Chapter 1

Stochastic Dominance

The concept of stochastic dominance is designed to capture the technical properties of statistical distributions for lotteries that enable broad rankings of those lotteries (with only limited information about the utility function of a particular consumer). Practically speaking, it is a way of comparing different lotteries or distributions of outcomes.

Let L_1 be a lottery with cumulative distribution $F(x)$ and L_2 be a lottery with cumulative distribution $G(x)$. One approach to comparing these lotteries (and thus examining stochastic dominance) is to ask the following two questions:

1) When can we say that everyone will prefer L_1 to L_2?

2) When can we say that anyone who is risk averse will prefer L_1 to L_2?

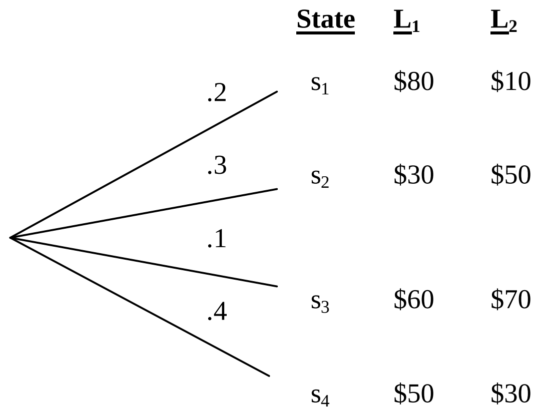
The answer to the first question is defined as the property of **First-Order Stochastic Dominance (FOSD)**, while the answer to the second question is the property of **Second-Order Stochastic Dominance (SOSD)**.

A second approach to stochastic dominance asks two related questions:

1a) Can we write $L_1 = L_2 +$ "something good"? If we can do so, then everyone should prefer L_1 to L_2 for the right definition of "something good."

2a) Can we write $L_2 = L_1 +$ "risk"? If we can do so, then every risk averse person should prefer L_1 to L_2 (and every risk loving person should prefer L_2 to L_1) for the right definition of "risk."

This section explains the definitions of "something good" and "risk," and then shows how the two approaches to stochastic dominance are equivalent for these definitions. There is also a separate set of technical conditions that can be used to check for FOSD and SOSD, but they are just simplified versions of the conditions for (1a) and (2a).



A final important general point is that FOSD and SOSD require only weak preference for L_1 vs. L_2, corresponding to weak conditions on utility functions (e.g. weak rather than strict concavity for risk aversion).

1.1 First-Order Stochastic Dominance (FOSD)

We want to find conditions where we can write $L_1 = L_2+$ "something good," and we want to find the appropriate definition (so that everyone will prefer $L_1 = L_2$) of "something good." We will impose only the most minimal restriction on the utility function, specifying that $u(x)$ is non-decreasing. This means that more wealth is at least as good as less wealth. For our definition, it must be that every person at least weakly prefers L_1 to L_2. No matter how strange the utility function, if it nondecreasing, it must be true that $L_1 \succeq L_2$.

In line with this restriction on $u(x)$, if we can match up the outcomes in L_1 and L_2 so that the outcomes in L_1 are at least as good as the outcomes in L_2 (in pairwise fashion) and L_1 is sometimes strictly better than L_2, then everyone will prefer L_1 to L_2.[1]

Example 1 *Convert a simple lottery into percentile terms. There are four states with the results for L_1 and L_2 as shown below.*

[1] If L_1 and L_2 are identical, then technically speaking, L_1 FOSD's L_2, and L_2 also FOSD's L_1, but this is not very interesting!

Percentile	L_1	L_2
0%-10%	*\$30*	*\$10*
10%-20%	*\$30*	*\$10*
20%-30%	*\$30*	*\$30*
30%-40%	*\$50*	*\$30*
40%-50%	*\$50*	*\$30*
50%-60%	*\$50*	*\$30*
60%-70%	*\$50*	*\$50*
70%-80%	*\$60*	*\$50*
80%-90%	*\$80*	*\$50*
90%-100%	*\$80*	*\$70*

We can make two immediate observations: First, L_2 is better in some states than L_1 (e.g. s_2 and s_3). Second, L_1 is at least as good as L_2 at every percentile, and L_1 is strictly better than L_2 in 8 of the 10 percentiles. The second observation is important to the comparison of the lotteries in terms of expected utility because expected utility relies on a comparison of distributions of outcomes, NOT a comparison of outcomes in individual states.

