Given some RNA-seq alignments in BAM format and a isoform annotation table, find the relevant equivalence classes, then estimate relative abundance of the isoforms. You can learn about the BAM format here. You can use whatever library you choose to parse the reads. In python, PySam is pretty good and in R Rsamtools is decent. If you are using C/C++ there are official libraries as well (the actual BAM format is in C). If you are a masochist, there is probably a Java implementation floating around. You may implement the RSEM version[3] which has a likelihood in read-space or the IsoEM/kallisto[1] version which is in equivalence class space. The kallisto version is a bit more work because it will require computing equivalence classes from the reads, whereas theRSEM version can be done directly from the reads. The kallisto version will be drastically faster than the RSEM version because of the reduced complexity of the likelihood function. Please provide the transcript abundance table outputting the following formant: transcript length effective length expected counts tpm ENST000003679 1219 1019 517 2.93 The reads provided are single-end reads, so you will have to assume a fragment length distribution. Treat it as N(200, 10) (with appropriate discretization/normalizations). In your report you might plot some metric against the various iterations of the EM. Take your pick. Extra credit: output the final posterior probability of each read mapping to a particular transcript.

#### steps:

- · parse the rna fragment reads
- · parse the transcriptome file

# Preparations: import packages & parse bam file

```
In [9]: import numpy as np
         import pandas as pd
         from scipy import stats
         from scipy.special import logsumexp
         from sklearn.mixture import GaussianMixture
         from matplotlib import pyplot as plt
In [10]: import itertools
         from collections import Counter
In [11]: from Bio import SeqIO
In [12]: import pysam
In [13]: alignments = []
In [14]: def parse_bam(path):
             given a path of a bam file, parse it
             bamfile = pysam.AlignmentFile(path, "rb")
             return bamfile
In [15]: bfile = parse bam("aligned.bam")
In [16]: bfile.count(until_eof=True)
Out[16]: 6439964
```

# Fetch alignment results for each read

### **Get Equivalence Class**

```
In [21]: def get_EC_count(bamfile, read_ids):
            get equivalence class
               bamfile: a parsed bam
               read_IDs: a list of query_names
            return:
            EC_count: a dict with key: equivalence class(tuple of transcript IDs); value: equivalence class count
            #bamfile = pysam.AlignmentFile(path, "rb")#default bam file
            #build by index
            bam indexed = pysam.IndexedReads(bamfile)
            bam_indexed.build()
            #get the alignment(multimapped) for each read, according to id
            for u in unique IDs:
                  cur_aligned_transcipts = [x.reference_name for x in bam_indexed.find(u)]
                 EC_list.append(cur_aligned_transcipts)
            EC_count = Counter( map(tuple, EC_list) )# a dict with key as equivalent classes, value as class counts
            return EC count
In [22]: ec = get EC count(bfile, rids)
        <ipvthon-input-21-0a8b22f9ab25>:18: VisibleDeprecationWarning: Creating an ndarray from ragged nested sequences (which is a
        list-or-tuple of lists-or-tuples-or ndarrays with different lengths or shapes) is deprecated. If you meant to do this, you
        must specify 'dtype=object' when creating the ndarray
          EC_list = np.array([np.sort(np.array([x.reference_name for x in bam_indexed.find(u)])) for u in read_ids])
In [23]: len(ec)
Out[23]: 13566
```

### Initial abundance calculation

#### transcriptome informations:

```
In [24]: transcriptomes = []
         for record in SeqIO.parse("chr11_transcriptome.fasta", "fasta"):
             transcriptomes.append([record.id, len(record.seq)])
In [25]: transcriptomes = dict(transcriptomes)
In [26]: def get_initial_abundance(transcriptomes, EC): #E step actually
             generate initial uniform abundance for each transcriptome
             params:
                 transcriptomes: a dict of key being transcript ID, value being transcript length
                 abundance: a dict of uniformly setted abundance(with respect to length of the transcript)
                             { key: transcript ID; value: abundance }
             #np.sum(df['occurence'])
             transcriptomes_count = len(transcriptomes)
             abundance = {}
             for i in transcriptomes:
                 #print(i)
                 abundance[i] = ( 1/len(transcriptomes) ) / transcriptomes[i] #i[1] is the length of the current isoform
             #print(abundance)
             #normalize with respect to length
             factor=1.0/sum(abundance.values())
             for k in abundance:
                 abundance[k] = abundance[k]*factor
             return abundance
In [27]: abu = get_initial_abundance(transcriptomes, ec)
In [32]: len(abu)
Out[32]: 7138
```

