ENGT101

Introduction to Physics

Lab report

Snell's Law

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Title: Determining the refractive index of a glass block using Snell's Law

1. Abstract:

The refractive index of a material shows how much a beam of light is bent or refracted when it enters is a medium. The purpose of this lab is to determine the refractive index of a glass block using a specific law (Snell's Law). A light source (ray box) passing through a slit is directed towards the glass block positioned on a graph paper. Using 4 different methods, the angles are measured with a protractor and the light rays with a ruler. The different refractive indexes are then compared to the expected result. Importantly assumptions, approximations and limitations are considered as they are key factors for this experiment. The results demonstrate that it is possible to find the refractive index with every method. Further recommendations to conduct this experiment are given to improve the accuracy of the refractive index.

2. Introduction:

The aim of this experiment is to determine the refractive index of a glass block using Snell's Law.

Snell's law was founded by Snell in 1621 when he discovered a relation between a beam of light incident on a boundary with glass. The angle of bending (angle of refraction) of the beam depends on the incident angle(angle of incidence) of that beam of light. A light ray moving in a straight line into the glass will not bend, but at an angle, the light ray is bent to a degree proportional to the angle of inclination.

Snell found a characteristic ratio between the angle of incidence and the angle of refraction. Snell's law demonstrates that every substance has a refractive index, a specific bending ratio.

The refractive index of a material is the ratio between the speed of light in the material(vm) and the speed of light in a vacuum (c):

$$\text{refractive index , n}=\frac{c}{Vm}$$

(dimensionless,DN)

$$c \simeq 3 \times 10^{8} ms^{-1}$$

$$Vm \longrightarrow always\ < 3 \times 10^{8} ms^{-1}$$

- The speed of light in the material is never bigger than the speed of light in a vacuum, therefore:

$$n = \frac{c}{Vm} > 1$$

For example, in this experiment the refractive index of air is required in order to find the refractive index of glass.

- The speed of light is slightly slower in the air than in the vacuum ($c \simeq Vm$) hence n_{air} = 1.000277.

- Through this experiment we are assuming that refractive index of air is 1, as it is only a tiny bit larger than 1.

- The refractive index of glass, instead, is about 1.5; that means the light travels significantly slower in glass than it does in a vacuum ($Vm \simeq 2 \times 10^8 ms^{-1}$).

- Therefore, to achieve the experiment's aim, it is expected a result around 1.5.

- In order to find that, Snell's law is needed:
- Snell's law relates the angle of incidence and refraction to the material properties and the speed of wave;

$$\frac{\sin\theta 1}{\sin\theta 2} = \frac{V1}{V2} = \frac{n2}{n1}$$

therefore:

$$n1 \, sin\, \theta 1 = n2 \, sin\, \theta 2$$

(Snell's Law)

n1 (DN) = refractive index of the incident material
n2 (DN) = refractive index of the refracted material
θ1 (°C) = angle of incidence (the angle to the normal)
θ2 (°C) = angle of refraction (it is different to θi)

- As the aim is to find the refractive index of the refracted material, it is possible to rearrange the equation:

$$n2 = \frac{n1 \sin\theta 1}{\sin\theta 2} \longrightarrow n1 \text{ is known } (n1 = 1), \theta 1 \text{ and } \theta 2 \text{ will be measured directly } (1^{st} \text{ M})$$

- Writing down the Snell's law equation, it is noticeable how this is similar to the equation of a straight line:

$$n1 \sin\theta 1 = n2 \sin\theta 2 \xrightarrow{\text{n1 = 1 (assumption)}} \sin\theta 1 = n2 \cdot \sin\theta 2$$

$$y = mx + c \longrightarrow y = m \cdot x$$

$$m = \frac{\Delta y}{\Delta x} \rightarrow n2$$

- A graph with sinθ1 on the y-axis and sinθ2 on the x-axis is drawn in order to find the gradient (m), and it is expected that the graph shows a straight line (in the middle of the points). As the y - intercept is not present in the equation (therefore c=0), it is expected that the line of best fit passes through the origin.

- During the experiment, there could be a phenomenon called total internal reflection (TIR).
  As the angle of incidence is increased, the angle of refraction increases as well, until it hits 90 degrees. The angle of incidence at this point is called the critical angle (θc). If the angle of incidence is greater than the critical angle, no refraction will occur. The light will just be reflected back, leading to the tir. However, θc only exists if n1>n2, therefore in this experiment does not exist and after a certain angle, there will be TIR.
  Even if TIR is experienced during the experiment, there is no issue as the angles can still be measured. In order to execute this experiment, it is only needed the light ray going into the glass (incident ray) and then where this ray goes when it is inside the medium (refracted ray). [refer to figure 1] ;

- Apart from TIR, when it comes to a single slit experiment, it is important to introduce the principle known as diffraction. When light passes through a small opening (e.g., a narrow slit), it bends and it spreads out [3].
  However, in this experiment it is expected that the slits will not cause diffraction, as they must be much narrower. Knowing the wavelength of visible light [2] that is between 380 - 700 nanometres and the slits opening being too wide, light simply travels forward in a straight line.

