

A new color image secret sharing protocol

José Ignacio Farrán and David Cerezo

Abstract—Visual cryptography aims to protect images against their possible illegitimate use. Thus, one can cipher, hash, or add watermarks for protecting copyright, among others. In this paper we provide a new solution to the problem of secret sharing for the case when the secret is an image. Our method combines the Shamir scheme for secret sharing using finite fields of characteristic 2 with the CBC mode of operation of a secure symmetric cryptographic scheme like AES, so that the security relies on that of the mentioned techniques. The resulting shares have the same resolution as that of the original image. The idea of the method could be generalized to other multimedia formats like audio or video, adapting the method to the corresponding encoded information.

Index Terms—Visual cryptography, finite fields, secret sharing, Shamir scheme, AES cryptosystem, CBC mode.

I. INTRODUCTION

The purpose of secret sharing schemes in cryptography [8] is to divide a secret into pieces among a set of users, in such a way that these pieces separately do not have any information about the original secret, and so that only authorized sets of participants can retrieve the secret if such users share their pieces, called shares. Moreover, a set of users sharing their shares can not obtain the secret if they do not constitute an authorized set.

The simplest secret sharing schemes are the so-called threshold schemes, where the total number of participants is n and there is a minimum number $1 \leq t \leq n$, called *threshold*, so that any set of t or more participants can obtain the secret, whereas any set of less than t users can not obtain any information about the secret. The most popular threshold schemes are based on the Chinese remainder theorem [12], or Lagrange interpolation (the Shamir scheme [18]). In fact, the Shamir's secret sharing scheme is very simple to apply, and has some extra properties like linearity, that allows us to use it in other applications like secure multiparty computation [4].

If the secret to share is an image, that is called Visual Secret Sharing Schemes or simply Visual Cryptography, there are some papers to build shares of an image with the same purpose. The first paper was by Naor and Shamir [14], where their method works only for black and white images, the size of the shares is doubled larger with respect to that of the original image, and the shares are grayscale images. Other contributions work for more general images, normally changing also the size of the shared images. So the ideas of Naor and Shamir were generalized for grayscale and colour images in [19], with the same drawbacks as in the previous paper. In [20] the secret sharing is done with the aid of

J.I. Farrán: Departamento de Matemática Aplicada, Universidad de Valladolid, Spain. E-mail: jifarran@uva.es

D. Cerezo: Computer Science Engineer. E-mail: dacema97@gmail.com

Partially supported by MCIN PID2019-105306RB-I00.

steganography, namely by using an extra image as covering of the secret image. In the paper [1] the sharing is done by dividing the color channels (Red-Green-Blue) and combining it with a covering extra image. Finally, the paper [13] is a survey on visual cryptography with the above techniques.

On the other hand, the paper [2] applies the Shamir scheme for color image secret sharing by interpreting the pixel as the coefficients of polynomials. The result is that the shares have small size than the original image, one has to generate a lot of random constant terms for those polynomials, and moreover in order to retrieve the original image one has to solve a lot of linear systems, depending on the size of the shared image. Our contribution uses Shamir on the pixels in an alternative way, so that the shares have the same size as the original image, we only generate a few random random polynomials, and consequently we only have to solve a few linear systems, and expanding the result with the use of a symmetric cryptosystem.

Concerning encryption, in order to cipher an image it is not a good idea to do it with the mode of operation ECB (Electronic Code Book), that ciphers the blocks independently, and that allows us to obtain an image similar to the original one, just by applying some visual filters [5]. In fact, it is better to use other modes of operation, like CBC [16].

In this paper, we propose a visual secret sharing scheme, on the basis of the Shamir's scheme over a finite field of characteristic 2, by combining this scheme with a symmetric encryption scheme like AES [15] in mode CBC. The security of the method is the same as that of the Shamir's scheme, and in addition that of the AES cryptosystem. The advantage of our scheme is that it can be applied to any type of image (color or grayscale), and the shares have the same size as the original one. Moreover, we claim that the same idea can be used for sharing other types of multimedia files, like audio or video.

