- Let 2 numbers be X & Y.
- Both 3-bit length → X : $x_2\ x_1\ x_0$ ← LSB
  Y : $y_2\ y_1\ y_0$ ← LSB
- Therefore, multiplication would look something like.

$X * Y$ = $y_2\ y_1\ y_0$

$x_2\ x_1\ x_0$

Partial Products → $a_2\ a_1\ a_0$

$b_2\ b_1\ b_0$

$c_2\ c_1\ c_0$

$R_5\ R_4\ R_3\ R_2\ R_1\ R_0$

where $a_0 = x_0 y_0$ $b_0 = x_1 y_0$ $c_0 = x_2 y_0$

$a_1 = x_0 y_1$ $b_1 = x_1 y_1$ $c_1 = x_2 y_1$

$a_2 = x_0 y_2$ $b_2 = x_1 y_2$ $c_3 = x_2 y_2$

Some nomenclature →

* The final 6-bit result is $R_5\ R_4\ R_3\ R_2\ R_1\ R_0$ ← LSB
* There is no carry generated for R_0 and R_1 computation.
* The carry generated "at R_1" (ie, for R_2) has not been named as it is (a_1 & b_0) only.

* The carry generated at R_2 (ie, for R_3) is represented by P_1 & P_2.

=>

$\{P_2\}$	$\{a_1 \& b_0\}$
$\{P_1\}$	a_2
b_2	b_1
c_1	c_0
R_3	R_2

As there are total 4-bits in R_2 position if all are 1, there would be two carry bits carried over to R_3

that's why P_1 & P_2

* Similarly at R_3 there are 4-bits hence two carries can be generated for R_4, ∴ they are represented by Z_1 & Z_2

$\{Z_2\}$	$\{P_1\}\{P_2\}$
$\{Z_1\}$	b_2
C_2	C_1
R_4	R_3

* As there are only 3-bits at R_4, only 1 carry bit can be generated for R_5

∴ it is represented by R_5 itself and computation for R_5 would be done likewise because there are no partial product bits for R_5 anyways.

$P_1 \rightarrow$

$(b_0 \oplus a_1) \oplus a_2 \oplus b_1 \oplus c_0$

$\rightarrow$ ~~$(a_1 b_0) a_2 + a_1 b_0 b_1 + a_1 b_0 c_0 + a_2 b_1 + a_2 c_0 + b_1 c_0$~~

$\Rightarrow (a_1 b_0)(a_2 + b_1) + a_1 b_0 c_0 + a_2 b_1 + a_2 c_0 + b_1 c_0$

$\Rightarrow a_1 b_0 (a_2 + b_1) + c_0 (a_2 + b_1) + a_1 b_0 c_0$

$\Rightarrow (a_2 + b_1)(a_1 b_0 + c_0) + a_1 b_0 c_0 \rightarrow 4$

~~$(a_1 b_0)(a_2 + b_1 + c_0)$~~

$x_0 y_1 x_1 y_0 x_0 y_2 + x_0 y_1 x_1 y_0 x_1 y_1 + x_0 y_1 x_1 y_0 x_2 y_0 +$
$x_0 y_2 x_1 y_1 + x_0 y_2 x_2 y_0 + x_1 y_1 x_2 y_0$

$\Rightarrow x_0 x_1 y_0 y_1 y_2 + x_0 x_1 y_0 y_1 + x_0 x_1 x_2 y_0 y_1 y_2$
$+ x_0 x_1 y_1 y_2 + x_0 x_2 y_0 y_2 + x_1 x_2 y_0 y_1$

$\Rightarrow x_0 x_1 y_1 y_2 + x_0 x_2 y_0 y_2 + x_1 x_2 y_0 y_1 +$
$x_0 x_1 y_0 y_1 (y_2 + 1 + x_2 y_2)$

$\Rightarrow x_0 x_1 y_1 y_2 + x_0 x_2 y_0 y_2 + x_1 x_2 y_0 y_1 + x_0 x_1 y_0 y$

$\Rightarrow$ ~~$x_0 x_1 (y_1 y_2 + y_0 y_2) + y_0 y_1 (x_1 x_2 + x_0 x_1)$~~

$\Rightarrow x_0 y_1 (x_1 y_2 + x_1 y_0) + x_2 y_0 (x_0 y_2 + x_1 y_1)$

$P_2 \Rightarrow a_1 (b_2 + b_0) + c_0 (a_2 + b_1) \rightarrow 3 \text{ bg}$

$P_1 = a_1 a_2 b_0 b_1 c_0$

$P_2 = a_1(b_2 + b_0) + c_0(a_2 + b_1)$

~~$a_1 a_2 b_0 b_1 c_0 b_2 c_1$~~

$Z_1 = P_1 P_2 b_2 c_1$

$= a_1 a_2 b_0 b_1 c_0 b_2 c_1 (a_1 b_2 + a_1 b_0 + c_0 a_2 + c_0 b_1)$

$= (x_0 y_1 x_1 y_2 + x_0 y_1 x_1 y_0 + x_2 y_0 x_0 y_2 + x_2 y_0 x_1 y_1)$ AND.
$(x_0 y_1 x_0 y_2 x_1 y_0 x_1 y_1 x_2 y_0 x_1 y_2 x_2 y_1)$

