# **TABLE OF FOURIER TRANSFORMS**

## 1. Functions of a Single Variable

| Entry no. | Generalized function $f$  | Fourier transform $F[f]$   |
|-----------|---|--|
| 1         | Ordinary summable function $f(x)$   | $F[f] = \int_{-\infty}^{\infty} f(x) e^{ix\sigma} dx$  |
| 2         | $\delta(x)$   | 1  |
| 3         | 1   | $2\pi \ \delta(\sigma)$  |
| 4         | Polynomial $P(x)$   | $2\pi P\left(-irac{d}{d\sigma} ight)\delta(\sigma)$   |
| 5         | $\delta^{(2m)}(x)$  | $(-1)^m\sigma^{2m}$  |
| 6         | $\delta^{(2m+1)}(x)$  | $(-1)^{m+1} i\sigma^{2m+1}$  |
| 7         | $e^{bx}$  | $2\pi \delta(s-ib)$  |
| 8         | sin bx  | $-i\pi[\delta(s+b)-\delta(s-b)]$   |
| 9         | $\cos bx$   | $\pi[\delta(s+b)+\delta(s-b)]$   |
| 10        | $\sinh bx$  | $\pi[\delta(s-ib)-\delta(s+ib)$  |
| 11        | $\coshbx$   | $\pi[\delta(s-ib)+\delta(s+ib)]$   |
| 12        | $\exp\left(\frac{x^2}{2}\right)$  | Analytic functional $i\sqrt{2\pi} \exp(s^2/2)$ (integration along the imaginary axis)  |
| 13        | $ x ^{\lambda}$ $(\lambda \neq -1, -3,)$  | $-2\sin\frac{\lambda\pi}{2}\varGamma(\lambda+1)\mid\sigma\mid^{-\lambda-1}$  |
| 14        | $ x ^{\lambda} \qquad (\lambda \neq -1, -3,)$ $f_{\lambda}(x) = 2^{-\frac{1}{2}\lambda} \frac{ x ^{\lambda}}{\Gamma\left(\frac{\lambda+1}{2}\right)}$ | $\sqrt{2\pi}f_{-\lambda-1}(\sigma) = \sqrt{2\pi} \frac{2^{\frac{1}{2}(\lambda+1)}  \sigma ^{-\lambda-1}}{\Gamma\left(-\frac{\lambda}{2}\right)}$                                 |
| 15        | $ x ^{\lambda} \operatorname{sgn} x$ $(\lambda \neq -2, -4,)$   | $2i\cos\frac{\lambda\pi}{2}\Gamma(\lambda+1) \sigma ^{-\lambda-1}\operatorname{sgn}\sigma$   |
| 16        | $g_{\lambda}(x) = 2^{-\frac{1}{2}\lambda} \frac{ x ^{\lambda} \operatorname{sgn} x}{\Gamma\left(\frac{\lambda+2}{2}\right)}$                          | $\sqrt{2\pi} i g_{-\lambda-1}(\sigma) = \sqrt{2\pi} i \frac{2^{\frac{1}{2}(\lambda+1)}  \sigma ^{-\lambda-1} \operatorname{sgn} \sigma}{\Gamma\left(\frac{1-\lambda}{2}\right)}$ |

| Entry no. | Generalized function f                     | Fourier transform $F[f]$   |
|-----------|--|--|
|           |  |  |
| 17        | $x^m$                                      | $2(-i)^m \pi \delta^{(m)}(\sigma)$   |
| 18        | $x^{-m}$                                   | $i^m \frac{\pi}{(m-1)!} \sigma^{m-1} \operatorname{sgn} \sigma$  |
| 19        | $x^{-1}$                                   | $i\pi \ { m sgn} \ \sigma$   |
| 20        | $x^{-2}$                                   | $-\pi \mid \sigma \mid$  |
| 21        | $x_+^{\lambda}$ $(\lambda \neq -1, -2,)$   | $ie^{i\lambda(\pi/2)}\Gamma(\lambda+1)(\sigma+i0)^{-\lambda-1}$  |
|           |  | $= i\Gamma(\lambda + 1) \times \left[e^{\lambda i(\pi/2)} \sigma_{+}^{-\lambda-1} - e^{-i\lambda(\pi/2)} \sigma_{-}^{-\lambda-1}\right]^*$ |
| 22        | $x_+^n$                                    | $i^{n+1}n! \ \sigma^{-n-1} + (-i)^n \pi \ \delta^{(n)}(\sigma)$  |
| 23        | $\theta(x)$                                | $i\sigma^{-1} + \pi\delta(\sigma)$   |
| 24        | $x_{-}^{\lambda}$ $(\lambda \neq -1, -2,)$ | $-ie^{-i\lambda(\pi/2)}\Gamma(\lambda+1)(\sigma-i0)^{-\lambda-1}$  |
|           |  | $= i\Gamma(\lambda + 1) \times \left[e^{i\lambda(\pi/2)} \sigma_{-}^{-\lambda-1} - e^{-i\lambda(\pi/2)} \sigma_{+}^{-\lambda-1}\right]^*$  |
| 25        | $(x+i0)^{\lambda}$                         | $rac{2\pi e^{i\lambda(\pi/2)}}{\Gamma(-\lambda)}\sigma^{-\lambda-1}$  |
| 26        | $(x-i0)^{\lambda}$                         | $\frac{2\pi e^{-i\lambda(\pi/2)}}{\Gamma(-\lambda)}\sigma_+^{-\lambda-1}$  |