- During the experiment, there are several factors that need to be considered, as they affect the refractive index of a medium: [5]
  - Speed of light in the medium: higher the speed of light, lower will be the refractive index;
  - Optical density: it differs according to the density of the medium; the denser is the medium, the slower the speed of light will be and therefore the refractive index will be more;
  - Temperature: the refractive index of glass at its standard temperature is 1.5. If the temperature of a medium increases, the speed of light increases as well, decreasing the refractive index;
  - Colour or wavelength of light: the refractive index varies slightly depending on the wavelength: greater the wavelength of light in the given medium, less the refractive index will be. Therefore, knowing the wavelength of red light (650-700nm) and violet light(380nm), the refractive index is expected to be the least for red light and the greatest for violet light. [4]

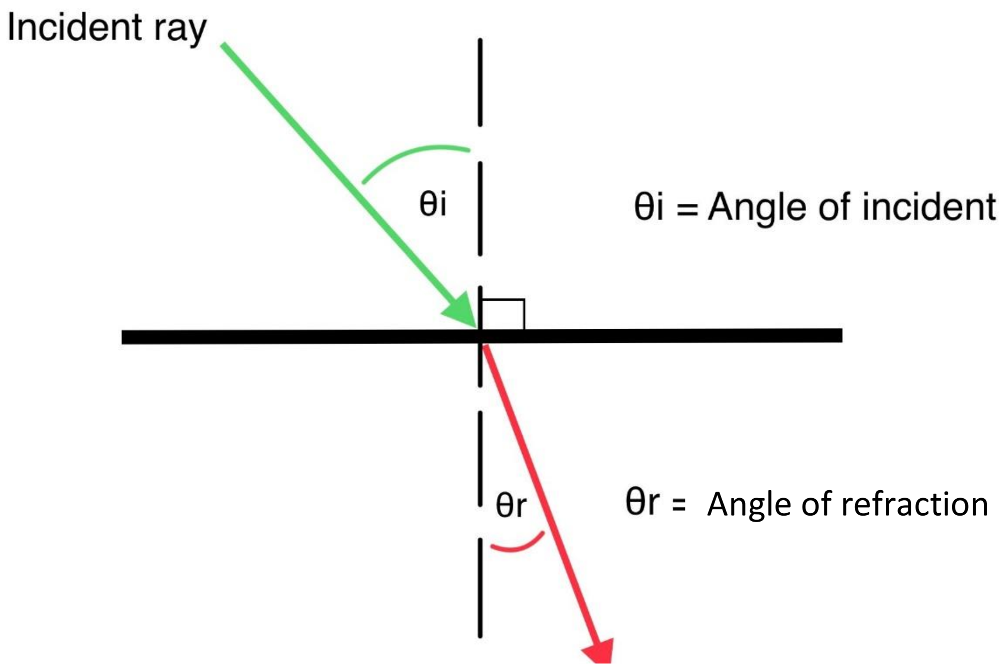


FIGURE 1 (REPRESENTATION OF A BEAM OF LIGHT INCIDENT ON A BOUNDARY WITH GLASS)

