Search Trees

Data and File Structures Laboratory

http://www.isical.ac.in/~dfslab/2018/index.html

Binary search trees

Definition

Binary tree in which following property holds for all nodes:

- *key values in left subtree are less than key value in the node*
- *key values in right subtree are greater than key value in the node*

Main operations

- **Insertion**
- **Search**
- **Deletion**

Auxiliary operations

- **Find successor**
- **Find predecessor**

Typedefs and helper function (bst.h)

```
typedef int DATA;

typedef struct node {
    DATA data;
    struct node *left, *right;
} NODE;

extern int compare(NODE *n, DATA d);
extern void inorder(NODE *root);
extern void print_tree(NODE *root, int indent);
extern void print_pstree(NODE *root);
extern NODE *search(NODE *root, DATA d);
extern NODE *detach_successor(NODE *node);
```

BST Insertion I

```
/**
 * Arguments: pointer to root, data
 * Returns: possibly modified pointer to root
 * If "root" is NULL (empty tree), it will be changed to point to
 *     newly inserted node.
 * This (possibly changed) value of root is returned.
 * Caller is responsible for updating to the new, returned value (see
 * recursive calls below, for example).
 */
NODE *insert(NODE *root, DATA d) {
    /* Base case */
    if (root == NULL) {
        root = Malloc(1, NODE); /* should check return value */
        root->data = d;
        root->left = root->right = NULL;
        return root;
    }
```

BST Insertion I (contd.)

```c
    /* Recurse */
    int cmp = compare(root, d);
    if (cmp < 0)
        root->left = insert(root->left, d);
    else if (cmp > 0)
        root->right = insert(root->right, d);
    return root;
}
```

BST Insertion II

- `insert` and `delete` routines take pointer to the root, and change the root via this pointer as and when necessary.
- Since `root` is changed within these routines (if necessary), both routines return `void`.

BST Insertion II (contd.)

```
void insert(NODE **rootptr, DATA d) {
    NODE *root = *rootptr;
    if (root == NULL) {
        root = Malloc(1, NODE); /* check return value */
        root->data = d;
        root->left = root->right = NULL;
        *rootptr = root;
    }
    int cmp = compare(root, d);
    if (cmp < 0) insert(&(root->left), d);
    else if (cmp > 0) insert(&(root->right), d);
    return;
}
```

BST Searching

```
NODE *search(NODE *root, DATA d) {
    if (root == NULL)
        return NULL;
    int cmp = compare(root, d);
    if (cmp < 0)
        return search(root->left, d);
    else if (cmp > 0)
        return search(root->right, d);
    else
        return root;
}
```

BST Deletion

Let X be the node to be deleted.

Case I X is a leaf node.
Simply delete X.

Case II X has one child.
Replace the link to X with a link to its only child.

Case III X has 2 children.

1. Find S, the successor of X (node with smallest key in right subtree of X).
2. Replace the value in X by the value in S.
3. Delete node S from the tree (see Cases I and II above).

May also use X's predecessor, the largest key in left subtree of X in a similar fasion.

BST Deletion

Helper function

```
NODE *detach_successor(NODE *node) {
    NODE *nptr;
    assert(node != NULL);
    /* Go to right child, then as far left as possible */
    nptr = node->right;
    if (nptr == NULL) /* no successors */
        return NULL;
    if (nptr->left == NULL) {
        node->right = nptr->right;
        return nptr;
    }
    while (nptr->left != NULL) {
        node = nptr;
        nptr = nptr->left;
    }
    node->left = nptr->right;
    return nptr;
}
```

BST Deletion II I

```
void delete(NODE **nodeptr, DATA d) {
    NODE *node, *s;

    assert(nodeptr != NULL);
    node = *nodeptr;
    if (node == NULL) return;

    int cmp = compare(node, d);
    if (cmp < 0) delete(&(node->left), d);
    else if (cmp > 0) delete(&(node->right), d);
    else {
        if (node->left == NULL &&
            node->right == NULL) {
            /* Case I: leaf, just delete */
            *nodeptr = NULL;
            free(node);
            return;
        }
```