## **Expectation Maximization Model**

### expectation

```
In [33]: def E_step(last_abundance , EC):
                "transcriptome
                  last_abundance: a dict: key: transcriptome ID; value: abundance from last step
                  EC: a dict: key: equivalent class; value: class count
             return:
                 update: a dict: key: ec
                                  value: a dict of probability to be updated with
                                              key: transcript;
                                               value: probability within that ec
                                               (each EC sums to 1)
              ##check if want to terminate
             update ={}
             for r in EC.keys(): #for each equivalence class
                 cur_update = {}
for t in r: # for
                     cur update[t] = last abundance[t]
                 norm=1.0/sum(cur_update.values())#normalize within ec
                  for k in cur_update:
                     cur_update[k] = cur_update[k]*norm
                  #append to update
                 update[r] = cur_update
             return update
```

### maximization

```
In [136]: def M_step(update, transcriptome, EC):
              maximization step: calculate new abundance based on given probability within each equivalence class
                               for each transcriptome
              param:
                   update: a dict: key: ec (wqe, das, qwe)
                                    value: a dict of probability with
                                                key: transcript; {wqe: 09,das: 09, qwe04}
value: probability within that ec
(each EC sums to 1)
                   transcriptome: a dict: key: transcriptome ID; value: length of the transcriptome reference
               return:
               new_abundance: a dict: key: transcriptome ID; value: updated abundance
              new abundance = {}
               for key in update.keys():#loop over the dict, key-> ec
                   cur_update = update[key]# cur_update -> dict of current ec
                   for t in key: #t-> transcript
                       if t not in new_abundance:
                           new_abundance[t] = (cur_update[t]/ transcriptomes[t] )* EC[key]
                       else:
                           cur_new_abun = new_abundance[t]
                           new_abundance.update({t: ((cur_update[t]/ transcriptomes[t])* EC[key] + cur_new_abun) })
               #add ec count -added
               #add fragment length factor:
               #print(new_abundance)
                 for n in new_abundance.keys(): -added
                     new_abundance[n] = new_abundance[n] / transcriptomes[n]
               #normalize updated abundance:
               nm=1.0/sum(new_abundance.values())#normalize within ec
               for ab in new_abundance:
                   new abundance[ab] = new abundance[ab]*nm
               return new_abundance
```

## Run EM, generate graphs, output

```
In [170]: def run_EM(transcriptomes, EC, steps = 10, verbose = 100):
               #updated_abundance = {}
              newest_abundance = {}
               init_data = get_initial_abundance(transcriptomes, EC)
              first_update = E_step(init_data, EC)
#newest_abundance = {}
              s=0
              for step in range(steps+1):
                   if step ==0:
                      update = E_step(init_data, EC)
                      update = E_step(newest_abundance, EC)
                  newest_abundance = M_step(update, transcriptomes, EC)
                   if s %verbose == 0:
                       print("current step:"+ str(s) )
                       plot_EM(newest_abundance)
                   #latest_abundance = new_abundance
              return newest abundance
```

```
In [172]: def plot_EM(abu):
    fig, ax = plt.subplots()
    ax.pie(abu.values(),

    #labels=pieLabels,

    #autopct='%1.2f',
    #autopct ='%1.1f%%',
    autopct=lambda p: format(p, '.2f') if p > 3 else None,
    normalize=True,
    startangle=90)

# Aspect ratio - equal means pie is a circle

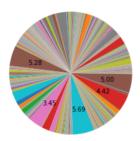
ax.axis('equal')
    plt.show()
```

In [173]: re = run\_EM(transcriptomes, ec, steps = 100, verbose = 20)