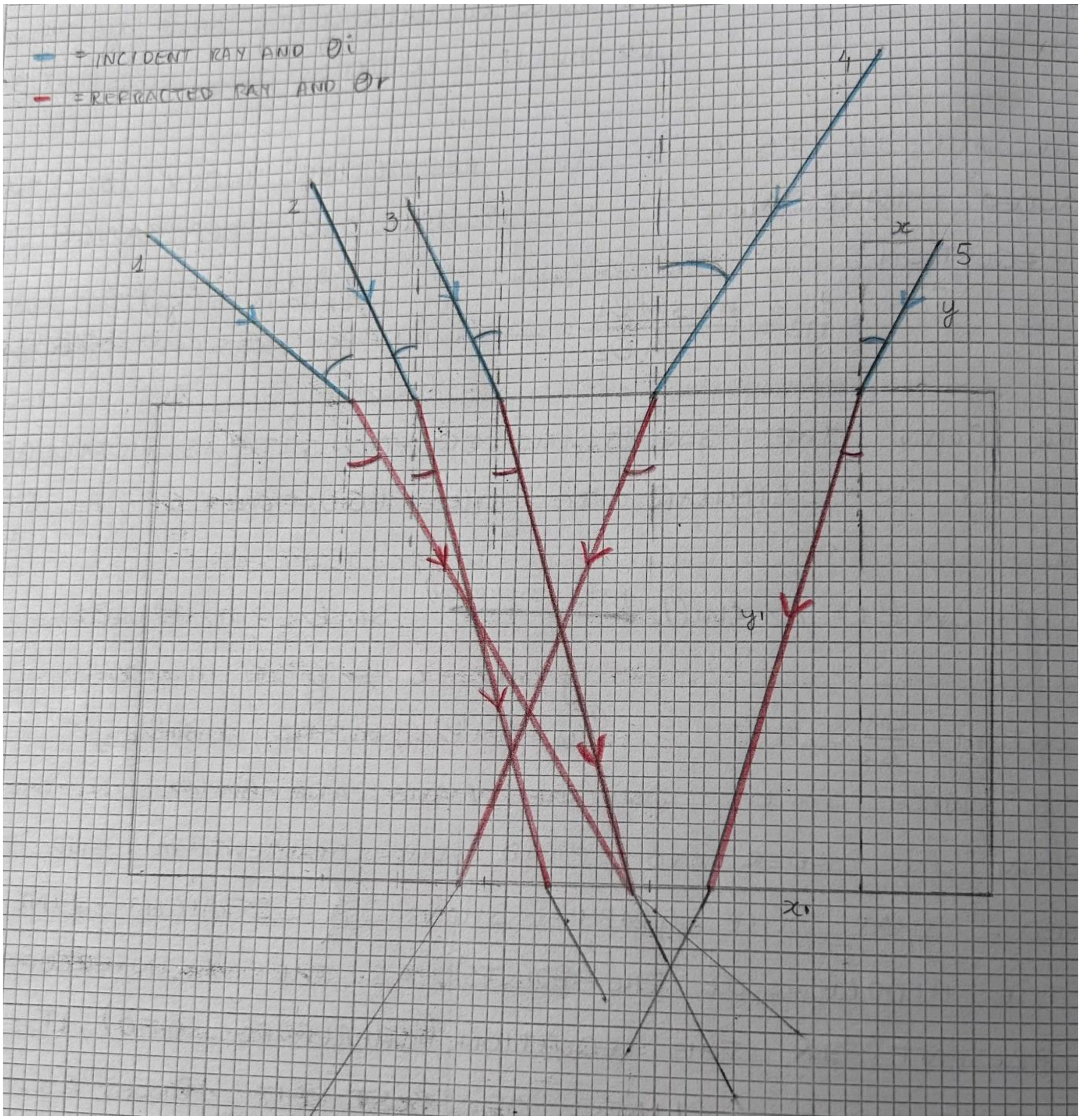
3. Experimental method:

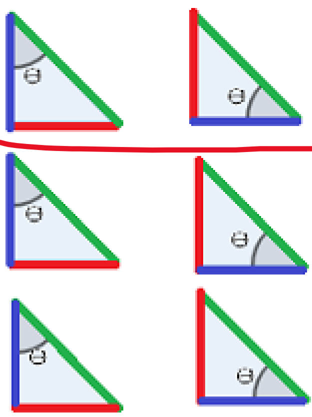
Equipment:

- Glass block
- Light source (ray box)
- Ruler
- Protractor
- Graph paper
- Slits

There are actually 4 methods to determine the refractive index (n2). The 1st and 3rd methods are independent from the others, so they can be fully done individually. The 2nd method uses data from the 1st method, and the 4th method relies on the 3rd method. Therefore, the 1st method and 2nd method are slightly different from each other and the same is valid for the 3rd and the 4th method.

Method [refer to figure 2a, 2b]:

