Monetary Stabilization of Export Shocks, Revisited

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Abstract

We study how monetary policy shapes macroeconomic outcomes in a two-sector small open economy hit by export shocks —due, e.g., to export tariffs, geopolitical tensions, or a recession in destination countries— allowing the shock to have both *aggregate* and *distributional* effects. Imperfect worker mobility across sectors, coupled with incomplete markets against aggregate and idiosyncratic shocks, implies that export contractions (i) spill over across sectors due to households' precautionary response and (ii) affect income and consumption inequalities within and across sectors —in addition to their usual asymmetric effects on sectoral outputs and wages. In this context, exchange-rate flexibility provides insurance against inefficient fluctuations in consumption inequality, which increases the social value of floating-rate regimes. Relative to a nominal exchange-rate peg, flexible inflation targeting helps mitigate the rise in consumption inequality after an export contraction, especially among tradable-sector workers. However, even flexible inflation targeting does not in general provide sufficient exchange-rate flexibility relative to the *optimal* monetary policy.

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1 Introduction

The escalation of tariff announcements in 2025 has revamped the debate on uncertainty as a driver of the business cycle. In this paper, we construct a two-sector Heterogeneous-Agent New Keynesian ("HANK") model of a small open economy, where shocks to the foreign demand for domestic goods generate fluctuations in idiosyncratic income risk and, consequently, precautionary saving among workers in the tradable sector, with significant aggregate demand spillovers to the entire economy. Following a global recession or a trade war, domestic income uncertainty rises while the increase in domestic income inequality weighs on aggregate economic activity. We show that the optimal response to foreign demand shocks contains fluctuations in idiosyncratic risk and inequality, consistent with the joint objective of stabilizing aggregate demand and enhancing risk sharing among domestic households. Implementing the optimal monetary stance requires significant exchange rate variability, thereby increasing the social value of floating-rate regimes.

The key assumptions underlying our analysis are that (i) workers are not mobile across sectors in the short run —so that sectoral wage incomes transitorily diverge from each other after an export contraction that hits the tradables sector, and (ii) markets are incomplete not only internationally but also domestically, implying that aggregate and idiosyncratic shocks generate consumption inequalities across and within sectors. To evaluate the performance of alternative policy regimes in this environment, we proceed in two steps. First, we construct welfare-based indices of within-sector inequality and examine how they respond to export shocks —jointly with other macroeconomic variables such as domestic sectoral wages, output levels, and inflation rates— under a particular monetary policy regime. Intuitively, the indices capture the welfare cost of consumption dispersion within a sector coming from the pass-through from idiosyncratic, uninsured income shocks to individual consumption. Comparing fixed and floating exchange-rate regimes, we find the former particularly detrimental not only for sector-wide variables but also for within-sector inequality. The reason of this is as follows. First, after an export contraction (say) the relative price of home tradables falls, leading to a real depreciation. Maintaining a nominal exchange-rate peg then requires a substantial deflation of the consumer price index, which in turn requires strong internal deflation. This can only be brought about by a sharp monetary policy *contraction*, which mitigates the real depreciation while contracting both external and internal demand. Output eventually falls sharply in both sectors, while counter-cyclical risk implies that the economic contraction raises domestic income risk and consumption inequality. Macroeconomic adjustments are much smoother under flexible inflation targeting, because this regime tolerates *some* imported inflation following the real depreciation induced by the import contraction (unlike the nominal peg), and thus commands a more accommodative monetary policy response. Eventually, the real depreciation is stronger and eventually contributes to stabilizing aggregate demand, output and consumption inequality in both sectors.

In a second step, we exploit our utility-based inequality index to compute the *optimal* monetary policy response to an export shock in HANK, and to compare the associated aggregate and distributional outcomes with those under the previously studied ad hoc policy regimes. While computing optimal monetary policy in HANK model is generally challenging —and especially so for open-economy HANK models, which feature a larger endogenous state vector than their closed-economy counterparts—, our parametric assumptions make the planner's social welfare function tractable, in such a way that the solution to its problem admits a standard state-space representation (with the inequality index as one of the endogenous variables). Examination of the optimal monetary policy response to export shocks shows that both a nominal exchange-rate peg and flexible inflation targeting produce *too little nominal exchange-rate flexibility.* After a contraction in exports, optimal monetary policy commands a *fall* in the real interest rate —unlike the peg or flexible inflation targeting— and is accompanied by a spike in the consumer price index and the real exchange rate. Intuitively, optimal monetary policy mitigates the contraction of the tradables sector by moderating the expansion of the nontradables sector, so that output and inflation eventually rise in the latter. While optimal monetary policy fails to eliminate consumption differences between sectors —since the policy instrument operates on the overall economy, not asymmetrically—, the way it buffers the falls in sectoral output levels implies that within-sector inequalities are strongly mitigated. To summarize, the optimal monetary policy response of a small open economy hit by an export contraction is to let go of imported inflation in order to stabilise aggregate demand and domestic consumption inequalities. From this perspective, nominal exchange-rate pegs are particularly costly, and even flexible inflation-targeting regimes do not, in general, allow sufficient variation in CPI inflation.

1.1 Literature review

The transmission of foreign demand shocks. The papers most closely related to ours are Druedahl et al. (2024) and Guo et al. (2023), which study the propagation of foreign demand shocks in two-sector SOE-HANK models from a positive perspective. Druedahl et al. (2024)'s key empirical finding is that foreign demand shocks sharply contract both the tradables and non-tradable sectors under a standard Taylor rule, a property they replicate in their quantitative model via the presence of high-MPC households and the associated general-equilibrium effects. We similarly find that an export contraction leads to a contraction in both sectors under flexible inflation targeting (and even worse so under strict inflation targeting or an exchange-rate peg) – though in our model, domestic cross-sectional propagation operates via countercyclical risk rather than high MPCs. We also find, however, that *optimal* monetary policy can and should mitigate such negative outcomes: by responding aggressively to the export contraction, it ultimately expands the non-tradable sector, thereby buffering the shock's impact on the country's output and inequality, at the cost of high inflation. Guo et al. (2023) study how financial and goods-market integration shapes the impact of foreign demand shocks on inequality under flexible inflation targeting; we confirm their finding that tradable-sector workers are highly exposed to export contractions, and we further elicit the optimal monetary policy response to such shocks. In contrast with Guo et al. (2023), we find pegs to be highly detrimental to consumption inequality; this is because they abstract from the countercylicality of income risk, which is a central feature of our analysis; it implies that the more a sector contracts, the greater the variance of idiosyncratic income shocks, while this greater variance is partly pass-through to the distribution of consumption.

One of the key motivations for the analysis of foreign demand shocks in HANK models is the inability of Representative-Agent ("SOE-RANK") models to match their strong propagation observed in the data – see, e.g., Justiniano and Preston (2010), Adolfson et al. (2007) and Christiano et al. (2011). On the normative side, Lombardo and Ravenna (2014) study optimal monetary policy in a two-sector SOE model with a Representative Agent and perfect international risk sharing. By construction, all such models abstract both from cyclical risk as a propagation mechanism and from inequality as a potential concern for monetary policy.

The macroeconomics of export tariffs. Our paper relates to the recent literature studying export tariffs as a source of foreign demand shocks in open-economy New Keynesian models. The first paper to compute the optimal monetary policy response to export tariffs is by Bergin and Corsetti (2023), who show that it is expansionary and inflationary. They make this point using a one-sector, two-country economy with Representative Agents in each country and under the assumption of international policy coordination.[1] Monacelli (2025) extends their analysis to the case of an SOE facing export tariffs and in the absence of policy coordination, while Hamano et al. (2025) introduces heterogeneous firms and endogenous export decisions. Like the latter two studies, we examine the optimal monetary policy response of an SOE operating in a non-cooperative environment. Different from all three papers, we introduce (i) tradables versus nontradables sectors, which are accordingly differentially exposed to tariff shocks and (ii) within-country incomplete markets leading to time-varying inequality between households. We confirm the recommendation that a rise in export tariffs commands and expansionary monetary policy response, while emphasising household inequality as an additional motive for monetary accommodation.