In Entries 27-38 we write:

$$ie^{i\lambda(\pi/2)} \Gamma(\lambda+1) = \frac{a_{-1}^{(n)}}{\lambda+n} + a_0^{(n)} + a_1^{(n)}(\lambda+n) + ...,$$

$$-ie^{-i\lambda(\pi/2)} \Gamma(\lambda+1) = \frac{b_{-1}^{(n)}}{\lambda+n} + b_0^{(n)} + b_1^{(n)}(\lambda+n) + ...,$$

$$-2\sin\frac{\lambda\pi}{2} \Gamma(\lambda+1) = \frac{c_{-1}^{(n)}}{\lambda+n} + c_0^{(n)} + c_1^{(n)}(\lambda+n) + ...,$$

$$2\cos\frac{\lambda\pi}{2} \Gamma(\lambda+1) = \frac{d_{-1}^{(n)}}{\lambda+n} + d_0^{(n)} + d_1^{(n)}(\lambda+n).$$

<sup>\*</sup> Second expression for  $\lambda \neq 0, \pm 1, \pm 2, \pm 3, \dots$ 

| Entry | Generalized | Fourier transform |
|-------|-------------|-------------------|
| no.   | function f  | F[f]              |

The  $a_{-1}^{(n)}$ ,  $a_0^{(n)}$ , ... are given by:

$$\begin{split} a_{-1}^{(n)} &= \frac{i^{n-1}}{(n-1)!}\,; \\ a_0^{(n)} &= \frac{i^{n-1}}{(n-1)!} \left[ 1 + \frac{1}{2} + \ldots + \frac{1}{n-1} + \Gamma'(1) + i \, \frac{\pi}{2} \right]\,; \\ a_1^{(n)} &= \frac{i^{n-1}}{(n-1)!} \left\{ \sum_{j,k=1}^{n-1} \frac{1}{jk} - \frac{\pi^2}{8} + \left( 1 + \frac{1}{2} + \ldots + \frac{1}{n-1} \right) \Gamma'(1) + \Gamma''(1) \right. \\ &\qquad \qquad + i \, \frac{\pi}{2} \left[ 1 + \frac{1}{2} + \ldots + \frac{1}{n-1} + \Gamma'(1) \right] \right\}\,; \\ b_i^{(n)} &= \bar{a}_i^{(n)}; \qquad c_i^{(n)} = 2 \, \operatorname{Re} \, a_i^{(n)}; \qquad d_i^{(n)} = 2 \, \operatorname{Im} \, a_i^{(n)}. \end{split}$$

In particular,

$$b_{-1}^{(n)} = \frac{(-i)^{n-1}}{(n-1)!}; c_{-1}^{(n)} = \frac{2(-1)^{n-1}}{(n-1)!} \cos(n-1) \frac{\pi}{2};$$