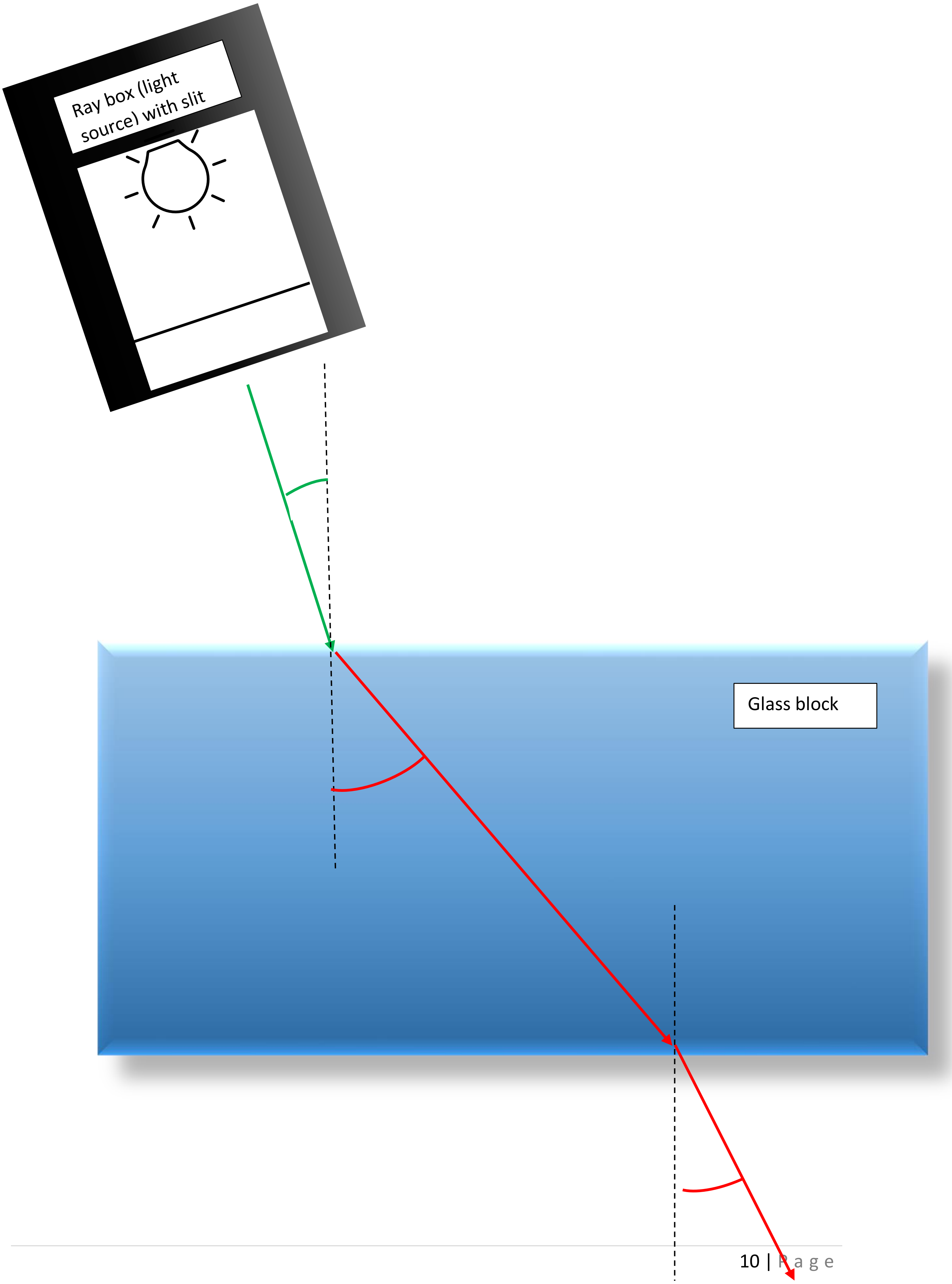




SOH $Sin\Theta = \frac{Opposite}{Hypotenuse}$

CAH $Cosine\Theta = \frac{Adjacent}{Hypotenuse}$

TOA $Tangent\Theta = \frac{Opposite}{Adjecent}$


Ray box (light source) with slit
Glass block

- The glass block was placed on a sheet of paper, and it was drown round it;
- One of the slits was placed into the ray box, checking that the ray box was positioned at a relatable distance (a distance where you can best see the beam of light through the experiment);
- A light ray was shone into the block and it was marked where it starts, enters, leaves, and finishes;
- These 4 points of each different light ray were then connected using a ruler to form the incident ray, the angle of incidence, the refracted ray, and the angle of refraction;
- Two lines to the normal were drawn (at the point where the light ray enters and leaves the block) in order to measure the angles as accurate as possible;
- Table 1 was drawn (with headings θi (°C), θr (°C), sinθi, sinθr, n2 ($\frac{\sin\theta i}{\sin\theta r}$));

➢ 1st method (1st M):

- In each light ray θ1(θi) and θ2(θr) were measured directly with the protractor and recorded on the table;
- Sinθi and sinθr were then calculated;
- As stated in the background, n2 was then calculated using Snell's law equation rearranged:

$$n2 = \frac{n1\,\mathrm{Sin}\,\theta 1}{\mathrm{Sin}\,\theta 2} \rightarrow 1^{st}\ \text{method to calculate n2}$$

➢ 2nd method (2nd M):

- Using sinθi and sinθr from the table 1, a scatter graph of sinθi – sinθr was plotted and the results were marked [figure…] ;
- Once the line of best fit was drawn, it was possible to find m, therefore the refractive index of the glass block (n2).

➢ 3rd method (3rd M) [refer to figure … of sohcahto explaining, and figure where light ray 5 is present]:

- During the experiment, it was noticeable that the normal line and the light ray formed a right-angle triangle, therefore it was possible to use the three main trigonometric ratios (SOHCAHTOA);

- Table 2 was drawn (with headings OPP$_{(x;\, x1)}$, HYP$_{(y;\, y1)}$, sinθi ($\frac{x}{y}$) , sinθr ($\frac{x_1}{y_1}$) , n2 ($\frac{\sin \theta i}{\sin \theta 2}$) , error in length) ;

- First measure x, x_1, y, y_1;

- Once sinθi ($\frac{OPP(x)}{HYP(y)}$) and sinθr ($\frac{OPP(x_1)}{HYP(y_1)}$) were calculated, n2 was determined using the Snell's law rearranged:

$$n2 = \frac{n1 \operatorname{Sin} \theta 1}{\operatorname{Sin} \theta 2}$$

➢ 4th method (4th M):

- Using sinθi ($\frac{x}{y}$) and sinθr ($\frac{x_1}{y_1}$) from the table 2, a scatter chart of sinθi – sinθr was plotted and the results were marked [figure …];

- Once the line of best fit was drawn, it was possible to find m, therefore the refractive index of the glass block (n2).