A related but distinct strand of the tariff shock literature focuses on the optimal monetary-policy response to *import* rather than export tariffs – asking, say, what the Federal Reserve's response to "liberation day" should be (see, e.g., Auclert et al. 2025; Auray et al. 2025; Bianchi and Coulibaly 2025; Werning et al. 2025). The discussion there centres on how to address best the imported inflation caused by import tariffs, and how this concern weighs against the central bank's other objectives. Our analysis differs in purpose and substance: we seek to understand how a small economy that is a target of liberation-day tariffs (say) should respond, and, in so doing, take into account both the aggregate and distributional effects of the tariff shock.

Open-economy HANK models. More generally, our analysis complements the nascent open-economy HANK literature, which revisits traditional international macroeconomic questions through the lens of household heterogeneity. De Ferra et al. (2020) inaugurated this approach, focusing on household balance-sheet effects of currency crisis under currency mismatch. Auclert et al. (2024) study how household heterogeneity and high-MPC households magnify the real-income channels associated with capital-flow and monetary-policy shocks. Closer to our paper, Oskolkov (2023) studies how the exchange-rate regime of a two-sector SOE-HANK economy responds to capital-flow shocks and uncovers a tradeoff between exchange-rate and inequality sabilization after such shocks. We find a similar trade-off after export shocks – a peg is the most inequality-conducive policy we consider – and, in fact, demonstrate that optimal policy entails substantial exchange-rate volatility. Acharya and Challe (2025) study optimal exchange-rate sabilization in an SOE-HANK model to uncover the conditions under which the "open-economy divine coincidence" in Galí and Monacelli (2005) extends to incomplete markets and heterogeneous agents. The present paper shares with Acharya and Challe (2025) the CARA-Normal structure instrumental for solv-

[1] Bergin and Corsetti (2025) extend that model to the case of multiple traded sectors and allow tariffs to be targeted at specific sectors.

ing the optimal policy problem,[2] but differs in every other dimension. In particular, here we (i) consider a two-sector economy and focus on intersectoral imbalances, (ii) assume wage rather than price rigidities, (iii) incorporate a broader range of sources of inequalities (within sector-cohort, within sector/between cohort and between sectors), and (iv) study the impact of global (export) shocks rather than domestic (productivity) shocks.

All the above-cited papers formulate SOE-HANK models. A complementary class of papers examines spillovers (and spillbacks) in two-country HANK models – e.g., Acharya and Pesenti (2024), Bayer et al. (2024), and Chen et al. (2023) – all from a positive rather than normative perspective.

2 The Model

Our framework of analysis is a two-country model a la Corsetti and Pesenti (2001, 2005), where the masses of households populating the Home and Foreign economies are $m \in (0, 1)$ and $1 - m$, respectively. Eventually, we will let $m \to 0$ (as in, e.g., Faia and Monacelli 2008 or De Paoli 2009a,b), so that the Home economy becomes an SOE facing a large Foreign economy and treating all its variables as given. The two economies produce both tradable and non-tradable goods. Good markets are competitive, with nominal prices fully flexible, but nominal wages are not. Moreover, our model does not admit a representative agent: markets are incomplete (both internationally and domestically), so households working in different sectors are differently exposed to aggregate shocks and are also hit by uninsured idiosyncratic shocks, as in Aiyagari (1994). While idiosyncratic risk is anticipated, we only consider once-occurring time-0 aggregate shocks (aka "MIT shocks"), after which perfect foresight with respect to aggregate variables prevails.

2.1 Firms

All firms are perfectly competitive. In each country, there are two layers of firms: "Retailers" produce the final consumption basket by combining domestically-produced nontradables, domestically-produced tradables, and imported tradables. "Producers" produce domestic tradables and nontradables using labor as the sole input.

2.1.1 Retailers

Home retailers. Home retailers produce c_t units of the Home composite consumption basket according to the following production function:

$$c_t = \Omega c_t (N)^{1-\phi} c_t (H)^{\phi(1-\alpha)} c_t (F)^{\phi\alpha}, \quad 0 < \alpha, \phi < 1 \tag{1}$$

where $c_t(N)$, $c_t(H)$ and $c_t(F)$ denote, respectively, the quantities of Home-produced nontradables, Home-produced tradables, and Foreign-produced tradables used in production. Retailers' productivity is normalised to $\Omega = (1 - \phi)^{\phi-1} [(1 - \alpha) \phi]^{(\alpha-1)\phi} (\phi\alpha)^{-\phi\alpha}$.

[2] While the development of numerical algorithms for solving optimal fiscal-monetary problems in HANK models is underway (see, e.g., Bhandari et al. 2021; Dávila and Schaab 2023; Le Grand et al. 2025), they typically apply to models with a smaller state space than open-economy models with imperfect international risk sharing.

Let P_t denote the Home-currency price of the Home consumption basket, and $P_{N,t}$, $P_{H,t}$ and $P_{F,t}$ the Home-currency prices of Home-produced nontradables, Home-produced tradables and Foreign-produced tradables, respectively. Last, define $p_{N,t} \equiv P_{N,t}/P_t$, $p_{H,t} \equiv P_{H,t}/P_t$ and $p_{F,t} \equiv P_{F,t}/P_t$ as the corresponding real prices (i.e., in units of the home consumption basket). Retailers' profit maximisation yields the following input demand functions:

$$c_t(N) = \frac{1-\phi}{p_{N,t}} c_t, \qquad c_t(H) = \frac{(1-\alpha)\,\phi}{p_{H,t}} c_t, \qquad c_t(F) = \frac{\alpha\phi}{p_{F,t}} c_t \tag{2}$$

while zero profit in the retail sector implies that retailers' real marginal cost $(p_{N,t}^{1-\phi} p_{H,t}^{\phi(1-\alpha)} p_{F,t}^{\phi\alpha})$ equals their selling price ($=1$) so that:

$$p_{N,t}^{1-\phi} p_{H,t}^{\phi(1-\alpha)} p_{F,t}^{\phi\alpha} = 1 \tag{3}$$

Foreign retailers. Foreign retailers operate symmetrically. Accordingly, the production function for the Foreign consumption basket is:

$$c_t^* = \Omega^* c_t^*\left(N\right)^{1-\phi} c_t^*\left(F\right)^{\phi(1-\alpha^*)} c_t^*\left(H\right)^{\phi\alpha^*} \tag{4}$$

where $c_t^*(N)$ and $c_t^*(F)$ denote the quantities of domestically-produced nontradables and tradables used in the production of the Foreign consumption basket, $c^*(H)$ is the demand by Foreign retailers for Home-produced goods – i.e., Home exports –, and $\Omega^* \equiv (1-\phi)^{\phi-1}\left[(1-\alpha^*)\,\phi\right]^{(\alpha-1)\phi}(\phi\alpha^*)^{-\phi\alpha^*}$ is Foreign retailers' productivity. This gives the demand system

$$c_t^*(N) = \frac{1-\phi}{p_{N,t}^*} c_t^*, \qquad c_t^*(F) = \frac{(1-\alpha^*)\,\phi}{p_{F,t}^*} c_t^*, \qquad c_t^*(H) = \frac{\alpha^*\phi}{p_{H,t}^*} c_t^* \tag{5}$$

together with the zero-profit condition

$$\left(p_{N,t}^*\right)^{1-\phi}\left(p_{F,t}^*\right)^{\phi(1-\alpha^*)}\left(p_{H,t}^*\right)^{\phi\alpha^*} = 1 \tag{6}$$

In equations (5)-(6), $p_{F,t}^*$ and $p_{N,t}^*$ respectively denote the price of Foreign-produced tradables and nontradables in units of the Foreign consumption basket, while $p_{H,t}^*$ is the price of Home-produced tradables in units of the Foreign consumption basket.

