1 Introduction

Big picture – land is being rapidly developed by provides many ecosystem services in its current state. Signficant externalities imply that private land use decisions are optimal. In this setting, conservation payments represent a potentially important policy tool.

PES payment plans have both "promise and peril". The peril comes from the chaDespite their promise, PES payment plans come with many challenges (Kinzig et al., 2011). Two particularly important issues are asymmetric information between the landowner and the purchaser of services and uncertainty about the relationship between conservation decisions and the ecoystem benefits buyers ultimately care about. In particular, the relationship between ecosystem service provision and landuse is both important and imprecisely understood so that ecologists are rarely able to provide precise and specific estimates of the service provided by individual parcels. As a result, buyers seeking to implement payment for ecosystem service approaches are left uncertain about what they might be purchasing. In many cases, this uncertainty is due to the inherently stochastic nature of ecoystems, but in other cases, improved scientific undertanding in the future may partially reduce this uncertainty. While this potential for improved information might indicate the benefit of a wait-and-see approach, development pressures are current and many forms of development are irreversible or costly to reverse. If buyers wait until they are sure of the services provided, it may be too late to avoid development. Instead, buyers may need to provide conservation incentives today to provide the option to conserve tomorrow. In this paper, I develop a stylized model to capture the impact of this potential for improved information on current PES plans.

There is an extensive literature investigating the role of asymmetric information between landowners and buyers of ecosystem service. *Cite lots of people with no specific info about the papers?* Ferraro (2008) considers the impact of asymmetric inforation on PES programs and notes that strategies to reduce the cost of providing services generally fall into three categories: (1) acquiring more information, (2) offering a menu of contracts, or (3) conducting procurement auctions. He notes that no one approach dominates.

Talk about mechanism design literature.*What important messages do I want coming out of this?* All of these strategies rely on the buyer using a policy that has two components: eg. a price and a quantity. In this paper, I consider a buyer that makes decisions about the price to offer only.

Talk about auction literature. *Important messages? – potential to reduce the information rents associated with uniform pricing, challenges about design, market power concerns. Duke, Messer, etc paper that talks about problem with non-additionality in the auction context.*

End by citing article pointing out that uniform price contracts are much more common. I thus focus on how to adjust a uniform price policy given the possibility of improved information about ecosystem service value.

– Summarize three or four main points

Key points:

– partially reducible uncertainty about both the private net benefit of different land uses and the public benefit of conservation. Both vary across sites

– Voluntary and temporary incentives to conserve land

– Irreversibility has cross-cutting effects:

  – Money spent encouraging conservation today is irrecoverable

  – Development is irreversible

– Race between development and information gathering (someone said this)

– Model approaching begins by specifying in detail what offers the buyer should make in a single period given uncertainty about both values and possible correlation between them. Also explore how this relationship changes if buyers believe all land who private net benefit of development exceeds some threshhold has already converted.

  – Using this function, explore how the cost of achieving a given expected environmental service level depends on the quality of information (and the correlations).

– Model then steps back and asks how landowners should respond to prior period offers knowing that if they do not develop their land, buyers will learn more information and make more specific offers next period.

– Step back 1 more period and ask what offers buyers should make now given the partially reducible uncertainty

– Particularly interested in comparative statics:

  – How valuable is information? How much does the buyer's payoff increase as the quality of the signals increases? Does this depend on the degree of correlation? In monotonic or non-monotonic ways?

  – How much should the buyer invest in PES payments while waiting for info?

    • How many parcels will it induce to conserve in period 1 that would immediately convert if there were no program? (e.g. if we set tempPay and period 2 offer to 0)

    • How many parcels will it induce to conserve in period 1 that wouldn't if we did nothing now and responded optimally to the info in period 2 (e.g. if we set tempPay to 0 but set period 2 offer using optOffer)

    • How many parcels will it induce to conserve in period 1 that it wouldn't induce to conserve if we weren't expecting better info? (Or might it induce fewer/different parcels to conserve in period 1 given the info than it would if we weren't getting more info) (e.g. set sd of signals to 0 – no info gained).

    • How do the answers to these questions depend on our information parameters?

Note: in general the sign of single variable comparative statics is clear a priori. The more interesting questions are the cross-partials

1.1 Previous Literature

There is an extensive literature on selecting sites for conservation. Newburn et al. (2005) and Newburn et al. (2006) emphasize the importance of considering both the cost of acquiring a particular parcel and the likelihoood that a parcel will be developed in the absence of policy.

Wunder et al. (2008) compare a broad set of PES case studies and conclude that programs in which the users of the provided ecosystem services are directly purchasing the services tend to perform better on a number of metrics than those where the government purchases services on behalf of the eventual users. Engel et al. (2008) reach a similar conclusion and note that in many government-financed PES programs, the buyers "have no first-hand information on its value, and generally cannot observe directly whether it is being provided" (pg. 666).

The analysis in this paper focuses on a PES buyer who has adopted strategy 1.

Wunder & Albán (2008) describe two decentralized PES programs in Ecaudor, one of which paid landowners to preserve natural páramos and forest to improve water quality and flow downstream. They note that the precise hydrology linking land use changes to desirable water outcomes was unavailable so the program contracted on the land use changes themselves rather than service provision.

Vedel et al. (2015) use a choice experiment to demonstrate significance variation in landowners' willingness to accept payment for adopting particular forest management strategies based on their current practices and suggest that many services provided by uniform contracts would be non-additional.

Polasky et al. (2011) use a simulation modeling tool to demonstrate significant divergence between privately optimal and socially optimal land use decisions in Minnesota. Shah & Ando (2016) develop an infinite horizon model to assess the minimum temporary and permanaent payments need to induce landowners to conserve their land when landowners face uncertainty in the future benefits from both conservation and conversion. They find that accounting for both sources of uncertainty delays the point at which landowners will decide to convert land. Moreover, they compare temporary versus permanent payments for conservation and identify situations where these payments differ substantially. The analysis in this paper has a similar flavor to Miao et al. (2016) who model grassland easement acquisition when farmers face substantial uncertainty in the relative returns of cropping and grazing. They

My analysis differs in that it focuses on partially reducible uncertainty about the ecosystem services provided by a parcel.

1.2 Key results

The possibility that buyers will make more targeted offers in the future will not induce more parcels to conserve today. This result may well be

2 Model

There is a buyer who wishes to maximize the social value of services provided by a given parcel of land over two time periods. The land parcel has two possible states: developed and conserved. The decision to develop a parcel of land is irreversible. The net private development value of the parcel is p and the net public benefit of conservation for the parcel is e. The socially optimal use of the parcel would be to develop the land if $e < 0$ and to conserve the land if $e \geq 0$, but development decisions are made by the landowner who will choose to develop if $p > 0$ and to conserve if $p \leq 0$.[1] In the absence of externalities, $p = -e$ and private development decisions are socially optimal. The presence of a variety of positive and negative externalities associated with development and conservation implies that $p \neq e$; for the parcels with $e > 0$ and $p > 0$, there is a mismatch in incentives and the buyer could improve social welfare by inducing the landowner to conserve the land. The buyer views payments to the landowner as a cost, so with perfect information, the buyer would offer the landowner a payment $\phi = p$ if $e \geq p > 0$ and no payment otherwise.[2][3] However, the buyer has imperfect information about the values of e and p. In particular, in the first period, the buyer has little information about how much, if anything, the parcel needs to be paid to remain undeveloped or the actual benefit the parcel provides if conserved. Waiting for additional information that is expected to arrive in the future carries a substantial risk that the land will be irreversibly developed before its conservation value is known. The possibility of new information creates an option value of conservation that will increase the buyer's willingness to pay for conservation today. The goal of the model is to describe how this option value influences both offers and social welfare today.

To focus on this possibility, I assume that p is fixed over time. At time 0, the buyer has an initial estimate of the distribution of e. At time T, the buyer observes a signal s that reduces uncertainty about the true conservation value. For analytical tractability, I assume that the variables have a multivariate

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[1] Throughout the model, I assume that indifference is resolved in favor of conservation.

[2] I assume that the buyer views payments as costs rather than redistributive transfers. This is consistent with a buyer who represents a direct user of the services provided by conservation or with a government entity whose objective is to maximize the social benefit from land use, one who faces a fixed budget, or one who assumes there is a substantial social cost associated with raising funds.

[3] This model setup is similar in many ways to the framework in Engel et al. (2008) and Fig. 2 in their paper demonstrates similar regions to those depicted in Figures 4 and 5. There are two important differences to note. Come back and think about this more.

normal distribution with the form

$$
\begin{bmatrix} s \\ p \\ e \end{bmatrix} \sim N_4 \left(\begin{bmatrix} 0 \\ \mu_p \\ \mu_e \end{bmatrix}, \begin{bmatrix} 1 & \rho_{es}\rho_{ep}\sigma_p & \rho_{es}\sigma_e \\ \rho_{es}\rho_{ep}\sigma_p & \sigma_p^2 & \rho_{ep}\sigma_e\sigma_p \\ \rho_{es}\sigma_e & \rho_{ep}\sigma_e\sigma_p & \sigma_e^2 \end{bmatrix} \right).
$$

With this structure $\rho_{es} \in (0,1)$ measures the quality of the signal and ρ_{ep} measures the correlation between environmental serice value and private development value. In this model, ρ_{ep} refers to correlation between the unknown elements of e and p, not a statement about their expected values. If the buyer knows that a particular parcel is likely to provide high environmental service value, but is also likely to provide high benefits to the landowner if it is developed, this implies that both μ_e and μ_p are high, but does not imply that $\rho_{ep} > 0$. The natural cause for a non-zero value of ρ_{ep} in this model is a difficult to observe or verify land characteristic that influences both development value and ecosystem service value. Moreover, $\mu_{p|e,s} = \mu_{p|e}$ for all values of s and e, implying that the only information the signal provides about private development value is derived from the information it reveals about conservation value.

