AI Robotics

Bayesian Filtering

Overview

Robot Model

Sensor Measurements:
$Z_{0:t} = \{z_0, \dots, z_t\}$

Robot State (or pose):
$X_{0:t} = \{X_0, \dots, X_t\}$

sense

act

Landmark or Map:
$m = \{m_1, \dots, m_n\}$

Robot Controls:
$u_{0:t} = \{u_0, u_1, \dots, u_t\}$

State Estimation

Robot State (or pose):
$X_{0:t} = \{X_0, \ldots, X_t\}$

Sensor Measurements:
$Z_{0:t} = \{z_0, \ldots, z_t\}$

sense

Known

Unknown

act

Robot Controls:
$u_{0:t} = \{u_0, u_1, \ldots, u_t\}$

Motivation

State estimation is important for:

- ❑ **Ensuring that the robot follows the pre-determined trajectory**
- ❑ **Ensuring that the robot reaches the destination safely**

State Estimation

How to estimate robot state based on sensor measurements and control data?

Robot State (or pose):
$X_{0:t} = \{X_0, \ldots, X_t\}$

Sensor Measurements

sense

act

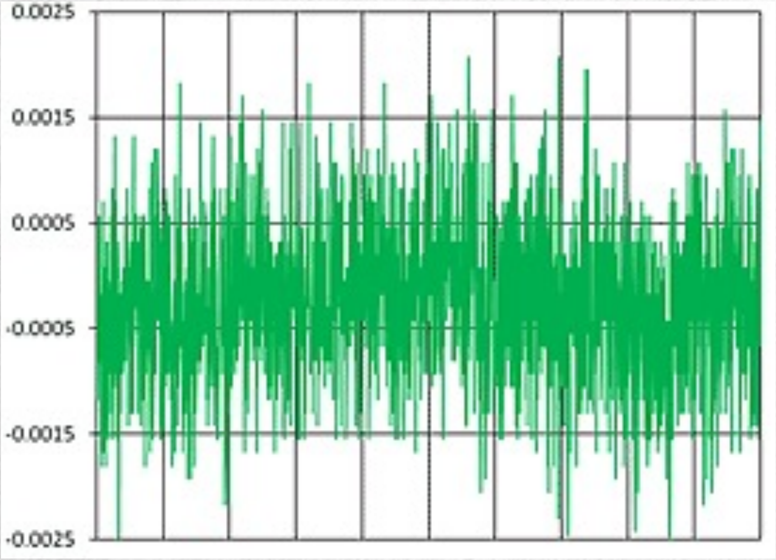
Robot Controls

1) Predict the robot position by observing the surroundings (e.g. with cameras, LiDARs)

2) Predict the robot position by tracking our control inputs (i.e. motor commands)