The paper is organized as follows: Section 2 recalls some preliminary concepts on digital images and finite fields, the threshold scheme of Shamir based on polynomial interpolation, and presents an overview on symmetric cryptography, with special emphasis on the Advanced Encryption System (AES) with the Cipher Block Chaining (CBC) mode of operation. In the main section 3 we propose a new visual secret sharing scheme for color images, and in section 4 we present some examples and experiments, together with some details about our implementation in pure Python [10]. We end the paper with some conclusions and remarks in section 5.

II. PRELIMINARES

This section is devoted to recall some basics on digital images, finite fields, secret sharing and symmetric cryptography.

The reader having this background could go directly to section III.

A. Digital images

Bitmap images consist of a matrix of pixels, where the number of rows is the vertical resolution of the image and the number of columns is its horizontal resolution (see [6]). In the RGB system for color images each pixel is encoded by three integers, each of them representing the level of (R)ed, (G)reen and (B)lue. For example, the Python package scikit-image represents a bitmap color image as a 3D array, where the internal arrays are lists of three integers.

The normal depth of the color channels is 8 bits, so that these integers belong to the interval $[0, 255]$, and hence each pixel is encoded by 24 bits. In the case of grayscale images, each pixel is encoded only by an integer representing the corresponding level of gray, normally also in the range $[0, 255]$, so that in this case scikit-image represents a grayscale image just as a 2D array.

Several formats contain the information about the pixels in raw, that is, without any compression, like BMP, PPM, XPM. In practice, there exist many compressed format, with or without loss of information, like JPEG, PNG, GIF or TIFF. In this case, an algorithm of decompression reconstructs the original matrix of pixels.

All the formats have a header containing basic information about the image, like the image type, the resolution, the depth of color, etc. The rest of the file contains a stream of bits corresponding to the integers associated to the pixels in binary, once the image is read or uncompressed, according to the case. For our purposes, we will separate the header on one hand, and on the other hand we will work with the bitstream as a long array of bits.

B. Finite Fields

Since the images we will work with are just streams of bits, we are interested in representing the images as a sequence of arrays of bits with constant size m , so that we can identify these bit arrays as elements of a finite field of characteristic 2, namely the Galois field with 2^m elements. We recall that such a finite field is constructed from the binary field $\mathbb{F}_2 = \{0, 1\}$ as the field extension generated by an irreducible polynomial of degree m .

In this way, the elements of \mathbb{F}_{2^m} can be regarded as polynomials of degree smaller than m with binary coefficients, and thus they can be encoded precisely as arrays of m bits. As a consequence, the sum in \mathbb{F}_{2^m} can be performed as the bitwise XOR of bit arrays, where as the multiplication is usually implemented by a table, containing the correspondence between the bit arrays and the powers of a primitive element (see [11]).

In case of the irreducible polynomial being a Conway polynomial, the computations are simpler and faster (see [7] for further details). However, these Conway polynomials are very hard to compute. A list of available Conway polynomials can be seen in [9]. For the sake of efficiency, the possible lengths of bit arrays are restricted to correspond to the degree

of a Conway polynomial in characteristic 2. In our Python implementation we have chosen $m = 64$, so that the chosen Conway polynomial is

$$\begin{aligned} P(X) = & X^{64} + X^{33} + X^{30} + X^{26} + X^{25} + X^{24} + X^{23} + \\ & X^{22} + X^{21} + X^{20} + X^{18} + X^{13} + X^{12} + X^{11} + \\ & + X^{10} + X^7 + X^5 + X^4 + X^2 + X + 1 \end{aligned}$$

This field is efficiently implemented in Python with the aid of the package `galois`. The use of finite fields is required because of the fact that, in the next paragraph, we need to perform linear algebra in order to solve systems of linear equations, so that we cannot just use integer or modular arithmetic.