The buyer can make an offer to the landowner in both periods in exchange for conserving the land. Accepting an offer at time 0 requires the landowner to conserve the land until time T, at which point the landowner is free to develop the land or to continue conservation. Any parcel that is developed at time 0 will provide a per period benefit of p to the landowner in both periods, while land that is conserved will provide a per period benefit of e to society. The relative value of payoffs in the second time period (starting at time T) to those in the first time period (from time 0 to time T) is given by δ.[4]

2.1 Buyer's final period offer

If the parcel remains undeveloped at time T, the buyer will observe a signal s about the conservation value provided by the parcel. This signal could be the result of improved information about the parcel itself or could result from improved information about the processes linking land use decisions or parcel characteristics to ecosystem services. Based on this information, the buyer can offer a payment $\phi > 0$ to the landowner in exchange for conserving the land.[5] Although the buyer does not receive a direct signal about the value of p, some information may be inferred from the landowner's previous action. In particular, under conditions described below, the buyer infers that there is an upper bound $\bar{p} \in [0, \infty]$ for the private development value of the parcel, given that the landowner chose to conserve the land

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[4] Since the model is designed to capture long time frames, there is no a priori assumption that the two time periods have equal length. As a result, I require $\delta > 0$, but do not impose $\delta < 1$. If the time until information is received is relatively short and the parties expect decisions in the second period to persist for a long time frame, $\delta > 1$ is reasonable. The simulations place equal weight on the two periods.

[5] Given the multivariate normal assumption, there is a probability that the buyer might want to set $\phi < 0$ to induce a landowner to develop land the landowner would prefer to conserve. Such a situation is outside the typical scope of payment for ecosystem service plans and is not considered in this analysis. The probability a parcel falls in this categoy is low in all of my simulations.

until period T. Since this is the last period, the landowner will accept the offer if $p \leq \phi$. The buyer's goal is to maximize the expected value from conservation less any payments made to the landowner. An offer greater than $\bar{p}$ cannot be optimal because the buyer could lower the offer to $\bar{p}$ and maintain perfect certainty that the parcel will be conserved, while lowering the expected cost. The buyer's expected payoff for a parcel as a function of ϕ, given that $\phi \leq \bar{p}$, is the probability the parcel is conserved in the second period times the expected payoff conditional on conservation or

$$W(\phi, s) = \int\limits_{-\infty}^{\phi} \int\limits_{-\infty}^{\infty} (e - \phi) f_{e|s,p}(e|s,p) \, de f_{p|s}(p|s) \, dp \tag{1}$$

Since this expression is independent of $\bar{p}$, the upper bound affects the problem only through the constraint that $\phi \leq \bar{p}$. As shown in the appendix, under the multivariate normal assumption, this expression can be simplified to

$$W(\phi, s) = -\sigma_e \sigma_p \rho_{ep} \left(1 - \rho_{es}^2\right) f_{p|s}(\phi|s) + \left(\mu_e + \rho_{es}\sigma_e s - \phi\right) F_{p|s}(\phi|s). \tag{2}$$

The buyer's goal is to identify the value of $\phi \in [0, \bar{p}]$ that maximizes this expression. The following two propositions are proved in the appendix.

Proposition 1 *$W(\phi, s)$ reaches a maximum on $[0, \bar{p}]$ for all values of s and for any $\bar{p} \geq 0$.*

Proposition 2 *There is at most one interior local maximum of $W(\phi, s)$ on $[0, \infty)$.*

An immediate consequence of Propositions 1 and 2 is that we could have a solution at $\phi = 0$, an interior solution, or a solution at the upper bound $\bar{p}$. If multiple solutions provide the same payoff, I assume the buyer will resolve the indifference in favor of maximizing conservation. At an interior solution, the first order condition

$$f_{p|s}(\phi|s) \left[\mu_e + \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2\right) s - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p - \left(1 - \gamma \frac{\sigma_e \rho_{ep}}{\sigma_p}\right) \phi \right] = F_{p|s}(\phi|s) \tag{3}$$

where $\gamma = \frac{1 - \rho_{es}^2}{1 - \rho_{es}^2 \rho_{ep}^2}$ must hold.[6] The right hand side is the marginal cost of increasing the offer: the buyer must make larger payments if the landowner accepts the offer, which happens with probability $F_{p|s}(\phi|s)$. This cost is illustrated for a specific example by the solid line in Figure 1. The left hand side of (3) is the marginal benefit of increasing the offer, which is the product of two quantites: the probability that increasing the offer changes the landowner's conservation decision and the expected net gain from changing the parcel's choice. This later term is the expected gain conditional on the parcel being indifferent between conservation and development and is the term in brackets on the the left-hand side of (3). In Figure 1, the dotted line is the proability that increasing the offer changes the parcel's decision

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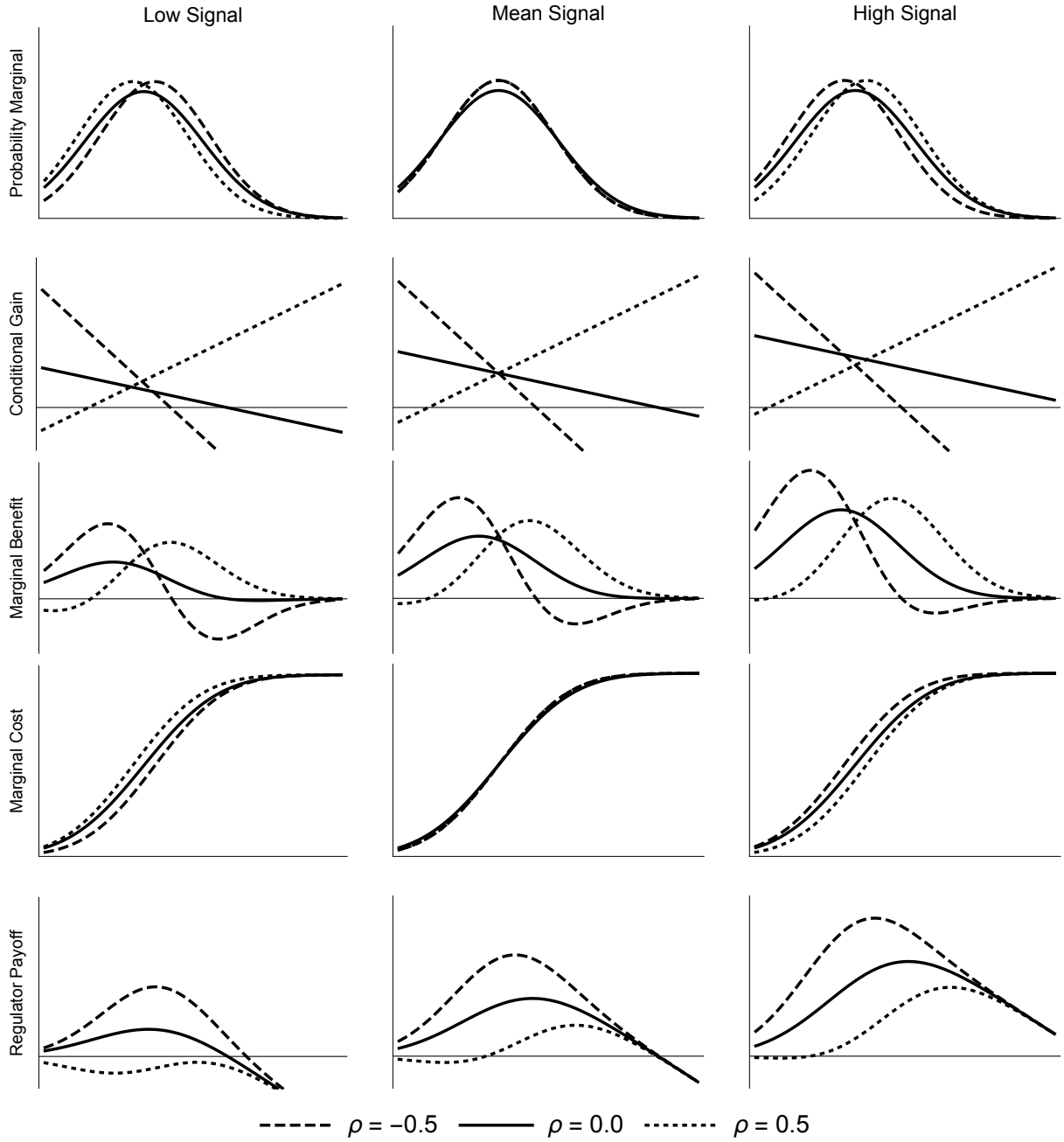[6] For the derivation of this condition, see Appendix C.