BST Deletion II II

```
        /* Case II: only one child */
        if (node->left == NULL) {
            *nodeptr = node->right;
            free(node);
            return;
        }
        if (node->right == NULL) {
            *nodeptr = node->left;
            free(node);
            return;
        }
        /* Case III: both sub-trees present */
        s = detach_successor(node);
        node->data = s->data;
        free(s);
    }
    return;
}
```

BST interface

```
NODE *root = NULL; // root of the tree

/* INSERTION */
for (i = 0; i < num; i++) {
    data = rand() % 100;
    insert(&root, data);
}

...

/* DELETION */
delete(&root, data);
```

Viewing trees

```
$ ./bst1 10 2> bst1.tex
$ latex tree.tex; dvips -o tree.ps tree; ps2pdf tree.ps
$ <view tree.ps using installed document viewer>
```

“Alternative” implementation

```
typedef struct node {
    DATA data;
    int left, right;
} NODE;

typedef struct {
    unsigned int num_nodes, max_nodes;
    int root, free_list;
    NODE *nodelist;
} TREE;

extern void inorder(TREE *, int root);
extern void print_pstree(TREE*, int root);
extern int search(TREE *, int root, DATA d);
extern int detach_successor(TREE *, int node);
```

Balanced BSTs

Time for insertion / search

Data Structure	Worst case	Average case
Ordinary binary search trees	$O(N)$	$O(\lg N)$
Balanced binary search trees	$O(\lg N)$	

More on balanced BSTs next week + after the break.

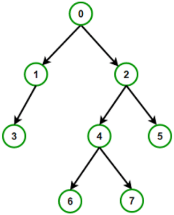
Problems I

1. Complete the “alternative” implementation of binary search trees, a skeleton of which is provided in `bst-alt.c`. You may choose either option I or II discussed above.
2. Write a program that computes the maximum width of a given binary tree. The maximum width of a tree is the maximum number of nodes at any level, where a level correspond to all nodes that are at the same distance from the root.

   `http://www.techiedelight.com/find-maximum-width-given-binary-tree/`

Problems II

3. You are given an array A which represents a binary tree in the following way: the parent of node i is given by $A[i]$. For the root node, the parent is denoted by -1. Construct the conventional representation of the binary tree from the above representation.
   For example, if $A = \{-1, 0, 0, 1, 2, 2, 4, 4\}$, then the tree is:



http://www.techiedelight.com/build-binary-tree-given-parent-array/

Problems III

4. Write a recursive function `treeToList(NODE root)` that takes a BST and *only rearranges the internal pointers* to make a circular doubly linked list out of the tree nodes. The `previous` pointers should be stored in the `left` field and the `next` pointers should be stored in the `right` field. The list should be arranged so that the nodes are in increasing order. Return the head pointer to the new list.
   Target complexity: $O(n)$ time. Your program should *reuse* the tree nodes, *without* creating a separate node.
   `http://cslibrary.stanford.edu/109/TreeListRecursion.html`
5. Write a function that takes a `NODE` as argument, and returns 1 if the argument is the root of a BST, 0 otherwise.
   See `https://www.hackerrank.com/challenges/is-binary-search-tree` for more details.
   Target complexity: $O(n)$ time
   Also, see SEDGEWICK AND WAYNE, problem 3.2.32.

Problems IV

6. https://www.hackerrank.com/challenges/tree-huffman-decoding
7. Given a BST, and two numbers $\min, \max$ with $\min \leq \max$, trim the tree so that all its elements lie in $[\min, \max]$. Return the root of the new, trimmed tree. Note that the root of the tree may change.
   leetcode.com/problems/trim-a-binary-search-tree/description/
8. http://www.spoj.com/problems/THREECOL/
9. https://www.hackerrank.com/challenges/balanced-forest