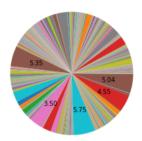
current step:0



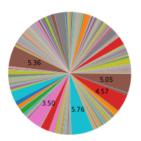
current step:20



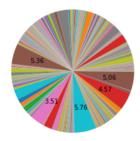
current step:40



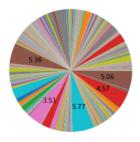
current step:60



current step:80



current step:100



#### output result:

```
In [174]: def make output file(raw bamfile, result abundance, transcriptomes, write out = False):
                     raw_bamfile: a parsed raw bam file
                    result abundance:
                    transcriptomes df: contacts information about the transcript length, occurence
                    write out: if want quantified csv result
                df = pd.DataFrame.from_dict(result_abundance, orient = 'index')
                df = df.reset_index()
                transcriptomes df = pd.DataFrame.from dict(transcriptomes, orient = 'index')
                transcriptomes_df =transcriptomes_df.reset_index()
                transcriptomes_df = transcriptomes_df.rename(columns={"index":"ID", 0: "length"})
                df = df.rename(columns={"index":"ID", 0: "abundance"})
df = transcriptomes_df.merge(df, on = 'ID')
                df['effective length'] = abs(df['length'] - int(np.random.normal(200, 10) ) + 1 )
#df['expected_counts'] = (transcriptomes_df['occurence'] * df['length']) / df['effective length']
                #raw_bamfile = parse_bam("aligned.bam")
                num reads = raw bamfile.count(until eof=True)
                df['expected_counts'] = df['abundance']* num_reads* (df['length'] / df['effective length'])
                #print(num_reads)
                df['expected_counts'] = df['expected_counts'].astype(int)
                #df['TPM intermediate df'] = transcriptomes df['occurence']/df['length']
                df['TPM_intermediate_df'] = df['abundance']* num_reads /df['length']
                df['TPM']= ( df['TPM intermediate df'] / np.sum(df['TPM intermediate df']) ) *1000000
                df['TPM'] = df['TPM'].round(2)
                df = df.rename(columns = {"ID": "transcript",})
df = df.drop(columns=[#'occurence',
                                          abundance'
                                         #'abundance'
                                         'TPM_intermediate_df'])
                #df = df.sort_values(by=['TPM'])
                if write_out:
                    df.to_csv('EM_quantified.csv')
                return df
In [175]: o = make output file(parse bam("aligned.bam"), re, transcriptomes)
In [176]: o
Out[176]:
                        transcript length effective length
                                                               2023 294.69
               o ENST00000410108
                                   637
                                                 415
               1 ENST00000325147
                                  2916
                                                2694
                                                                444
                                                                     20.06
               2 ENST00000382762
                                                2570
                                  2792
                                                               4118 193.33
               3 ENST00000529614
                                   560
                                                 338
                                                                 0
                                                                     0.00
               4 ENST00000332865
                                   533
                                                 311
                                                                 0
                                                                      0.00
            6319 ENST00000339772 3152
                                                2930
                                                                 31
                                                                     1.34
            6320 ENST00000535456
                                  3251
                                                3029
                                                                 1
                                                                      0.06
            6321 ENST00000392580
                                  3522
                                                3300
                                                                 0
                                                                      0.00
            6322 ENST00000312527
                                  3976
                                                3754
                                                                 0
                                                                      0.00
            6323 ENST00000524765
                                                5515
                                                                 16
                                                                      0.39
  In [ ]:
  In [ ]:
```

```
In []:
In []:
In []:
```

```
Drafts
In [165]: aligned = aligned.rename(columns={"index":"ID", 0: "occurence"})
In [663]: aligned[aligned['ID'] == 'ENST00000327804']
Out[663]:
              ID occurence
In [665]: max(aligned['occurence'])
Out[665]: 63018
In [182]: #np.sum(init data['occurence'])
           init_dat_trail = init_data
           init_dat_trail['ab'] = init_data['occurence'] / np.sum(init_data['occurence'])
In [183]: init_dat_trail
Out[183]:
                             ID length occurence
                                                         ab
              0 ENST00000410108
                                  637
                                            191 2.965855e-05
               1 ENST00000325147 2916
                                            1569 2.436349e-04
               2 ENST00000382762 2792
                                            1591 2.470511e-04
               3 ENST00000529614
                                  560
                                            243 3.773313e-05
               4 ENST00000332865
                                   533
                                            258 4.006234e-05
            6319 ENST00000339772 3152
                                             13 2.018645e-06
            6320 ENST00000535456 3251
                                             13 2.018645e-06
            6321 ENST00000392580 3522
                                              3 4.658411e-07
            6322 ENST00000312527
                                              3 4.658411e-07
            6323 ENST00000524765 5737
                                             12 1.863364e-06
In [371]: ecd = pd.DataFrame.from_dict(ec,orient = 'index')
In [372]: ecd = ecd.reset_index()
In [375]: ec df = ecd.rename(columns={ 'index' :"EC", 0: "EC count"})
In [376]: ec df
Out[376]:
                o (ENST00000325147, ENST00000332865, ENST0000038...
                                                                   55
                                            (ENST00000410108,)
                              (ENST00000325147, ENST00000382762)
                2
                                                                 1262
                3 (ENST00000328221, ENST00000399808, ENST0000039...
                4 (ENST00000300737, ENST00000526596, ENST0000061...
                                                                  816
            13561 (ENST00000422465, ENST00000525690, ENST0000052...
                  (ENST00000278572, ENST00000422465, ENST0000052...
                                            (ENST00000526608.)
                                                                   6