Fig. 1 Marginal Benefit and Cost of Increasing the Final Period Offer

Δ Expected Payoff/$
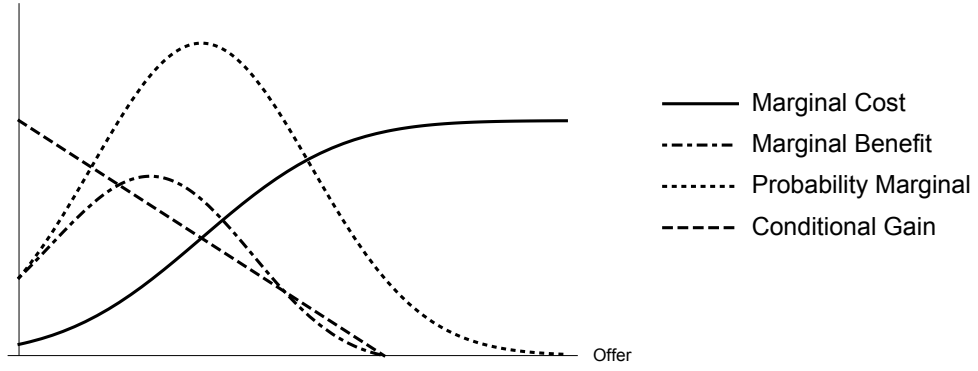
(given by $f_{p|s}(\phi|s)$), while the dashed line is the conditional expected gain. In this example,$\rho_{ep} = 0$. Since there is no correlation, knowing that the parcel has accepted the offer reveals nothing about the expected benefit of conserving the parcel and increasing the offer decreases the conditional gain dollar for dollar. The dot-dashed line is the product of the dashed and dotted lines and represents the marginal benefit. In this example, with zero correlation, increasing the expected conservation value μ_e or the signal would increase the benefit of changing a parcel's choice, tending to increase the optimal offer. Increasing uncertainty about the conservation value or the quality of the signal magnifies the impact of the signal on the optimal offer. Changing the mean and variance of the private development value distribution impact both the marginal benefit and marginal cost in potentially different ways above and below μ_p so their impacts are harder to identify immediately. Section 3.2 explores these impacts numerically.

Figure 2 illustrates the impact of correlation and the observed signal on each of the lines from Figure 1. In the middle column, the buyer observes a signal of zero, while the left column and right columns illustrate negative and positive signals respectively and thus correspond to learning that the parcel is likely to provide more or less benefit when conserved than originally expected. The solid black lines show the curves when there is no correlation, the dashed lines illustrate negative correlation between public conservation value and private development value and the dotted lines illustrates positive correlation. From this figure, we observe several effects.

- As the observed signal increases, the conditional gain rises because we expect the parcel to provide more value. Thus each of the lines in the conditional gain panels on the second row shift up as we move from a negative to zero to positive signal.
- When there is correlation, (either negative or positive), the buyer's uncertainty about the private value is reduced through observation of the signal s, leading to changes in the probability increasing the offer will change the landowner's choice and to changes in the costs. When the buyer observes a signal of 0 in the middle column, the mean of the subjective distribution remains unchanged but its variance falls as illustrated with the dotted lines.

Fig. 2 Impact of Changing Parameters

Low Signal | Mean Signal | High Signal

Probability Marginal / Conditional Gain / Marginal Benefit / Marginal Cost / Regulator Payoff

$- - - - ~ \rho = -0.5$ ⸺ $\rho = 0.0$ $\cdots\cdots ~ \rho = 0.5$

– In the left and right columns we see that in addition to reducing the variance of the conditional distribution of p , positive and negative correlation will cause its mean to shift if a non-zero signal is observed. High signals increase the mean with positive correlation and decrease the mean with negative correlation, while low signals do the reverse.

– The most dramatic effect of correlation is on the conditional expected gain lines, shown in the second row of Figure 2. As the offer increases, the expected gain on the marginal parcel falls due to the increase in payment. If $\rho_{ep} = 0$ (e.g. the solid lines), this is the only impact of changing the offer so the conditional gain has a slope of -1 with respect to the offer, as with the solid lines. With correlation, increasing the offer not only increases the cost at the marginal parcel, it also influences

the expected conservation value of the marginal parcel. These effects are driven by how correlation changes the value of γ. We know that that $1 - \rho_{es}^2 \leq \gamma \leq 1$ and γ increases as ρ_{ep}^2 increases and decreases as ρ_{es} increases. If $\rho_{ep} < 0$, the negative correlation implies that as we increase the offer, the conditional expected conservation value falls because the highest conservation value parcels are the ones most likely to conserve even with a low offer. As a result, the slope of the conditional gain will be less than -1 and the dashed lines slope down more steeply than the solid lines. The effect is amplified as the negative correlation gets stronger. With positive correlation, the conditional expected conservation value increases as the offer increases, so the slope will increase and can be positive if $\sigma_e > \sigma_p$, especially when the correlation is strong.

– Any correlation dampens the impact of the signal on the conditional gain (because $\gamma \rho_{ep}^2 < 1$), so the dashed and dotted lines shift up and down less as we move across the columns than the solid lines do. Intuitively, since knowing the parcel was induced to change its choice by the change in offer reveals some information about the expected conservation value when there is correlation, there is less remaining uncertainty for the singal to reveal.

– Finally, since γ falls as ρ_{es} increases, all of the impacts of changing correlation are dampened as the signal reveals more information about the environmental benefit so the conditional gain cannot vary as much. In constrast, as ρ_{es} increases, the impact of the signal is magnified since it reveals more information.

The combined effect of the changes on the first two rows is shown on the marginal benefit curve in the middle row of Figure 2. For both negative and zero correlation, the marginal benefit first rises, then falls, and eventually rises again, approaching zero asymptotically. This pattern is reversed for positive correlation if $\sigma_p < \sigma_e$ as in the case in this example. The final row illustrates the buyer's payoff function for the different cases and illustrate the different possible solution types described above. With the parameters illustrated here and negative or zero correlation, there is a single local (and thus global) maximum of $W(\phi, s)$, while with positive correlation there are two local maxima of $W(\phi, s)$ in the feasible set: one at zero and one with a positive offer. This reflects an important cost that positive correlation between e and p imposes on the buyer: the land is likely to be ecologically valuable only when it will also require a high payment to induce conservation. Unfortunately, making such a high offer will also induce the landowner to conserve when the land is not ecologically valuable. In this situation, the buyer is better off making no offer than a low offer and must carefully assess whether a zero offer or a high offer is preferable. In the example here, the positive solution is preferred with the mean or high signal, but no offer is preferred with a low signal. Moreover, this implies that the optimal offer may change discontinuously as problem parameters change, if the change induces a shift from preferring a positive offer to preferring a zero offer.

2.2 Landowner's prior period decision

The upper bound on p consistent with the landowner conserving the land until time T is found by considering the landowners' decision at time 0. At this time, the landowner must decide whether to develop the land immediately and receive a payoff of p in every period or to accept the buyer's offered conservation payment τ for this period and retain the option to conserve or develop the land at time T. The landowner knows p, which will give private information about the likelihood of possible values of s when $\rho_{es} > 0$ and $\rho_{ep} \neq 0$. In particular, $s|p \sim N\left(\sigma_p \rho_{es} \rho_{ep}\left(p - \mu_p\right), \sigma_p \sqrt{1 - \rho_{es}^2 \rho_{ep}^2}\right)$. This induces a subjective distribution of possible offers with $f_{\phi|p}\left(\phi|p\right) = \sum_{s:\phi^*(s,\bar{p})=\phi} f_{s|p}\left(s|p\right)$. The expected benefit of conservation for the landowner is

$$B\left(p\right) = \tau + \delta \left(\int\limits_p^\infty \left(\phi - p\right) f_{\phi|p}\left(\phi|p\right) d\phi\right) - p.$$

Consistent with the second period decision, the landowner is assumed to conserve the land when indifferent. Although there is no closed form solution for $\phi^*\left(s\right)$, several conclusions about the landowner's decision can be reached. First, since ϕ is bounded below by zero, the landowner will conserve the land if it was privately optimal to do so without a policy, e.g. if $p \leq 0$. Second, when $\rho_{ep} = 0$, $B\left(0\right) = \tau + \delta\left(\int_p^\infty \left(\phi - p\right) f_{\phi|p}\left(\phi|p\right) d\phi\right) \geq 0$ and $f_{\phi|p}\left(\phi|p\right)$ is constant for all values of p, implying that $\frac{dB}{dp} = -1$. In this case, there is a unique threshhold $\bar{p}$ such that the landowner will conserve at $t = 0$ if $p \leq \bar{p}$ and will develop immediately otherwise. As the correlation grows stronger, the impact of changes in p on the integral increases. It is possible, although unlikely, that there will be multiple solutions to $B\left(p\right) = 0$.

When a unique threshhold exists, its value solves the equation $B\left(\bar{p}\right) = 0$. If the landowner knows that the parcel is at this threshhold, $p = \bar{p}$ and $f_{\phi|p}\left(\phi|p\right) = 0$ for any value of $\phi > \bar{p}$ since the landowner knows that the buyer will never make an offer higher than this. As a result, the landowner's second period payoff will be $\bar{p}$ with certainty − either because the buyer offers exactly $\bar{p}$ and the landowner accepts or because the buyer offers $\phi < \bar{p}$ and the landowner chooses to develop. As a result

$$B\left(\bar{p}\right) = \tau + \delta\bar{p} - \bar{p}\left(1 + \delta\right) = 0$$

implying that the threshhold value is $\bar{p} = \tau$. If the private value is below the cutoff the landowner might receive more than the private value in the second period, but if $p = \bar{p}$, the landowner will only conserve today if the payment today exceeds the private development value. As a result, while the value of ρ_{es} may affect the landowner payoff if $p \leq \bar{p}$, the value of ρ_{es} has no impact on the likelihood the parcel will convert or on the value of $\bar{p}\left(\tau\right)$ as long as $B\left(p\right) = 0$ has a unique solution. The simulations discussed

below confirm that this is true under a wide varitey of parameter values. I thus focus on this situation and discuss the possibility of multiple threshholds in the simulation section below.