C. Shamir secret sharing

When a piece of information is too valuable to be accessible to an only person, there exist protocols to divide this secret information among several parts, so that only authorized sets of participants can get the secret if they share their parts of the information. These protocols are called secret sharing schemes. The simplest secret sharing protocols are called (t, n) -threshold schemes, where the secret information can be retrieved if at least t out of n participants share their partial informations (called shares), so that any set with less than t participant cannot get any information about the original secret.

The Shamir secret sharing scheme [18] is the one that is used the most in many situations, because its linearity, meaning that a linear combination of some secrets can be obtained as the corresponding linear combination of the shares. The Shamir scheme works as follows:

- The secret S is a number, and it is hidden into a polynomial $f(x)$ of degree $t - 1$. For example, S can be the constant term, that is the polynomial evaluated at the origin, and the other coefficients of f are generated at random.
- Each participant has a distinct identification number I , and then the share given to this participant is precisely $f(I)$.
- Since the polynomial f can be determined by the values at t different points, if t participants share their couples $(I, f(I))$ they can determine f , and hence $S = f(0)$, by simple Lagrange interpolation over a (finite) field. Notice that, since the identification numbers are different, the matrix of the corresponding linear system is of Vandermonde type, so that the solution always exists and it is unique.
- Note also that less than t shares lead to an underdetermined system, so that there are at least as many solutions as elements in the corresponding base field, and hence a brute-force attack is not feasible as long as the size q of the underlying finite field \mathbb{F}_q is large enough.

In our case, we apply the above scheme over the finite field $\mathbb{F}_{2^{64}}$, so that the identification numbers, as well as the random coefficients, are all elements of this field.

Example 1. We share the secret number $S = 1234$ with the Shamir (3, 6)-threshold scheme, over the field of real numbers. We generate at random the polynomial

$$P(x) = 1234 + 166x + 94x^2$$

and distribute the shares

$$(1, 1494), (3, 2578), (4, 3402), (6, 5614), (8, 8578), (11, 14434)$$

Now, if the users 2, 5 and 6 join their shares, they obtain the linear system

$$\begin{cases} s + 3a_1 + 9a_2 = 2578 \\ s + 8a_1 + 64a_2 = 8578 \\ s + 11a_1 + 121a_2 = 14434 \end{cases}$$

whose unique solution determines $P(x)$, obtaining then the secret $S = P(0) = 1234$.

D. AES and CBC

The last ingredient of our algorithm is the use of a secure enough symmetric cryptosystem like the Advanced Encryption Algorithm (AES). The standard specification of AES is given by the Federal Information Processing Standards Publication 197 [15]. This is a block cipher, operating on blocks of bits with size 128, and using keys of bit size 128, 192 or 256 (AES128, AES192 and AES256, respectively). There is no known efficient attack to AES, and the security increases as the key size is larger. Moreover, the algorithm is efficient in time and space. The details of the algorithm is referred to the cited FIPS 197 publication [15].

The idea of our algorithm is to use the first bits of the image as a key for the AES encryption, to apply the Shamir secret sharing protocol to share the key, and encrypt the rest of the image with this key. The problem is that, when ciphering an image with the ECB mode of operation, the aspect of the encrypted image shows patterns that can be seen with the aid of some visual filters (see [5] for further details). Thus, it is better to use alternative modes of operations [16], like CBC, CFB or OFB. In all these three cases we need an initialization vector, that can be computed from the identification number i used in the Shamir scheme. This makes the shared images completely different for each of the participants.

In our Python implementation we have chosen the CBC mode, with the AES128 version, but these are parameters that can be changed in order to optimize the security of the algorithm. If we want to use the AES192 or the AES256 version, then we need to use the corresponding number of bits of the original image as the key of the algorithm.

In order to increase the security, instead of choosing the first bits of the image as key for the AES cipher, we could used the identification number i (or a hash of it) to set the position of the consecutive bits that are used for the cipher key.