✓ The four methods were then compared to see which one was more accurate according to the expected result (n2 = 1.5).

4. Results & calculations

θi (°C)	θr (°C)	sinθi	sinθr	n2 ($sin\theta i/sin\theta r$)
51	31.5	0.777146	0.522499	1.487364778
27	17.5	0.45399	0.300706	1.50974973
26.5	17	0.446198	0.292372	1.526131995
32	20	0.529919	0.34202	1.549380277
22	16	0.374607	0.275637	1.359055968
26	16	0.438371	0.275637	1.590390384
33	21	0.544639	0.358368	1.519776185
27	17	0.45399	0.292372	1.55278535
53	31	0.798635	0.515038	1.550633157
25	16	0.42661	0.275637	1.547722001
36	22	0.587785	0.374607	1.569072756
32.590909	**20.454545**	**0.530172**	**0.347717**	**1.523823871**

TABLE 1 (TABLE OF 1ST & 2ND METHOD WITH OUTLIER)

θi (°C)	θr (°C)	sinθi	sinθr	n2 ($sin\theta i/sin\theta r$)
51	31.5	0.777146	0.522499	1.487364778
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53	31	0.798635	0.515038	1.550633157
25	16	0.42661	0.275637	1.547722001
36	22	0.587785	0.374607	1.569072756
33.65	**20.9**	**0.545728**	**0.354926**	**1.540300661**

TABLE 2 (TABLE OF 1ST AND 2ND METHOD WITHOUT OUTLIER)

OPP(x)	OPP(x1)	HYP(y)	HYP(y1)	sinθi (x/y)	Uncertainty in y-axis (±)	sinθr (x1/y1)	n2 ($sin\theta i/sin\theta r$)
3.15	4.1	4.05	7.7	0.777777778	0.0938	0.532467532	1.460704607
1.6	2	3.6	6.9	0.444444444	0.0912	0.289855072	1.533333333
1.4	1.95	3.15	6.9	0.444444444	0.1042	0.282608696	1.572649573
2.95	2.6	5.65	7.1	0.522123894	0.06	0.366197183	1.425799864
1	1.95	2.3	6.85	0.434782609	0.14223	0.284671533	1.527313266
2.6	2.7	4.4	7.1	0.590909091	0.0792	0.38028169	1.553872054
1.2	1.9	2.9	6.8	0.413793103	0.112	0.279411765	1.480943739
1.85	2	4.2	6.85	0.44047619	0.078	0.291970803	1.508630952
3.75	4	4.7	7.65	0.79787234	0.0816	0.522875817	1.525930851
3	2.5	5.45	7	0.550458716	0.0628	0.357142857	1.541284404
1.55	2	3.4	6.85	0.455882353	0.097	0.291970803	1.561397059
2.186363636	2.518181818	3.981818182	7.063636364	0.533905906	0.091093636	0.352677614	1.517441791

TABLE 3 (TABLE OF 3RD AND 4TH METHOD)