Uncertainty

Sensor uncertainty: sensor measurements are noisy



Motion uncertainty: robot control is noisy due to vibration, external forces

Probabilistic State Estimation

$$P(x_t|u_1, z_1, u_2, z_2, \cdots, u_t, z_t) = P(x_t|z_{1:t}, u_{1:t})$$

Robot State

sense

Sensor Model

$$P(z_t|x_t)$$

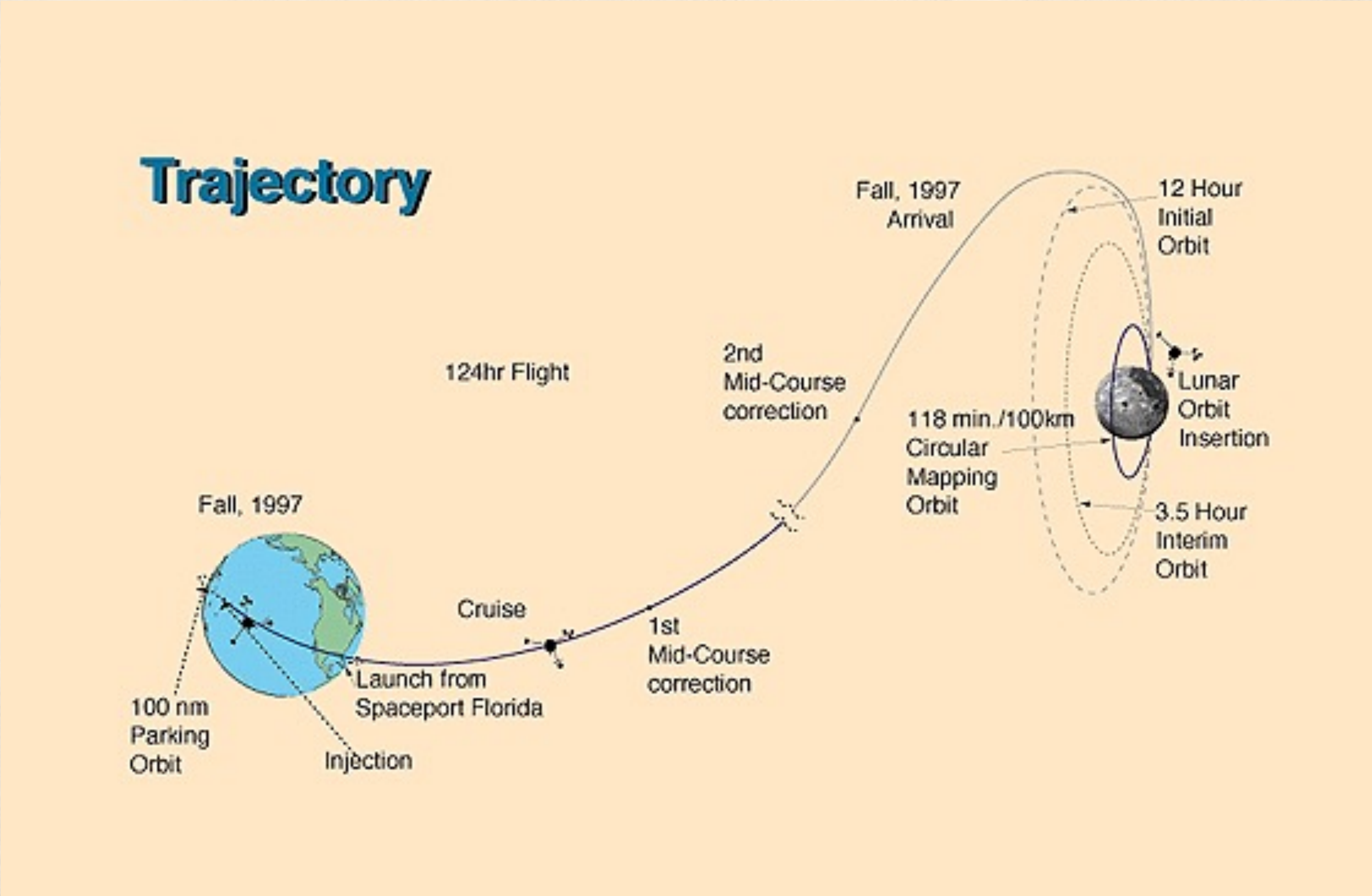
act

Motion Model

$$P(x_t|x_{t-1}, u_t)$$

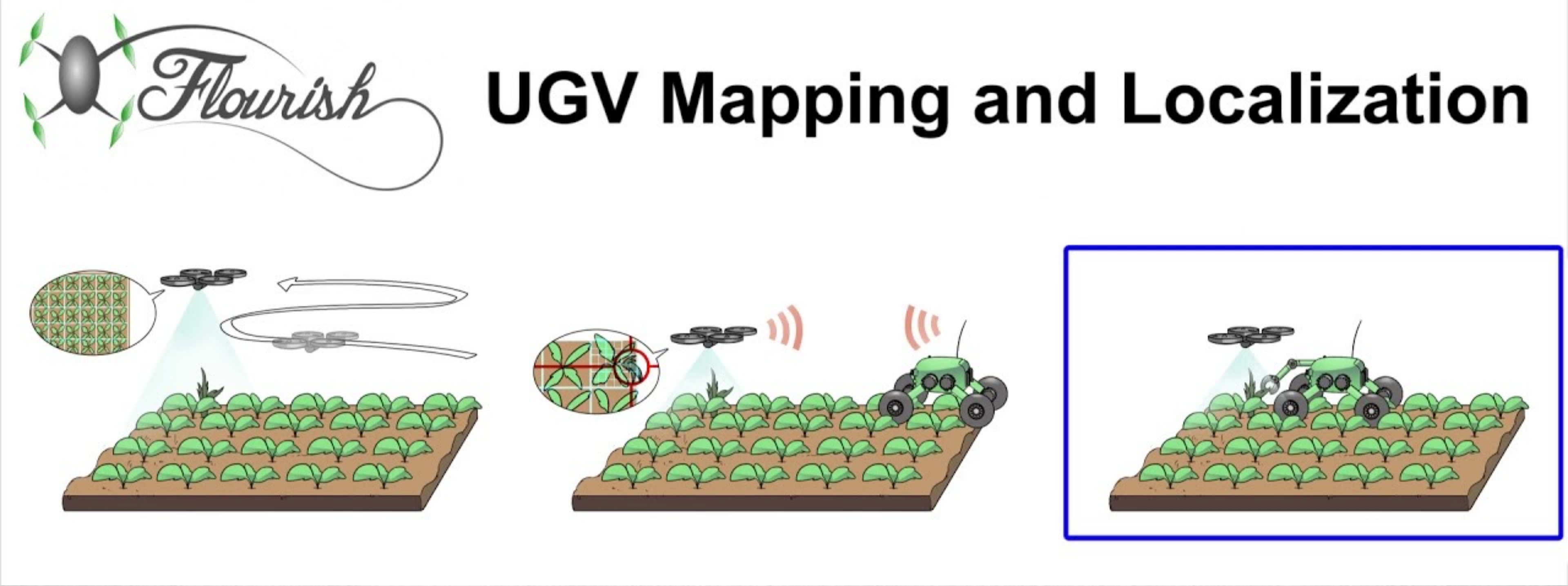
