* Bit manipulation
Operators
① AND

a	b	a & b
0	0	0
0	1	0
1	0	0
1	1	1

* When you &1 with any number, digits remains the same

```
  1 1 0 0 1 0 1 0 0
& 1 1 1 1 1 1 1 1 1
-------------------
  1 1 0 0 1 0 1 0 0
```

② OR

a	b	a OR b
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0	1	1
1	0	1
1	1	1

③ XOR (^) (if and only if)
exclusive OR

a	b	a ^ b
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0	1	1
1	0	1
1	1	0

observations

$a \wedge 1 = \bar{a}$

$a \wedge 0 = a$

$a \wedge a = 0$

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(4) complement (~)

a = 10110

$\bar{a}$ = 01001

* Number systems

(1) Decimal ⟶ 0, 1, 2, ..., 9 Base 10

$(357)_{10}$, $(10)_{10}$

(2) Binary ⟶ 0 & 1 Base 2

$(10)_{10} = (1010)_2$

$(7)_{10} = (111)_2$

(3) Octal ⟶ 0, 1, 2, 3, ... 7 Base 8

Decimal: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Octal: 0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 20

$(9)_{10} = (11)_8$

(4) Hexadecimal :-

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Base : 16

$(10)_{10} = (A)_{16}$ $(12)_{10} = (C)_{16}$

Conversation

① Decimal to base b

Q Convert $(17)_{10}$ to base 2

Keep dividing by base, take remainder, write in opposite

$$\begin{array}{r|l l} 2 & 17 & \\ \hline 2 & 8 & 1 \\ \hline 2 & 4 & 0 \\ \hline 2 & 2 & 0 \\ \hline & 1 & 0 \end{array}$$

$(10001)_2 = (17)_{10}$

$(17)_{10} = (\overset{2}{\cancel{8}})_8$

21

$$\begin{array}{r|l l} 8 & 17 & \\ \hline & 2 & 1 \end{array}$$

② Convert any base b to decimal

$(10001)_2 = (\quad)_{10}$?

Steps :- multiply & add the power of base with digits

$= 1 \times 2^{\cancel{5}4} + 0 \times 2^{\cancel{4}3} + 0 \times 2^{\cancel{3}2} + 0 \times 2^{\cancel{2}1} + 1 \times 2^0$

$= 16 + 1$

$= 17$

9 $(21)_8 = (\ )_{10}$

$= 2 * 8^1 + 1 * 8^0$

$= 2 \times 8 + 1$

$= (17)_{10}$

* continuing with operators

⑤ left shift operator (<<)

$(10)_{10} = (1010)_2$ 10 << 1

step : 1010 << 1 = 10100

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2$

$= 16 + 4$

$= 20$

$\boxed{\therefore a << 1 = 2a}$

General point :

$\boxed{a << b = a \times 2^b}$

⑥ Right shift >>

0011001 >> 1 $\Rightarrow$ 001100 $\Rightarrow$ (1100)

$(00011234)_{16} = (11234)_{10}$

(Ignored) $\longrightarrow$ same for all number systems

Genral point: $\boxed{a >> b = \frac{a}{2^b}}$

Questions:-

① Given a no. n find if it is odd or even.
Point:- Every no is calculated in binary form internally

$12 + 7 \Rightarrow$

$$\begin{array}{r} 1100 \\ +\,0111 \\ \hline 10011 \end{array}$$

$(19)_{10} = (10011)_2$

$16 + 2 + 1$

Note:- $\boxed{1001}$ 1 $\rightarrow$ leaving this every other is a power of 2

$\boxed{1001}$ $\rightarrow$ This will always be even.

1 $\rightarrow 2^0$

Hence if 2^0 place $== 1 \Rightarrow$ odd
otherwise $\Rightarrow$ even

code

```
static boolean isOdd(int n) {
        return (n & 1) == 1;
}
```

② arr = [2, 3, 4, 1, 2, 1, 3, 6, 4]

Ans

Find unique element?
ans: XOR all the nos.

Code

```
static int ans(int[] arr){
    int unique = 0;

    for(int n : arr){
        unique ^= n;
    }
    return unique
}
```

Q Find i^{th} bit of a no.

8	7	6	5	4	3	2	1
1	0	1	1	0	1	1	0
			?				

Ans

1 0 1 1 0 1 1 0
0 0 0 1 0 0 0 0
0 0 0 1 0 0 0 0 Ans

This is called a mask

n ⇒ mask with n-1 zero → 1<<(n-1)

1<<4 ⇒ 10000

Ans: n & (1<<(n-1))

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Q Set the ith bit

turn it to 1 $\longrightarrow$ $0 \longrightarrow 1$

$1 \longrightarrow 1$

4th

```
   1 0 1 0 1 1 1 0
OR 0 0 0 0 1 0 0 0   --> mask
   ---------------
   1 0 1 1 1 1 1 0
```

Q Reset ith bit $1 \longrightarrow 0$

$0 \longrightarrow 0$

```
  1 0 1 0 1 1 1 0
& 1 1 0 1 1 1 1 1   --> How to get this mask?
  ---------------
  1 0 0 0 1 1 1 0   Ans
```

mask : $!(1 << (n-1))$

Q Find the position of the right most set bit

Ex :- $\underbrace{101101}_{a}\,1\,\underbrace{00}_{b}$ Ans = 4

$N = a1b$ $a = 101101$

$b = 00$

$-N = \bar{a}1b$ How?

Ans = N & (-N)

* Negative of a number in Binary form :-

1 byte = 8 bits

MSB LSB

10 => $\boxed{0|0|0|0|1|0|1|0}$ → value of no.

→ Tells us if no is +ve or −ve

−10 = ?

