Problem 1:

(a). $P(diagnosed\ early\ |assists\ to\ routine\ consulations) = \frac{(0.7*0.6)}{(0.7*0.6+0.4*0.1)} = 93.333\%$

(b). $P(x|y) = \frac{P(x,y)}{P(y)} = \frac{\sum_z P(x,z,y)}{P(y)} = \sum_z \frac{P(x,z,y)}{P(y)} = \sum_z P(x,z|y)$

Problem 2:

(a).

there are 2^7 entries in this joint distribution. The joint distribution is over binary variables, each node can only have 2 variables. There are 7 nodes in total.

(b).

$$P(a,b,c,d,e,f,g) = P(a)P(b)P(c|a,d)P(d|b)P(e|b)P(g|d,e)P(f|c)$$

(c).

(i).

$$P(g, A=0) = \sum_{B,D,C,F,E} \phi_A(A)\phi_B(B)\phi_C(C,A,D)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\phi_F(F,C)$$

$$= \sum_{D,C,F,E} \phi_A(A)\left(\sum_B \phi_B(B)\phi_D(D,B)\phi_E(E,B)\right)\phi_C(C,A,D)\phi_G(G,D,E)\phi_F(F,C)$$

$$= \sum_{D,C,F,E} \phi_A(A)\, f_1(D,E)\, \phi_C(C,A,D)\phi_G(G,D,E)\phi_F(F,C)$$

$$= \sum_{C,F,E} \phi_A(A)\ \phi_F(F,C)\left(\sum_D \phi_C(C,A,D) f_1(D,E)\phi_G(G,D,E)\right)$$

$$= \sum_{C,F,E} \phi_A(A)\, \phi_F(F,C) f_2(A,C,E,G)$$

$$= \sum_{F,E} \phi_A(A)\left(\sum_C \phi_F(F,C) f_2(A,C,E,G)\right)$$

$$= \sum_{F,E} \phi_A(A)\ f_3(A,E,F,G)$$

$$= \sum_F f_3(A,E,F,G) \sum_E \phi_A(A)$$

$$= f_4(A,E,G) \sum_E \phi_A(A)$$

$$= \sum_E f_4(A,E,G)\, \phi_A(A)$$

$$= f_5(A,G)\, \phi_A(A)$$

$$= f_6(A,G)$$

(ii).

f_2 and f_3 are the largest factor with 3 variables, because g is evidence we don't need to count it.
The order should be: E,B,D,F,C with only 2 variables in the scope.

d.
(i).

$$P(f,b) = \sum_{A,C,D,E,G} \phi_A(A)\phi_B(B)\phi_C(C,A,D)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\phi_F(F,C)$$

$$= \sum_{C,D,E,G} \left(\sum_A \phi_A(A)\phi_C(C,A,D)\right)\phi_B(B)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\phi_F(F,C)$$

$$= \sum_{C,D,E,G} f_1(C,D)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\phi_F(F,C)\phi_B(B)$$

$$= \sum_{D,E,G} \left(\sum_C f_1(C,D)\phi_F(F,C)\right)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\phi_B(B)$$

$$= \sum_{D,E,G} f_2(D,F)\phi_D(D,B)\phi_E(E,B)\phi_G(G,D,E)\,\phi_B(B)$$

$$= \sum_{E,G} \left(\sum_D f_2(D,F)\phi_D(D,B)\phi_G(G,D,E)\right)\phi_E(E,B)\,\phi_B(B)$$

$$= \sum_{E,G} f_3(B,E,F,G)\phi_E(E,B)\phi_B(B)$$

$$= \sum_G (\sum_E f_3(B,E,F,G)\phi_E(E,B))\phi_B(B)$$

$$= \sum_G f_4(B,F,G)\phi_B(B)$$

$$= f_5(B,F)\phi_B(B)$$

$$= f_6(B,F)$$

(ii).
There are 4 variables in f_3 including evidence F, so that it is the largest factor.
Using order, A, C, G, E, D it only has the largest factor with 2 variables.

Problem 3:
(a)
1.not possible because there is no any probability of $P(x_1,x_2,y)$
2.

$$P(y|x_1,x_2) = \frac{P(y,x_1|x_2)}{P(x_1|x_2)} = \frac{P(y,x_1,x_2)}{P(x_1,x_2)} = P(x_1,x_2|y) * \frac{P(y)}{P(x_1,x_2)}$$

3. not possible same as reason of 1.
4. not possible because cannot get joint probability $P(x_1,x_2)$.
5. not possible because cannot get joint probability $P(x_1,x_2)$.

(b)

1. $P(y|x_1, x_2) = \frac{P(x_1,x_2,y)}{P(x_1,x_2)} = \frac{P(y)*P(x_1|y)*P(x_2|y)}{P(x_1,x_2)}$

2. $P(y|x_1, x_2) = \frac{P(x_1,x_2,y)}{P(x_1,x_2)} = \frac{P(x_1,x_2|y)}{P(y)P(x_1,x_2)}$

3. no. cannot find $P(x_1, x_2)$
4. no, cannot find $P(x_1, x_2)$
5. no cannot find $P(x_1, x_2)$

Problem 4:
(a).
C, F, H, I, E are d-connected to A given S={B}

(b).
G, D, B, E, I are d-connected to A given S={J}