We can compare the expected utility for each lottery:

$$\begin{aligned} EU(L_1) &= .2 \cdot u(80) + .3 \cdot u(30) + .1 \cdot u(60) + .4 \cdot u(50) \\ EU(L_2) &= .2 \cdot u(10) + .3 \cdot u(50) + .1 \cdot u(70) + .4 \cdot u(30) \end{aligned}$$

Reordering these terms,

$$\begin{aligned} EU(L_1) &= .3 \cdot u(30) + .4 \cdot u(50) + .1 \cdot u(60) + .2 \cdot u(80) \\ EU(L_2) &= .2 \cdot u(10) + .4 \cdot u(30) + .3 \cdot u(50) + .1 \cdot u(70) \end{aligned}$$

Now we can compare them directly by rewriting:

$$\begin{aligned} EU(L_1) - EU(L_2) &= .2[u(30) - u(10)] + .3[u(50) - u(30)] + .1[u(60) - u(50)] \\ &\quad + .1[u(80) - u(50)] + .1[u(80) - u(70)] \\ EU(L_1) - EU(L_2) &\geq 0. \end{aligned}$$

Note that each percentile contributes a term to the calculation of $EU(L_1) - EU(L_2)$. In fact, it is not necessary to calculate $EU(L_1)$ and $EU(L_2)$ once we can rank order the terms from both lotteries and show that L_1 is at least as good at every possible percentile.

With a finite number of outcomes in each lottery (a discrete distribution), we would have to find the least common denominator of probability outcomes in order to find the relevant percentiles that will enable comparison of L_1 and L_2. For example, if the L_1 probabilities are in $\frac{1}{5}$'s, and the L_2 probabilities are in $\frac{1}{6}$'s. then the relevant percentiles will be in $\frac{1}{30}$'s.

Formalizing this discussion, for a finite number of outcomes, if we can divide the distribution functions for L_1 and L_2 into probability ranges of size $\frac{1}{M}$ (where $\frac{1}{M}$ is the least common denominator) and L_1 is at least as good as L_2 for each range, then $L_1 \succeq L_2$.

With a continuous distribution of outcomes, where L_1 is given by the cumulative density function (cdf) $F(x)$, and L_2 is given by the cdf $G(x)$, the same condition would be that for each p between 0 and 1, for the values x_1 and x_2 such that $F(x_1) = G(x_2) = p$, then $x_1 \geq x_2$. Since the cdf $F(x)$ is non-decreasing, this condition is equivalent to $F(x_2) \leq G(x_2)$ for x_2 such that $G(x_2) = p$. But there is nothing special about the particular value of p - this statement must hold for each and every p between 0 and 1, and therefore for each x.

Definition 2 *L_1 **first-order stochastically dominates** L_2 if $F(x) \leq G(x)$ for all x, where F and G give the distributions of L_1 and L_2, respectively.*

This discussion has shown that if this condition holds, then $EU(L_1) \geq EU(L_2)$. There are two things left to do: 1) Show how this definition corresponds to $L_1 = L_2 +$ "something good." 2) Show that for any pair of lotteries L_1, L_2 where this condition does not hold, there is at least one person (i.e. one utility function) who will not weakly prefer L_1 to L_2.

1.2 What is "Something Good"?

Returning to Example 1, let's try to convert L_2 into L_1 with the addition of sublotteries. We'll also return to the rank order in percentiles to see what must be added to L_2 to complete this conversion process.