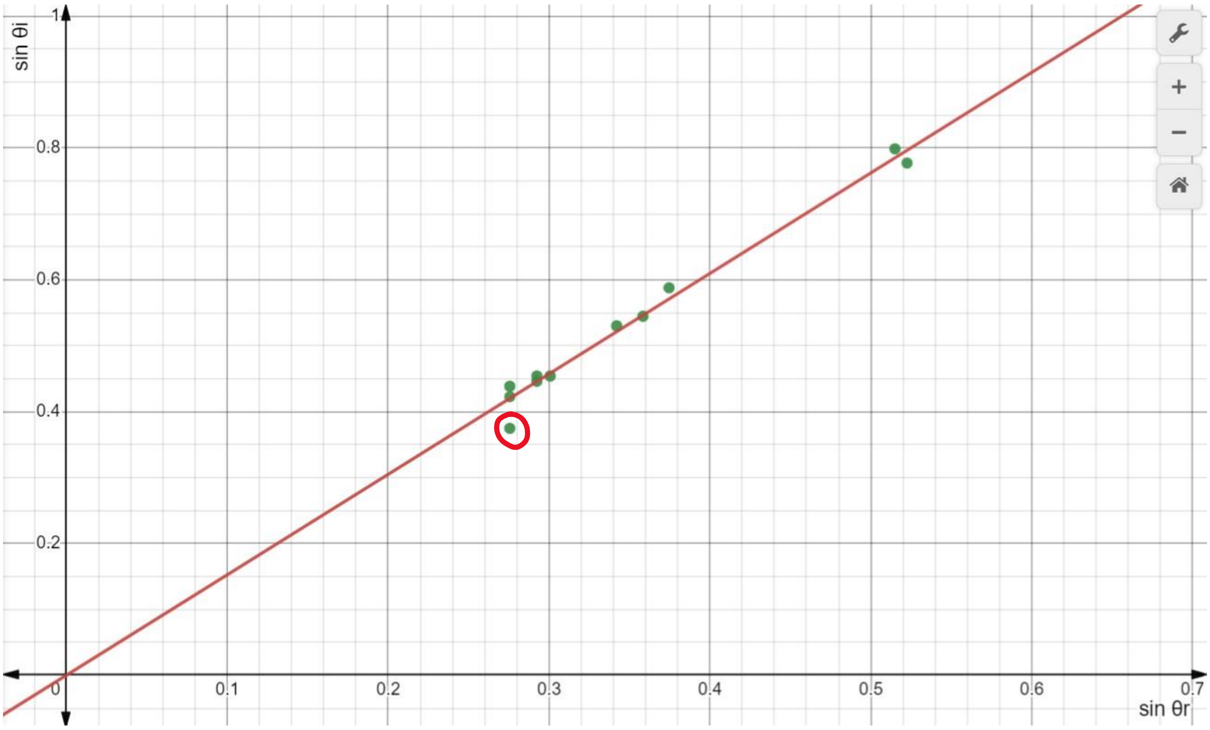

FIGURE 2 (GRAPH OF 2ND METHOD WITH OUTLIER)

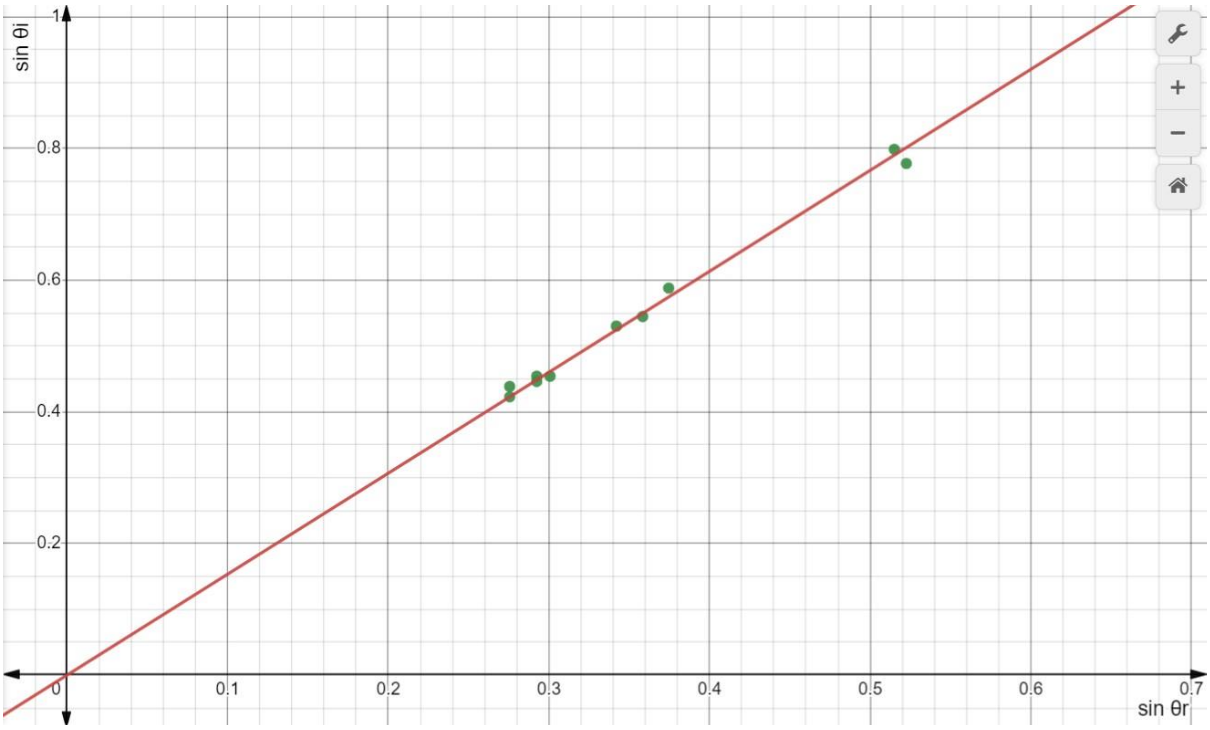

FIGURE 3(GRAPH OF 2ND METHOD WITHOUT OUTLIER)