2.2.1 Buyer's initial period offer with unique threshhold

At time 0, the buyer knows better information will arrive at time T and must decide how aggressive to be in inducing conservation today. If the buyer makes a large offer today, the parcel is likely to conserve, but the buyer risks both paying for non-additional conservation and conserving land that provides conservation benefits below the cost. On the other hand, if the buyer makes no offer today, any opportunity to alter landowner development decisions is lost since all land with $p > 0$ will be developed immediately. The buyer must balance these concerns in deciding how aggressive to be with payments today. When there is a unique threshhold as described in the previous section, the buyer knows that the landowner will develop if $p > \tau$ and will conserve otherwise, implying that $\bar{p} = \tau$. This means that the buyer's full payoff is given by

$$V\left(\tau\right) = \int_{-\infty}^{\tau} \left(\mu_{e|p}\left(p\right) - \tau\right) f_p\left(p\right) dp + \delta \int_{-\infty}^{\infty} W^*\left(s, \tau\right) f_s\left(s\right) ds.$$

As shown in the appendix, this can be simplified to

$$F_p\left(\tau\right)\left(\mu_e - \tau\right) - \sigma_e \sigma_p \rho_{ep} f_p\left(\tau\right) + \delta \int_{-\infty}^{\infty} W^*\left(s, \tau\right)^*\left(s, \tau\right) f_s\left(s\right) ds$$

The first-order condition for an interior solution is

$$f_p\left(\tau\right)\left(\mu_e - \tau + \frac{\sigma_e}{\sigma_p}\rho_{ep}\left(\tau - \mu_p\right)\right) + \delta \int_{-\infty}^{\infty} \frac{\partial W^*}{\partial \bar{p}}\left(s, \tau\right) f_s\left(s\right) ds = F_p\left(\tau\right)$$

This condition mirrors (3) in that the marginal cost of increasing the offer is the probability it will be accepted. The first term of the marginal benefit of increasing the offer is the same as in (3): the expected gain in the initial period conditional on the parcel's decision being changed times the probability of its choice is changed. The new element is the benefit of increasing the probability that the parcel will be undeveloped at time T when the signal is observed. By the envelope theorem, $\frac{\partial W^*}{\partial \bar{p}}\left(s, \tau\right) = 0$ if the upper bound is not binding in the second period problem and $\frac{\partial W^*}{\partial \bar{p}}\left(s, \tau\right) = \frac{\partial W}{\partial \phi}\left(\bar{p}, s\right) > 0$ when the upper bound is binding. Therefore, the new term is necessarily positive, as the possibility of new information increases the marginal benefit of making an offer and will thus increase the optimal offer. While we know that the optimal offer will (weakly) increase with information, the central question of interest is how much it will increase at different values. To answer these questions, I turn to numerical simulation since the model has no analytical solution.

## 3 Impact of Information without Correlation

To explore the impact of different parameters on the model solution, I begin by normalizing the expected conservation value (μ_e) to 1 and measure all other values relative to this value. Throughout the main results, I also set $\delta = 1$ so the decision makers place equal weight on the time before and after additional information is received.[7] The numerical results presented here proceed in several steps. First, I describe in detail the impact of signal quality on the optimal last period policy for a single example, assuming that $\rho_{ep} = 0$. I then describe the two period solution for that case and how it varies with signal quality. Next, I discuss the impact of the distribution of possible values of p on those solutions, still keeping $\rho_{ep} = 0$. Finally, I explore the impact of allowing ρ_{ep} to vary. Throughout the results, the emphasis is on identifying when the possibility of improved information will have the largest impacts on policy today. At the end of the section, I briefly describe the parameters that would lead to multiple solutions to $B(p) = 0$ and would require a different first period strategy for the buyer.

3.1 Optimal policy with no correlation

For this simulation, I set $\mu_p = 0.5$, $\sigma_p = 0.25$, and $\sigma_e = 0.75$. These parameters imply that the buyer believes that the parcel provides twice as much value to society if it is conserved than it provides to the landowner when developed suggesting that a PES offer has a strong chance of being viable. The standard deviations imply that the buyer believes there is a roughly an 84% chance that the parcel will be developed in the absence of a payment and that there is a roughly 91% chance that the net benefit of conservation is positive.

Final period policy In the last period, the buyer has a signal of environmental service value that allows some degree of targeting of offers based on the expected environmental service value conditional on the signal observed. Based on the offer received, a parcel will fall into one of four categories: (I) conserved despite a zero offer because it is privately optimal to do so ($\phi = 0$ and $p \leq 0$), (II) paid for non-additional conservation ($\phi > 0$ and $p < 0$) (III) induced to conserve by the program because $0 < p \leq \phi$, and (IV) developed despite the offer because $p > \phi$. As the offer increases, the buyer must weigh the cost of additional losses on parcels in category II against increased gains for moving more parcels from category IV to category III. Figure 3 illustrates the optimal offers and identifies which parcels will fall into each category for three different values of ρ_{es} if all parcels remain undeveloped at the start of the second period. In each panel, the private development value increases along the vertical axis while the signal increases along the horizontal axis. The shaded contours in the background indicate the likelihood that

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[7] Increasing or decreasing the value of δ has the expected impact of increasing or decreasing the impact of improved information on today's outcomes. Also note that as long as $B(p) = 0$ has a unique solution, the value of δ does not influence the landowner's decision. Question for Vis – should I make a comparative static picture and put it in an appendix or is this too obvious to matter?

Fig. 3 Optimal offers and parcel categorization (baseline scenario)
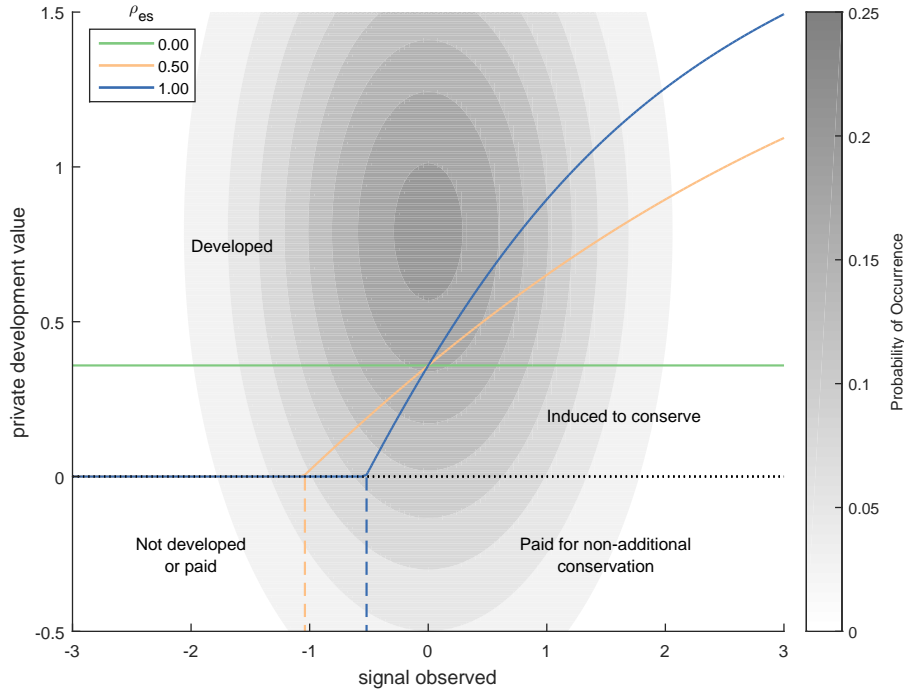
the parcel is located at that point. The solid line corresponding to the shading for the given ρ_{es} value is the optimal second period offer. When $\rho_{es} = 0$, the signal provides no useful information and the buyer will make the same offer to all parcels regardless of signal because the expected conservation value is the same. When $\rho_{es} > 0$, the conditional gain will shift with the signal as illustrated in Figure 2, resulting in different offers. As ρ_{es} increases, the minimum signal that will induce the buyer to make a positive offer increases. Above this threshhold, increasing the signal increases the offer at a decreasing rate, with the rate of decrease falling as the signal quality increases.

The parcel will be developed if it is located above the offer line corresponding to the given ρ_{es} value, since $p > \phi$ in this region. The offer will result in additional conservation if the parcel is located above the y-axis and below the solid line. If the parcel is below the y-axis it would have conserved regardless. If it is located below the y-axis and to the right of the dashed line corresponding to the given ρ_{es} value, it will be paid for non-additional conservation, while if it lies to the left of this line, it will conserve despite a zero offer. Since all parcels are made a positive offer when $\rho_{es} = 0$, there is no dashed line corresponding to $\rho_{es} = 0$ and all parcels below the y-axis would be paid for non-additional conservation. As the minimum signal needed to induce a positive offer increases with signal quality, the area in which the parcel is paid for non-additional conservation shrinks. Moreover, since the offer rises more rapidly as the signal quality increases, the area in which the offer induces the parcel to provide additional conservation also grows.