$$d_{-1}^{(n)} = \frac{2(-1)^n}{(n-1)!} \sin(n-1) \frac{\pi}{2}.$$

| Entry | Generalized                                  | Fourier transform  |
|-------|--|--|
| no.   | function f                                   | F[f]   |
| 31    | $\ln x_+$                                    | $i\left\{\left(\Gamma'(1)+i\frac{\pi}{2}\right)(\sigma+i0)^{-1}\right\}$   |
|       |  | $-(\sigma+i0)^{-1}\ln(\sigma+i0)$  |
| 32    | ln x_  | $-i\left\{\left(\Gamma'(1)-i\frac{\pi}{2}\right)(\sigma-i0)^{-1}\right\}$  |
|       |  | $-(\sigma-i0)^{-1}\ln{(\sigma-i0)}$  |
| 33    | $ x ^{\lambda} \ln  x $                      | $ie^{i\lambda(\pi/2)}\left\{\left[\Gamma'(\lambda+1)+i\frac{\pi}{2}\Gamma(\lambda+1)\right]\right\}$                         |
|       | $(\lambda \neq -1, -2,)$                     | $\times (\sigma + i0)^{-\lambda-1}$  |
|       |  | $-\Gamma(\lambda+1)\left(\sigma+i0\right)^{-\lambda-1}\ln\left(\sigma+i0\right)$   |
|       |  | $-ie^{-i\lambda(\pi/2)}\left\{\left[\Gamma'(\lambda+1)-i\frac{\pi}{2}\Gamma(\lambda+1)\right]\right\}$                       |
|       |  | $\times (\sigma - i0)^{-\lambda - 1}$  |
|       |  | $-\Gamma(\lambda+1)\left(\sigma-i0\right)^{-\lambda-1}\ln\left(\sigma-i0\right)$   |
| 34    | $ x ^{\lambda} \ln  x  \operatorname{sgn} x$ | $ie^{i\lambda(\pi/2)}\left\{\left[\Gamma'(\lambda+1)+i\frac{\pi}{2}\Gamma(\lambda+1)\right]\right\}$                         |
|       | $(\lambda \neq -1, -2,)$                     | $\times (\sigma + i0)^{-\lambda-1}$  |
|       |  | $-\Gamma(\lambda+1)\left(\sigma+i0\right)^{-\lambda-1}\ln\left(\sigma+i0\right)$   |
|       |  | $+ie^{-i\lambda(\pi/2)}\left\{\left[\Gamma'(\lambda+1)-i\frac{\pi}{2}\Gamma(\lambda+1)\right]\right\}$                       |
|       |  | $\times (\sigma - i0)^{-\lambda - 1}$  |
|       |  | $-\Gamma(\lambda+1)\left(\sigma-i0\right)^{-\lambda-1}\ln\left(\sigma-i0\right)$   |
| 35    | $x^{-2m} \ln  x $                            | $ c_1^{(2m)} \sigma ^{2m-1}-c_0^{(2m)} \sigma ^{2m-1}\ln \sigma $  |
| 36    | $x^{-2m-1} \ln  x $                          | $id_1^{(2m+1)}\sigma^{2m}\operatorname{sgn}\sigma-id_0^{(2m+1)}\sigma^{2m}\operatorname{ln} \sigma \operatorname{sgn}\sigma$ |
| 37    | $ x ^{-2m-1} \ln  x $                        | $c_1^{(2m+1)}\sigma^{2m} - c_0^{(2m+1)}\sigma^{2m} \ln  \sigma $   |
|       |  | $+\frac{1}{2}c_{-1}^{(2m+1)}\sigma^{2m}\ln^2 \sigma $  |