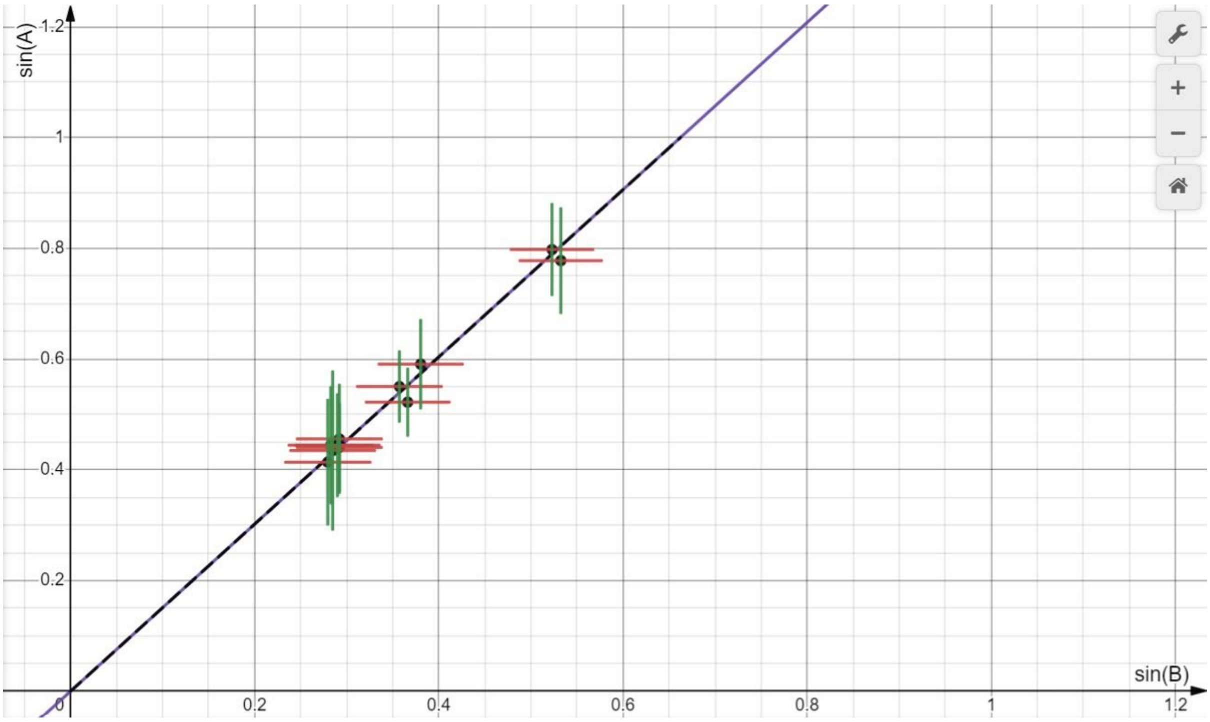

FIGURE 4(GRAPH OF 4TH METHOD)

- 1st method:

- 1st reading of θi (°C) with protractor [refer to Table 1]:

θi = 51°C → sinθi ≃ 0.7771

θr = 31.5°C → sinθr ≃ 0.5225

$$n2 = \frac{n1 \cdot sin\theta i}{sin\theta r} = \frac{sin\theta i}{sin\theta r} = \frac{\sin(51)}{\sin(31.5)} \simeq 1.49$$

- The rest of sinθi, sinθr and n2 were found following the same procedure and using the same calculation.

- In order to verify the accuracy of the 1st method, an average value of n2 is calculated:

$$n2_{\ average} = \frac{n2(r1 + r2 + r3 + r4 + r5 + r6 + r7 + r8 + r9 + r10 + r11)}{11} \simeq 1.52$$

- 2nd method:

- Now that the line of best fit is drawn, it is possible to find m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y2 - y1}{x2 - x1} =$$

y = sinθi
x = sinθr
m = n2 (1.52)
c = 0 (passes through origin)

- As stated earlier:

$$1 \cdot sin\theta i = n2 \ \cdot sin\theta 2 \rightarrow y = mx + c$$

$$1 \cdot sin\theta i = 1.52 \cdot sin\theta r$$

After plotting the graph, it was verified if the value circled in figure 2 could be considered an outlier.

Using desmos values, these are the values of the gradient (refractive index):

Graph with outlier: m1 = 1.52428
Graph without outlier: m2 = 1.53365

Difference in m = m2 – m1 = 0.00937

Moreover, the average of the n2 values with outlier is more accurate than the the average without the outlier (Table 1, table 2).

$$n2_{\substack{avg.without \\ outlier}} = \frac{n2(r1 + r2 + r3 + r4 + r5 + r6 + r7 + r8 + r9 + r10 + r11)}{11} \simeq 1.54$$

Therefore after verifying, the value circled is not considered an outlier and for the 2nd method we will take the average of table 1 (≃1.52).

An outlier is something that is really significantly far away. In this case, its probably just writing down the result wrong.

In both methods an error estimate of ±2 degrees is considered as the points and lines are not marked perfectly. More details can be found in the discussion.

➢ 3rd method:

5th light ray calculations [refer to Table 2 & figure where light ray 5 is present] :

- x, x_1 , y, y_1 are labelled on light ray 5, as it can be easier for the reader to understand the measurements.

x = 1.00cm ± 0.3 ; y = 2.30cm ± 0.3 → $sin\theta i = \frac{x}{y} = \frac{1.00}{2.30} = \frac{10}{23}$

x_1 = 1.95cm ± 0.3 ; y_1 = 6.85cm ± 0.3 → $sin\theta r = \frac{x_1}{y_1} = \frac{1.95}{6.85} = \frac{39}{137}$

therefore:

$$n2 = \frac{sin\theta i}{\sin\theta r} = \frac{\frac{10}{23}}{\frac{39}{137}} \simeq 1.53$$

- The rest of sinθi, sinθr and n2 were found following the same procedure and using the same calculation.
- The error is ±0.3cm as we are not measuring and drawing exactly the light rays. It is better to oversize rather than undersize.
  More details about the errors are found in the discussion.