Localization

Localization in Space



Localization in Warehouse

Localization in Precision Agriculture

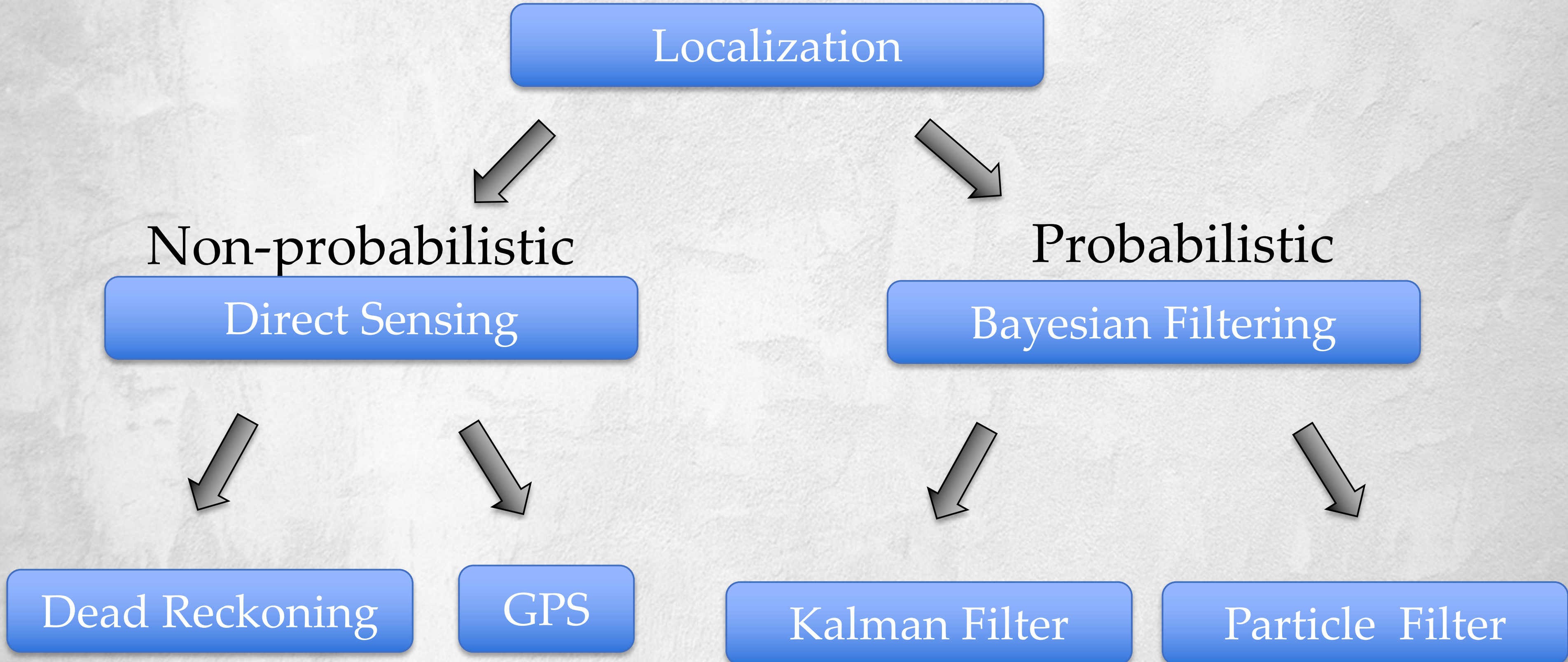


Localization on the Road

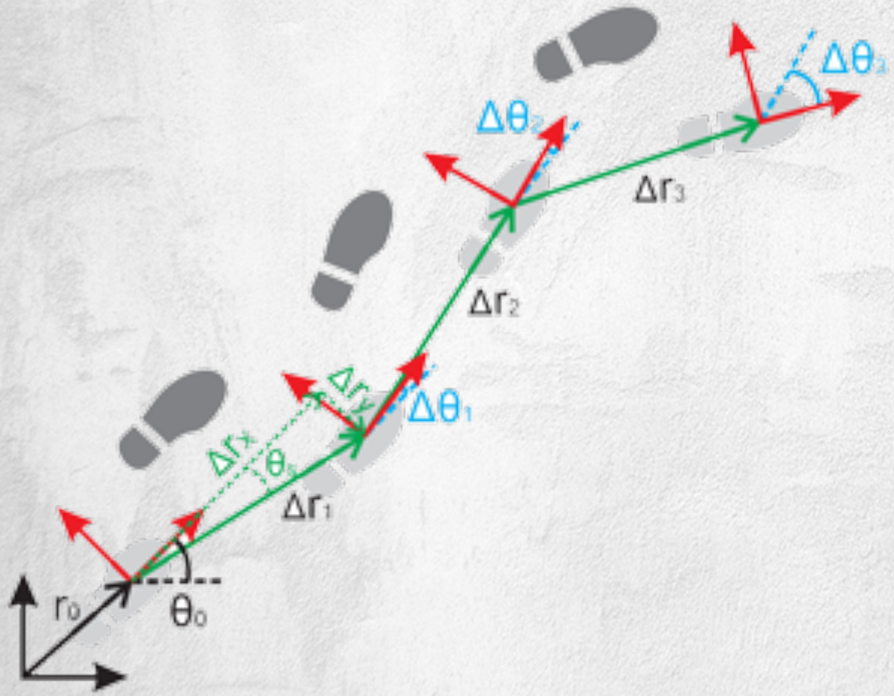
Localization on the Road

Algorithms

Types of Localization Algorithms

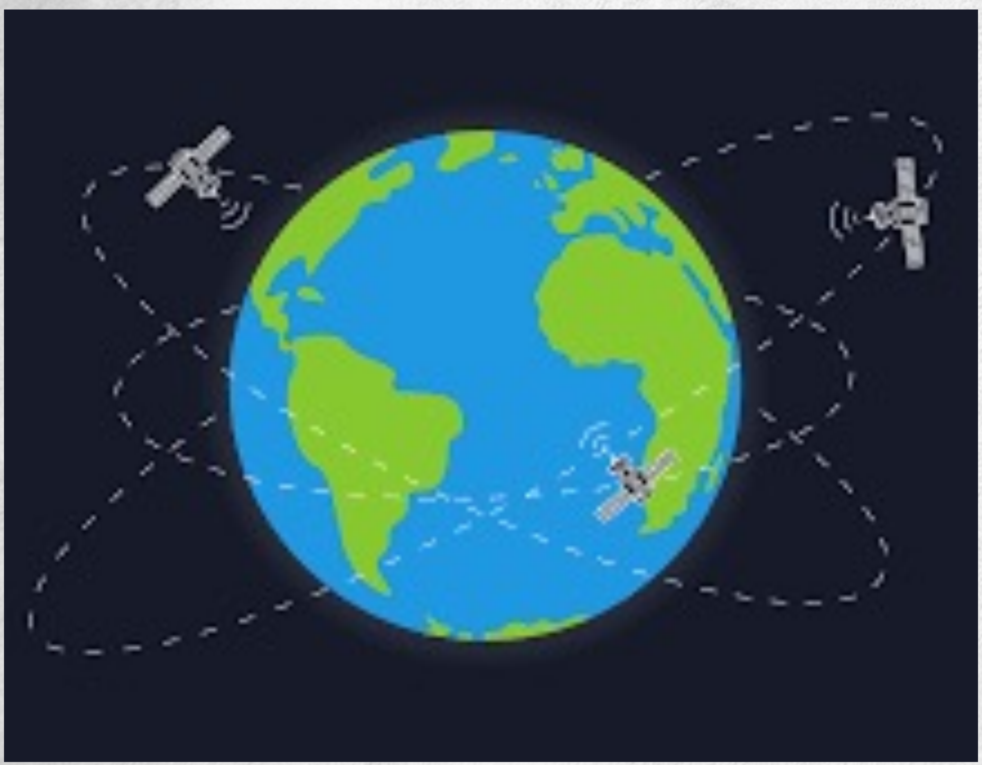


Dead Reckoning



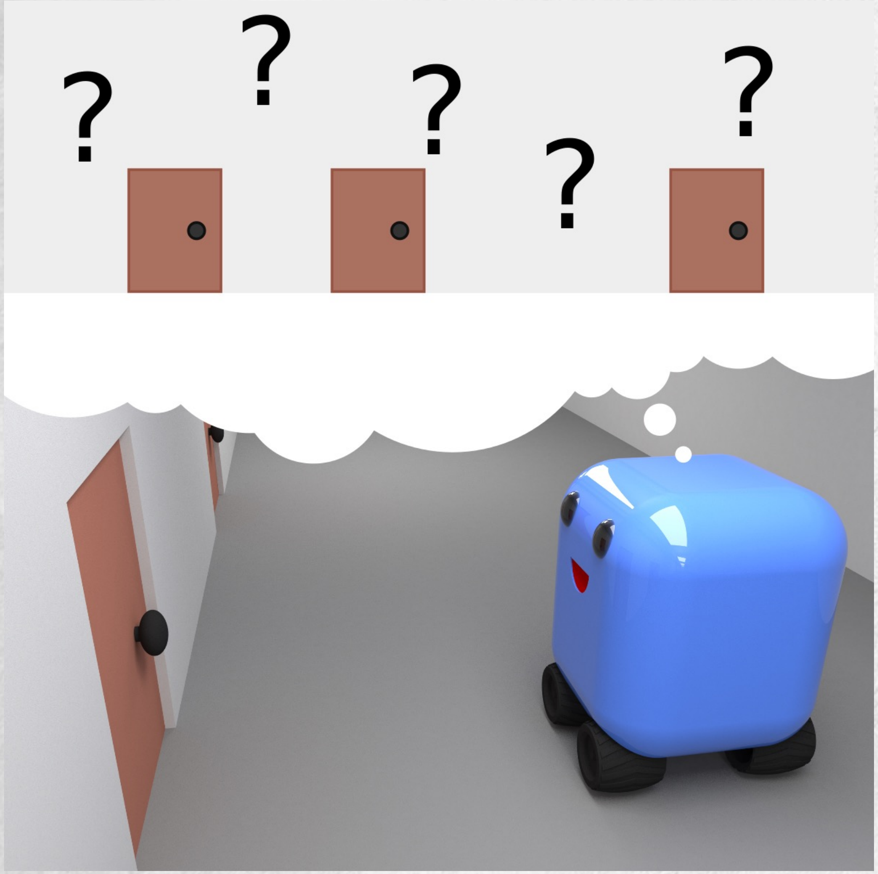
- Simple to implement
- Errors accumulate