If the buyer observed the signal before any development occurred, the conclusions of Figure 3 would be straightforward: better information about conservation value would allow the buyer to target purchases more effectively. By limiting offers to parcels that are likely to provide high conservation value, the buyer

Fig. 4 Expected period 2 outcomes (baseline scenario)
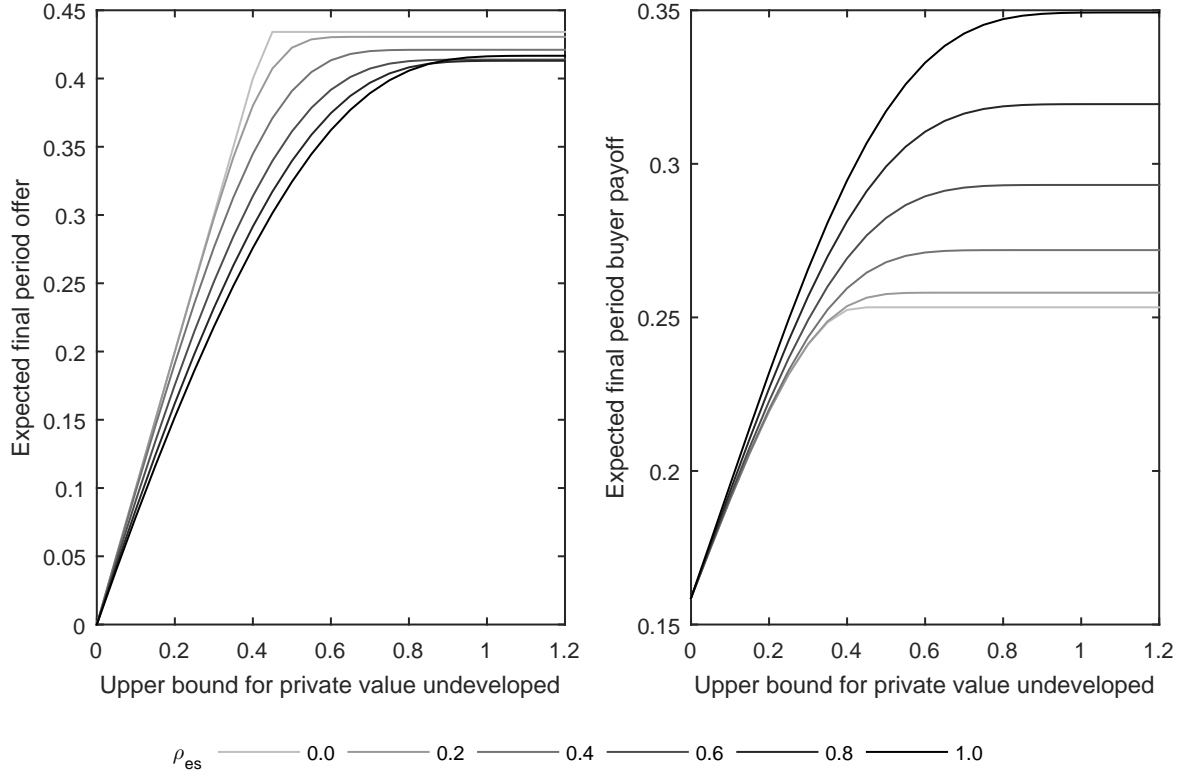
would limit non-additional payments and increase the expected gain on the parcels in category III. This figure readily demonstrates that if the parcel has a high value of p, it will eventually develop in period 2, implying its owner will develop at $t = 0$ before the signal is observed unless offered an incentive to conserve. In fact, since $\rho_{ep} = 0$ in this scenario, we know from Section 2.2 that $B(p) = 0$ has a unique solution and therefore any parcel in category III would develop at $t = 0$. The buyer must pay in the first period to have any option to purchase additional conservation in the second period.

As $\bar{p}$ falls, category III shrinks because if the parcel is in the upper right of this region, it has a p value and would have developed at $t = 0$. As this region shrinks, the buyer will reduce offers made after observing high signals, thus reducing the expected offer. Figure 4 illustrates the impact of changes in the upper bound remaining undeveloped on the expected second period offer and the buyer's optimzed second period payoff for several different signal quality levels. The shading of the lines indicates the signal quality with darker lines representing better information. As expected, lowering the upper bound reduces the mean offer as long as $\rho_{es} > 0$. For relatively high upper bounds, increasing signal quality increases the expected offer since improved information leads to bigger increases in offers when the signal is high than decreases in offers when the signal is low as shown in Figure 3. However, as $\bar{p}$ falls, this effect can be reversed. Since the optimal offer is bounded above by $\bar{p}$, the increases in offer when the signal is high fall while the decreases in offers for low signals remain unchanged. As a result, when $\bar{p}$ is sufficiently low, increasing ρ_{es} will lower the expected offer. Regardless of the impact on signal, increasing

information naturally increases the buyer's optimized payoff, with the impact increasing with the quality of the signal.

Initial period policy Since the conditions described in Section 2.1 hold when there is no correlation, we know that the parcel will only remain undeveloped if the private development value is less than or equal to the conservation payment offered in the initial period. Thus in the first period, the buyer will weigh the cost of paying to make the parcel more likely to conserve in period 1 against the gain from it being more likely that the parcel will remain undeveloped in the second period, allowing the buyer to achieve a higher payoff as illustrated in Figure 4. In the two period model, a parcel will be in one of five categories: (I) conserved in both periods despite a zero offer at time T because it is privately optimal to do so ($\phi = 0$ and $p \leq 0$), (II) paid for non-additional conservation in both periods ($\phi > 0$ and $p \leq 0$), and (III) induced to conserve in both periods by the program because $0 < p \leq \phi$, (IV) developed at time T because $\phi < p < \tau$, (V) developed immediately because $p > \tau$.[8]

The right hand column of Figure 5 illustrates the location of these categories in s, p space for $\rho_{es} = (0, 0.5, 1)$, in a manner similar to Figure 3. In each panel, the dashed line is the initial payment offered to all parties, which becomes the upper bound for private development value of the parcels remaining undeveloped at the start of the second period. The dot-dashed line is the optimal second period offer. If the buyer does not expect improved information as in the top right panel, the second period is simply a repeat of the first; in this case, no parcels are developed in the second period and no parcel is made a zero offer in the second period so there are no parcels in either category I or category IV. The buyer expects half of the uncertainty about e to be resolved in the middle panel and all of the uncertainty to be resolved in the bottom panel. In these figures, the dashed line marking the inital period offer and the dot-dashed line marking the second period offer diverge for low signals. As the signal quality improves, two effects occur. First, the first period offers increase, decreasing the size of category V. Second, the the buyer targets the second period offers more carefully, reducing the number of parcels in Category II and increasing the parcels in Category I. This targeting also shift parcels from Category III to Category IV. Category III may thus increase or decrease with more information.

The right column illustrates the probability and gain for each of these categories. The top right hand panel shows the probability that the parcel is in Categories I-IV.[9] The height of the stacked bar gives the probability the parcel will be conserved in period 1. The light purple section at the top of the bar is the probability the parcel will conserve in period 1 but then develop in period 2, so the top of the dark purple bar gives the probability the parcel will remain conserved at the end of the second period. The dark orange bars indicate non-additional conservation that is paid for in both periods, while the light orange bars indicate non-additional conservation that is only paid for in the first period. As the figure

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[8] Assuming all parcels remain undeveloped at time T is equivalently to assuming that category V is empty.

[9] The probability the parcel is developed immediately (Category V) is the space between the top of the bar and 1.

demonstrates, improving the quality of the signal increases the offer made initially and thus increases the probability the parcel will conserve initially. In the second period, the better information lowers the probability of conservation because the buyer targets high conservation value parcels only.

The middle panel shows the expected gain relative to no PES policy, conditional on the parcel being in each category. Categories I and II necessarily imply losses since they represent non-additional conservation. For Category I, these losses increase slightly with improved information as the initial offer increases because they will be paid more in that period. For Category II, the losses increase more noticeably because the parcel is likely to made larger offers in both periods. The value of the information is seen in the substantial increase in the gain for the parcels in Category III. Because the buyer's offer at T is targeted toward high-conservation value parcels, the gain on parcels in this category is substantially higher.

The bottom row illustrates the buyer's expected gain across both periods relative to no policy. The height of the stacked bar gives the expected gain, while the shading indicates the contribution to this gain from each category (e.g. the conditional gain from the middle panel times the probability the parcel will be in that category from the top panel). Since the buyer will on parcels in Categories I, II, and possibly IV, the bars for these categories are below the axis (representing losses) and the wide purple bars show much of the gain from Category III offsets these losses and how much remains.