III. THE PROPOSED VISUAL SECRET SHARING SCHEME

First we describe the process of generating the shares of a given image, for a fixed finite field \mathbb{F}_{2^m} . Since we are going to use AES and the corresponding key is going to be shared, the optimal m would be either 128 or 256. Since a Conway

polynomial for these finite fields is not available, an alternative is to use $m = 64$, as we saw in section II-B. Denote by M the number of bits of the AES version we choose, that is $M \in \{128, 192, 256\}$, and assume from now on that $m = 64$. In case we can handle the finite fields with $m = 128$ or $m = 256$, the algorithms can be easily adapted to the cases $M = 128, 256$.

In all the cases, we are using the CBC or the OFB modes of operation with AES (see [16]). Then we need an initialization vector IV of 128 bits for the encryption, that for each participant will be the first 128 bits of a secure hash like SHA [17] of the corresponding identification number I , generated in the Shamir secret sharing. In the case that we use the finite field with $m = 128$, this hashing is not necessary.

Assume that the information about the pixels is stored in an array B of bits, and that the number N of bits of B is a multiple of 128, since AES works on blocks of bits of length 128. If this is not the case, the last remaining bits of B may not be ciphered, since the information they contain is not relevant compared to the whole image. Nevertheless, since normally each pixel is encoded by 24 bits, the above condition is easily satisfied as long as the total number of pixels is a multiple of 16; this is the case when both the horizontal and vertical resolution are multiples of 4. Otherwise we can fill the original image with random pixels at the margins in order to get this condition, and add this information to the file header. Other alternative is to use the CFB mode [16], adapting the size of the blocks in B .

On the other hand, we need to save apart the essential information contained in the header of the image, namely the horizontal and vertical resolution, and the depth of color channels (by default 255, meaning that each pixel is represented by 24 bits). This will be necessary to retrieve the original image from the shares.

Thus, assume in the sequel that there are n participants, and that the threshold is t . Denote $q = 2^{64}$, and $M = 64\ell$. Note that a block of 64 bits can be interpreted as an element of $I \in \mathbb{F}_q$ when we are applying the Shamir secret sharing method. The procedure to generate the shares is described in Algorithm 1.

Remark 1. In order to increase the security, instead of taking the first M bits of B as the key K for the AES encryption, we could take M consecutive bits from a random position of B . This position should be known for all he participants in order to reconstruct the original image.

In this case we remove these bits from B , cipher the remaining bits as in Algorithm 1, and place the shares S_j^i concatenated at the beginning of C^i . The random position of the key in B must be kept in mind in order to insert the bits of K at the right position in the original image.

In fact, the position where the selected bits start can be dependent on the identifier I , in terms of a hash for instance, so that this position is different for each user in the scheme.

Now we are going to describe the process to retrieve the original image B from the shares C^i of at least t participants. Each of the t participants must share with the others the

Algorithm 1 Procedure to generate the shares of a color image.

Input: The bit array B of N bits, the length M , and the parameters n and t .

- For each participant i :
 - 1) Generate at random an identifier $I \in \mathbb{F}_q$.
 - 2) Compute IV^i from the hash SHA of I .
- Get K the first M bits of B .
- Divide K into ℓ pieces of 64 bits M_j .
- For each M_j : generate n shares S_j^i with the Shamir secret sharing, i.e. $S_j^i = f_j(I)$ for a certain random polynomial f_j of degree $t - 1$ with $f_j(0) = M_j$.
- For each participant generate the image-share C^i as follows:
 - 1) The first M bits are the concatenation of the shares S_j^i .
 - 2) The remaining bits are the result of applying AES in CBC/OFB mode to the last $N - M$ bits of B , with the key K and initialization vector IV^i .

Output: The n shares C^i together with the identifiers I .

identifier I and the first M bits of C^i , that is, the concatenation of $S_j^i = f_j(I)$. The procedure of reconstructing the original image is described in Algorithm 2.

Algorithm 2 Procedure to reconstruct the original color image.

Input: The data I and C^i of t participants.