GPS



- Only accurate up to ~1m
- Poor signal in indoor / underground environments

Probabilistic Localization



- Express location in terms of probabilities / beliefs
- Reason about motion noise
- Reason about sensor noise

Bayes Rule

Terminology

LIKELIHOOD
The probability of "B" being True, given "A" is True

PRIOR
The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

POSTERIOR
The probability of "A" being True, given "B" is True

MARGINALIZATION
The probability "B" being True.

Terminology

Likelihood

How probable is the evidence given that our hypothesis is true?

Prior

How probable was our hypothesis *before* observing the evidence?

$$P(H \mid e) = \frac{P(e \mid H)\, P(H)}{P(e)}$$

Posterior

How probable is our hypothesis given the observed evidence?
(Not directly computable)

Marginal

How probable is the new evidence under all possible hypotheses?
$P(e) = \sum P(e \mid H_i)\, P(H_i)$

Derivation

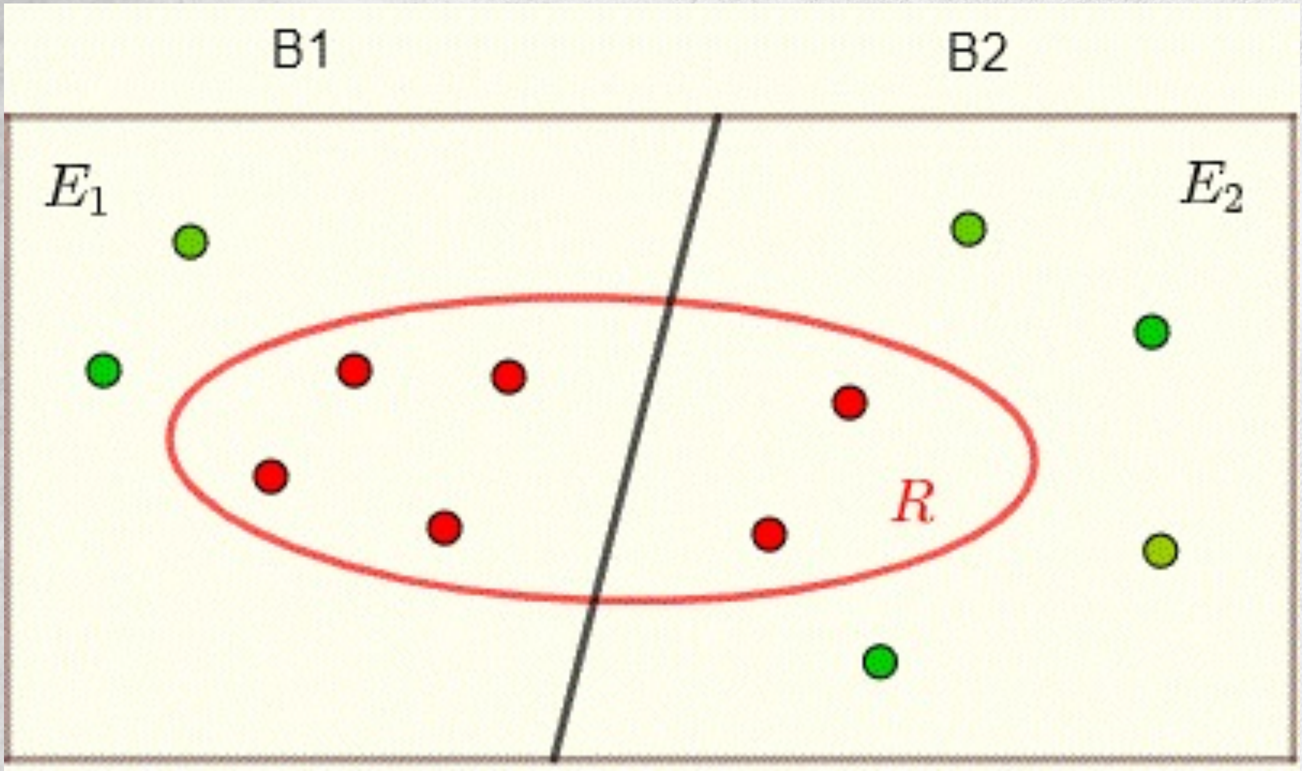
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|B)\,P(B) = P(A \cap B) = P(B|A)\,P(A)$$

