Insurance

In section today, we will be learning why people buy insurance. Instead of presenting the graph and analysis of insurance, this handout will walk you through how to analyze insurance using tools I will present at the beginning of section. This exercise will serve as both a demonstration of how insurance works (a topic you are responsible for on the exams and problem sets) **and** a demonstration of how to apply a model you have learned in one context to a slightly new situation. Your problem set asks you to go through a similar exercise applying the same ideas to another new situation but doesn't give you as many hints about how to proceed. You may find reviewing this exercise helpful when working on that problem. I anticipate that you will probably not have time to complete the entire exercise in section.

How Insurance Works

Many of you may already be familiar with how insurance operates, but as a brief summary for those who are not:

Consumers who are facing a possible loss (due to theft, fire, accident, illness, etc.) purchase an insurance policy against the loss. In insurance markets, the price paid for the insurance is called a **premium**. The consumer pays the premium up front. If there a loss occurs, the insurance company will reimburse the consumer for the losses. If a loss does not occur, the insurance company keeps the premium.

Today, we will focus on **full insurance** which completely eliminates risk because the insurance company will completely cover the potential loss. For example, suppose there is a risk that you will be robbed. If you purchase full insurance on your belongings and are robbed, the insurance company will completely compensate you for the loss and you will be no worse off having been robbed. (For simplicity in our analysis today, we will ignore any psychic or sentimental losses that result from being robbed, having your house burn down, etc.).

Before we get started, we need one more definition. **Actuarially fair** insurance is insurance whose premium is equal to the expected loss. For example, suppose there is a 10% chance that you will be robbed and lose \$3000. **Full insurance** would pay you \$3000 if you are robbed and nothing if you are not robbed. Your expected loss is $.9\,(\$0) + .1\,(\$3000) = \$300$ because 90% of the time you will lose nothing and 10% of the time you will lose \$3000. Actuarially fair insurance against this loss will have a premium of \$300.

Understanding the Purchase of Insurance

We will be using a basic set of facts throughout this example. The total value of David's possessions is \$10,000 (note that $10,000^{1/2} = 100$). There is a 20% chance that David will be robbed. If he is robbed, he will lose \$7500. David's utility function is defined over his total wealth and is given by $u\,(w) = w^{\frac{1}{2}}$. For our purposes, we can treat a utility function defined over wealth just like we treated one defined over income.

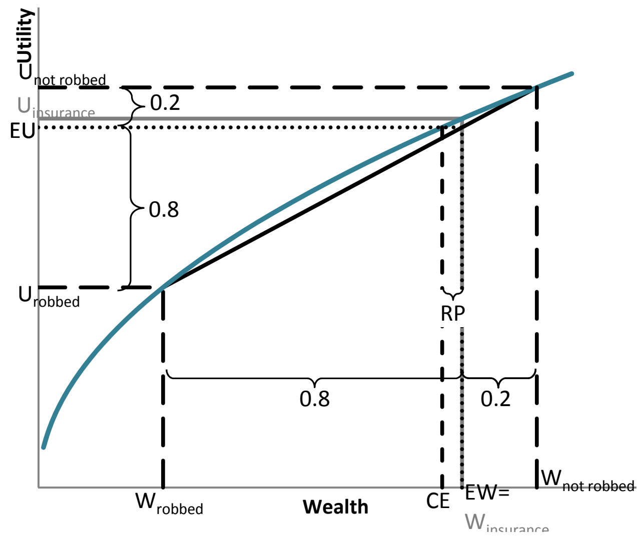
Questions 1-4 use the model I will present at the beginning of section while the remaining questions add insurance into the picture.

1. Calculate David's expected wealth and his expected utility using the approach I presented at the beginning of class.

$$EW = .8\,(10,000) + .2\,(2500) = 8,000 + 500 = 8500$$
$$EU = .8\left(10,000^{1/2}\right) + .2\left(2500^{1/2}\right) = .8\,(100) + .2\,(50) = 80 + 10 = 90$$

2. Draw a diagram illustrating David's current wealth, his wealth in the event of a loss, his expected wealth and his expected utility.

   When drawing one of these diagrams, you do not need to make it to scale (although the diagram shown here **is** to scale).