- Each user shares I and the first M bits of C^i .
- Slice the above bits to obtain the shares $S_j^i = f_j(I)$.
- Retrieve K by ℓ interpolation procedures, as in the Shamir method.
- Each of the t participants reconstruct the original image B as follows:
 - 1) The first M bits of B are those of K .
 - 2) Compute IV^i from the hash SHA of I .
 - 3) The remaining bits are the result of deciphering with AES in CBC/OFB mode the last $N - M$ bits of C^i , with the key K and initialization vector IV^i .

Output: The original color image B .

Remark 2. Following the idea of remark 1, if the selected bits for the encryption key are not at the beginning of the original image, we have to insert these bits in the correct position, once we have deciphered the tail of the bitarray.

Remark 3. Note that the generated shared images have the following properties:

- 1) The shares have the same resolution as the original images, and they are all color images.

- 2) All the shares look random because of the CBC/OFB mode operation of AES, so that no information can be extracted from them without the key K used for the encryption.
- 3) All the shares are completely different, since in the encryption distinct initialization vectors are used by each participant.

Finally, we note that the security of these procedures relies of the Shamir secret sharing protocol, together with that of the AES cipher and the corresponding mode of operation.

IV. EXAMPLES AND EXPERIMENTS

We have a Python implementation of the algorithms available in [3]

https://github.com/jifarran/color_visual_crypto

The required packages that we have used for the implementation are, among others `skimage`, `bitarray`, `galois`, `numpy` and `Crypto`.

Example 2. As an example, you can see a test image in figure 1, and the shared images by a $(3, 2)$ -threshold scheme in table I. On the other hand, you can see in figure 2 the reconstruction of the original image from the first two shares.



Fig. 1. Original image to be shared.



Fig. 2. Reconstructed image from two shares.

We have done several tests with images of different size, on a MAC with processor 2,3 GHz Intel dual Core i5 and 8GB of RAM. In tables II and III the sizes of the images are expressed in Megabytes on the left column. The execution times to compute the image shares are shown in table II. In our implementation, $M = 128$, $q = 2^{64}$, we use AES128 in CBC mode, and we do not use the hashing SHA, but for the sake of

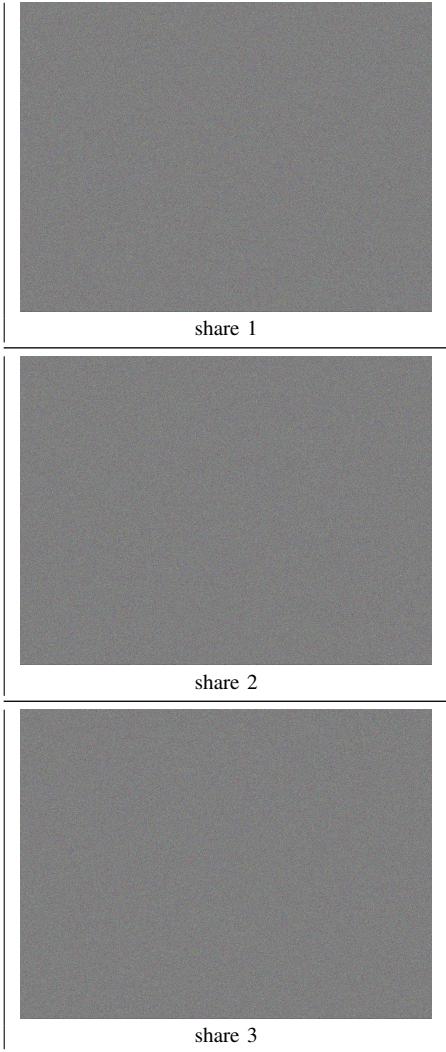


TABLE I

SHARES OF IMAGE IN FIGURE 1.

(t,n)	(2,3)	(3,5)	(5,5)	(6,10)
4.9MB	51.07s.	84.37s.	85.31s.	166.03s.
9.4MB	101.02s.	160.87s.	164.38s.	308.05s.
24.9MB	277.38s.	419.96s.	422.41s.	840.93s.
49.2MB	564.61s.	893.06s.	885.27s.	1813.78s.