1 → −ve
0 → +ve

steps :- ① Compliment of no. ② +1 to it] => aka 2's compliment method

$(10)_{10} = (00001010)_2$

① 11110101

② 11110101
\+ 1
$(11110110)_2$ => $(-10)_{10}$

Why tho?

This will be stored

10 $\boxed{10111011\,1}$

discarded

* When we subtract any number by 0 then we will get −ve of that number

10 in binary 00001010

$$\begin{array}{r} 100000000 \\ -\ 00001010 \\ \hline \end{array}$$

What is this

$10000000 = 1111111 + 1$

Now: $1111111 + 1 - 00001010$

$\Rightarrow \underbrace{11111111 - 00001010}_{\text{complement}} \underbrace{+1}_{\text{step 2}}$

step 1

$$\begin{array}{r} 11111111 \\ -\ 00001010 \\ \hline 11110101 \end{array}$$ complement

$1000 = 111 + 1$
$8 = 7 + 1$

$10000 = 1111 + 1$
$16 = 15 + 1$

$$\begin{array}{r} 1111 \\ +\ 0001 \\ \hline 10000 \end{array}$$

* Range of numbers :

① 1 byte :

0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1

128

sign of number

Total $= 2 \times 2 \times 2 \times 2 \ldots$ 8 times
$= 2^8 = 256$

actual no is stored in bits $= n-1$
total bits

In 1 byte : 7 bits

Total I can make from 7 bits
= $2^7 = 128$

-128 to 127
↳ 0 is here as well

* -0 is 0
proof 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 compliment

Adding 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1
\+ 1
1 $\underbrace{0\,0\,0\,0\,0\,0\,0\,0}$ ~~discarde~~ ~~discarded~~
9-bit

discarded

* Range formula: for n bits

$$\boxed{-2^{n-1} \text{ to } 2^{n-1}-1}$$

Q. arr = [2,2,3,2,7,7,8,7,8,8]

					1	0

1 0
1 0
1 1
1 0
1 1 1
1 1 1
1 0 0 0
1 1 1
1 ← 0 1 0 0 0

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| 3 | 3 | 7 | 4 | % 3

0 0 1 1 = 3

Q Find the n^{th} magic no.

	5^3	5^2	5^1		magic no
1 =	0	0	1	⟶	5
2 =	0	1	0	⟶	25
3 =	0	1	1	⟶	30 (5+25)
4 =	1	0	0	⟶	125
5 =	1	0	1	⟶	130

⋮

n n = 6 110

loop [n & 1 ⟹ This will give me last digit in binary
n >> 1]

$0 \times 5^1 + 1 \times 5^2 + 1 \times 5^3$

Q Find no of digit in base 6

$(6)_{10} \Rightarrow 1$

$(6)_{10} \Rightarrow (110)_2 = 3$ (By Keeping counter)

Formula :- No of digits in base b of no. x

$= \text{int}(\log_b n) + 1$

$$\boxed{\log_b a = \frac{\log_x a}{\log_x b}}$$

$\log_b a = x$

$a = b^x$

$\log_2 6 = x$

$6 = 2^x$

$\log_2 10 = 3.32$

$10 = 2^{3.32}$

int + 1

= no of digits

1 1

1 2 1

1 3 3 1

1 4 6 4 1

* Pascal's Triangle

∴ Find the sum of n^{th} row

Sum of each row = ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$

$= 2^n$

For n^{th} row, sum $= 2^{n-1}$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Ans : $1 << (n-1) = 1 * 2^{n-1}$

Ans

Q You are given a number. Find out if its power of 2 or not.

100000, 100010

10000000 = 1111111 + 1

if its power of 2

10000000	10010
& 01111111	& 01111
0	00010

Ans :- If $n \& (n-1) = 0$ // It is power of 2

Q. Calculate a^b

// $O(b)$

$3^6 \Rightarrow 3 * 3 * 3 * 3 * 3 * 3$

$3^6 = 3^{110} = 3^{2+4} = 3^2 * 3^4$

ans = 1 base = 3 base = 9 base = base * base	n = 110 n&1 ⇒ 0 n >> 1 ⇒ 11 & 1 ⇒ 1 n = 11 >> 1 ⇒ 1 & 1 = 1 n = 1 >> 1 = 0	ans = ans * base

$O(\log(b))$

$3^{110} \Rightarrow 3 * 1^4 * 3 * 1^2 * 3 * 0^0$

Imp to understand this question

Q Given a number n, find the no of set bits in it.

ans $n = 9$

$n = 1001$ Ans = 2

$n \& (-n) = 0001$

$n - [n \& (-n)] = 1000 \Rightarrow$ ①

$n = 1001$
$\& \ 1000$
1000 — ①

$8 \& 7 \Rightarrow 1000$
$\& \ 111$
0 ②

No of set bits = no of iterations

Q Find XOR of nos from 0 to a

a	XOR from 0 to a		
0	0	6	7
1	0^1 = 1	7	0
2	0^1^2 = 3	8	8
3	0	9	10
4	4		
5	1		
6	7		

1 $a\,n\,! = 0$

- order →
- as
- d

0 → a

If $a \% 4 = 0$	a
$a \% 4 = 1$	1
$a \% 4 = 2$	$a+1$
$a \% 4 = 3$	0

Q XOR of all nos between a & b

$a = 3$ & $b = 9$

$3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9$

$0 \wedge 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \wedge 9$

These are the extras

This is $0 \to (a-1)$

Ans $f(b) \wedge f(a-1)$

$f(n) \to$ XOR of $0 \to x$

up = row

down = ~~2n-1~~ N - row

right = N - col

left = col

	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	1
3	1	2	3	2	1
4	1	2	2	2	1
5	1	1	1	1	1