Percentile	L_1	L_2	$L_1 - L_2$ Difference
0%-10%	\$30	\$10	\$20
10%-20%	\$30	\$10	\$20
20%-30%	\$30	\$30	\$0
30%-40%	\$50	\$30	\$20
40%-50%	\$50	\$30	\$20
50%-60%	\$50	\$30	\$20
60%-70%	\$50	\$50	\$0
70%-80%	\$60	\$50	\$10
80%-90%	\$80	\$50	\$30
90%-100%	\$80	\$70	\$10

If $L_2 = 10$, then $L_1 - L_2 = 20$.

If $L_2 = 30$, then $L_1 - L_2 = 0$ in 1 of 4 cases, and $L_1 - L_2 = 20$ in 3 of 4 cases.

If $L_2 = 50$, then $L_1 - L_2 = 0$ in 1 of 3 cases, $L_1 - L_2 = 10$ in 1 of 3 cases, and $L_1 - L_2 = 30$ in 1 of 3 cases.

If $L_2 = 70$, then $L_1 - L_2 = 10$.

To convert L_2 to L_1, simply add the $L_1 - L_2$ terms to L_2 using conditional probabilities for each value of in L_2:

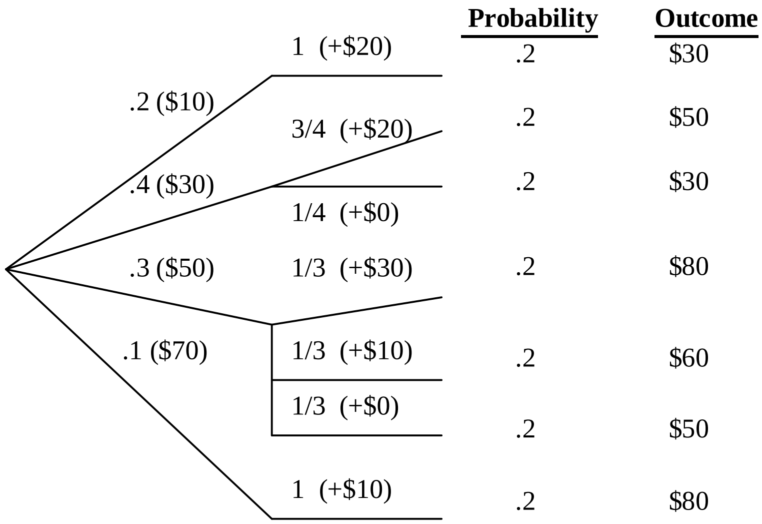
L_3 consists of the set of conditional lotteries appended to L_2. The definition for L_3 to be "something good" is simply that it never has any outcome < 0. An important point is that L_3 need not be the same for each value of L_2; for instance, it is not the same conditional on the result for L_2 in this example.

Thus, $L_1 = L_2 +$"something good," where "something good" $= L_3 =$ conditional distribution for each value in L_2, where no outcome in L_3 is negative.

1.2.1 FOSD is a Necessary Condition for $EU(L_1) \geq EU(L_2)$ for All Bernoulli Utility Functions

The discussion so far has shown that percentile comparison is sufficient for FOSD, and that this can be shown in two ways: 1) $L_1 = L_2 + L_3$ where L_3 is always 0 or better; 2) $F(w) \leq G(w)$ for all w.

But there is still nothing in the derivation thus far to show that these properties are necessary



L2 + L3 = L1

for every person to prefer L_1 to L_2. We can prove this by contradiction. Suppose that there is a pair of lotteries L_1 and L_2 where every person prefers L_1 to L_2 and yet $F(w) \leq G(w)$ does not hold for each w. Then there is a wealth level w^* so that $F(w^*) > G(w^*)$, which is the same as saying that $P(L_1 \leq w^*) > P(L_2 \leq w^*)$. Our only restriction on utility functions is that we require u to be non-decreasing. Can there be a utility function that does not lead to $L_1 \succeq L_2$ under these conditions?