| Entry<br>no. | Generalized function $f$                            | Fourier transform $F[f]$  |
|--------------|---|---|
| 38           | $ x ^{-2m} \ln  x  \operatorname{sgn} x$            | $\left id_1^{(2m)}\sigma^{2m-1}-id_0^{(2m)}\sigma^{2m-1}\ln \sigma \right $   |
|              |   | $+\frac{i}{2} d_{-1}^{(2m)} \sigma^{2m-1} \ln^2  \sigma $   |
| 39           | $(1-x^2)^{\lambda}_+$<br>$(\lambda \neq -1, -2,)$   | $\sqrt{\pi}\Gamma(\lambda+1)\left(\frac{\sigma}{2}\right)^{-\lambda-\frac{1}{2}}J_{\lambda+\frac{1}{2}}(\sigma)$  |
| 40           | $(1+x^2)^{\lambda}_+$                               | $\frac{2\sqrt{\pi}}{\Gamma(-\lambda)}\left \frac{\sigma}{2}\right ^{-\lambda-\frac{1}{2}}K_{-\lambda-\frac{1}{2}}(\mid\sigma\mid)$                                    |
| 41           | $(x^2-1)^{\lambda}_{+}$<br>$(\lambda \neq -1, -2,)$ | $-\Gamma(\lambda+1)\sqrt{\pi}\left \frac{\sigma}{2}\right ^{-\lambda-\frac{1}{2}}N_{-\lambda-\frac{1}{2}}(\mid\sigma\mid)$  |
|              | (* / _, _, _,                                       | $= \Gamma(\lambda+1) \sqrt{\pi} \left  \frac{\sigma}{2} \right ^{-\lambda-\frac{1}{2}}$   |
|              |   | $\times \frac{\cos \pi (\lambda + \frac{1}{2}) \int_{-\lambda - \frac{1}{2}} ( \sigma ) - \int_{\lambda + \frac{1}{2}} ( \sigma )}{\sin \pi (\lambda + \frac{1}{2})}$ |
| 42           | $(x^2-1)^n_+$                                       | $(-1)^n 2\pi \left(1 + \frac{d^2}{d\sigma^2}\right)^n \delta(\sigma)$   |
|              |   | $+ (-1)^{n+1} \sqrt{\pi} \left(\frac{\sigma}{2}\right)^{-n-\frac{1}{2}} J_{n+\frac{1}{2}}(\sigma)$  |

#### 2. Functions of Several Variables

| 1        | $\delta(x_1,, x_n)$   | 1   |
|----------|---|---|
| 2        | 1   | $(2\pi)^n\delta(\sigma_1,,\sigma_n)$  |
| <b>3</b> | Polynomial $P(x_1,, x_n)$   | $(2\pi)^n P\left(-i\frac{\partial}{\partial\sigma_1},,-i\frac{\partial}{\partial\sigma_n}\right)\delta(\sigma)$   |
| 4        | $r^{\lambda} \qquad \left(r = \sqrt{\sum_{j} x_{j}^{2}}\right)$   | $2^{\lambda+n} \pi^{\frac{1}{2}n} \frac{\Gamma\left(\frac{\lambda+n}{2}\right)}{\Gamma\left(-\frac{\lambda}{2}\right)} \rho^{-\lambda-n}  \left(\rho = \sqrt{\sum \sigma_j^2}\right)$ |
| 5        | $f_{\lambda}(r) = \frac{2^{-\frac{1}{2}\lambda} r^{\lambda}}{\Gamma\left(\frac{\lambda + n}{2}\right)}$ | $(2\pi)^{\frac{1}{2}n} f_{-\lambda-n}(\rho) = (2\pi)^{\frac{1}{2}n} \frac{2^{\frac{1}{2}(\lambda+n)} r^{-\lambda-n}}{\Gamma\left(-\frac{\lambda}{2}\right)}$                          |

| Entry | Generalized       | Fourier transform |
|-------|-------------------|-------------------|
| no.   | ${\rm function}f$ | F[f]              |

In Entries 6-9 we write:

$$egin{align} C_{\lambda} &= 2^{\lambda+n} \, \pi^{rac{1}{2}n} \, rac{\Gamma\left(rac{\lambda+n}{2}
ight)}{\Gamma\left(-rac{\lambda}{2}
ight)} \ &= rac{c_{-1}^{(n+2m)}}{\lambda+n+2m} + c_{0}^{(n+2m)} + c_{1}^{(n+2m)}(\lambda+n+2m) + ...; \end{split}$$

the right-hand side is the Laurent expansion of this function about  $\lambda = -n-2m$ . Further,

$$\Omega_n = \frac{2\pi^{\frac{1}{2}n}}{\Gamma\left(\frac{n}{2}\right)}$$

is the hypersurface area of the unit sphere in n dimensions.