TABLE II

COMPUTATION TIMES TO OBTAIN THE SHARES OF THE TEST IMAGES.

simplicity we concatenate twice the identifier I in algorithm 1 instead.

Note that the computation time to obtain the shares S_j^i only depends on the parameters n , t and M in the algorithm 1, so that it will be irrelevant compared to the encryption time, that depends on the size of the image. Thus, from the results of this table, we can conclude that the time to compute the image shares seems to be linear with respect to the size of the uncompressed data. In fact, on our computer turns out to be approximately around 3.5 seconds per MB of data (note that in each case we have to obtain n images). The total times includes the time to open the file, do some internal conversions, and apply the AES in CBC mode.

We show now the computation times for the reconstruction of the original image in table III. Note that, in this case, we only have to reconstruct one image, and not n images as in the sharing algorithm.

(t,n)	(2,3)	(3,5)	(5,5)	(6,10)
4.9MB	24.80s.	33.32s.	53.60s.	64.25s.
9.4MB	43.05s.	62.54s.	105.71s.	125.17s.
24.9MB	119.90s.	174.23s.	289.60s.	346.43s.
49.2MB	263.79s.	371.40s.	614.21s.	800.00s.

TABLE III

COMPUTATION TIMES TO OBTAIN THE ORIGINALS FOR THE TEST IMAGES.

Note that, in this case, by comparing the cases $(t, n) = (3, 5)$ and $(t, n) = (5, 5)$, the parameter t has an influence in the time of computation, since we have to solve a linear system of size t in the finite field \mathbb{F}_q , and this has a complexity $\mathcal{O}(t^3)$ with Gaussian elimination. In fact, most of the time in algorithm 2 is devoted to the interpolation task in Shamir secret sharing procedure.

In this case, the time of computation does not seem to be linear with respect to the size of the data, but one easily checks that the computation time per MB of data do not differ very much, for t and n fixed. Anyway, we have experimented with large images, but for small images the times of computation are quite reasonable, although we could optimize a lot the implementation to speed-up the algorithms.

Remark 4 (File formats). *We finally notice that, in our implementation, two formats of images are admitted: PPM and PNG. In the second case there is a lossless compression, while in the first case there is no compression at all. Lossy compression make no sense in this context, since this type of compression may downgrade the quality of the image.*

All the above computation times are calculated with respect to the size of the uncompressed image, although in the implementation the shares and the reconstructed images are generated in PNG format, in order to save space. The skimage Python package automatically compresses and uncompresses the corresponding images, without loss of information.

V. CONCLUSIONS

A new and efficient method to perform secret sharing with color image has been presented, on the basis of the Shamir threshold secret sharing scheme over binary finite fields, combined with a secure symmetric cipher like AES in CBC/OFB mode of operation. Thus, the security of the method is based on that of the above techniques, which have been proven secure so far.

The advantages of our method, compared to the existing ones, are the following:

(1)

- 1) The shares are images of the same type (color images).
- 2) The shares have all the same resolution.
- 3) The shares look quite random, showing no visual pattern.
- 4) The shares are completely different each other.

Concerning future work, the authors propose to apply the same idea to do secret sharing with other types of multimedia

or binary formats (audio, video, etc.). In the case of video, or other kinds of lossy compressed data, one should study carefully the impact of the loss of information in the secret sharing process, but if the compression is reliable enough, our method may work properly.