            13563
                                            (ENST00000527273,)
            13565 (ENST00000300737, ENST00000525055, ENST0000052...
           13566 rows × 2 columns
In [346]: transcriptome_df = pd.DataFrame.from_dict(transcriptomes)
In [347]: transcriptome_df = transcriptome_df.rename(columns={0:"ID", 1: "length"})
```

```
In [348]: transcriptome_df
              0 ENST00000410108
              1 ENST00000325147
                                 2916
              2 ENST00000382762
              3 ENST00000529614
                                 560
              4 ENST00000332865
                                 533
            7133 ENST00000339772
                                 3152
           7134 FNST00000535456
                                 3251
                ENST00000392580
           7136 ENST00000312527
                                 3976
            7137 ENST00000524765
           7138 rows x 2 columns
In [396]: len(transcriptomes)
Out[396]: 7138
In [489]: re = run_EM(transcriptomes, ec, steps = 20, verbose = 100)
In [493]: len(re.keys() )
Out[493]: 6324
In [147]: fig, ax = plt.subplots()
           ax.pie(re.values(),
                   #labels=pieLabels,
                   autopct='%1.2f',
                   startangle=90)
           # Aspect ratio - equal means pie is a circle
           ax.axis('equal')
           plt.show()
```



```
In [134]:
Out[134]:
                                             abundance effective length expected_counts TPM_intermediate_df
                                                                                                             TPM
                          transcript length
                o ENST00000410108
                                     637 9.115317e-268
                                                                 438
                                                                        8.537300e-261
                                                                                           9.215433e-264
                                                                                                              0.0
             4221 ENST00000541951
                                     1383 1.896416e-259
                                                                1184
                                                                        1.426552e-252
                                                                                           8.830696e-256
                                                                                                              0.0
                                                                383
                                                                                           2.086712e-265
             4220
                  ENST00000537289
                                     582 1.885828e-269
                                                                        1.845482e-262
                                                                                                              0.0
                  ENST00000541455
                                     1038 2.407134e-268
                                                                 839
                                                                        1.917870e-261
                                                                                           1.493435e-264
                                                                                                              0.0
             4218 ENST00000545127
                                     922 1.861062e-266
                                                                        1.528400e-259
                                                                                           1.299910e-262
                                                                 723
                                                                                                              0.0
             2102 ENST00000399249 4217 2.883072e-274
                                                                4018
                                                                       1.948644e-267
                                                                                           4.402865e-271
                                                                                                              0.0
             2101 ENST00000634938
                                     658 6.410969e-270
                                                                 459
                                                                        5.918618e-263
                                                                                           6.274531e-266
                                                                                                              0.0
             2110 ENST00000531974
                                     856 6.598389e-273
                                                                 657
                                                                        5.536429e-266
                                                                                           4.964180e-269
                                                                                                              0.0
             6323 ENST00000524765 5737 7.417684e-274
                                                                5538
                                                                        4.948615e-267
                                                                                           8.326585e-271
                                                                                                              0.0
             2732 ENST00000378024 18761 1.000000e+00
                                                               18562
                                                                         6.509006e+06
                                                                                            3.432634e+02 1000000.0
 In [64]: o [o['transcript'] == 'ENST00000327804']
 Out[64]:
               transcript length abundance effective length expected_counts TPM_intermediate_df TPM
 In [98]: max(o["abundance"])
 Out[98]: 1.0
```

```
In [66]: o.iloc[o['abundance'].idxmax() ]
                                 ENST00000378024
Out[66]: transcript
         length
                                            18761
         abundance
         effective length
                                            18562
         expected_counts
                                      6.50901e+06
         {\tt TPM\_intermediate\_df}
                                          343.263
         TPM
                                            1e+06
         Name: 2732, dtype: object
 In [ ]:
 In [ ]:
 In [ ]: transcriptomes_data
 In [ ]:
         get transcripts with length
 In [ ]:
 In [ ]:
 In [ ]:
 In [ ]:
 In [ ]:
In [43]: transcriptomes
Out[43]: {'ENST00000410108': 637,
           'ENST00000325147': 2916,
           'ENST00000382762': 2792,
           'ENST00000529614': 560,
           'ENST00000332865': 533,
           'ENST00000486280': 665,
           'ENST00000342878': 435,
           'ENST00000325113': 1284,
           'ENST00000525282': 764,
           'ENST00000526104': 3506,
           'ENST00000325207': 2702,
           'ENST00000528357': 500,
           'ENST00000526982': 965,
           'ENST00000530889': 563,
           'ENST00000626818': 159.
           'ENST00000527696': 2565,
           'ENST00000527468': 571,
           'ENST00000527728': 859,
           'ENST00000524854': 649,
```