$$\begin{array}{lll}
6 & r^{\lambda} \ln r \\
(\lambda \neq -n, -n - 2, ...) & \frac{dC_{\lambda}}{d\lambda} \rho^{-\lambda - n} + C_{\lambda} \rho^{-\lambda - n} \ln \rho \\
7 & r^{\lambda} \ln^{2} r \\
(\lambda \neq -n, -n - 2, ...) & \frac{d^{2}C_{\lambda}}{d\lambda^{2}} \rho^{-\lambda - n} + 2 \frac{dC_{\lambda}}{d\lambda} \rho^{-\lambda - n} \\
8 & \Omega_{n} r^{-2m-n} & \times \ln \rho + C_{\lambda} \rho^{-\lambda - n} \ln^{2} \rho \\
9 & \Omega_{n} r^{-2m-n} \ln r & \frac{1}{2} c_{-1}^{(n+2m)} \rho^{2m} \ln \rho + c_{0}^{(n+2m)} \rho^{2m} \ln \rho \\
& + c_{1}^{(n+2m)} \rho^{2m} \\
10 & \delta(r - a) & (n \geqslant 1) & \times \int_{\frac{1}{2}(n-2)} (a\rho) \\
11 & \text{The same, for } n = 3 & 2^{\frac{1}{2}n-1} \Gamma\left(\frac{1}{2}n - \frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \Omega_{n-1} \alpha^{\frac{1}{2}n} \rho^{1-\frac{1}{2}n} \\
& \times \int_{\frac{1}{2}(n-2)} (a\rho) \\
12 & \left(\frac{d}{a da}\right)^{m} \frac{\delta(r - a)}{a} & 2^{\frac{1}{2}n-1} \Gamma\left(\frac{1}{2}n - \frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) \Omega_{n-1} \sqrt{\frac{2}{\pi}} \frac{\sin a\rho}{\rho}
\end{array}$$

For the notation in Entries 13-25 see the Summary of Fundamental Definitions and Equations for Chapter III, Section 2.

| Entry | Generalized            | Fourier transform   |
|-------|------------------------|---|
| no.   | function f             | F[f]  |
| 13    | $(P+i0)^{\lambda}$     | $\frac{1}{\Gamma(-\lambda)} e^{-(\pi/2)qi} 2^{n+2\lambda} \pi^{\frac{1}{2}n} \Gamma(\lambda + \frac{1}{2}n) (Q - i0)^{-\lambda - \frac{1}{2}n}$   |
| 14    | $(P-i0)^{\lambda}$     | $\frac{e^{(\pi/2)qi} 2^{n+2\lambda} \pi^{\frac{1}{2}n} \Gamma(\lambda+\frac{1}{2}n)}{\Gamma(-\lambda)} (Q+i0)^{-\lambda-\frac{1}{2}n}$  |
| 15    | $P_+^{\lambda}$        | $2^{n+2\lambda} \pi^{\frac{1}{2}n-1} \Gamma(\lambda+1) \Gamma(\lambda+\frac{1}{2}n)$  |
|       |                        | $2^{n+2\lambda} \pi^{\frac{1}{2}n-1} \Gamma(\lambda+1) \Gamma(\lambda+\frac{1}{2}n)$ $\times \frac{1}{2i} \left[ e^{-i(\frac{1}{2}q+\lambda)\pi} (Q-i0)^{-\lambda-\frac{1}{2}n} - e^{i(\frac{1}{2}q+\lambda)\pi} (Q+i0)^{-\lambda-\frac{1}{2}n} \right]$    |
| 16    | P <u>\</u>             | $-2^{n+2\lambda} \pi^{\frac{1}{2}n-1} \Gamma(\lambda+1) \Gamma(\lambda+\frac{1}{2}n)$   |
|       |                        | $\times \frac{1}{2i} \left[ e^{-(\pi/2)qi} (Q - i0)^{-\lambda - \frac{1}{2}n} - e^{(\pi/2)qi} (Q + i0)^{-\lambda - \frac{1}{2}n} \right]$   |
| 17    | $(c^2+P+i0)^{\lambda}$ | $\frac{2^{\lambda+1}(\sqrt{2\pi})^nc^{\frac{1}{2}n+\lambda}}{\Gamma(-\lambda)\sqrt{\Delta}}\frac{K_{\frac{1}{2}n+\lambda}\left[c(Q-i0)^{\frac{1}{2}}\right]}{(Q-i0)^{\frac{1}{2}(\frac{1}{2}n+\lambda)}}$   |
|       |                        | $=\frac{2^{\lambda+\frac{1}{2}n+1}\pi^{\frac{1}{2}n}e^{-\frac{1}{2}q\pi i}c^{\lambda+\frac{1}{2}n}}{\Gamma(-\lambda)\sqrt{ \Delta }}$   |
|       |                        | $\times \left[ \frac{K_{\lambda + \frac{1}{2}n}(cQ^{\frac{1}{2}})}{O^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} + \frac{\pi i}{2} \frac{H_{-\lambda - \frac{1}{2}n}^{(1)}(cQ^{\frac{1}{2}})}{O^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right].$                 |
|       |                        |   |
| 18    | $(c^2+P-i0)^{\lambda}$ | $\frac{2^{\lambda+1}(\sqrt{2\pi})^n c^{\frac{1}{2}n+\lambda}}{\Gamma(-\lambda) \sqrt{\Delta}} \frac{K_{\frac{1}{2}n+\lambda} \left[c(Q+i0)^{\frac{1}{2}}\right]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n+\lambda)}}$   |
|       |                        | $=\frac{2^{\lambda+\frac{1}{2}n+1}\pi^{\frac{1}{2}n}e^{\frac{1}{2}q\pi i}c^{\lambda+\frac{1}{2}n}}{\Gamma(-\lambda)\sqrt{\mid\varDelta\mid}}$   |
|       |                        | $\times \left[ \frac{K_{\lambda + \frac{1}{2}n}(cQ_{+}^{\frac{1}{2}})}{Q_{+}^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} - \frac{\pi i}{2} \frac{H_{-\lambda - \frac{1}{2}n}^{(2)}(cQ_{-}^{\frac{1}{2}})}{Q_{-}^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right].$ |