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

From definition of conditional probability



P(B1) = 0.5
P(B2) = 0.5
P(red | B1) = 0.67
P(red | B2) = 0.33
P(red) = 0.5

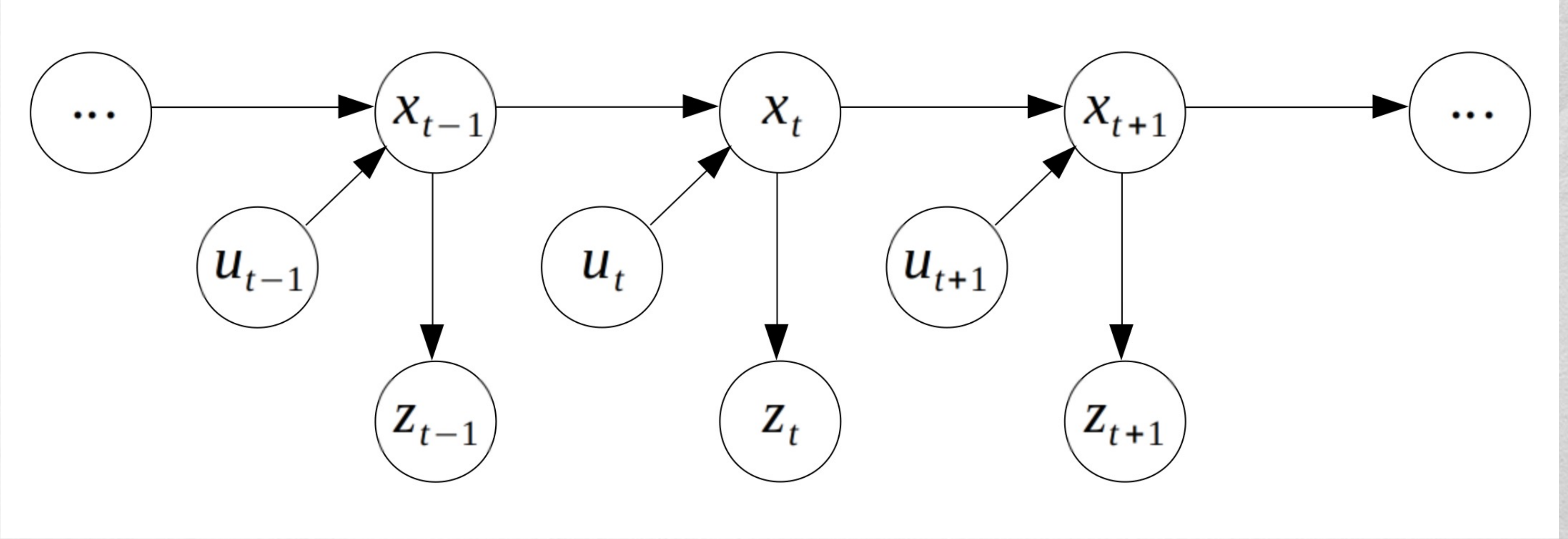
$$P(B1 \mid red) = \frac{P(red \mid B1)\, P(B1)}{P(red)}$$

$$= \frac{(0.67)\,(0.5)}{(0.5)} = 0.67$$

Bayes Net

Graphical Model

Robot State (or pose):
$X_{0:t} = \{X_0, \dots, X_t\}$



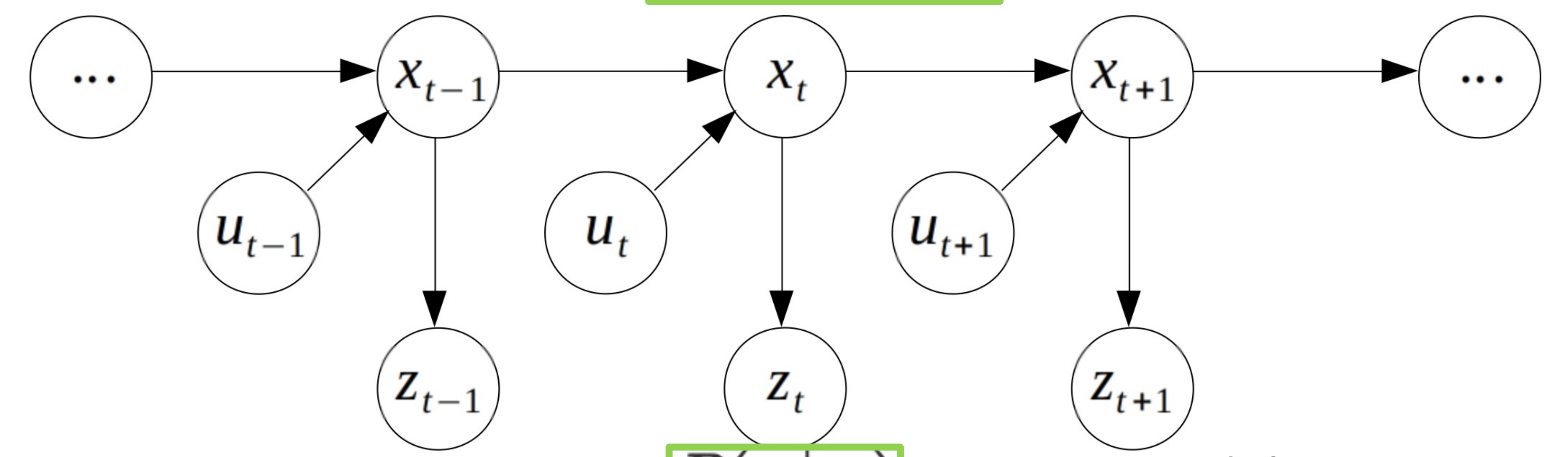
Robot Controls:
$u_{0:t} = \{u_0, u_1, \dots, u_t\}$

Sensor Measurements:
$Z_{0:t} = \{z_0, \dots, z_t\}$

Graphical Model

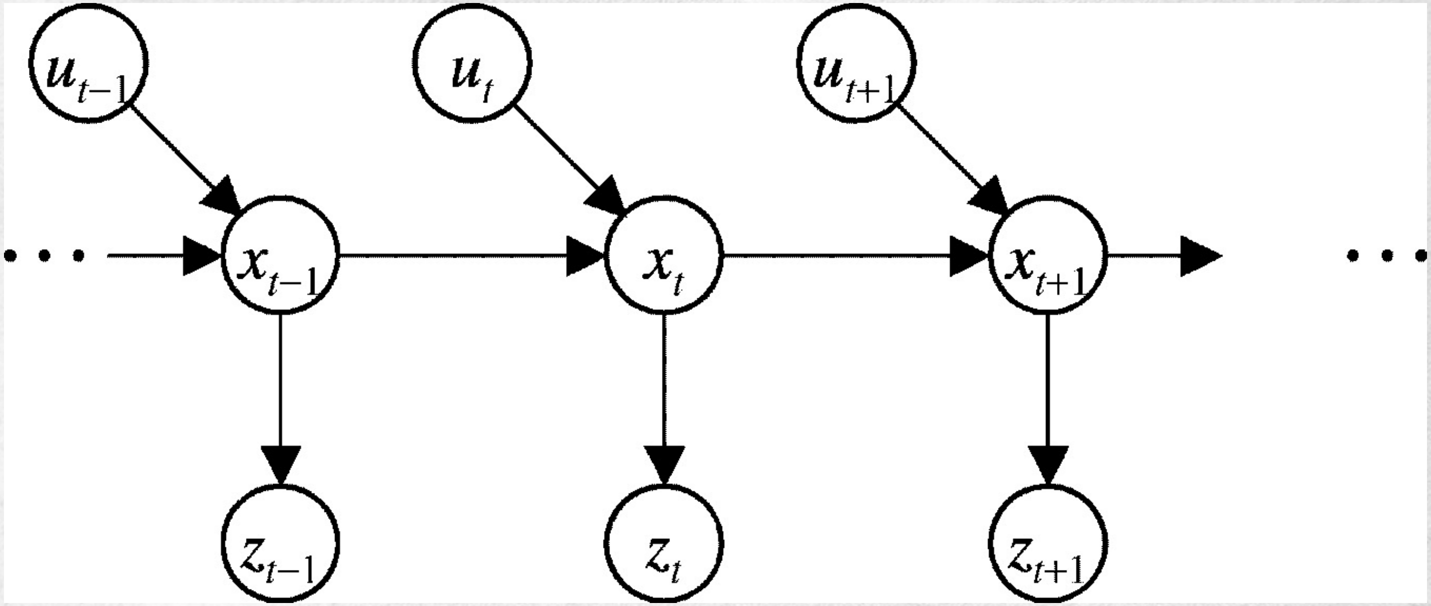
$$P(x_t|u_1, z_1, u_2, z_2, \cdots, u_t, z_t) = P(x_t|z_{1:t}, u_{1:t})$$