3.2 Impact of changing the distribution of development value

Figure 6 illustrates the impact of changing the mean and variance of private development value on the offers made today, the probability of conservation today, and the expected payoff across both periods. The dashed lines in the second and third row represent the outcome if the buyer adopted no policy, while the different color lines indicate the value for various levels of ρ_{es}. As the mean development value increases along the x-axis in each panel, two effects occur. First, it is less likely that the parcel has a negative private development value and will conserve without payment. This reduction in the probability of paying for non-additional conservation will lead the buyer to increase payments as shown on the top row. Second, the cost of inducing a given probability of conservation increases because the probability of a high private development value increases. This leads to a smaller optimal conservation probability as shown on the second row. Since the probability of conservation falls to zero as μ_p increases, the buyer's payoff (which is measured relative to the value of development) also falls to zero. Changing the variance of private development while holding the mean constant as we move from panel to panel similarly has a number of impacts. It increases the probability of both very high and very low private development values, while decreasing the probability the parcel is close to the mean. This increases the probability the parcel will be paid for non-additional service and thus lowers the optimal offer. For a high standard deviation
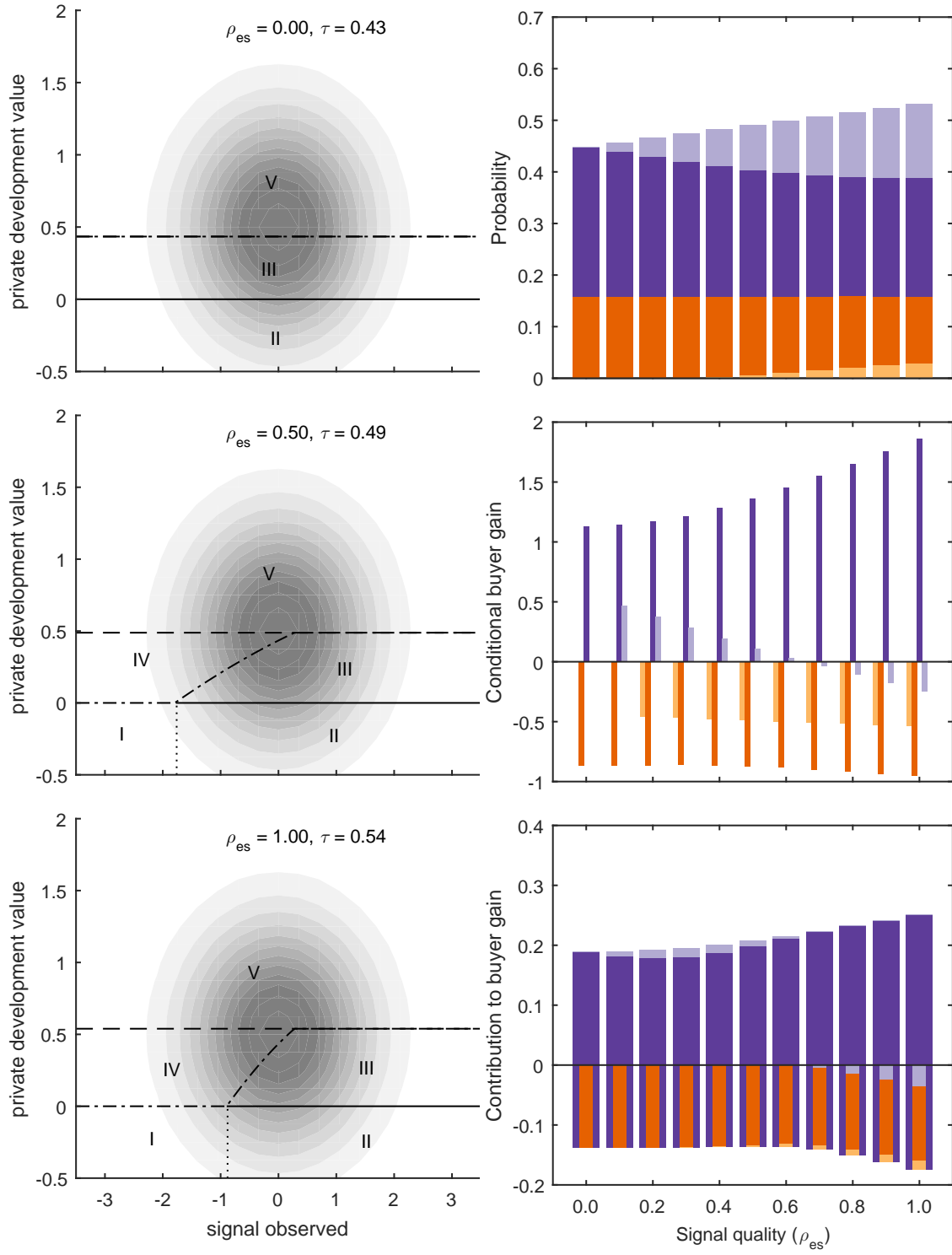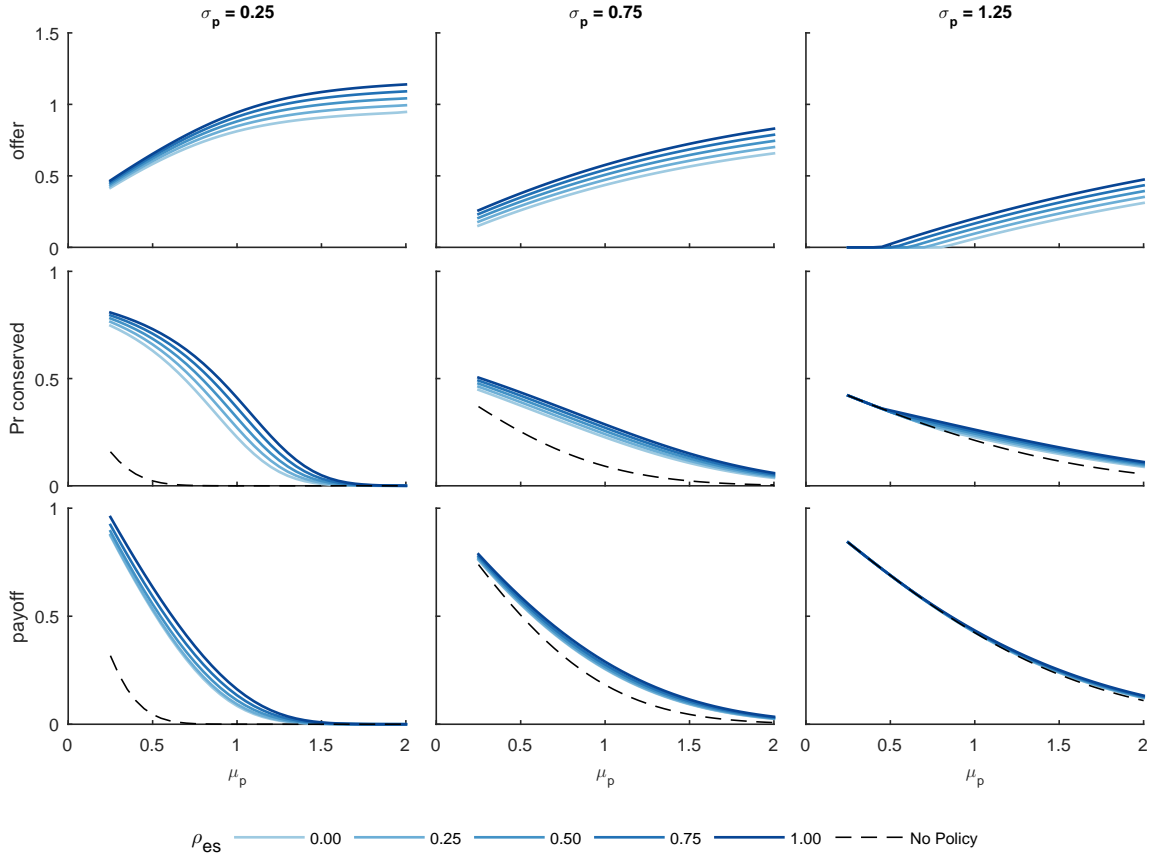
Fig. 5 Optimal outcomes for buyer by signal quality (baseline scenario)

Fig. 6 Impact of mean and variance of private development value

and low mean, the high probability of paying for non-additional services drives optimal payments to zero. As optimal offers fall, the probability of conservation and the buyers payoff both fall as well.

The effect of improved future information on the outcomes is illustrated by the different lines, with the lightest line representing no improved future information and the darkest line representing the resolution of all uncertainty about e by T. Looking at the top row, we see that the lines diverge more as information improves, implying that improving information leads to progressively larger changes in offers as μ_p increases. In other words, the buyer should modify today's policy more in response to the possibility of improved information when the private development value is expected to be high. At the same time, the value of modifying the policy, both in terms of the probability that land is conserved and overall payoff first rises and then falls as expected private development value increases. Ultimately, this implies that buyers should pay the most attention to the possibility of improved information when the uncertainty about private development value is smaller than the uncertainty about conservation value and the expected development value is neither too small or too large. When the expected development value is relatively small, the benefit of adopting some conservation payment policy is large because the required payments are low, but adjusting that policy to account for improved future information is less important. When the expected development value is relatively high, the buyer is unlikely to be able to profitably induce conservation whether or not improved information arrives in the future. The buyer

could marginally increase her payoff by making a large offer that is still almost certain to be rejected, but the value of doing so is small. This pattern remains similar as the variance increases, with one notable addition. In the top right panel, we see that as the quality of the expected information improves (moving from lighter to darker lines), the minimum value of μ_p that induces the buyer to make an offer falls. Holding the variance fixed, lowering the mean will increase the probability that the landonwer finds conservation privately optimal and will thus increase the probability that the buyer will end up paying for non-additional conservation. Information thus plays an important role in determining how large a probability of non-additional payments in period 1 the buyer is willing to tolerate when making an offer.

4 Impact of future information with correlation

The prior section assumes that conditional on the information known at time 0, there is no correlation between e and p. Due to the interpretation of e and p, the zero correlation case covers many of the scenarios often described as correlation between development and conservation value. If the buyer knows a particular parcel is likely to have both a high value to the landowner if developed and a high value to society if conserved, this suggests high values of μ_e and μ_p and relatively low values of σ_e and σ_p, but does not imply correlation. This section considers the possibility of correlation between the deviations of e and p from their means. As described above, this could occur if the parcel of land possesses a difficult to verify or observe characteristic that influences both e and p.