   The key points on the diagram are:

   - his current wealth is high at $W_{not\,robbed}$
   - his wealth in the event of a loss is much lower at W_{robbed}
   - you have drawn a chord (i.e. a straight line) from U_{robbed} to $U_{not\,robbed}$ (you did not need to label U_{robbed} and $U_{not\,robbed}$ because the question did not ask you to)
   - EW is found by dropping down to the horizontal axis 80% of the way along your chord from $robbed$ to $not\,robbed$ because there is an 80% chance that you will NOT be robbed. On a test, you would not need to explicitly label the 80% and the 20% but you would need to have the point noticeably closer to $not\,robbed$.
   - EU is found by going over to the vertical axis 80% of the way along your chord from $robbed$ to $not\,robbed$. Again, you would not need to explicitly label the 80% and the 20% but the lines identifying EU and EW must intersect along the chord, not along the utility function. In the diagram, these lines are shown with small black dots (note that the vertical one is on top of the gray solid line described below).

3. On your diagram, identify David's certainty equivalent and his risk premium.

   David's certainty equivalent is found by determining where the line showing his expected utility (i.e. the horizontal line of small dots) intersects his utility function and going down to the horizontal axis. On the diagram this line is shown with short black dashes. His risk premium is positive and is the difference between his expected wealth and his certainty equivalent.

4. Calculate David's certainty equivalent and risk premium.

   David's certainty equivalent is the amount of money which will make exactly as well off as the risky situation he is facing. In (1), we calculated that his expected utility is 90. Therefore, we must have $U(CE) = 90$. Since his utility function is $w^{1/2}$, this means $CE^{1/2} = 90$ or $CE = 90^2 = 8100$.

   David's risk premium is the difference between his expected wealth and his CE. $RP = EW - CE = 8500 - 8100 = \$400$.

5. Suppose David is offered actuarially fair insurance against the full value of his potential loss. What is the premium that would be charged?

   The premium for actuarially fair insurance is equal to the expected loss. David's expected loss is $EL = .8(0) + .2(7500) = 1500$. Therefore, the premium for actuarially fair insurance will be \$1500.

6. What is David's expected wealth if he buys the insurance? His expected utility? Add these amounts to your diagram.

   If David buys the insurance, he gives up \$1500 today and faces no further risk. Therefore,

   $EW = 1(10,000 - 1500) = 8500$

   $EU = 1\left(8500^{1/2}\right) \approx 92.2$

   These amounts were added to the diagram using solid gray lines. Notice that David's expected wealth with insurance is exactly equal to his expected wealth without insurance, but his expected utility is higher.

7. Will David buy the insurance? How do you know?

   David will buy the insurance because it gives him higher utility than not buying it. We can see this from either the diagram or our calculations. (And referring to either would be a sufficient explanation on a problem set or exam.) Moreover, as we said in (6), David's expected wealth is the same with or without insurance, but insurance eliminates all risk.

   We can tell from David's utility function that he is risk averse. His marginal utility of wealth is $MU(W) = \frac{1}{2}w^{-1/2} = \frac{1}{2w^{1/2}}$ which decreases as wealth increases implying he has diminishing marginal utility of wealth which means he is risk averse. Since David is risk averse, he prefers the certain wealth with insurance to the risky situation without insurance.

8. Suppose the insurance premium was \$2000. Would David buy the insurance?

   If the insurance premium was \$2000 and David bought the insurance, his (certain) wealth would be \$8000. His utility would be $EU = 8000^{1/2} \approx 89.4$. This is less than his expected utility of 90 without insurance so David would not buy the insurance.

9. At what premium would David be indifferent between buying the insurance and not buying it? How much does this amount exceed the actuarially fair premium?

   For David to be indifferent between buying and not buying the insurance, his expected utility must be the same in either case. Suppose the insurance premium is x. If David buys insurance, his wealth will be $10,000 - x$. Since we need his utility equal to 90 (his expected utility w/o insurance), we must have $U(10,000 - x) = 90$ or $(10,000 - x)^{1/2} = 90$. Squaring both sides gives us $10,000 - x = 8100$ or $x = 1900$. This is $400 more than the actuarially fair premium.

10. What is the relationship between your answer to (9) and David's certainty equivalent and risk premium?

    Notice that with a premium of $1900 David's wealth will be $8100 which is his certainty equivalent. Thus David is indifferent between buying insurance and not buying insurance when the premium leaves him at his certainty equivalent if he buys the insurance.

    Also notice that David was willing to pay $400 more than the actuarially fair premium, which is exactly equal to his risk premium. The actuarially fair premium offers insurance against risk that doesn't change expected wealth/income. David's risk premium tells us the amount of expected wealth/income David is willing to give up to avoid risk. By paying an extra $400 over the expected loss, David gives up $400 in expected wealth to avoid risk.

11. What is the maximum amount David would be willing to pay for insurance?

    This question is just designed to help you realize that if David is indifferent between buying the insurance or not when the premium is $1900, this is the maximum amount he will pay. If it cost a tiny bit more, he would be better off not buying it.