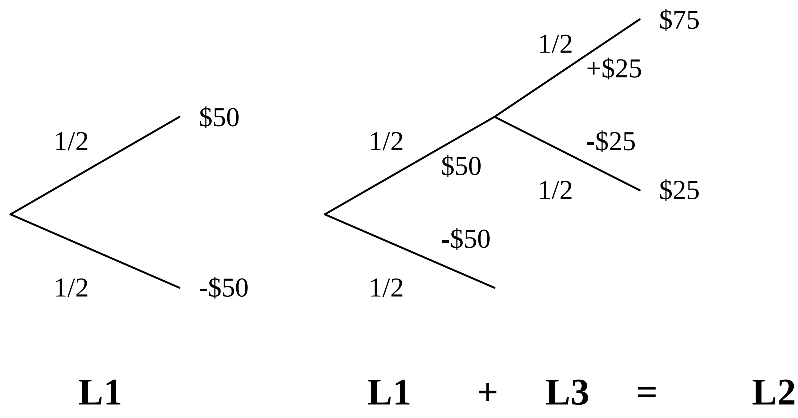
Consider the utility function:

$$
\begin{aligned}
u(w) &= 0 \text{ for } w \leq w^* \\
u(w) &= 1 \text{ for } w > w^*
\end{aligned}
$$

This is a non-decreasing utility function. Though it is discontinuous at w^*, this doesn't violate our conditions on u.

Thus, $EU(L_1) = P(L_1 > w^*) = 1 - F(w^*)$ and $EU(L_2) = P(L_2 > w^*) = 1 - G(w^*)$. This implies $EU(L_2) > EU(L_1)$.

Regardless of how L_1 and L_2 differ apart from $F(w^*)$ vs. $G(w^*)$, a person with this particular



Example 4

utility function will prefer L_2 to L_1. This contradicts the assumption that there could be an L_1 and L_2 where everyone prefers L_1 to L_2 and yet $F(w^*) > G(w^*)$ for some w^*. Therefore, this proves that $F(w^*) \leq G(w^*)$ is a necessary and sufficient condition for $L_1 \succeq L_2$ for anyone with non-decreasing utility.

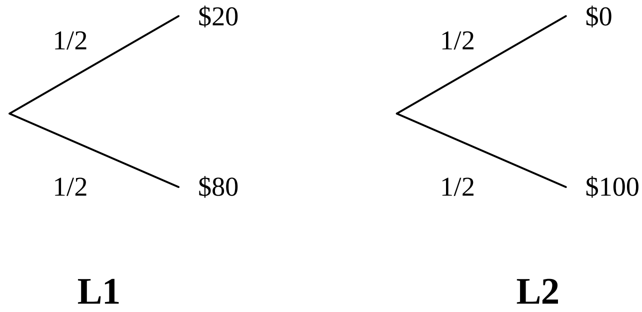
1.3 Second-Order Stochastic Dominance (SOSD)

We want to write $L_2 = L_1+$"risk," and then find the appropriate definition for "risk" so that every risk averse person will prefer L_1 to L_2. Here we assume weak preference and weak concavity, so $u"(x) \leq 0$ is the condition for risk aversion. To isolate the effect of risk, we want the two distributions to have the same mean. This can be accomplished by specifying $L_2 = L_1 + L_3$, where L_3 represents "risk" and has a conditional mean of 0 for each value of L_1.

Definition 3 *A **mean-preserving spread** is a lottery with mean 0 and some variation, meaning that it is not a degenerate lottery with 0 as the only possible outcome.*

We will use this as our definition for "risk": $L_2 = L_1 + L_3$, where L_3 is a mean-preserving spread for each possible value in L_1.