$P(x_t|x_{t-1}, u_t)$ Motion Model



$P(z_t|x_t)$ Sensor Model

Markov Assumption



$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Understanding the Markov Assumptions

$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = \boxed{p(z_t \mid x_t)}$$

Sensor Model

- Sensor Markov Assumption – The current sensor reading is conditionally independent of past states, sensor readings, and actions, given the current state
- i.e. The sensor reading only depends on current state

$$p(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t}) = \boxed{p(x_t \mid x_{t-1}, u_t)}$$

Motion Model

- Motion Markov Assumption – The current state is conditionally independent of past sensor readings, past states, and past actions, given the most recent state and action
- i.e. The current state depends only on last state and action

Bayesian Update

Bayes Filters

z = observation
u = action
x = state

$$\boxed{Bel(x_t)} = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes $$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \, P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov $$= \eta \; P(z_t \mid x_t) \, P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob. $$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \, dx_{t-1}$$

Markov $$= \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \, P(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1}) \, dx_{t-1}$$

Sensor Model

$$= \eta \, \boxed{P(z_t \mid x_t)} \int \boxed{P(x_t \mid u_t, x_{t-1})} \, Bel(x_{t-1}) \, dx_{t-1}$$

Motion Model

Bayes Filter Algorithm

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Previous State

Current control input

Current sensor measurement

Algorithm **Bayes_filter**($Bel(x_{t-1})$, u_t, z_t):

