true with respect to a model and interpretation.
- Models contain objects and relations among them.
- Interpretation specifies referents for
  - *constant symbols* $\rightarrow$ objects
  - *predicate symbols* $\rightarrow$ logical relations
  - *function symbols* $\rightarrow$ functional relations
- An atomic sentence *predicate*($term_1$,…,$term_n$) is true *iff* the objects referred to by $term_1$,…,$term_n$ are in the relation referred to by *predicate*.

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Quantifiers

- Universals are like conjunction
  - $\forall x\ P(x)$ means P holds for all values x
  - Usually used with implication
- Existentials are like disjunction
  - $\exists x\ P(x)$ means P holds for some value x
  - Usually used with conjunction
- Switching the order of like quantifiers does not change meaning.
- Switching the order of different quantifiers does.

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Universal Quantification

- $\forall\langle variables\rangle\ \langle sentence\rangle$
- Everyone at MSU is smart.
  - $\forall x\ At(x,MSU) \Rightarrow Smart(x)$
- Remember, $\forall x\ P(x)$ is equivalent to the conjunction of *all* instantiations of $P(x)$.
  - $At(KingJohn,MSU) \Rightarrow Smart(KingJohn) \wedge At(Richard,MSU) \Rightarrow Smart(Richard) \wedge At(KingJohn,MSU) \Rightarrow Smart(MSU) \wedge \ldots$
- Typically, $\Rightarrow$ is the main connective with $\forall$.
- Common mistake: using $\wedge$ as the main connective with $\forall$:
  - $\forall x\ At(x,MSU) \wedge Smart(x)$ means "everyone is at MSU and everyone is smart."
  - This is *not* the same!

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Existential Quantification

- $\exists\langle variables\rangle\ \langle sentence\rangle$
- Someone at UM is smart.
  - $\exists x\ At(x,UM) \wedge Smart(x)$
- Remember, $\exists x\ P(x)$ is equivalent to the disjunction of instantiations of $P(x)$.
  - $At(KingJohn,UM) \wedge Smart(KingJohn) \vee At(Richard,UM) \wedge Smart(Richard) \vee At(KingJohn,UM) \wedge Smart(UM) \vee \ldots$
- Typically, $\wedge$ is the main connective with $\exists$.
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:
  - $\exists x\ At(x,UM) \Rightarrow Smart(x)$ means "if there is some person at UM, then that person is smart."
  - This is *not* the same!

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Properties of Quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$.
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$.
- $\exists x\ \forall y$ is *not* the same as $\forall y\ \exists x$.
  - $\exists x\ \forall y\ Loves(x,y)$ means "There is a person who loves everyone in the world."
  - $\forall y\ \exists x\ Loves(x,y)$ means "Everyone in the world is loved by at least one person."
- Duality of Quantifiers (express each as the other)
  - $\forall x\ Likes(x,IceCream) \equiv \neg\exists x\ \neg Likes(x,IceCream)$
  - $\exists x\ Likes(x,Broccoli) \equiv \neg\forall x\ \neg Likes(x,Broccoli)$

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