Tariffs shocks. We allow the Foreign demand for Home-produced tradables $c_t^*(H)$ to be affected by a *export tariff* shock τ_t^X imposed by the Foreign government. The tariff creates a wedge $1+\tau_t^X$ between the Home currency price of Home-produced goods sold domestically $(P_{H,t})$ versus sold abroad – i.e., $\mathcal{E}_t P_{H,t}^*$, where $P_{H,t}^*$ is the cum-tariff price of the good in Foreign currency units and $\mathcal{E}_t$ is the direct nominal exchange rate for the Home economy. That is, we have:

$$\mathcal{E}_t P_{H,t}^* = (1+\tau_t^X) P_{H,t}$$

Define the real exchange rate Q_t as the price of Foreign consumption in terms of Home consumption. The Law of one price applies strictly to Foreign tradables, but with the tariff wedge on Home tradables, so that:

$$p^*_{H,t} = (1+\tau^X_t)p_{H,t}/Q_t \quad \text{and} \quad p_{F,t} = Q_t p^*_{F,t} \tag{7}$$

The tariff is possibly persistent but, for simplicity, assumed to be transitory and to have a zero mean. It is paid by Foreign consumers and the tax revenue $mc^*_t(H)\tau^X_t p_{H,t}/Q_t$ it generates is rebated lump-sum to them.

Finally, we let retailers' production function depend on the relative size of the Home economy, m, so that each economy's imports are proportional to its own size. To do so, define $\lambda \in (0,1)$ as the underlying degree of trade openness, and assume that $\alpha = (1-m)\lambda$ and $\alpha^* = m\lambda$. Therefore, as $m \to 0$ (the Home economy becomes an SOE), we have $\alpha \to \lambda$ (the import share of the Home economy approaches the degree of openness) and $\alpha^* \to 0$ (the import share of the large economy vanishes).

2.1.2 Producers

Home producers. In the Home economy, there are two sectors of production k, namely tradables ($k = T$) and nontradables ($k = N$). Let $\varphi \in (0,1)$ and $1-\varphi$ denote the shares of the Home population working in the tradables and nontradables sectors, respectively. Those shares are exogenous and constant, reflecting the limited mobility of workers across sectors in the short run.[3] Finaly, let $n_{T,t}$ and $n_{N,t}$ respectively denote hours per worker in the tradables and nontradables sectors, $y_{H,t}$ tradables output per capita in the Home economy (so that total tradables output is $my_{H,t}$), and $y_{N,t}$ nontradable output per capita (so that total nontradables output is $my_{N,t}$). Assuming each sector turns one unit of labor into one unit of goods, the per capita levels of tradables and nontradables output are given by:

$$y_{H,t} = \varphi n_{T,t}, \qquad y_{N,t} = (1-\varphi)n_{N,t} \tag{8}$$

Since production in both sectors is perfectly competitive, prices merely reflect real marginal costs. Sectoral real marginal costs, in turn, have two components. First, they depend on the real wage per unit of effective labor paid by firms in that sector, namely $w_{k,t}$, $k = T, N$ – which generically differ across sectors due to imperfect labor mobility. Second, they depend on the payroll taxes that the government may impose on producers. Since there is no benefit in distorting the production of nontradable goods, we assume that the payroll tax is at its optimal value of zero in that sector so that:

$$p_{N,t} = w_{N,t} \tag{9}$$

In the absence of international coordination – our maintained assumption –, the government may, however, find it optimal to limit the production of *tradable* goods (relative to laissez-faire) in order to manipulate the terms of trade *in steady state* (see, e.g., Galí and Monacelli 2005; Farhi and Werning 2012). In the presence of a payroll tax τ^T, the real price of Home-produced tradables is given by:

$$p_{H,t} = (1+\tau^T)w_{T,t} \tag{10}$$

[3] In Section 4.2 we will parameterise population shares such that steady-state wages are equalised, consistent with the notion that worker reallocation eventually occurs in the long run and eliminates wage differentials.

Foreign producers. Foreign producers operate symmetrically, pricing their own tradable and nontradable outputs according to the relevant real marginal costs so that $p^*_{N,t} = w^*_{N,t}$ and $p^*_{F,t} = (1 + \tau^{T*})w^*_{T,t}$, where τ^{T*} is the Foreign payroll tax and where the Foreign real wages $w^*_{k,t}$ are measured in units of the Foreign consumption basket.

2.2 Home households

The demographics follow a perpetual-youth structure à la Blanchard (1985)-Yaari (1965), in which all households face a survival probability of $\vartheta < 1$ at any date t. To ensure that the Home population stays constant (normalized to 1), there are $1 - \vartheta$ "newborn" households, who replace an equal measure of households that did not survive. The perpetual youth structure serves two purposes here. First, it ensures the existence of a stationary distribution of wealth within the Home economy despite the repeated occurrence of idiosyncratic shocks, as in Acharya et al. (2023) and Acharya and Challe (2025). Second, it ensures that the net foreign asset position of the economy returns to a well-defined steady state after an aggregate shock, as explained by Ghironi (2006).[4]

Households are heterogeneous in three dimensions. First, they work in a specific sector (tradable versus non-tradable) and are not mobile across sectors in the short run, while transitory wage differences may occur following aggregate shocks. Second, they differ *across cohorts* within each sector, since every household is born with no wealth and must gradually accumulate it over their lifetime. Last, households differ *within sector-cohort* due to the repeated occurrence of uninsured idiosyncratic labor-productivity shocks.

Preferences and constraints. The intertemporal utility of a Home household i born at time s and working in sector k is given by:

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left[-e^{-c^s_{k,t}(i)} - \kappa n_{k,t} - \frac{\Psi}{2} \left(\ln \Pi^w_{k,t} \right)^2 \right], \quad \kappa, \Psi > 0, \tag{11}$$

where $\beta \in [0, 1)$ is the subjective discount factor common to all households, $c^s_{k,t}(i)$ their consumption of the composite consumption good, $n_{k,t}$ their labor supply (common to all households working in the same sector), and $\Pi^w_{k,t} = W_{k,t}/W_{k,t-1}$ is gross sector-k nominal wage inflation ($W_{k,t}$ is the nominal wage in sector k). As explained below, $n_{k,t}$ and $\Pi^w_{k,t}$ are not directly chosen by households but delegated to unions operating at the sector level and demanding the same number of hours worked from all workers in the sector.

Our functional specification for the disutility of labor and wage inflation (respectively linear and log-quadratic) in equation (11) is for analytical convenience only: all our results would carry over under increasing marginal disutility from labor and convex disutility from wage inflation. Our constant absolute risk aversion (CARA) specification for consumption utility, in contrast, is not innocuous: as we will explain further below, this assumption, combined with conditionally normally distributed idiosyncratic shocks,

[4] An alternative, popular way of ensuring NFA stationarity in SOEs is to assume a convex transaction cost as in Schmitt-Grohé and Uribe (2003). In the present case, however, this would stationarize *total* NFA but not *sectoral* assets, which would both have a unit root. The perpetual youth structure operates within sectors and hence stationarises both sectoral assets.

provides sufficient tractability to formulate and solve our optimal monetary policy problem. Note that (11) has a unit absolute risk aversion coefficient (for conciseness only). When we normalise steady-state mean consumption to 1 later on, so that the relative and absolute risk aversion coefficients of the median household are identical, this will imply that the elasticity of intertemporal substitution (the inverse of the RRA coefficient) of the median household is also equal to 1.

In every period, the household decides how much to consume and how much to save. Markets are incomplete: the only assets available to Home households are actuarial bonds that pay one unit of the Home consumption good in the next period upon survival (and zero upon death). The time-t budget constraint of the household is:

$$c_{k,t}^s(i) + (1+\tau^\star)\frac{\vartheta}{R_t}a_{k,t+1}^s(i) = a_{k,t}^s(i) + (1-\tau^w)\, w_{k,t}n_{k,t}e_{k,t}^s(i) + T_{k,t}, \tag{12}$$

where ϑ/R_t is the pre-tax date-t price of an actuarial bond paying off at date $t+1$, $a_t^s(i)$ is the date-t wealth of household i born at date $s \leq t$, and $e_t^s(i)$ is (uninsured) idiosyncratic household productivity. That is, a household i that works $n_{k,t}$ hours supplies $n_{k,t}e_t^s(i)$ effective labor units, each of which is paid $w_{k,t}$ pre-tax.