$$Bel'(x_t) = \int P(x_t \mid u_t, x_{t-1}) \, Bel\,(x_{t-1}) dx_{t-1}$$

Prediction Step: Motion Update

$$Bel(x_t) = \eta \, P(z_t \mid x_t) \, Bel'(x_t)$$

Correction Step: Measurement Update

$return \quad Bel(x_t)$

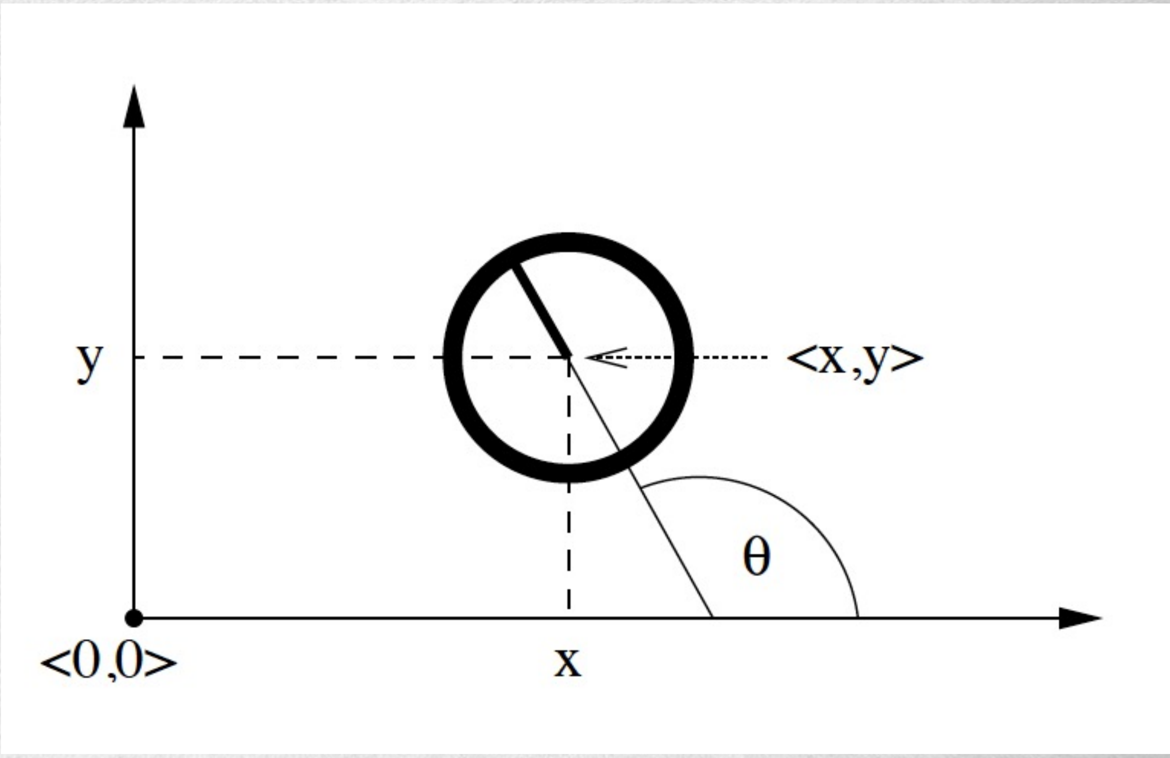
Current State

Motion Model

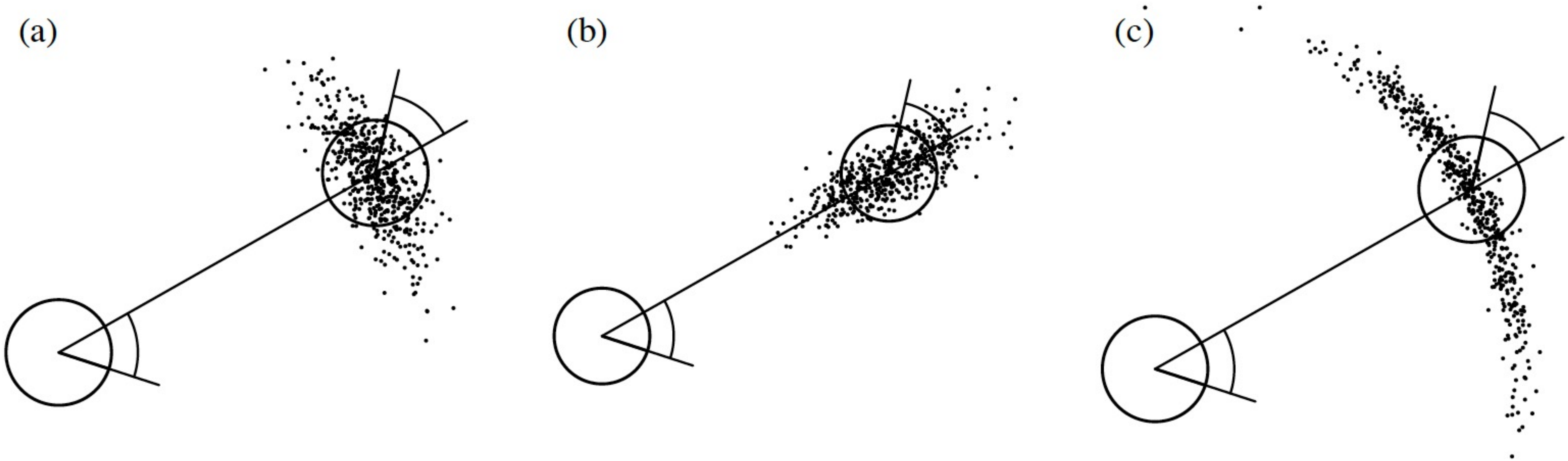
Robot State

Robot State = position & orientation

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$



Motion Model



Motion Model

Algorithm motion_model_odometry(x_t, u_t, x_{t-1})**:**

$$\delta_{\text{rot1}} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
$$\delta_{\text{trans}} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^s}$$
$$\delta_{\text{rot2}} = \bar{\theta}' - \bar{\theta} - \delta_{\text{rot1}}$$

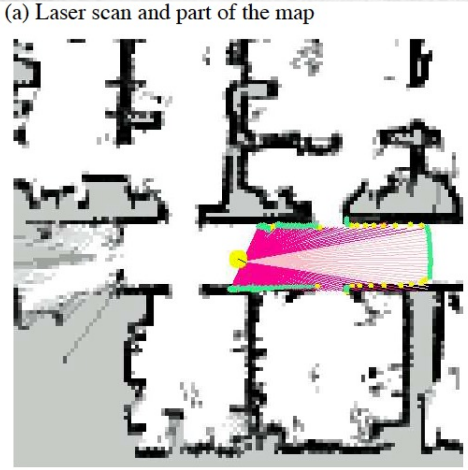
$$\hat{\delta}_{\text{rot1}} = \text{atan2}(y' - y, x' - x) - \theta$$
$$\hat{\delta}_{\text{trans}} = \sqrt{(x - x')^2 + (y - y')^2}$$
$$\hat{\delta}_{\text{rot2}} = \theta' - \theta - \hat{\delta}_{\text{rot1}}$$

$$p_1 = \mathbf{prob}(\delta_{\text{rot1}} - \hat{\delta}_{\text{rot1}}, \alpha_1 \hat{\delta}_{\text{rot1}} + \alpha_2 \hat{\delta}_{\text{trans}})$$
$$p_2 = \mathbf{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \alpha_3\, \hat{\delta}_{\text{trans}} + \alpha_4(\hat{\delta}_{\text{rot1}} + \hat{\delta}_{\text{rot2}}))$$
$$p_3 = \mathbf{prob}(\delta_{\text{rot2}} - \hat{\delta}_{\text{rot2}}, \alpha_1 \hat{\delta}_{\text{rot2}} + \alpha_2 \hat{\delta}_{\text{trans}})$$