Final Period with Correlation Figure 7 replicates the left hand panels of Figure 5 with negative (left column) and positive (right column) correlation. Positive correlation makes the situation worse the buyer in all cases because the buyer will want to induce the parcel to conserve precisely when it is also the most expensive to induce to conserve. Moreover, since the buyer cannot reliably identify when this is the case, attempts to induce conservation will require high offers that will also be accepted when the parcel provides low gains. In contrast, negative correlation improves the situation for the buyer. The parcel is most likely to provide high conservation value when it is also most likely to be conserved without payment or to conserve with only a small payment. At the same time, information is far more valuable when there is positive correlation. With positive correlation and current levels of uncertainty about the conservation value, the buyer will make only a very small offer today. The probability of non-additionality and the likelihood of paying a parcel more than the services it ultimately provides are too high to raise the offer further. As a result, it is highly likely that the
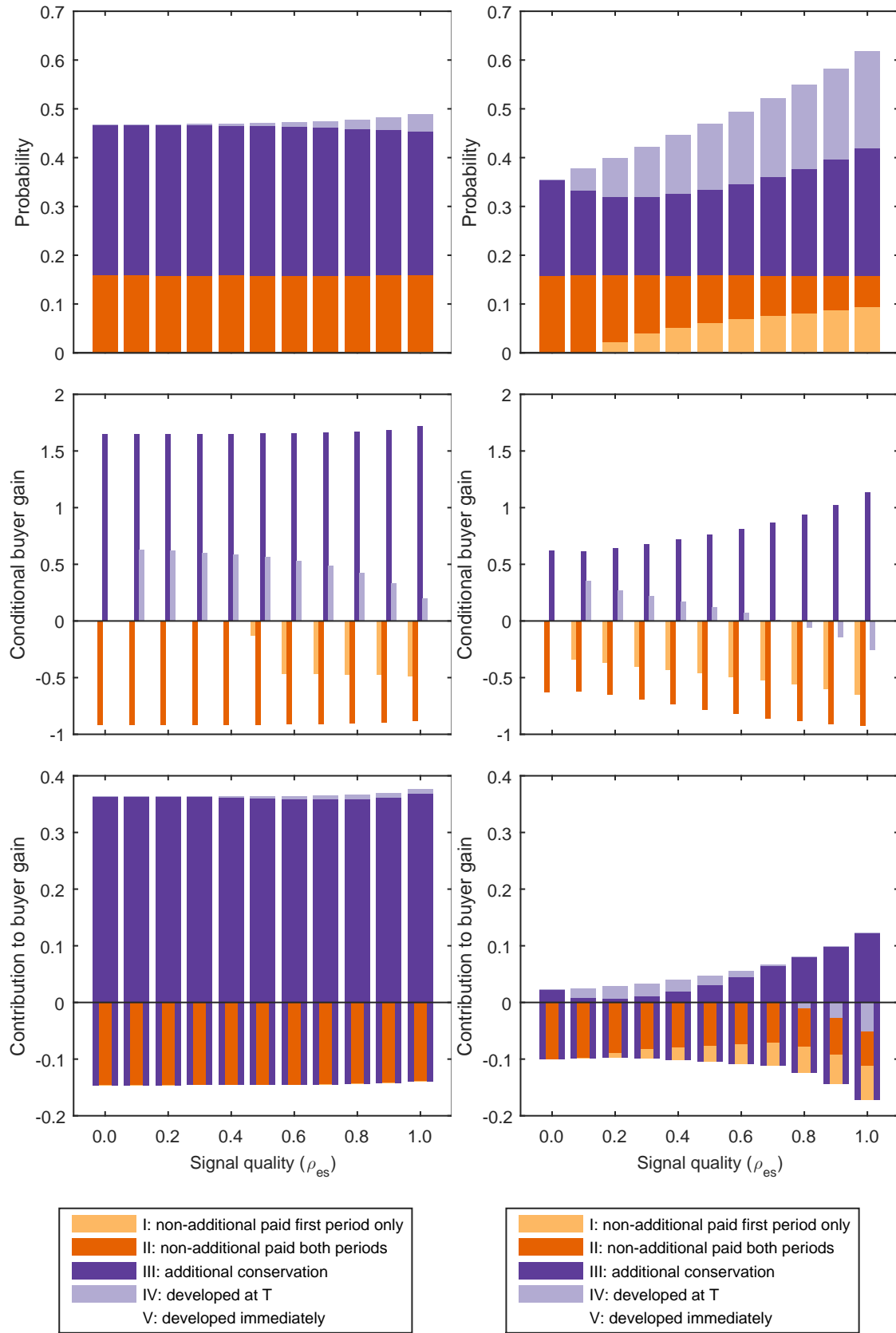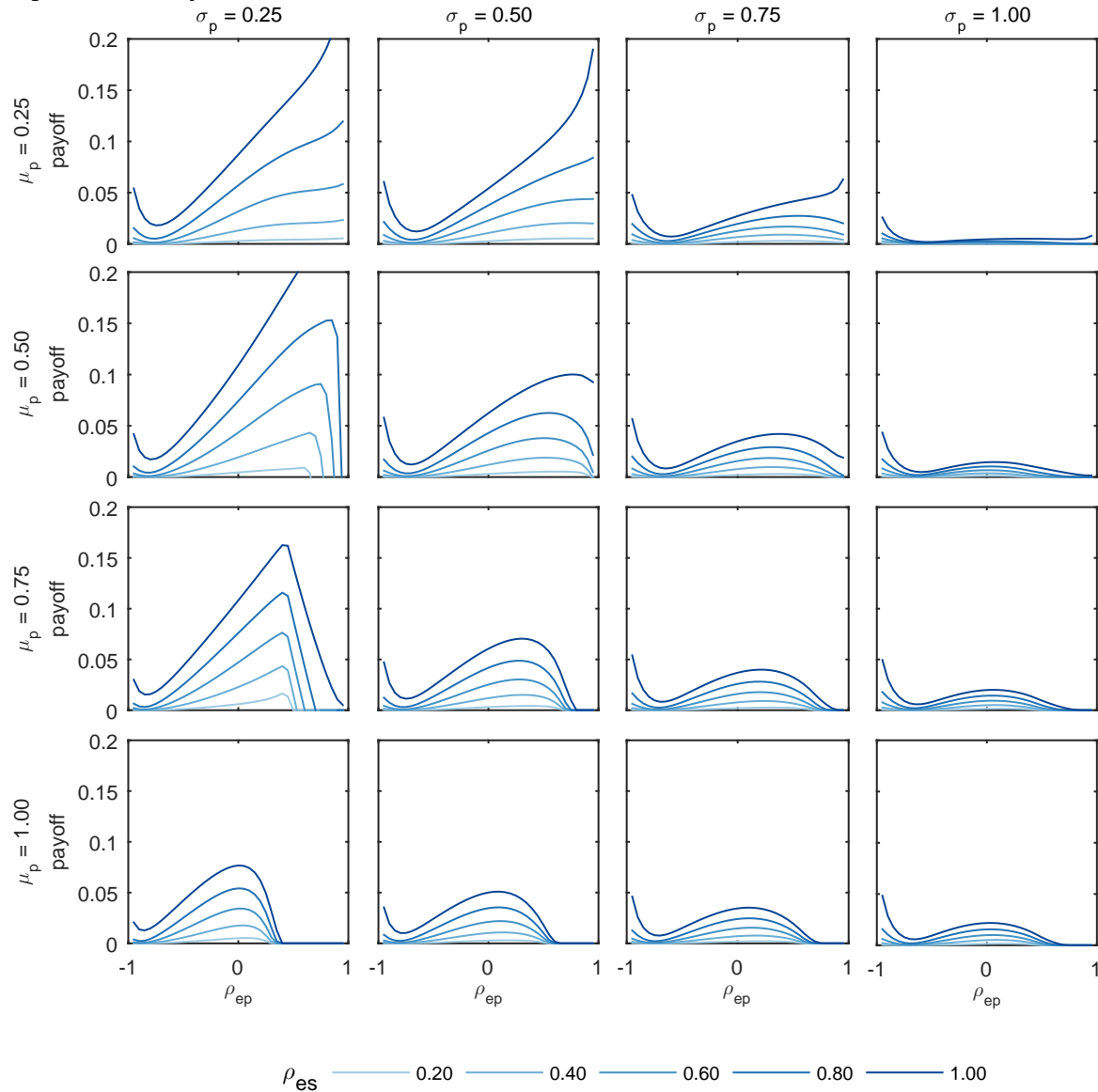
Fig. 7 Parcel Categories with Correlation

Fig. 8 Value of Improved Information

5 Conclusions

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A Detailed Mathematical Derivations

A.1 Simplification of buyer's single period payoff

The buyer's single period payoff as a function of the payment offered is given by

$$W\left(\phi,s\right)=\int\limits_{-\infty}^{\phi}\int\limits_{-\infty}^{\infty}\left(e-\phi\right)f_{e|s,p}\left(e|s,p\right)de f_{p|s}\left(p|s\right)dp$$

The multivariate normal assumption allows us to simplify this expression considerably. First, note that

$$\mu_{e|s,p}\left(s,p\right)=\frac{\sigma_{e}\left(\rho_{ep}-\rho_{ep}\rho_{sp}\right)}{\sigma_{p}\left(1-\rho_{sp}^{2}\right)}p+\mu_{e}+\frac{\sigma_{e}\left(\rho_{es}-\rho_{ep}\rho_{sp}\right)}{\left(1-\rho_{sp}^{2}\right)}s-\frac{\sigma_{e}\left(\rho_{ep}-\rho_{ep}\rho_{sp}\right)}{\sigma_{p}\left(1-\rho_{sp}^{2}\right)}\mu_{p}$$

so the inner integral can be rewritten as

$$\begin{aligned}\int\limits_{-\infty}^{\infty}\left(e-\phi\right)f_{e|s,p}\left(e|s,p\right)de &= \int\limits_{-\infty}^{\infty}ef_{e|s,p}\left(e|s,p\right)de-\int\limits_{-\infty}^{\infty}\phi f_{e|s,p}\left(e|s,p\right)de\\ &=\mu_{e|s,p}\left(s,p\right)-\phi\\ &=\frac{\sigma_{e}\left(\rho_{ep}-\rho_{es}\rho_{sp}\right)}{\sigma_{p}\left(1-\rho_{es}^{2}\rho_{ep}^{2}\right)}p+\mu_{e}+\frac{\sigma_{e}\left(\rho_{es}-\rho_{ep}\rho_{sp}\right)}{\left(1-\rho_{es}^{2}\rho_{ep}^{2}\right)}s-\frac{\sigma_{e}\left(\rho_{ep}-\rho_{ep}\rho_{sp}\right)}{\sigma_{p}\left(1-\rho_{es}^{2}\rho_{ep}^{2}\right)}\mu_{p}-\phi\end{aligned}$$

To simplify derivations, let $\alpha = \mu_e + \frac{\sigma_e\left(\rho_{es}-\rho_{ep}\rho_{sp}\right)}{\left(1-\rho_{es}^2\rho_{ep}^2\right)}s - \frac{\sigma_e\left(\rho_{ep}-\rho_{ep}\rho_{sp}\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}\mu_p - \phi$ and let $\beta = \frac{\sigma_e\left(\rho_{ep}\left(1-\rho_{es}^2\right)\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}$. The full first integral is thus

$$\int_{-\infty}^{\phi} \left(\alpha + \beta p\right) f_{p|s}\left(p|s\right) dp$$

or

$$\alpha F_{p|s}\left(\phi|s\right) + \beta \int_{-\infty}^{\phi} p f_{p|s}\left(p|s\right) dp.$$