Two constant tax rates appear in equation (12), namely a *savings tax* $\tau^\star$ and a *labor-income tax* τ^w. The former affects the expected returns on savings and thus distorts savings decisions, while the latter affects post-tax labor earnings and thus distorts labor-supply decisions. As explained in Section 5 below, both tax rates are set optimally by the fiscal authority and are necessary for the constrained efficiency of the steady state – and eventually the time-consistency of optimal monetary policy –, given the underlying frictions plaguing the economy. All taxes are rebated lump-sum, giving rise to the transfer $T_{k,t}$, which is sector-specific and uniform across households of the same sector.

Idioysncratic risk. Individual productivity $e_{k,t}^s(i)$ evolves stochastically as follows:

$$e_{k,t}^s(i) = 1 + \sigma_{k,t}\xi_{k,t}^s(i), \qquad \xi_t^s(i) = \xi_{t-1}^s(i) + \upsilon_t^s(i), \qquad \upsilon_t^s(i) \sim \mathcal{N}(0,1), \tag{13}$$

This stochastic process concisely captures the observation that idiosyncratic income shocks are highly persistent, while their size depends on the stage of the business cycle – recessions typically being associated with greater earnings risk. In equation (13), the average size of innovations to individual productivity is commanded by the sector-specific, time-varying standard deviation $\sigma_{k,t}$. Given $\sigma_{k,t}$, equation (12) implies that the conditional standard deviation (as of time-$t-1$) of individual income ("income risk") is sector-specific and given by:

$$\sigma_{k,t}^y = (1-\tau^w)w_{k,t}n_{k,t}\sigma_{k,t}, \quad k = T, N \tag{14}$$

Importantly, we assume that income risk is possibly cyclical, so that the distribution of labor earnings within a sector fluctuates with the business cycle. For simplicity, we directly specify the behaviour of income risk $\sigma_{k,t}^y$, and then let $\sigma_{k,t}$ (a free variable in equation (14)) adjust in the background to materialise $\sigma_{k,t}^y$, given $n_{k,t}, w_{k,t}$). More specifically, we assume that

$$\sigma_{T,t}^y = \sigma^y \exp\left\{-\Theta\left(\frac{y_{H,t}}{y_H} - 1\right)\right\} \quad \text{and} \quad \sigma_{N,t}^y = \sigma^y \exp\left\{-\Theta\left(\frac{y_{N,t}}{y_N} - 1\right)\right\}, \tag{15}$$

where $\sigma^y \geq 0$ denotes steady-state labor-earnings risk and Θ indexes the cyclicality of labor-earnings risk: risk is countercyclical if $\Theta > 0$, acyclical if $\Theta = 0$, and procyclical if $\Theta < 0$). Consistent with the evidence (e.g., Storesletten et al. 2004), we will henceforth focus our discussion on weakly countercyclical idiosyncratic risk, i.e. $\Theta \leq 0$.

Optimal consumption and savings. Households maximise (11) subject to (12) and a no-Ponzi condition preventing exploding private debt. The consumption-saving plan of a generic household i of cohort s working in sector k must thus satisfy the following individual Euler condition:

$$e^{-c^s_{k,t}(i)} = \left(\frac{\beta R_t}{1+\tau^\star}\right)\mathbb{E}_t e^{-c^s_{k,t+1}(i)} \tag{16}$$

In general, uninsured idiosyncratic risk prevents the existence of closed-form expressions for *aggregate* consumption demand. In this paper, we leverage the CARA-Normal structure of the model to linearly aggregate individual consumption decisions at the sector level (that, is both across individual and across cohorts). As shown in Appendix C, sectoral consumption demand is eventually summarised by the following Euler condition:

$$c_{k,t} = \underbrace{c_{k,t+1}}_{\text{PI behavior}} - \underbrace{\ln\left(\frac{\beta R_t}{1+\tau^\star}\right)}_{\text{intertemp. substitution}} + \underbrace{\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{k,t+1}}_{\text{Blanchard-Yaari term}} - \underbrace{\frac{1}{2}(\sigma^c_{k,t+1})^2}_{\text{precautionary savings}}, \quad k = T, N, \tag{17}$$

where $c_{k,t}$ and $a_{k,t+1}$ respectively denotes the average consumption and end-of-period assets of households working in sector k.[5]

The first two terms in the right-hand side of (17) – permanent-income behavior and intertemporal substitution in consumption – are familiar. The third term, which features average sectoral assets $a_{k,t+1}$, arises in perpetual youth models a la Blanchard (1985) and is a manifestation of between-cohort heterogeneity and generational turnover. (If there are positive sectoral assets to accumulate, recently-born households are poorer than the average cohort member and their asset accumulation drags down overall sectoral consumption growth). The variable μ_t also showing up in the Blanchard-Yaari term is the marginal propensity to consume out of wealth and is shown in Appendix C to obey the following forward recursion:

$$\mu_t^{-1} = 1 + \frac{\vartheta\,(1+\tau^\star)}{R_t}\mu_{t+1}^{-1} \tag{18}$$

Finally, Appendix A.2 shows that sector-k average bond holdings in equation (17) (once individual savings are aggregated across all individuals from all cohorts) evolve as follows:

$$a_{k,t+1} = \frac{R_t}{1+\tau^\star}\left\{a_{k,t} + (1-\tau^w)w_{k,t}n_{k,t} + T_{k,t} - c_{k,t}\right\}, \quad k = T, N. \tag{19}$$

The last term in (17) reflects precautionary savings against consumption risk in sector k, as measured by the conditional standard deviation of individual consumption in that sector, $\sigma^c_{k,t+1}$. As derived in Appendix C, consumption risk is itself a function of income risk ($\sigma^y_{k,t}$ in equation (15)) going forward, as

[5] To be clear: $c_t(H)$ and $c_t(N)$ in (1) are the consumption *of* the Home tradables and nontradables (by anyone, given a consumption basket c_t), while the $c_{k,t}$ are the average consumption of households *in sector* k.

is summarised by the following recursion:

$$\sigma^c_{k,t} = \mu_t \sigma^y_{k,t} + (1-\mu_t)\,\sigma^c_{k,t+1}, \quad k = T, N \tag{20}$$

Equations (17) to (20), for $k = T, N$, fully characterise economy-wide consumption and savings decisions. Given that φ households work in the tradables sector and $1-\varphi$ in the nontradable sector, total per capita consumption and end-of-period assets (the country's net foreign asset position) are $c_t = \varphi c_{T,t} + (1-\varphi)c_{N,t}$ and $a_{t+1} = \varphi a_{T,t+1} + (1-\varphi)a_{N,t+1}$, respectively.

Optimal labor supply. Our modelling of labor supply follows Auclert et al. (2024). More specifically, in each sector, individual labor-supply decisions are delegated to sector-specific, competitive unions, which maximise the intertemporal welfare of the average union member and demand an equal number of hours from each of them. As shown in Appendix B, this delegation ultimately gives rise to the following sectoral New Keynesian wage Phillips curves:

$$\ln \Pi^w_{k,t} = \frac{n_{k,t}}{\Psi\left(\mathcal{M}_w - 1\right)}\left[\kappa\mathcal{M}_w - (1-\tau^w)w_{k,t}e^{-c_{k,t}}\right] + \beta \ln \Pi^w_{k,t+1}, \quad k = T, N, \tag{21}$$

where $n_{k,t}$ and $c_{k,t}$ are sectoral labor supply and consumption and $\mathcal{M}_w > 1$ denotes the gross steady-state wage markup. The term in square brackets captures deviations of the sectoral post-tax wage from the sectoral marginal rate of substitution between consumption and leisure. When this deviation falls below the steady state in a particular sector, unions ration labor supply to put upward pressure on real wages, and wage inflation follows. Note that equation (21) also characterises sectoral nominal price inflation, since there is no nominal price rigidity and hence full pass-through from marginal labour cost to prices.