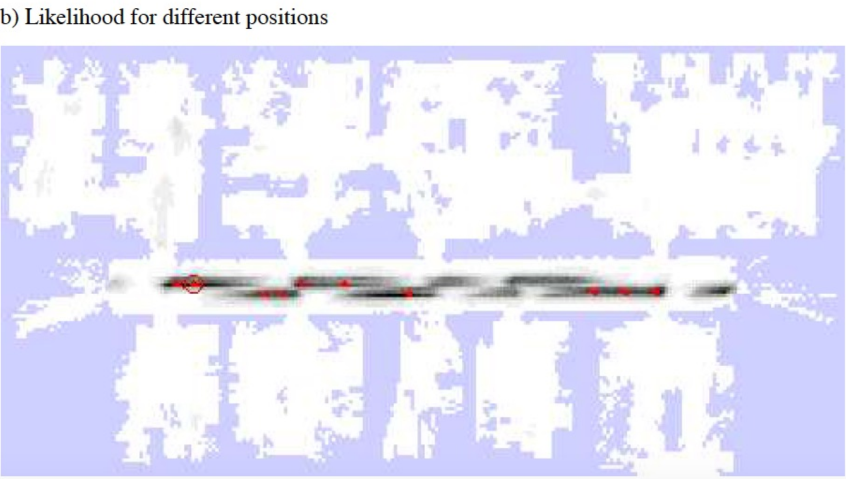
return $p_1 \cdot p_2 \cdot p_3$

Sensor Model

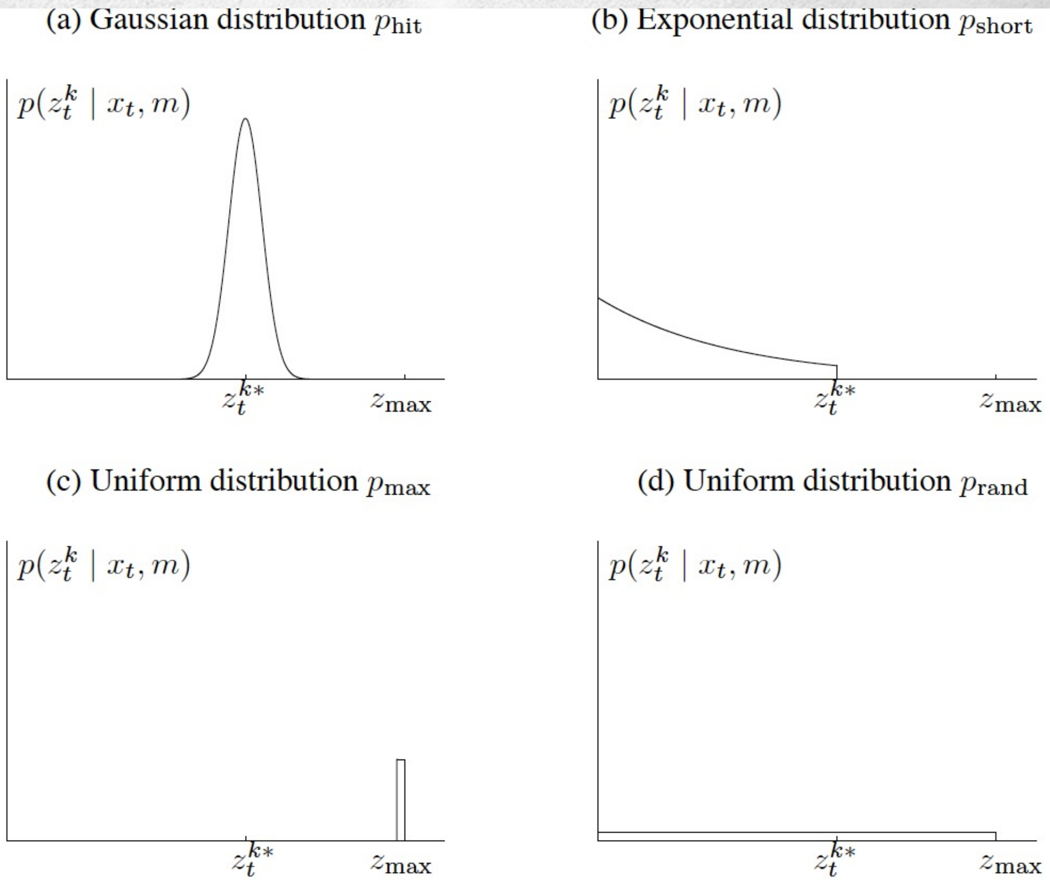
Beam Model



(a) Laser scan and part of the map

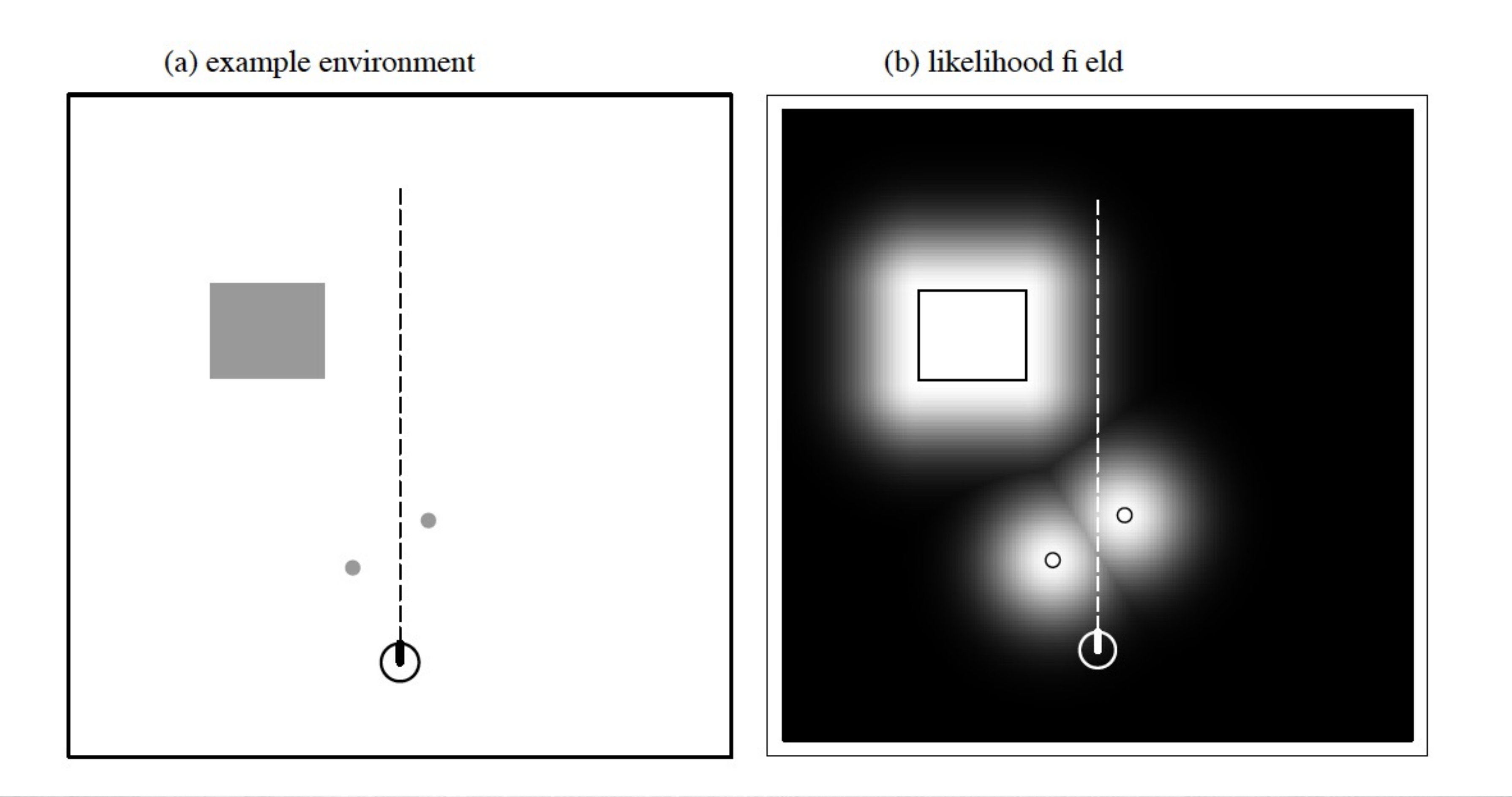


b) Likelihood for different positions



(a) Gaussian distribution p_{hit}
$p(z_t^k \mid x_t, m)$
z_t^{k*}
$z_{\max}$
(b) Exponential distribution p_{short}
$p(z_t^k \mid x_t, m)$
z_t^{k*}
$z_{\max}$
(c) Uniform distribution $p_{\max}$
$p(z_t^k \mid x_t, m)$
z_t^{k*}
$z_{\max}$
(d) Uniform distribution p_{rand}
$p(z_t^k \mid x_t, m)$
z_t^{k*}
$z_{\max}$

Likelihood Field Model



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