REFERENCES

- [1] S. Abdulla (2010): New Visual Cryptography Algorithm For Colored Image. *Journal of Computing*, vol. 2/4, pp. 21-25. <https://sites.google.com/site/journalofcomputing>
- [2] S. Calkavur, F. Molla (2018): An image secret sharing method based on Shamir secret sharing. *Current Trends in Computer Sciences & Applications*, vol. 1, 2. DOI: 10.32474/CTCSA.2018.01.000106
- [3] D. Cerezo, J. I. Farrán (2023): Color Visual Cryptography, Python implementation. Github repository https://github.com/jifarran/color_visual_crypto
- [4] R. Cramer, I. Damgård, J. Nielsen (2015): *Secure Multiparty Computation and Secret Sharing*. Cambridge University Press. DOI: 10.1017/CBO9781107337756
- [5] I. F. Elashry, O. S. Faragallah, A. M. Abbas, S. El-Rabaie, Fathi E. Abd El-Samie (2012): A New Method for Encrypting Images with Few Details Using Rijndael and RC6 Block Ciphers in the Electronic Code Book Mode. *Information Security Journal: A Global Perspective*, vol. 21/4, pp. 193-205. DOI: 10.1080/19393555.2011.654319
- [6] R. C. Gonzalez, R. E. Woods (2017): *Digital Image Processing*. Ed. Pearson.
- [7] L. S. Heath, N. A. Loehr (2004): New algorithms for generating Conway polynomials over finite fields. *Journal of Symbolic Computation*, vol. 38/2, pp. 1003-1024.
- [8] H. Krawczyk (1994): Secret Sharing Made Short. *Advances in Cryptology – CRYPTO' 93. Lecture Notes in Computer Science*, vol 773, pp. 136-146, Springer.
- [9] F. Lübeck: Conway polynomials for finite fields. Last updated: July 2021. <http://www.math.rwth-aachen.de/~Frank.Luebeck/data/ConwayPol/index.html>
- [10] M. Lutz, D. Ascher (1999): *Learning Python*. Ed. O'Reilly.
- [11] R. J. McEliece (1987): *Finite Fields for Computer Scientists and Engineers*. Ed. Kluwer.
- [12] M. Mignotte (1983): How to Share a Secret. *Cryptography. EUROCRYPT 1982. Lecture Notes in Computer Science*, vol. 149, pp. 371-375, Springer.
- [13] J. Mohan, R. Rajesh (2015): Exploration of Color Visual Cryptography Schemes. *International Journal of Science and Research*, vol. 4/7, pp. 1033-1038. <https://www.ijsr.net/archive/v4i7/SUB156559.pdf>
- [14] M. Naor, A. Shamir (1995): Visual cryptography. *Advances in Cryptology – EUROCRYPT'94. Lecture Notes in Computer Science*, vol. 950, pp. 1-12, Springer.
- [15] NIST FIPS - 197 (2001): Advanced Encryption Standard (AES). <https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.197.pdf>
- [16] NIST Special Publication 800-38A (2001): Recommendation for Block Cipher Modes of Operation. <https://nvlpubs.nist.gov/nistpubs/Legacy/SP/nistspecialpublication800-38a.pdf>
- [17] NIST FIPS 180-4 (2015): Secure Hash Standard. <https://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.180-4.pdf>
- [18] A. Shamir (1979): How to share a secret. *Communications of the ACM*, vol. 22/11, pp. 612-613.
- [19] R. Verheul, H. C. A. van Tilborg (1997): Constructions and Properties of k out of n Visual Secret Sharing Schemes. *Designs, Codes and Cryptography*, vol. 11/2, pp. 179-196.
- [20] G. Wu, M. Wang, Q. Wang, Y. Yao, L. Yuan, G. Miao (2021): A Novel Threshold Changeable Secret Image Sharing Scheme. *Symmetry*, vol. 13, 286. <https://doi.org/10.3390/sym13020286>

J. I. Farrán He received his PhD in Mathematics from the University of Valladolid, Spain, in 1997. Since 2001 he is an associate professor at the Department of Applied Mathematics, in the University of Valladolid, being a lecturer in the Computer Science School of Segovia, and a member of the IMUVA (Institute for Mathematical Research of the University of Valladolid). His research interests are in algebraic geometry, computer algebra, numerical semigroups, information and coding theory, cryptography, and analysis of meteorological data. He is a contributor to the computer algebra system SINGULAR, from the Technische Universität Kaiserslautern.

D. Cerezo He received his degree in Computer Science Engineering from the University of Valladolid, Spain, in 2022. He is an expert in cybersecurity, network security and ethical hacking.