Here, L_3 is the 50%-50% lottery between +\$25 and -\$25 if $L_1 = \$50$, and $L_3 = 0$ for certain if $L_1 = -\$50$. In simple lottery form, L_2 can be written as: {+75, +25, -50; 1/4, 1/4, 1/2}. By Jensen's inequality, if $u(x)$ is concave: $u(50) \geq \frac{1}{2}u(75) + \frac{1}{2}u(25)$. That is, every risk averse person



Example 5

prefers \$50 for sure to \$50 plus the mean-preserving spread of L_3 (the lottery between an additional +\$25 or -\$25). The comparison between L_1 and L_2 depends only on the parts where they differ. The result then is that $L_1 \succeq L_2$ if the consumer is risk averse. In equation form:

$$\begin{aligned} EU(L_1) &= \frac{1}{2}u(50) + \frac{1}{2}u(-50) \\ &\geq \frac{1}{2}[\frac{1}{2}u(75) + \frac{1}{2}u(25)] + \frac{1}{2}u(-50) = EU(L_2) \end{aligned}$$

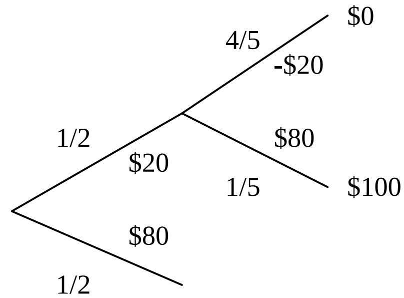
Again, the result is an application of Jensen's inequality. Thus, $L_1 \succeq L_2$ for every risk averse person.

If we can write $L_2 = L_1 + L_3$, where L_3 is mean-preserving spread, or 0, for each value of L_1 and there are always a finite number of values for each lottery, then repeated use of Jensen's inequality, as in the example above, will show that $L_1 \succeq L_2$ if $u(x)$ is concave.

Although the derivation is slightly trickier if the lotteries have continuous distributions of values, the result is the same and the idea of the derivation is the same: repeated application of Jensen's inequality to mean-preserving spreads shows that $L_1 \succeq L_2$.

The next step is to find a condition to check whether $L_2 = L_1 + L_3$, where L_3 is mean-preserving spread or 0. It will not always be obvious how to create an L_3 that transforms L_1 into L_2 even when L_2 is clearly riskier than L_1. The best way to proceed is to try some examples and see if we can discover, through experience, the appropriate conditions to check.

Here L_1 and L_2 each have the expected value \$50, and it seems clear that L_2 is riskier than L_1 since its outcomes vary more from \$50 than do the outcomes in L_2. Suppose we try to add



L1 + Risk

additional mean-preserving spreads to L_1 to create L_2.

Step 1: *Add a mean-preserving spread to +$20 to create outcomes $0 and $100. This will require a lottery with outcomes -$20 and +$80, so probabilities must be 4/5 and 1/5 to give the expected value of 0.In simplified form, this compound lottery reduces to: {$0, $100, $80; 2/5, 1/10, 1/2}. This step reduces expected utility because it adds risk to the certain outcome +$20.*

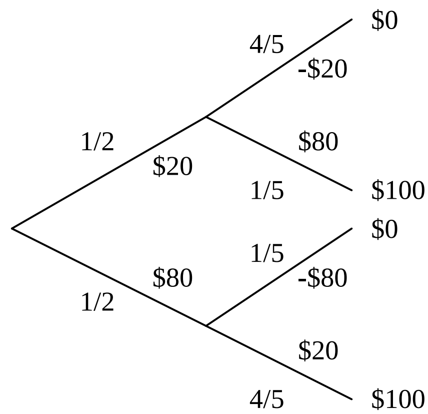
Step 2: *Now add a mean-preserving spread to +$80 to create outcomes $0 and $100.In simplified form, this compound lottery reduces to: {$0, $100; 1/2, 1/2}. Thus, we have recreated L_2 by adding this pair of lotteries to L_1.*

$$L_2 = L_1 + L_3$$

Since L_3 is a mean-preserving spread, $L_1 \succeq L_2$ for all concave $u(x)$.

The preceding example suggests an algorithm for trying to transform L_1 into L_2 when there are a finite number of outcomes. Start with the lowest outcome in L_1. Transform the lowest value in L_1 into the *two* lowest values in L_2. Then do the same for the second-lowest value in L_1, and continue through all values in L_1, subject to some checking.