2.3 Foreign households

The Foreign economy is populated by risk-neutral households who share the same subjective discount factor β as Home households. Foreign households may freely borrow and lend internationally in the form of Foreign-denominated bonds costing $1/R^* = \beta$ units of the Foreign consumption basket and delivering 1 unit of the Foreign consumption basket in the following period. Foreign households incur no disutility from wage inflation, which implies full nominal flexibility in the Foreign economy.

2.4 Fiscal and monetary authorities

The fiscal authority sets the payroll, labor-income and savings tax rates τ^T, τ^w and $\tau^\star$. It runs a balanced budget and thus rebates all tax revenues to households in every period. The tax base for direct household taxes (on labor-income and savings) depends on sector-specific variables (wages and assets), hence is rebated in lump-sum and uniformly at the sector level. In contrast, the payroll tax is indirectly borne by all households (since it is passed through to tradables prices) and is thus rebated in a lump-sum, uniform manner to all households in the economy.[6] We show in Section A.1 that the implied sector-k rebate is

[6] While other ways of rebating tax revenues are conceivable, ours eliminates any intersectoral redistribution via the tax and transfer system that may cloud the interpretation of our results.

given by:

$$T_{k,t} = \underbrace{\tau^w w_{k,t} n_{k,t}}_{\text{rebate of income tax}} + \underbrace{\varphi \tau^T w_{T,t} n_{T,t}}_{\text{rebate of payroll tax}} + \underbrace{\frac{\tau^\star a_{k,t+1}}{R_t}}_{\text{rebate of savings tax}}, \quad k = T, N \tag{22}$$

We assume (without loss of generality) that the monetary authority directly controls the sequence of real interest rate $\{R_t\}_{t=0}^{\infty}$. The optimal tax rates and optimal interest-rate sequence in response to export shocks are computed in Section 5 below.

3 Equilibrium

3.1 Market clearing and capital flows

On the goods market, there are four market-clearing conditions, depending on whether the goods are produced at Home or in the Foreign economy and whether they are tradable or non-tradable. Focusing on the Home economy (the Foreign economy operates symmetrically), the market-clearing condition for Home tradables is:

$$my_{H,t} = mc_t(H) + (1-m)\,c_t^*(H), \tag{23}$$

where m is the relative size of the Home economy and the consumption demands $c_t(H)$ and $c_t^*(H)$ are given by equations (2) and (5) above. On the other hand, the market-clearing condition for Home-produced non-tradable goods is:

$$my_{N,t} = mc_t(N), \tag{24}$$

where $c_t(N)$ is given by (2).

Assume all international borrowing and lending takes place in the form of bonds denominated in units of Foreign consumption.[7] Let a_{t+1}^* denote the Foreign holdings of Foreign bonds per capita, so that total Foreign end-of-period savings are $(1-m)a_{t+1}^*/R^*$. On the other and, total Home end-of-period net savings are $ma_{t+1} \times (\vartheta/R_t)$, where $a_{t+1} = \varphi a_{T,t+1} + (1-\varphi)a_{N,t+1}$ are per capita bond holdings. Those net savings are entirely invested in Foreign bonds, and bond market clearing must hold, so that:

$$m\frac{\vartheta}{R_t}a_{t+1} + Q_t(1-m)\frac{1}{R^*}a_{t+1}^* = 0$$

In the next period, all Foreign bonds pay off 1 units of the Foreign consumption basket, thereby delivering $-(1-m)Q_{t+1}a_{t+1}^*$ units of Home consumption; these are shared between the $m\vartheta$ surviving households, each of whom holds a_{t+1} claims so that

$$m\vartheta a_{t+1} = -(1-m)Q_{t+1}a_{t+1}^*$$

Combining the latter two expressions gives the real interest rate parity equation (under perfect foresight w.r.t. aggregates, our maintained assumption):

$$R_t = \frac{Q_{t+1}}{Q_t}R^* \tag{25}$$

[7] Since we will focus our analysis on the case where the Home economy is small relative to the Foreign economy, this assumption amounts to ruling out home bias in asset portfolios (i.e., Foreign bonds overwhelm the bond market).

The per capita net foreign assets (NFA) of the Home economy at the end of time t is a_{t+1}/R_t. We verify in Appendix A.3 that it evolves as:

$$\frac{a_{t+1}}{R_t} = a_t + \underbrace{p_{H,t}y_{H,t} + p_{N,t}y_{N,t}}_{\text{real Home income}} - c_t$$

3.2 Small open economy limit

From now on, we focus on the case where the Home economy becomes arbitrarily small relative to the Foreign economy, i.e. $m \to 0$. Substituting (2), (5), (7) and (8) into (23), taking account of the fact that $\alpha = (1-m)\lambda$ and $\alpha^* = m\lambda$ as stated above, dividing both sides by m, and letting $m \to 0$, we can rewrite the market-clearing condition for Home-produced tradables as follows:

$$\varphi n_{T,t} = \frac{\phi}{p_{H,t}} \left\{ (1-\lambda) \left[\varphi c_{T,t} + (1-\varphi) c_{N,t}\right] + \lambda \frac{Q_t c_t^*}{1+\tau_t^X} \right\}, \tag{26}$$

where $\varphi c_{T,t} + (1-\varphi)\, c_{N,t} = c_t$ is aggregate Home consumption per capita. Similarly, substituting (2) and (8) into (24) and dividing by m gives the following expression for the market-clearing condition for Home-produced non-tradables:

$$(1-\varphi) n_{N,t} = \frac{1-\phi}{p_{N,t}} \left[\varphi c_{T,t} + (1-\varphi)\, c_{N,t}\right] \tag{27}$$

Other properties of the SOE limit allow simplifying or specifying some of the expressions above. First, in the SOE limit, the Home economy's NFA becomes immaterial to the world economy; we thus need only keep track of its NFA per capita, a_{t+1}/R_t, disregarding its implications for the world asset market equilibrium. Second, we know from Galí and Monacelli (2005) and Farhi and Werning (2012) that for an SOE the optimal payroll tax under a unit elasticity of substitution between Home and Foreign tradables is

$$\tau^T = \frac{\lambda}{1-\lambda} > 0, \tag{28}$$

which is the value we will henceforth assume (for both the Home and Foreign economies, to maintain international price symmetry).[8]

Finally, under the SOE assumption, there are no spillovers from the Home to the Foreign economy, so Foreign prices can be taken as exogenous and unresponsive to domestic shocks and policies. Appendix D shows that, under these assumptions, changes in the (log) real exchange rate are only driven by changes in home prices, or equivalently wages (by equation (9)-(10)):

$$\ln Q_t = (\lambda^{-1} - \phi) \ln (1-\lambda) - \left(\frac{1-\phi}{\phi\lambda}\right) \ln w_{N,t} - \left(\frac{1-\lambda}{\lambda}\right) \ln w_{T,t} \tag{29}$$

The intuition underlying (29) is that low wages in either sector produce low relative prices, which,

[8] To be clear, $1+\tau^T = 1/(1-\lambda)$ is necessary for the constrained efficiency of the steady state in the Home economy, without which the central bank would face a time-inconsistency problem. Intuitively, a different level of the payroll tax would not achieve the efficient level of steady-state terms-of-trade manipulation, prompting the central bank to (temporarily) achieve this manipulation upon reoptimisation. We then impose that $\tau^{T*} = \tau^T$ so that all goods prices are internationally symmetric in the steady state (that is, $Q = 1$, $p_H = p_H^* = p_F = p_F^*$, and $p_N = p_N^*$).

under Home bias, depreciates the relative price of Home consumption (i.e., raises Q_t).

3.3 The dynamics of inequality

A distinctive feature of our model is the generation of cross-sectional heterogeneity both across sectors —due to unequal exposure to aggregate export shocks— and within sectors —due to uninsured cyclical idiosyncratic risk as well as gradual asset accumulation over household lifetime. In this section, we construct a welfare-based measure of household inequality that integrates those three dimensions of inequality into a single index. We will use this index to summarise how inequalities respond to export shocks, together with the other model aggregates.

