

Problem Set 1—Due in class on Friday, Feb 9

1. Fill in the missing prices in the following table of minimum wage levels:

	Nominal Wage (\$/hr)	CPI	Real Wage (\$ ₁₉₅₅ /hr)	Real Wage (\$ ₂₀₁₅ /hr)
1955	0.75	26.8		
1985		107.6		7.38
2015		237.0		7.25

2. Using separate diagrams for each subpart, sketch the impact of the following changes on the market for automobiles. (*Note:* You do not need to find a mathematical answer. Just sketch the qualitative impacts). You should identify the equilibrium both before and after the change and then briefly explain the change.
- An increase in the price of steel (a major input for automobile manufacturers)
 - An increase in consumer income
 - A decrease in the supply of gasoline (you can assume the market for gasoline is perfectly competitive)
3. Suppose the demand for bicycles is given by $Q_D = 10,000P^{-\frac{2}{3}}$ and the supply of bicycles is given by $Q_S = 0.01P^{\frac{4}{3}}$.
- Using calculus and the given supply and demand curves, find mathematical expressions for the elasticities of demand and supply as a function of the price. Use these expressions to demonstrate that both curves have constant elasticity along their entire length.
 - Find the equilibrium price and quantity in this market mathematically.
4. Suppose you run a factory whose short-run production function is given by

$$f(L) = 180L^2 - 3L^3.$$

- Find mathematical expressions for the total product of labor, the average product of labor and the marginal product of labor.
- On two separate axes, graph TP_L , AP_L , and MP_L . The diagrams do not need to be scale, but be sure they are internally consistent. In other words, make sure that all relevant relationships between the three lines are correct. If you have trouble figuring out what the graph looks like, feel free to use a graphing calculator or an online graphing tool to find the basic shape. Find the quantity of labor associated with each of the following points and label it on your diagram: (i) the maximum value of TP_L , (ii) the maximum value of AP_L , (iii) the maximum value of MP_L .
- For what values of L does the production function exhibit each of the following:
 - increasing marginal labor productivity
 - diminishing marginal labor productivity
 - negative marginal labor productivity

Your answers for part (c) should be in the form of an inequality.

5. Under what conditions will each of the following production functions exhibit (i) increasing returns to scale, (ii) decreasing returns to scale, and (iii) constant returns to scale? *Notes: your answer should be in the form of inequalities involving a and/or b . For some production functions it may not be possible to exhibit each type of returns to scale.*

- (a) $f(L, K) = L^a K^b$
- (b) $f(L, K) = a(K^b + L^b)$
- (c) $f(L, K) = (aK + bL)^{\frac{1}{2}}$
- (d) $f(L, K) = aKL^b$

6. Indira runs a bakery using labor (L , measured in hours of her time) and capital (K , measured in ovens). Indira's production function (measured in loaves of bread) is given by

$$f(L, K) = 4L^{\frac{1}{3}}K^{\frac{2}{3}}.$$

- (a) Indira currently has 8 ovens.
 - i. Find an expression for her (short-run) average and marginal product of labor.
 - ii. In two separate diagrams, graph Indira's total, average, and marginal product of labor. Do these curves follow all the rules we discussed about the relationships between curves? Does Indira's production technology satisfy the law of diminishing marginal returns?
 - (b) Suppose Indira is trying to decide whether to buy more ovens or sell some of the ones she has and wants to understand the tradeoffs between more ovens and her time. If Indira wants to produce 100 loaves a week, help her find out how many ovens she'll need for various amounts of her time. That is, find an expression for the number of ovens she'll need to produce 100 loaves as a function of the number of hours she works. (Hint: if you're stuck, try picking a couple specific numbers of hours she might work and see how many ovens she'd need to get 100 loaves).
 - (c) Plot your equation from part (b). What does this line represent?
 - (d) Find an expression for Indira's marginal rate of technical substitution between ovens and labor. What is the connection between your answer to this question and the picture you drew in part (c).
7. XYZ corporation produces widgets. Its short-run marginal cost curve is given by $MC(q) = 10 - 5q + q^2$ (this is a parabola whose minimum occurs at $q = 2.5$). XYZ's fixed costs are 10. In two separate diagrams, graph the following cost curves: (a) total cost, (b) total variable cost, (c) total fixed cost, (d) marginal cost, (e) average variable cost, and (f) average total cost. Your diagrams do not need to be scale, but must be internally consistent (i.e. the relationships between different curves must be correct). You **do not** need to find mathematical expressions for the other cost curves – you only need to sketch lines that are consistent with the shape of the marginal cost curve.

8. Zoya runs a cement pouring business. The rental rate for cement trucks is \$80 day. Zoya currently has 9 trucks in her fleet. Her drivers are paid \$18 an hour. The total number of cement driveways she can pave in a day is given by $f(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$ where K measures the number of trucks and L measures the total hours worked by all her drivers (Note that the production function is unrealistic because it assumes that 1 driver working 10 hours is the same as 10 drivers working 1 hour each). Throughout this question, you can think of Zoya renting fractional trucks if necessary.
- (a) Find Zoya's **short-run** total cost, average total cost, average variable cost and marginal cost as a function of the number of cement driveways she produces. Assume that labor is the variable input and capital is fixed in the short run (i.e. Zoya has long term leases on the cement trucks).
 - (b) Zoya currently has 1 driver for each of her 9 trucks and each driver works 4 hours per day for a total of 36 hours of labor. Is Zoya's current production regime minimizing her long-run cost of production? If yes, explain how you know. If not, explain a small change that Zoya could make to produce the same number of driveways at a lower cost. In either case, your answer should focus on the intuition and should not rely on either the method of Lagrange multipliers or an isoquant/isocost diagram.
9. Join our Slack workspace (see Moodle or email from me for the link) and answer the survey regarding how you meet the pre-requisites for this class.