The latter integral can also be simplified under the normality assumption to

$$\sigma_{p|s}^2 \left(-f_{p|s}\left(\phi|s\right)\right) + \mu_{p|s} F_{p|s}\left(\phi|s\right).$$

Combining elements, we have the full first integral as

$$\alpha F_{p|s}\left(\phi|s\right) + \beta \left(\sigma_{p|s}^2 \left(-f_{p|s}\left(\phi|s\right)\right) + \mu_{p|s} F_{p|s}\left(\phi|s\right)\right)$$

or

$$\left(\alpha + \beta\mu_{p|s}\right) F_{p|s}\left(\phi|s\right) - \beta\sigma_{p|s}^2 f_{p|s}\left(\phi|s\right).$$

We also know that

$$\mu_{p|s} = \mu_p + \sigma_p\rho_{sp}s$$

and

$$\begin{aligned}
\beta\mu_{p|s} &= \frac{\sigma_e\left(\rho_{ep}\left(1-\rho_{es}^2\right)\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}\left(\mu_p + \sigma_p\rho_{sp}s\right)\\
&= \frac{\sigma_e\left(\rho_{ep}\left(1-\rho_{es}^2\right)\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}\mu_p + \frac{\rho_{sp}\left(\rho_{ep}\left(1-\rho_{es}^2\right)\right)}{\left(1-\rho_{es}^2\rho_{ep}^2\right)}\sigma_e s
\end{aligned}$$

$$\begin{aligned}
\alpha + \beta\mu_{p|s} &= \mu_e + \frac{\sigma_e\rho_{es}\left(1-\rho_{ep}^2\right)}{\left(1-\rho_{es}^2\rho_{ep}^2\right)}s - \frac{\sigma_e\rho_{ep}\left(1-\rho_{es}^2\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}\mu_p - \phi + \frac{\sigma_e\rho_{ep}\left(1-\rho_{es}^2\right)}{\sigma_p\left(1-\rho_{es}^2\rho_{ep}^2\right)}\mu_p + \frac{\rho_{es}\rho_{ep}\rho_{ep}\left(1-\rho_{es}^2\right)}{\left(1-\rho_{es}^2\rho_{ep}^2\right)}\sigma_e s\\
&= \mu_e + \rho_{es}\sigma_e s - \phi
\end{aligned}$$

Finally, note that

$$\sigma_{p|s}^2 = \sigma_p^2\left(1-\rho_{es}^2\rho_{ep}^2\right)$$

so

$$\beta\sigma_{p|s}^2 = \sigma_e\sigma_p\rho_{ep}\left(1-\rho_{es}^2\right).$$

Putting all these pieces together, we have

$$W\left(\phi, s\right) = -\sigma_e\sigma_p\rho_{ep}\left(1-\rho_{es}^2\right)f_{p|s}\left(\phi|s\right) + \left(\mu_e + \rho_{es}\sigma_e s - \phi\right)F_{p|s}\left(\phi|s\right)$$

B Proof of Proposition 1

Case 1 $\bar{p}$ is finite.

If $\bar{p}$ is finite, then the interval $[0, \bar{p}]$ is closed and bounded and since $W(\phi, s)$ is continuous, it attains a maximum on this interval.

Case 2 $\bar{p} = \infty$

In this case, the buyer believes the landowner will choose to conserve in period 1 regardless of the value of p. The $\lim_{\phi \to \infty} W(\phi, s) = -\infty$ so there exists a value $\hat{\phi}$ such that $W(\phi, s) < W(\hat{\phi}, s)$ for any $\phi > \hat{\phi}$. Again, continuity of $W(\phi, s)$ implies that it attains a maximum on the closed and bounded interval $[0, \hat{\phi}]$, with the maximized value greater than or equal to $W(\hat{\phi}, s)$. Thus, this value also maximized $W(\phi, s)$ on $\phi \geq 0$.

C Proof of Proposition 2

A local interior maximum requires $\frac{\partial W}{\partial \phi} > 0$ and $\frac{\partial^2 W}{\partial \phi^2} \leq 0$.

$$\frac{\partial W}{\partial \phi}(\phi, s) = -\sigma_e \sigma_p \rho_{ep} \left(1 - \rho_{es}^2\right) \frac{df_{p|s}}{dp}(\phi|s) + (\mu_e + \rho_{es}\sigma_e s - \phi) f_{p|s}(\phi|s) - F_{p|s}(\phi|s)$$

Since $p|s \sim N\left(\mu_p + \sigma_p \rho_{es} \rho_{ep} s, \sigma_p^2 \left(1 - \rho_{es}^2 \rho_{ep}^2\right)\right)$, we know that

$$\frac{df_{p|s}}{dp}(\phi|s) = -\frac{\phi - \mu_p - \sigma_p \rho_{es} \rho_{ep} s}{\sigma_p^2 \left(1 - \rho_{es}^2 \rho_{ep}^2\right)} f_{p|s}(\phi|s).$$

giving

$$\frac{\partial W}{\partial \phi}(\phi, s) = f_{p|s}(\phi|s) \left[\left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right) \phi + \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2\right) s + \mu_e - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p\right] - F_{p|s}(\phi|s) \qquad (4)$$

where $\gamma = \frac{1 - \rho_{es}^2}{1 - \rho_{es}^2 \rho_{ep}^2}$. The second derivative is

$$\frac{\partial^2 W}{\partial \phi^2}(\phi, s) = f_{p|s}(\phi|s) \left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right) + \frac{df_{p|s}}{dp}(\phi|s) \left[\left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right) \phi + \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2\right) s + \mu_e - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p\right] - f_{p|s}(\phi|s).$$

Using the same substitution as above, we get

$$\frac{\partial^2 W}{\partial \phi^2} = f_{p|s}(\phi|s) \left[\left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right) + \left(\frac{\mu_p + \sigma_p \rho_{es} \rho_{ep} s - \phi}{\sigma_p^2 \left(1 - \rho_{es}^2 \rho_{ep}^2\right)}\right) \left[\left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right) \phi + \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2\right) s + \mu_e - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p\right] - 1\right].$$

Letting $a = \left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1\right)$, $b = \frac{1}{\sigma_p^2 (1 - \rho_{es}^2 \rho_{ep}^2)}$, $c = \frac{\mu_p + \sigma_p \rho_{es} \rho_{ep} s}{\sigma_p^2 (1 - \rho_{es}^2 \rho_{ep}^2)}$, and $d = \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2\right) s + \mu_e - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p$, we have

$$\frac{\partial^2 W}{\partial \phi^2} = f_{p|s}(\phi|s) \left[a + (c - b\phi)(a\phi + d) - 1\right]$$

or

$$\frac{\partial^2 W}{\partial \phi^2} = f_{p|s}(\phi|s) \left[a + \left(ca\phi + dc - ba\phi^2 - bd\phi\right) - 1\right]$$

or

$$\frac{\partial^2 W}{\partial \phi^2} = f_{p|s}(\phi|s) \left[-ba\phi^2 + (ca - bd)\phi + a + dc - 1\right].$$

Since $f_{p|s}(\phi|s) > 0$ for all ϕ, $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) = 0$ only when the term in brackets equals 0. This expression is quadratic in ϕ, implying that $\frac{\partial^2 W}{\partial \phi^2}(\phi, s)$ has at most two zeros. If there are multiple interior local maxima of $W(\phi, s)$, they must both have $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) \leq 0$ and be separated by an interior minimum where $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) \geq 0$. This implies that there must be two zeros of $\frac{\partial^2 W}{\partial \phi^2}(\phi, s)$ between the local maxima. Since $\frac{\partial^2 W}{\partial \phi^2}(\phi, s)$ has at most two zeros, this requires that the sign of $\frac{\partial^2 W}{\partial \phi^2}(\phi, s)$ is the same for any value of ϕ above the largest value of ϕ that results in a local maximum. Since we must

have $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) < 0$ and $\frac{\partial W}{\partial \phi}(\phi, s) = 0$ at the largest local maxium, this requires that $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) < 0$ and $\frac{\partial W}{\partial \phi}(\phi, s) = 0$ for any value of ϕ above the largest value of ϕ that results in a local maximum. Moreover, it requires that $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) < 0$ and $\frac{\partial W}{\partial \phi}(\phi, s) > 0$ for any value of ϕ below the smallest value of ϕ that results in a local maximum. Note that the $\lim_{\phi \to \infty} -ba\phi^2 + (ca - bd)\phi + a + dc - 1 = -ba(\infty)$ and $b > 0$, suggesting that the sign of $\frac{\partial^2 W}{\partial \phi^2}(\phi, s)$ at very high values of ϕ is the opposite of the sign of a. Since we must have $\frac{\partial^2 W}{\partial \phi^2}(\phi, s) < 0$ at high values of ϕ, we can only have two local maxima of $W(\phi, s)$ if $a > 0$. But if $a > 0$, then $\lim_{\phi \to -\infty} \left(\gamma \frac{\sigma_e \rho_{ep}}{\sigma_p} - 1 \right) \phi + \sigma_e \rho_{es} \left(1 - \gamma \rho_{ep}^2 \right) s + \mu_e - \frac{\sigma_e \rho_{ep}}{\sigma_p} \gamma \mu_p = -\infty$ which means that $\frac{\partial W}{\partial \phi}(\phi, s) < 0$ at very low values of ϕ, which contradicts the requirement that $\frac{\partial W}{\partial \phi}(\phi, s) > 0$ for values of ϕ below the smallest value of ϕ that results in a local maximum.

D Supplemental Graphs

Fig. 9 Optimal Policies with Correlation