To construct our welfare-based inequality index, we take the perspective of a utilitarian social planner who assigns equal weights to the lifetime utility of all households born at time $s \leq 0$ and a weight β^t to the lifetime utility of households born at time $t > 0$.[9] Under these assumptions, we can aggregate households' utility flows as follows. First, we break down the intertemporal utility of the planner $\mathbb{W}_t$ at time 0 (say) as follows:

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t (\varphi \mathbb{U}_{T,t} + (1-\varphi)\mathbb{U}_{N,t}) \tag{30}$$

where $\mathbb{U}_{k,t}$, is the total time-t flow utility brought by all households working in sector k:

$$\mathbb{U}_{k,t} = \underbrace{(1-\vartheta)\sum_{s=-\infty}^{t} \vartheta^{t-s} \int \left(-e^{-c^s_{k,t}(i)}\right) di}_{\text{heterogeneous within sector}} \underbrace{-\kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2}_{\text{homogeneous within sector}}, \quad k = T, N$$

The variables $n_{k,t}$ and $\Pi^w_{k,t}$ are common across all households working in sector k (see Section 2.2) and hence do not raise any distributional concerns. However, the individual consumption utility flows $-e^{-c^s_{k,t}(i)}$ are heterogeneous across and within cohorts and must be aggregated accordingly.[10] While this aggregation problem is, in general, not tractable, we again leverage the CARA-Normal structure of the model to obtain analytical expressions for within-sector consumption utility dispersion. We show in Appendix E that the latter expression can be simplified into

$$\mathbb{U}_{k,t} = -e^{-c_{k,t}}\Sigma_{k,t} - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2, \quad k = T, N$$

where $-e^{-c_{k,t}}$ is the utility of average consumption in sector k and the index $\Sigma_{k,t}$ measures the disutility to the planner associated with consumption dispersion within the sector (since utility is negative, an increase in $\Sigma_{k,t}$ is welfare-worsening). Intuitively, the term $-e^{-c_{k,t}}$ captures the welfare impact of average sectoral consumption fluctuations, while $\Sigma_{k,t}$ captures the welfare impact of fluctuations in consumption inequality within sector k. Crucially, within-sector utility dispersion is an endogenous state variable with

[9] That is, the planner discounts the intertemporal welfare of future generations at the same rate as individuals discount future utility flows. If it were not the case, the planner would have time-inconsistent preferences, as shown by Calvo and Obstfeld (1988).

[10] Intuitively, the $\int$-operator aggregates utility flows within cohort while the $\sum$-operator aggregates them across cohorts, where $(1-\vartheta)\vartheta^{t-s}$ is the size cohort s.

law of motion:

$$\Sigma_{k,t} = e^{\frac{1}{2}(\sigma^c_{k,t})^2}\left[(1-\vartheta)e^{\mu_t a_{k,t}} + \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}}\Sigma_{k,t-1}\right] \tag{31}$$

Equation (31) has a straightforward interpretation. First, within-sector consumption-utility dispersion is slow-moving; this is because individual consumption depends on cash-on-hand, while the distribution of wealth is itself an endogenous state variable.[11] Second, within-sector dispersion has two components, namely *between-cohort* and *within-cohort*. Between-cohort dispersion is reflected in the presence of $a_{k,t}$ in equation (31): since households gradually accumulate wealth over their lifetime, more sector-level wealth is associated with greater dispersion of wealth – hence consumption – between the average members of different cohorts. Within-cohort dispersion is driven by the distribution of idiosyncratic shocks and is reflected in the term $\sigma^c_{k,t}$ in (31).

Further aggregating utility flows, now *across sectors*, Appendix E shows that the total utility flow in equation (30) can be written as

$$\varphi\mathbb{U}_{T,t} + (1-\varphi)\mathbb{U}_{N,t} = -e^{-c_t}\Sigma_t - \kappa\left[\varphi n_{T,t} + (1-\varphi)n_{N,t}\right] - \frac{\Psi}{2}\left[\varphi\left(\ln\Pi^w_{T,t}\right)^2 + (1-\varphi)\left(\ln\Pi^w_{k,t}\right)^2\right]$$

where $-e^{-c_t}$ is the consumption utility of the average household in the Home country and Σ_t measures the overall dispersion of consumption utility across the population and is given by:

$$\Sigma_t = \varphi e^{-(1-\varphi)\left(c_{T,t}-c_{N,t}\right)}\Sigma_{T,t} + (1-\varphi)e^{\varphi\left(c_{T,t}-c_{N,t}\right)}\Sigma_{N,t} \tag{32}$$

The latter expression makes clear that there are two components to the utility loss associated with consumption dispersion, namely, *within-sector* dispersion – captured by the $\Sigma_{k,t}$s discussed above – and *between-sector* dispersion – captured by the difference in sectoral consumption levels $|c_{T,t} - c_{N,t}|$.[12] That is, even if idiosyncratic risk were fully insured (in which case $\Sigma_{T,t} = \Sigma_{T,t} = 1$ at all times), households in different sectors would still be unequally exposed to the aggregate export shock, so that the shock would still lead to socially costly differences in mean sectoral consumption levels. Both considerations enter the planning problem and, hence, affect optimal monetary policy, as we discuss below.

4 Aggregate and distributional dynamics under alternative monetary policy regimes

Before characterising optimal policy, we compare aggregate and distributional outcomes under two common popular monetary policy regimes for small open economies, namely, *(flexible) inflation targeting* and a *nominal exchange-rate peg*. We start by providing a brief summary of all equilibrium conditions and the implied model steady state.

[11]The consumption function is derived in Appendix C.

[12]For given $\Sigma_{k,t}$s, Σ_t is continuous and strictly convex in $c_{T,t} - c_{N,t}$ and reaches a minimum at $c_{T,t} = c_{N,t}$. Hence, both upward and downward deviations from symmetric sectoral consumption are socially costly.

4.1 Summary of aggregate dynamics

The "aggregate demand" block of the model is made up of the sectoral Euler equations (16) together with the recursions (20)-(18) and the dynamics of income risk (14). The sectoral Euler equations also feature end-of-period sectoral bond holdings, which evolve according to equations (38)-(39) in Appendix A.2.

The "aggregate supply" block of the model features the two wage Phillips curves (21), where the two wage inflation rates are (tautologically) tied to the evolution of sectoral real wages as follows:

$$\Pi_{T,t}^{w} = \left(\frac{w_{T,t} w_{N,t-1}}{w_{T,t-1} w_{N,t}} \right) \Pi_{N,t}^{w} \tag{33}$$

Next, the market-clearing conditions for Home tradables and nontradables are given by (26)-(27), with prices given by (9)-(10). Finally, the real exchange rate is tied to interest rates and wages via (25) and (29), while the dynamics of inequality are given by equations (31)–(32). All the structural equations listed in this section will enter the planning problem as constraints faced by the social planner.

4.2 Steady state

We undertake our optimal policy analysis around a steady state that features (i) zero wage inflation, (ii) symmetric wages, per-capita consumption and per-capita labor supply across sectors and countries, and (iii) symmetric prices across countries. The present section derives the restrictions that any steady state with these properties must satisfy. Section 5 below will impose further conditions related to the planner's optimal choice of tax instruments, thereby restricting candidate steady states to a unique steady state of the Ramsey plan consistent with (i)-(iii).

First, we parameterise the share of households working in the tradables sector (φ) in such a way that steady-state wages are equalised across sectors, i.e., $w_T = w_N \equiv w$.[13] As shown in Appendix D, this requires that

$$\varphi = \frac{\phi\,(1-\lambda)}{1-\lambda\phi} \in (0,\phi) \tag{34}$$

Intuitively, symmetric steady-state wages across sectors ($w_T = w_N$) together with marginal cost pricing (equations (9)-(10)) and an efficiently positive payroll tax (28) on tradables (but no payroll tax on non-tradables) imply that the steady-state price of tradables is higher than that of non-tradables, namely, $p_H = p_N/(1-\lambda) > p_N$. Such asymmetric prices twist down the steady-state consumption and production of tradables relative to their share in retailers' production function (the parameter ϕ in equation (1)).[14] At symmetric sectoral wages, symmetric (zero) inflation also implies symmetric per capita consumption ($c_T = c_N \equiv c$) and ultimately labor supply ($n_T = n_N \equiv n$) across sectors (by (21)).