|  |   | Continued  |
|--|---|--|
| Entry  | Generalized                                       | Fouirer transform  |
| no.  | function f  | F[f]   |
| 19   | $\frac{(c^2+P)_+^{\lambda}}{\Gamma(\lambda+1)}$   | $-\frac{2^{\lambda+\frac{1}{2}n}i\pi^{\frac{1}{2}n-1}c^{\frac{1}{2}n+\lambda}}{\sqrt{ \Delta }}$   |
| THE THE PARTY OF T |   | $\times \left\{ e^{-i(\lambda + \frac{1}{2}q)\pi} \frac{K_{\frac{1}{2}n + \lambda} \left[ c(Q - i0)^{\frac{1}{2}} \right]}{(Q - i0)^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right\}$   |
|  |   | $\left  -e^{i(\lambda + \frac{1}{2}q)n} \frac{K_{\frac{1}{2}n + \lambda}[c(Q + i0)^{\frac{1}{2}}]}{(Q + i0)^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right  = \frac{2^{\lambda + \frac{1}{2}n + 1} \pi^{\frac{1}{2}n - 1} c^{\frac{1}{2}n + \lambda}}{\sqrt{ \Delta }}$                           |
|  |   | $\times \left\{ -\sin\left(\lambda + \frac{1}{2}q\right)\pi \frac{K_{\lambda + \frac{1}{2}n}(cQ_{+}^{\frac{1}{2}})}{Q_{+}^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} + \frac{\pi}{2\sin\left(\lambda + \frac{1}{2}n\right)\pi} \right\}$   |
|  |   | $\times \left[ \sin \left( \lambda + \frac{1}{2} q \right) \pi \frac{J_{\lambda + \frac{1}{2}n} (cQ_{-}^{\frac{1}{2}})}{Q_{-}^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right]$  |
|  |   | $+\sin\frac{p\pi}{2}\frac{J_{-\lambda-\frac{1}{2}n}(cQ_{-}^{\frac{1}{2}})}{Q_{-}^{\frac{1}{2}(\lambda+\frac{1}{2}n)}}\bigg]\bigg\}.$   |
| 20   | $\frac{(c^2+P)^{\lambda}_{-}}{\Gamma(\lambda+1)}$ | $\frac{2^{\lambda + \frac{1}{2}n_{i\pi}\frac{1}{2}n - 1}c^{\frac{1}{2}n + \lambda}}{\sqrt{ \Delta }} \left\{ e^{-\frac{1}{2}q\pi i} \frac{K_{\frac{1}{2}n + \lambda}[c(Q - i0)^{\frac{1}{2}}]}{(Q - i0)^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right\}$   |
|  |   | $\left  -e^{\frac{1}{2}qni} \frac{K_{\frac{1}{2}n+\lambda}[c(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\lambda+\frac{1}{2}n)}} \right  = \frac{2^{\lambda+\frac{1}{2}n+1}\pi^{\frac{1}{2}n-1}c^{\frac{1}{2}n+\lambda}}{\sqrt{ \Delta }}$   |
|  |   | $\times \left\{ \sin \frac{q\pi}{2} \frac{K_{\lambda + \frac{1}{2}n}(cQ_{+}^{\frac{1}{2}})}{Q_{+}^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} - \frac{\pi}{2\sin(\lambda + \frac{1}{2}n)\pi} \right\}$  |
|  |   | $\times \left[ \sin \frac{q\pi}{2} \frac{J_{\lambda + \frac{1}{2}n}(cQ^{\frac{1}{2}})}{Q^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} + \sin \left( \lambda + \frac{p}{2} \right) \pi \frac{J_{-\lambda - \frac{1}{2}n}(cQ^{\frac{1}{2}})}{Q^{\frac{1}{2}(\lambda + \frac{1}{2}n)}} \right] \right\}.$ |
| 21   | $\delta^{(t-1)} \times (c^2 + P)$                 | $(-1)^{t+1} \frac{i}{\sqrt{ \Delta }} 2^{\frac{1}{2}n-t} \pi^{\frac{1}{2}n-1} c^{\frac{1}{2}n-t}$  |
|  |   | $\times \left[ e^{-\frac{1}{2}niq} \frac{K_{\frac{1}{2}n-t}[c(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{1}{2}n-t)}} \right]$   |
|  |   | $-e^{\frac{1}{2}niq}\frac{K_{\frac{1}{2}n-t}[c(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n-t)}}\right].$  |
|  |   | $(Q+i0)^{\frac{1}{2}(\frac{1}{2}n-t)}  ]$  |