So far, the two examples we've examined showed how to add mean-preserving spreads to L_1 to recreate L_2. Each addition of a mean-preserving spread makes L_1 less attractive to a risk-averse consumer. This shows that if we can translate L_1 into L_2 by the addition of mean-preserving spreads, then every consumer who is risk averse will prefer L_1 to L_2. This preference is strict if the consumer is strictly risk averse (i.e. $u"(x) < 0$).



L1 + Risk + Risk

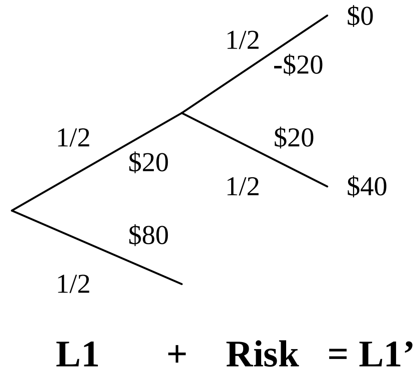
There are two things left to examine: First, what are the conditions to check to determine if L_1 can converted to L_2? Second, can we show that it is not only sufficient, but also necessary, that L_1 can be converted to L_2 with mean-preserving spreads in order for all risk-averse consumers to prefer L_1 to L_2?

1.3.1 Conditions for Converting L_1 to L_2 with Mean-Preserving Spreads (MPS's)

The previous section suggested an algorithm for converting L_1 to L_2 with MPS's. What could go wrong with this procedure? Right at the outset, if the smallest value in L_1 is less than the smallest value in L_2, then it is impossible to convert this value into outcomes in L_2.

Example 6 *Let* $L_1 = \{\$0, \$50, \$100;\ .01, .98, .01\}$, *and* $L_2 = \{\$10, \$90;\ .5, .5\}$. *Both lotteries have a mean = \$50, and L_2 seems much riskier than L_1 because L_1 is almost certain to give the mean value but L_2 is not. Indeed, L_2 has higher variance than L_1. But, the addition of a mean-preserving spread to the value \$0 in L_1 creates the possibility of a result less than \$0, which is not possible in L_2. It is impossible to translate the value \$0 in L_1 into values in L_2. Similarly, since \$100 in L_1 is a higher value than any of the values in L_2, it cannot be converted into values in L_2. So, neither of these two lotteries SOSD's the other.*

The same phenomenon can occur for values other than the lowest and higher in L_1:



Example 7 *Let $L_1 = \{\$20, \$80;\ 1/2, 1/2\}$, and $L_2 = \{\$0, \$50, \$60, \$90;\ 1/4, 1/4, 1/4, 1/4\}$. Both lotteries have a mean = \$50, and the extreme values are more extreme in L_2 than L_1 (\$0 is lower than \$20, \$90 is higher than \$80). Adding a mean-preserving spread to \$20 in L_1 converts this value to \$0 and \$40:L_1' in reduced form can be written as $\{\$0, \$40, \$80;\ 1/4, 1/4, 1/2\}$. Now the lowest value in L_1' matches the lowest value in L_2, but the second-lowest value in L_1' (\$40) is lower than the second lowest value in L_2 (\$50). Once again, the addition of MPS's cannot covert this value (\$40) into values in L_2, so it is impossible to convert L_1 into L_2 with MPS's. Here, the problem with converting L_1 into L_2 is because L_1 is too small on average, over the range of values from \$0 to \$50. L_1 and L_2 are each 50% likely to be between \$0 and \$50, but the average value of L_1 in this range is \$20 and the average value of L_2 in this range is \$25:*

$$
\begin{aligned}
P(L_1 &\leq 50) = P(L_2 \leq 50) = \frac{1}{2} \\
E(L_1|L_1 &\leq 50) = 20 < E(L_2|L_2 \leq 50) = 25
\end{aligned}
$$

The addition of MPS's to the values in L_1 between 0 and 50 maintains this value, $E(L_1|L_1 \leq 50)$, which is less than $E(L_2|L_2 \leq 50)$. Since $E(L_1'|L_1' \leq 50)$ is also less than $E(L_2|L_2 \leq 50)$, we cannot convert from L_1 to L_2.[2] So there is no SOSD relationship.