Next, assuming that the Foreign economy applies the same payroll taxes as the Home economy – so that relative domestic prices are the same in the Foreign economy as in the Home economy – pins down

[13] Our model takes φ as exogenous and wages as endogenous. However, setting φ so that $w_T = w_N$ can be interpreted as the outcome of free inter-sectoral worker mobility in the long run, whereby φ would endogenously adjust until wage differences across sectors are eliminated.

[14] While the Home economy produces and consumes fewer tradables than they would under symmetric prices, the consumption *shares*, which feature relative prices, are not affected and still given by $1-\phi$ (for nontradables), $(1-\lambda)\phi$ (for Home tradables) and $\lambda\phi$ (for Foreign tradables) – see (1), where $\alpha \to \lambda$ in the SOE limit.

all steady state prices and wages. As shown in Appendix D, we get:

$$p_N = p_N^* = w = w^* = (1-\lambda)^{\phi}, \qquad p_H = p_F^* = (1-\lambda)^{\phi-1}, \qquad \text{and} \quad Q = 1$$

Next, by equations (21) and (26)-(27) steady state consumption and labor supply per capita are:

$$c = \ln\left[\frac{(1-\tau^w)\,w}{\kappa\mathcal{M}_w}\right] \qquad \text{and} \qquad n = \frac{1-\lambda\phi}{(1-\lambda)^{\phi}}c$$

so that both depend on the labor-income tax rate τ^w; the latter will be set optimally in Section 5. The steady state values of $y_H = \varphi n$ and $y_N = (1-\varphi)n$ follow.

At the steady-state value of $Q = 1$, real interest-rate parity (25) and the fact that $R^* = 1/\beta$ imply that $\beta R = 1$. Moreover, the steady-state counterpart of (20) implies that $\sigma^c = \sigma^y$, while that of (18) implies that $\mu = 1 - \beta\vartheta(1+\tau^\star)$. Plugging these relations into the steady state counterpart of the sectoral Euler equations (17) and aggregating, we can write steady-state aggregate bond holdings by the Home economy $a = \varphi a_T + (1-\varphi)a_N$ as follows:

$$a = \vartheta\frac{\frac{1}{2}(\sigma^y)^2 - \ln(1+\tau^\star)}{(1-\vartheta)\left[1-\vartheta\beta(1+\tau^\star)\right]} \tag{35}$$

The latter expression illustrates the competing effects of the precautionary motive (indexed by σ^y) and the savings tax ($\tau^\star$) in determining the demand for savings. At a zero savings tax, the precautionary motive induces positive steady-state bond holdings; such savings can be mitigated and even eliminated with an appropriate savings tax. To ensure zero steady-state net foreign assets, we must set the savings tax to

$$\tau^\star = e^{\frac{1}{2}(\sigma^y)^2} - 1$$

We will merely impose the latter value of the savings tax in the current section, and will verify in the next section that it corresponds to the *optimal* savings tax along the steady state of the Ramsey plan.[15]

4.3 Flexible inflation targeting versus nominal exchange-rate peg in HANK

We are now in a position to compare aggregate and distributional outcomes under alternative monetary-policy regimes, and consider two such regimes for conciseness.

Flexible inflation targeting. Under flexible inflation targeting, the central bank merely obeys a Taylor rule of the form

$$i_t = R - 1 + \phi_\pi(\Pi_t - 1)$$

where i_t denotes the nominal interest rate and Π_t denotes CPI inflation (so that $R_t = (1+i_t)/\Pi_{t+1}$). As shown in Appendix D, CPI inflation in our model is given by:

$$\Pi_t = \left(\Pi_{N,t}^w\right)^{\frac{1-\phi}{1-\lambda\phi}}\left(\Pi_{T,t}^w\right)^{\frac{\phi(1-\lambda)}{1-\lambda\phi}}\left(\frac{Q_t}{Q_{t-1}}\right)^{\frac{\phi\lambda}{1-\lambda\phi}} \tag{36}$$

[15] As we discuss further below, the reason for this level of the tax to be the optimal one is that, under the Blanchard-Yaari demographics, nonzero steady-state net foreign assets generate inefficient inequalities between cohorts in the steady state.

Equation (36) summarises the determinants of CPI inflation in our economy, namely, "internal" inflation from nontradables and tradable wage inflation $\Pi^w_{T,t}$ and $\Pi^w_{N,t}$ – which mechanically push up the price of Home-produced goods according to (9)-(10) –, as well as an "imported" inflation component coming from changes in the Home-currency prices of imported tradables and captured by changes in the real exchange rate Q_t/Q_{t-1}. An export contraction can affect all three terms on the RHS of (36), by (i) directly contracting the tradables sector, (ii) contracting the nontradables sector through intersectoral spillovers, and (iii) raising import prices due to currency depreciation. Under inflation targeting, how much the shock affects those three components ultimately depends on how the central bank responds, as summarised by Taylor rule parameter ϕ_π. We set the latter to $\phi_\pi = 1.5$ in our simulations.

Nominal exchange-rate peg. Under a peg, the nominal exchange rate $\mathcal{E}_t$ is fixed at its steady state value $\overline{\mathcal{E}}$. By the mere definition of the real exchange rate and the fact that the Foreign price level P^* is exogenous and constant for the SOE, we have $Q_t = \overline{\mathcal{E}}P^*/P_t$, so that this policy implies CPI *deflation* in response to the devaluation caused by the export shock, i.e.

$$\Pi_t = \frac{P_t}{P_{t-1}} = \frac{Q_{t-1}}{Q_t} < 0$$

Substituting the latter expression into (36) are rearranging, we find the required *internal* deflation consistent with the nominal exchange-rate peg to be

$$\left(\Pi^w_{N,t}\right)^{1-\phi}\left(\Pi^w_{T,t}\right)^{\phi(1-\lambda)} = \frac{Q_{t-1}}{Q_t}$$

Parameterisation. When computing impulse-response functions, we impose the following parameters (our results are robust to reasonable variations around this central scenario). The tradables share in consumption is set to $\phi = 0.5$ and the openness parameter to $\lambda = 0.4$, implying an import share of $\lambda\phi = 0.2$. For preferences, we set β such that $R = \beta^{-1} = 1.04$, Ψ such that the slope of the wage Phillips curve is 0.05 at the gross wage markup $\mathcal{M}_w = 1.05$, and we normalise κ such that steady state consumption is $c = 1$. We set σ^y to 0.2, and the cyclicality parameter to $\Theta = 5$ following Acharya and Challe (2025). ϑ is set to .85 following Nisticò (2016)'s and Farhi and Werning (2019)'s interpretation.