| Entry | Generalized                      | Fourier transform   |
|-------|----------------------------------|---|
| no.   | function f                       | $m{F}[f]$   |
| 22    | $\delta(c^2+P)$                  | $-\frac{i}{\sqrt{ \Delta }} (2\pi c)^{\frac{1}{2}n-1} \left[ -e^{-\frac{1}{2}\pi q i} \frac{K_{\frac{1}{2}n-1}[c(Q-i0)^{\frac{1}{2}}]}{(Q-i0)^{\frac{1}{2}(\frac{1}{2}n-1)}} \right]$   |
|       |                                  | $+ e^{\frac{1}{2}\pi qi} \frac{K_{\frac{1}{2}n-1}[c(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n-1)}} \bigg].$  |
| 23    | $rac{(c^2+P)_+^t}{\Gamma(t+1)}$ | $(-1)^{t+1}i2^{t+\frac{1}{2}n}\pi^{\frac{1}{2}n-1}c^{\frac{1}{2}n+t}$   |
|       | - (* 1 -)                        | $(-1)^{t+1}i2^{t+rac{1}{2}n}\pi^{rac{1}{2}n-1}c^{rac{1}{2}n+t} \ 	imes \left[e^{-rac{1}{2}q\pi i}rac{K_{rac{1}{2}n+t}[c(Q-i0)^{rac{1}{2}}]}{(Q-i0)^{rac{1}{2}(rac{1}{2}n+t)}} ight.$   |
|       |                                  | $-e^{\frac{1}{2}q\pi i}\frac{K_{\frac{1}{2}n+t}[c(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n+t)}}\bigg]$  |
|       |                                  | $+ (2\pi)^n \sum_{m=0}^t \frac{(-1)^m (\frac{1}{2}c)^{2t-2m}}{4^m m! (t-m)!} L^m \delta(s),$  |
| 24    | $\frac{(c^2+P)^t}{\Gamma(t+1)}$  | $\frac{i \cdot 2^{t + \frac{1}{2}n_{\boldsymbol{\pi}} \frac{1}{2}n - 1} c^{\frac{1}{2}n + t}}{\sqrt{ \Delta }} \left[ e^{-\frac{1}{2}q\pi i} \frac{K_{\frac{1}{2}n + t} [c(Q - i0)^{\frac{1}{2}}]}{(Q - i0)^{\frac{1}{2}(\frac{1}{2}n + t)}} \right]$ |
|       |                                  | $-e^{\frac{1}{2}q\pi i}\frac{K_{\frac{1}{2}n+t}[c(Q+i0)^{\frac{1}{2}}]}{(Q+i0)^{\frac{1}{2}(\frac{1}{2}n+t)}}\bigg],$   |
| 25    | $\frac{(c^2+P)^t}{\Gamma(t+1)}$  | $(2\pi)^n \sum_{m=0}^t \frac{(-1)^m \left(\frac{1}{2} c\right)^{2t-2m}}{4^m m! (t-m)!} L^m \delta(s).$  |