To derive a mathematical condition for SOSD, suppose that L_1 has a cdf $F(z)$, and L_2 has a cdf $G(z)$, with associated pdf's $f(z)$ and $g(z)$ respectively. This analysis will assume continuous distributions of outcomes for L_1 and L_2, but the argument also holds for finite numbers of outcomes

[2] This assumes that the addition of MPS's does not create values greater than \$50.

in each lottery.

Assume further that for some value x, that $P(L_1 \leq x) = P(L_2 \leq x)$ and $E(L_1|L_1 \leq x) < E(L_2|L_2 \leq x)$. Lastly, let the outcomes of both lotteries be distributed among values greater than or equal to zero. So, for some value of x, $F(x) = G(x)$, and

$$\int_0^x z \cdot f(z)dz < \int_0^x z \cdot g(z)dz$$

Use integration by parts for $\int_0^x z \cdot f(z)dz$, with $u = z$, $du = dz$; and $v = F(z)$, $dv = f(z)dz$:

$$\begin{aligned} \int_0^x z \cdot f(z)dz &= zF(z)|_0^x - \int_0^x F(z)dz \\ &= xF(x) - \int_0^x F(z)dz \end{aligned}$$

This lets us rewrite $E(L_1|L_1 \leq x) < E(L_2|L_2 \leq x)$ as:

$$xF(x) - \int_0^x F(z)dz < xG(x) - \int_0^x G(z)dz$$

Since $F(x) = G(x)$ by assumption, this is equivalent to:

$$\int_0^x F(z)dz > \int_0^x G(z)dz$$

This condition turns out to be precisely the standard condition for SOSD *to fail* at the value x (with the generalization that the outcomes for other lotteries might be negative). In other words, as long as $\int_{-\infty}^x F(z)dz \leq \int_{-\infty}^x G(z)dz$ for each and every x, we are guaranteed that the problems with the conversion algorithm that occurred in the preceding examples will not occur.

Comment: This discussion does not constitute a proof, and for this reason, MWG opted not to include it at all in the discussion in stochastic dominance. The important concepts to take away from this discussion are:

1) SOSD cannot hold if $F(x) = G(x)$ and $E(L_1|L_1 \leq x) < E(L_2|L_2 \leq x)$ for any x;

2) Statement #1 is equivalent to saying that $F(z)$ cannot SOSD $G(z)$ if, for any x, $\int_0^x F(z)dz > \int_0^x G(z)dz$.

Thus, to determine whether $F(z)$ SOSD's $G(z)$, it is only necessary to check that:

$$\int_{-\infty}^x F(z)dz \leq \int_{-\infty}^x G(z)dz \quad \text{for all } x$$

1.3.2 Side Notes of Mathematical Proof:

The biggest hold in this proof is that the algorithm to convert L_1 to L_2 is a bit vaguely described. If this issue interests you, you might want to try to fill in the details of the algorithm and prove that it always works, as long as the condition in Inequality 1.3.1 above holds. The proof here also did not deal with the case where:

$$\int_0^x F(z)dz > \int_0^x G(z)dz \quad \text{where } F(x) \neq G(x).$$

Under these conditions, it is no longer certain that $E(L_1|L_1 \leq x) < E(L_2|L_2 \leq x)$ but the result that L_1 can't be converted into L_2 still holds - and thus, $F(z)$ does not SOSD $G(z)$.

Case 1: $\int_0^x F(z)dz > \int_0^x G(z)dz$ and $F(x) < G(x)$. Then you can ignore some of the lowest values in L_2; for the set of values in L_2 that have probability $F(z)$ are below x but closest to x, the probability is the same as $P(L_1 \leq x)$ and the expectation is higher than $E(L_1|L_1 \leq x)$.

Case 2: $\int_0^x F(z)dz > \int_0^x G(z)dz$ and $F(x) > G(x)$. Then you can make L_2 worse by shifting values for above x to just below x, until $F(x) = G(x)$. Then $\int_0^x F(z)dz > \int_0^x G(z)dz$ and $F(x) = G(x)$.