$= (\sim\sim\sim)$ AND. $(x_0 x_1 x_2 y_0 y_1 y_2)$

$Z_1 \Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 = x_0 y_0 x_1 y_1 x_2 y_2 = a_0 b_1 c_2$

$Z_2 \Rightarrow P_1 P_2 + P_1 b_2 + P_1 c_1 + P_2 b_2 + P_2 c_1 + b_2 c_1$

$[x_0 y_1 x_0 y_2 x_1 y_0 x_1 y_1 x_2 y_0 \Rightarrow P_1] \Rightarrow P_1 = x_0 x_1 x_2 y_0 y_1 y_2$

As P_1 has all 6 basic bits, ANDing P_1 with anything must only yeild

$Z_2 = x_0 x_1 x_2 y_0 y_1 y_2 + (x_0 x_2 y_0 y_1 + x_0 x_2 y_1 y_2 + x_2 y_0 x_0 y_2 + x_2 y_0 x_1 y_1)(x_1 y_2 + x_2 y_1$

$x_1 y_2 + x_2 y_1$

$\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 + (x_0 x_1 y_0 y_1 y_2 + x_0 x_1 y_0 y_1 y_2 + x_0 x_1 x_2 y_0 y_2 + x_1 x_2 y_0 y_1 y_2 + x_0 x_1 x_2 y_0 y_1 + x_0 x_1 x_2 y_1 y_2 + x_0 x_2 y_0 y_1 y_2 + x_2 y_0 x_1 y_1)$

$+ x_1 y_2 + x_2 y_1$

$\Rightarrow x_0 x_1 x_2 y_0 y_1 y_2 + x_1 y_2 + x_2 y_1 + (x_0 x_1 x_2 y_0 (y_1 + y_2) + x_1 y_0 y_1 y_2 (x_2 + x_0) + x_0 x_2 y_1 y_2 (y_0 + x_1) + x_1 y_1 (x_2 y_0 + x_0 y_0)$

Z_2 =

$R_5 = c_2 \& z_1 \& z_2 = z_1 (x_0x_1x_2y_0y_1y_2) \cdot a_0 b_1 c_2$

$x_0x_1x_2y_0y_1y_2 + \left(x_0x_2 [x_1y_0(y_1+y_2) + y_1y_2(y_0+x_1)] + x_1y_1 [y_0y_2(x_2+x_0) + (x_2y_0 + x_0y_2)] \right)$
$+ x_1y_2$
$+ x_2y_1$

$\Rightarrow$ 6 tg.

$R_5 = z_1z_2 + z_1c_2 + z_2c_2$

z_1 (as z_1 has all 6 basic bits $x_0x_1x_2y_0y_1y_2$)

i.e. for z_2c_2 $(c_2 = x_2y_2)$

$x_2y_2 \left[x_0x_1x_2y_0y_1y_2 + \left(x_0x_1y_0y_1y_2 + x_0x_1y_1y_2 + x_0x_1x_2y_0y_2 + x_1x_2y_0y_1y_2 + x_0x_1x_2y_0y_1 + x_0x_1x_2y_1y_2 + x_0x_2y_0y_1y_2 + x_2y_0x_1y_1 \right) \right]$

$\Rightarrow x_0x_1x_2y_0y_1y_2 + \left(x_0x_1x_2y_0y_1y_2 + x_0x_1x_2y_0y_1y_2 + x_0x_1x_2y_0y_1 + x_1x_2y_0y_1y_2 + x_0x_1x_2y_0y_1y_2 + x_0x_1x_2y_1y_2 + x_0x_2y_0y_1y_2 + x_2y_0x_1y_1y_2 \right)$
$+ x_1x_2y_2$
$+ x_2y_1y_2$

$\Rightarrow x_0x_1x_2y_0y_1y_2 + \left(x_0x_1x_2y_0y_1y_2 + x_1x_2y_0y_1y_2 + x_0x_1x_2y_0y_2 + x_0x_2y_0y_1y_2 \right)$
$+ x_1x_2y_2$
$+ x_2y_1y_2$

$\Rightarrow x_0x_1x_2y_0y_1y_2 + \left(x_2y_0y_2 (x_0x_1y_1 + x_1y_1 + x_0x_1 + x_0y_1) \right)$
$+ x_1x_2y_2 + x_2y_1y_2$