Teacher's Sign / Remarks

DSA Continue

- Prime Nos

2, 3, 5, 7, 13, - - - .

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

```
for (i = 2; i < N; i++) {
        if (N % i) {
                Not prime
        }
}
prime
```

Another example :- 36

1 x 36
2 x 18
3 x 12
4 x 9
6 x 6

Repeated Hence ignore

9 x 4
12 x 3
18 x 2
36 x 1

3 x 12
12 x 3

Hence, only make checks for numbers ≤ sqrt(n)

Q $N = 40$

2, 3, 5, 7, 10, 13, 17, 19, 25, 29, 31, 37

Time complexity:-

$$\frac{n}{2} + \frac{n}{3} + \frac{n}{5} + \frac{n}{7} + \ldots$$

$$\underbrace{n\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots\right)}_{}$$

Harmonni progression for primes

$\parallel$

$\log(\log N)$

Total time complexity $= O(N * \log(\log N))$

Q Find square root of a number

$$\underbrace{0 \qquad 18}_{} \qquad 36$$

```
if(m * m > n)
    e = m-1
else
    s = m+1
```

31, 37

sqrt(40) = 6 . 32 → ?

above way

root = 6.1
= 6.2
= 6.3 Ans
= 6.4

same thing for 0.01

* Newton Raphson method

$$root = \frac{\left(x + \frac{N}{x}\right)}{2}$$

error = |root - x|

actual sq root $= \sqrt{N}$

sqrt you have assumed

1. Assign x to N
2. You will find your ans when error < 1
3. Update the value of x = root

* Factor of a number

n = 20 ⟹ 1, 2, 4, 5, 10, 20

20 % 1	20 * 1 = 20
20 % 2	10 * 2 = 20
20 % 4	5 * 4 = 20
20 % 5	4 * 5 = 20
20 % 10 Repeated	2 * 10 = 20

Properties of modulo (%)

* $(a+b) \% m = ((a \% m) + (b \% m)) \% m$
* $(a-b) \% m = ((a \% m) - (b \% m) + m) \% m$
* $(a * b) \% m = ((a \% m) * (b \% m)) \% m$
* $\left(\frac{a}{b}\right) \% m = ((a \% m) * (b^{-1} \% m)) \% m$

$b^{-1} \% m \Rightarrow$ multiplicative modulo inverse (MMI)

Ex :- $(6 * y) \% 7 = 1$

$y =$ MMI for 6 & $y = 6$
$(6 * 6) \% 7 = 36 \% 7 = 1$

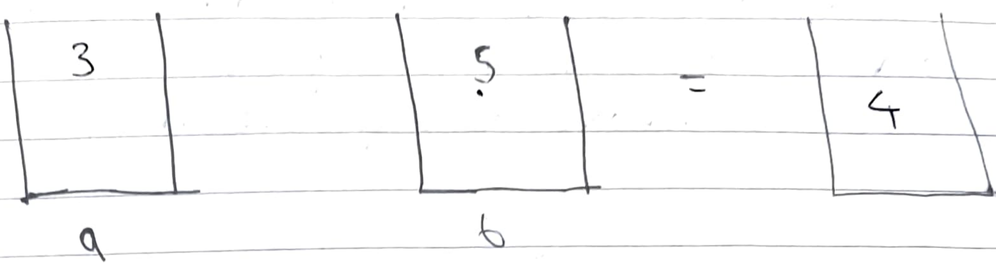
MMI $= b^{-1} \% m$ means that b & m & co-primes

* $(a \% m) \% m = a \% m$
* $m \% m = 0 \quad \forall x \in +ve$ Integers

Extra :- if p is prime no. which is not a divisor of b, then $a b^{p-1} \% p = a \% p$ due to fermets little theorem

How? will be covered in advance DS course.

* Die-hard Example:



a b

1st → (0,0) → (3,0) → (0,3)

2nd → (0,3) → (3,3) → (1,5)

↓

(0,1) ← (1,0)

3rd → (0,1) → (3,1) → (0,4)

↓

Ans

$r = ax + by$

$3x + 5y = 4$?

Put x & y as integer, what is the minimum +ve value you can have of equation

$x = -3, y = 2$

$3x + 5y = 1$ → minimum +ve value that I can form

☆ This is called HCF :

HCF/of a & b = min +ve value
GCD of eqⁿ (ax + yb)
where x & y are ints

HCF (4, 18) = 2

1, 2, 4 1, 2, 3, 6, 9, 18

HCF(3, 9) = 3

1, 3 1, 3, 9

min(3x + 9y) = 3

$3x + 9y$

$3(x + 3y)$

$= 3(-2 + 3) = 3$ ✓

a, b

$ax + by = L$

$2x + 4y = 5$

$2(x + 2y) = 5$

$x + 2y = 2.5$ ✗

Note :
whatever HCF you will get, that will come out as common

$3x + 6y = 9$

$3(x + 2y) = 9$

$x + 2y = 3$ ✓

$3x + 5y = 17$

$1(3x + 5y) = 17$

✯ Euclid's Algorithm :

$$\gcd(a,b) = \gcd(\text{rem}(b,a), a)$$

$$\gcd(105, 224) = \gcd(\text{rem}(224, 105), 105)$$
$$= \gcd(14, 105)$$

$$105x + 224y$$

why?

why subtract

$$14x + 105y$$

i.e becomes the gcd of (105, 224) also divides a linear combination of 105 & 214

Ex : $224 - 2 * 105 = 14$ (rem)

✯ LCM :-

lcm(a,b) = min no divisible by both a & b

$$\text{lcm}(2, 4) = 4$$
$$(3, 7) = 21$$

Note :-

say we have a, b

$$d = \gcd(a, b)$$

$$f = \frac{a}{d}, \quad g = \frac{b}{d}$$

$$\Rightarrow a = fd, \quad b = gd.$$

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LCM = c ☆ lcm (a, b) = lcm (fd, gd)

☆ We know that f & g will have no other common factor

$a = 9$, $b = 18$ $d = 9$
$f = 1$, $g = 2$

☆ $a = f d$ $b = g d$

$lcm = f * g * d$ => This is how above conditions are satisfied.

more info :- $= a * b$
$= fd * g * d$ -> d is repeating hence remove

$lcm = f * g * d$

☆ $a * b = fd * gd$
$= d * d f g$
$= hcf * lcm$

Formula

$$\boxed{LCM(a, b) = \frac{a \times b}{HCF(a, b)}}$$