In either case, it is not possible to convert L_1 into L_2 with the addition of MPS's, so $F(z)$ will not SOSD $G(z)$.

Is It Necessary that L_1 Can Be Converted to L_2 with MPS's to Show SOSD?

In this section, we will show that Inequality 1.3.1 is a necessary condition for every risk averse consumer to prefer L_1 (with cdf $F(z)$) to L_2 (with cdf $G(z)$). To show this, consider the set of utility functions based on the expected value up to w:

$$\begin{aligned} u_w(x) &= x \text{ if } x \leq w \\ u_w(x) &= w \text{ if } x > w \end{aligned}$$

This is a risk-averse function because $u_w'(x) = 1$ for $x < w$, and $u_w'(x) = 0$ for $x > w$. This consumer has a "target income" of w, and once she attains that income, additional money does not affect her utility.

If $\int_{-\infty}^w F(z)dz > \int_{-\infty}^w G(z)dz$ for some w, then $E(u_w)$ will be greater in L_2 (G) than in L_1 (F), meaning that a risk averse consumer with utility function $u_w(x)$ will prefer L_2 to L_1. It's easiest to see this in the case where $F(w) = G(w)$:

$$
\begin{aligned}
E[u_w(L_1)] &= \int_{-\infty}^{\infty} u_w(z) \cdot f(z)dz = \int_{-\infty}^{w} z \cdot f(z)dz + \int_{w}^{\infty} w \cdot f(z)dz \\
&= wF(w) - \int_{-\infty}^{w} z \cdot F(z)dz + [1 - F(w)]
\end{aligned}
$$

By assumption, $F(w) = G(w)$ and $\int_{-\infty}^{w} F(z)dz > \int_{-\infty}^{w} G(z)dz$. Therefore:

$$
\begin{aligned}
E[u_w(L_2)] &= wG(w) - \int_{-\infty}^{w} G(z)dz + [1 - G(w)] \\
&> wF(w) - \int_{-\infty}^{w} z \cdot F(z)dz + [1 - F(w)] = E[u_w(L_1)]
\end{aligned}
$$

Therefore, the consumer with utility function $u_w(x)$ prefers L_2 to L_1. So, if $\int_{-\infty}^{w} F(z)dz > \int_{-\infty}^{w} G(z)dz$ for some w, then there is at least one risk averse consumer who prefers L_2 to L_1, so L_1 cannot SOSD L_2.

1.4 Combining First- and Second-Order Stochastic Dominance

It is possible that every risk averse consumer prefers L_1 to L_2, and yet we cannot use SOSD directly. This occurs when L_1 has a higher mean than L_2, so it is not possible to apply the definition of SOSD, which assumes $E(L_1) = E(L_2)$. But we can compare the two lotteries if we use both FOSD and SOSD.

For instance, let's compare $L_1 = \{\$60; 1.0\}$, and $L_2 = \{\$0, \$100; 1/2, 1/2\}$. Here, $E(L_1) = \$60$, and $E(L_2) = \$50$, and it is clear that L_2 is riskier. The basic logic is that L_2 is worse than \$50 for certain, which is still worse than L_1.

To convert the logical argument into technical statements using stochastic dominance, we want to show: 1) The lottery L_3, which is \$50 for certain, SOSD's L_2. 2) L_1 FOSD's L_3, since $60 > 50$. By a form of transitivity, L_1 is better than L_2 for any risk averse consumer. Specifically, L_1 FOSD's L_3, so every consumer prefers L_1 to L_3, and L_3 SOSD's L_2, so every risk averse consumer prefers L_3 to L_2.

Thus, for any risk averse consumer:

$$
EU(L_1) \geq EU(L_3) \geq EU(L_2).
$$

Note that even though L_1 FOSD's L_3, it could be that a risk-loving consumer prefers L_2 to L_1. You can check this for yourself by creating a utility function such that $EU(L_2) > EU(L_1)$.