  The average of n2 in the 3rd method is more accurate than the 3rd and the 4th method, as it is 1.517.

➢ 4th method:

Plotting the skatter graph it is found that m = 1.50985.

$$\%\ error = \frac{|expected - calculated|}{expected} \times 100$$

% error 2nd method = 1.3% (n2 = 1.52)

% error 3rd method = 1.13%

% error 4th method = 0.656 %

5. Discussion

As it was stated initially, the expected result must have been around 1.5 in order to achieve the aim.

Every method gave very precise and accurate results, as the % errors were 1.3%, 1.13%, 0.656%

In the 1st method 2 average values were used (one with outlier, one without). In the 2nd method while plotting the line of best fit, it was noticeable that all the points were accurate and precise.

However , both methods can be used to determine n2, but in this case the last method gave a more accurate result .

As mentioned in the background, an important assumption we made the refractive index of air is not exactly 1, but more precisely 1.000277. Therefore, through the experiment we assumed that n.air was 1 to simplify the calculation, this tiny difference should also be considered in our experiment. It is very accurate but still is an approximation.

In order to increase the reliability and precision of the results, use the equipment properly ensuring that the hands and equipment are still while taking a measurement.

In this case, there are some considerations regarding the protractor, the ruler and glass block. It is unlikely that during the experiment this equipment were still and the 4 points of the light ray (where it starts, enters, leaves, and finishes) were marked perfectly.

In general, the angle of incidence is more likely to be accurate as the light ray is brighter and there is no medium in between. In spite of that, there are some considerations to make.

Although we tried to draw precisely with the ruler, it is not completely exact. The same applies when measuring the angles by using the protractor and marking the 4 points of the light ray. In both steps is a matter of judgement and that makes the results of θi and θr less accurate.

In order to be more accurate and save time, it is possible to draw the incident rays before the experiment with definite angles, preferably larger angles as mentioned earlier (e.g., 50,55.60,65). Doing that, there is no need to point where the light ray starts and enters the block. The light ray has to be positioned over the incident ray drawn, and then only two centres have to be marked (of where the light ray leaves and finishes). It is possible that with this method the angle of incidence is most likely to be more accurate than the angle of refraction, as the incident ray is drawn precisely.

However, even if only two centres of the light are meant to be marked, it will be still tough to point them correctly. That's because the exit ray is broad and less bright, especially because it passes through the glass block. It would be better to conduct the experiment in a darker room to make the marking more precisely, improving the results for the angles (especially θr) and consequently n2 more accurate.

Another way would be using a more intense light of source, resulting in a brighter light of ray, particularly the exit ray.

Therefore, there a lot of tiny errors accumulating during the experiment that must be considered. We can be more careful, still, and precise following the recommendations given.

There are other ways this experiment could be conducted:

1. Instead of using the lab book entirely for a paper, it is possible to use a single graph paper ensuring that the underground surface is flat. Doing that the lines are drawn more precisely as the ray box is on a uniform flat surface.
2. An equipment that could make a difference in this case would be a vernier calliper, preferably a digital one so there will not be any reading errors. These devices are designed to be significantly accurate to measure lengths and are a great alternative to rulers.

For measuring the angles instead, there is not much choice as protractor is one of the most accurate equipment if used properly.

As stated in the background, there are some factors that affect the refractive index, for example the colour or the wavelength of light.

A step further of this lab would be finding out if the refractive index is noticeably different using different colours of light (with the help of filters). The extreme would be using a red filter and a violet filter because that is the biggest difference in the wavelength and frequency. Using the same procedure of this experiment, it is possible to calculate the refractive index of a red light and then of a violet light. The final step would be observing if there is any noticeable difference between the two refractive indexes. It is expected a small difference, but the experiment would be useful to see if it is accurate enough to find out that difference.

6. Conclusion:

The precision of the graph is reliable (the best fit line) and the refractive index is nearer to the expected result.

Therefore the 4th method of calculation is better, and the aim of the experiment is achieved.

7. References

[1] https://micro.magnet.fsu.edu/optics/timeline/people/snell.html#:~:text=Snell%20discovered%20that%20a%20beam,to%20the%20angle%20of%20inclination.

[2] https://science.nasa.gov/ems/09_visiblelight#:~:text=The%20visible%20light%20spectrum%20is,from%20380%20to%20700%20nanometers.

[3] https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/diffraction/#:~:text=The%20amount%20of%20diffraction%20depends,is%20blue%20and%20violet%20light).

[4] https://www.britannica.com/science/color/The-visible-spectrum

[5] https://www.utsc.utoronto.ca/webapps/chemistryonline/production/refractive.php