Impulse responses Figure 1 plots the responses of relative prices (gross consumer price inflation Π_t, the real interest rate R_t and the real exchange rate Q_t) as well as sectoral variables (outputs, wages, inflation, consumption and asset levels, inequalities), to a persistent export contraction (a fall in $c^*_t/(1+\tau^X_t)$) depending on whether the central bank implements flexible inflation targeting or an exchange-rate peg. Because goods markets are competitive, sectoral wage inflation equals sectoral price inflation and thus reflects the domestic sources of CPI inflation. On the other hand, the real exchange rate reflects changes in the price of imports in terms of Home consumption (by (7)). Both regimes involve substantial output and consumption losses for the Home economy (Panels (d) and (e)), as well as substantial distributional costs (Panels (e) and (i)). Indeed, both between- and within-sector consumption inequalities persistently rise after the shock. The increase in *between-sector* inequality is due to the fact that tradables workers are disproportionately hit by the export contraction, regardless of the monetary policy response; to the

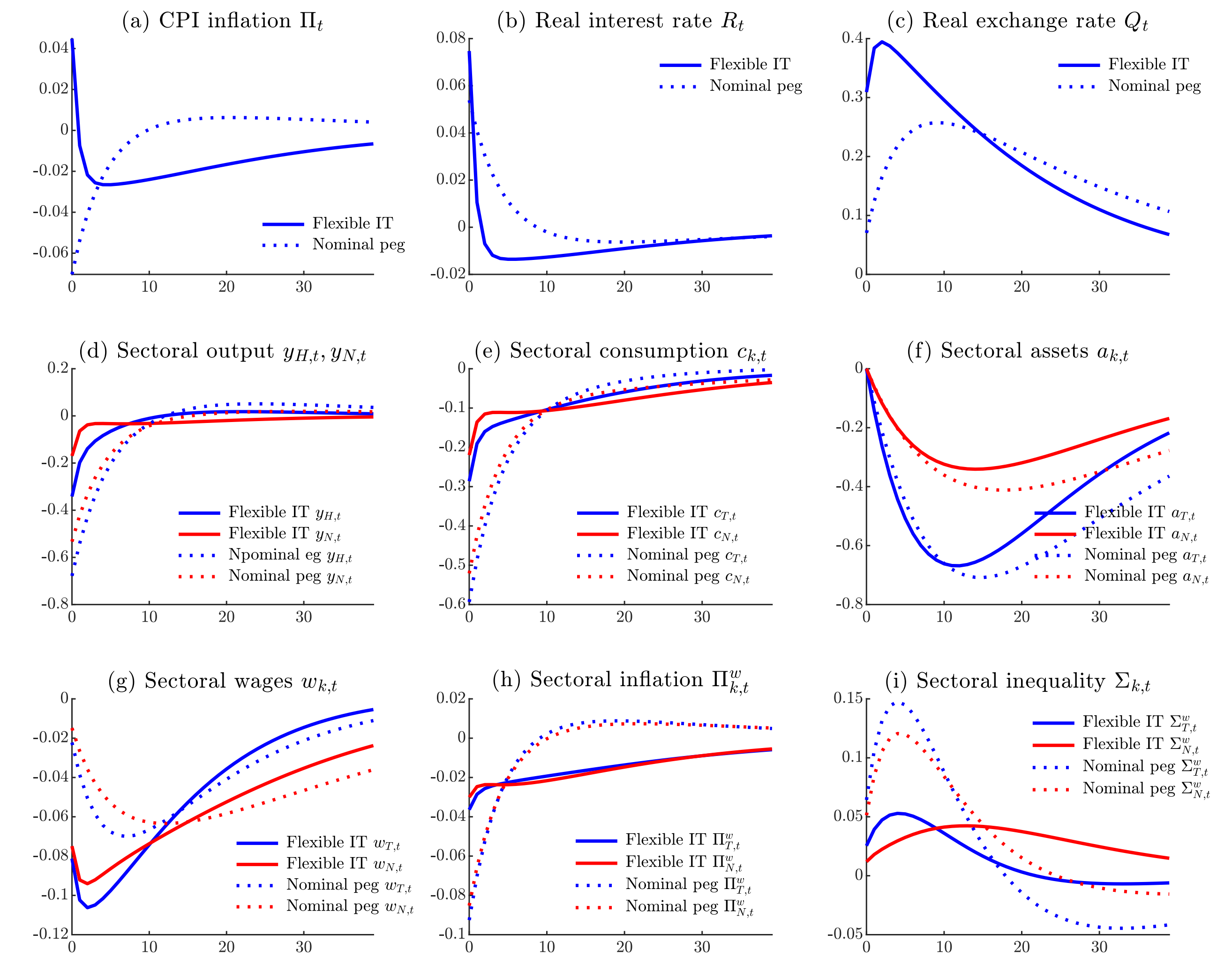

Figure 1: **Aggregate and distributional dynamics under alternative monetary policy regimes.** The solid lines show the economy's response to an export contraction under *flexible inflation targeting*, while the dashed lines show the same responses under a *nominal exchange-rate peg*. In panels (d) to (i), blue is for the tradables sector and red is for the nontradables sector.

extent that monetary policy involves a single instrument that "lifts all boats", it has limited ability to mitigate the asymmetric exposure of households across sectors to the shock. On the other hand, the increase in *within-sector* inequality has two distinct drivers here, namely (i) cyclical idiosyncratic risk and (ii) gradual cohort-level asset accumulation. Because of the first factor, the effect of the shock on the tradable sector (holding monetary policy unchanged) is to increase the dispersion of earnings within that sector, which directly contributes to increasing consumption inequalities there. To the extent that the contraction is persistent, so that tradables workers' consumption *risk* (and not only inequality) rises, this also strengthens tradables workers' precautionary motive, prompting them to cut their consumption of both goods, thereby spilling over to the nontradable sector and raising consumption risk and inequality also in the latter sector. The second driver of within-sector inequality is *between cohorts*. Indeed, the export contraction is partly buffered by foreign borrowing (see Panel (f)), which is associated with a gradual decumulation of assets that generates consumption dispersion between members of different cohorts –see equation (31). Ultimately, both cyclical earnings risk and cohort-specific assets contribute to the protracted rise in within-sector inequality apparent in Panel (i).

The overall impact of the shock on aggregates and inequality is mediated by monetary policy, and hence by the regime under which the central bank operates. In this respect, maintaining an exchange rate peg is particularly costly in both respects. This is because keeping the peg requires fending off downward pressure on the Home currency's value by sharply contracting domestic consumption (Panel (e)). Ultimately, the exchange-rate peg kills imported inflation altogether, but at the cost of substantial internal deflation (Panel (h)) and ultimately a contraction in the consumer price index (Panel (a)). By contrast, flexible inflation targeting tolerates *some* imported inflation, ultimately allowing the CPI to temporarily rise thanks to limited internal deflation. As a result, consumption and output levels contract less, and the rise in sectoral consumption inequality is much less pronounced.

4.4 Intersectoral spillovers from precautionary savings

In our economy, when the tradables sector is hit by an export contraction, idiosyncratic income risk rises for workers in that sector, triggering a precautionary saving response that contracts the demand for *both tradables and nontradables goods*, ultimately propagating the recession to the nontradables sector. While such intersectoral spillovers are at work in the dynamics depicted in Figure 1, they are obscured by the monetary policy responses and other general-equilibrium effects —both of which do not operate in the same way across policy regimes.

To isolate the strength of intersectoral spillovers, we perform the following experiment. First, we construct two alternative benchmark economies, namely an *acyclical risk* model and a *Representative-Agent* New Keynesian (RANK) model. The first benchmark is similar in every respect to the baseline HANK model except that risk is assumed acyclical (i.e., $\Theta = 0$ in (15)) rather than countercyclical ($\Theta > 0$). By construction, that model rules out idiosyncratic risk as a source of cyclical fluctuations in precautionary savings, though fluctuations in *inequality* are still possible between sectors (due to their asymmetric exposure to the export shock) and within sectors (due to the response of sectoral asset wealth and its impact on inter-cohort dispersion, see equation (31)).

The second benchmark model is the comparable RANK model. The latter mintains the Blanchard-Yaari demographics —to ensure stationarity of the Home economy's net foreign assets despite imperfect international risk sharing— as well as imperfect intersectoral workers' mobility —leading to heterogeneous hours worked across sectors. However, in RANK, idiosyncratic risk is shut down altogether ($\sigma^y = 0$), and a "cohort head" pools all cohort income and optimally chooses labour supply, consumption and assets of every cohort member at every point in time. For the RANK benchmark, we set the savings tax $\tau^\star$ to zero, consistent with zero steady state net foreign assets in the absence of idiosyncratic risk (see equation (35)).

The detailed derivation of the RANK benchmark is presented in Appendix G. In a nutshell, in RANK the representative cohort-s household solves

$$\max \mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left\{ -e^{-c_t^s} - \kappa \left(\varphi n_{T,t} + (1-\varphi)\, n_{N,t}\right) - \frac{\Psi}{2} \left[\varphi \left(\ln \Pi^w_{T,t}\right)^2 + (1-\varphi)\left(\ln \Pi^w_{N,t}\right)^2 \right] \right\}$$

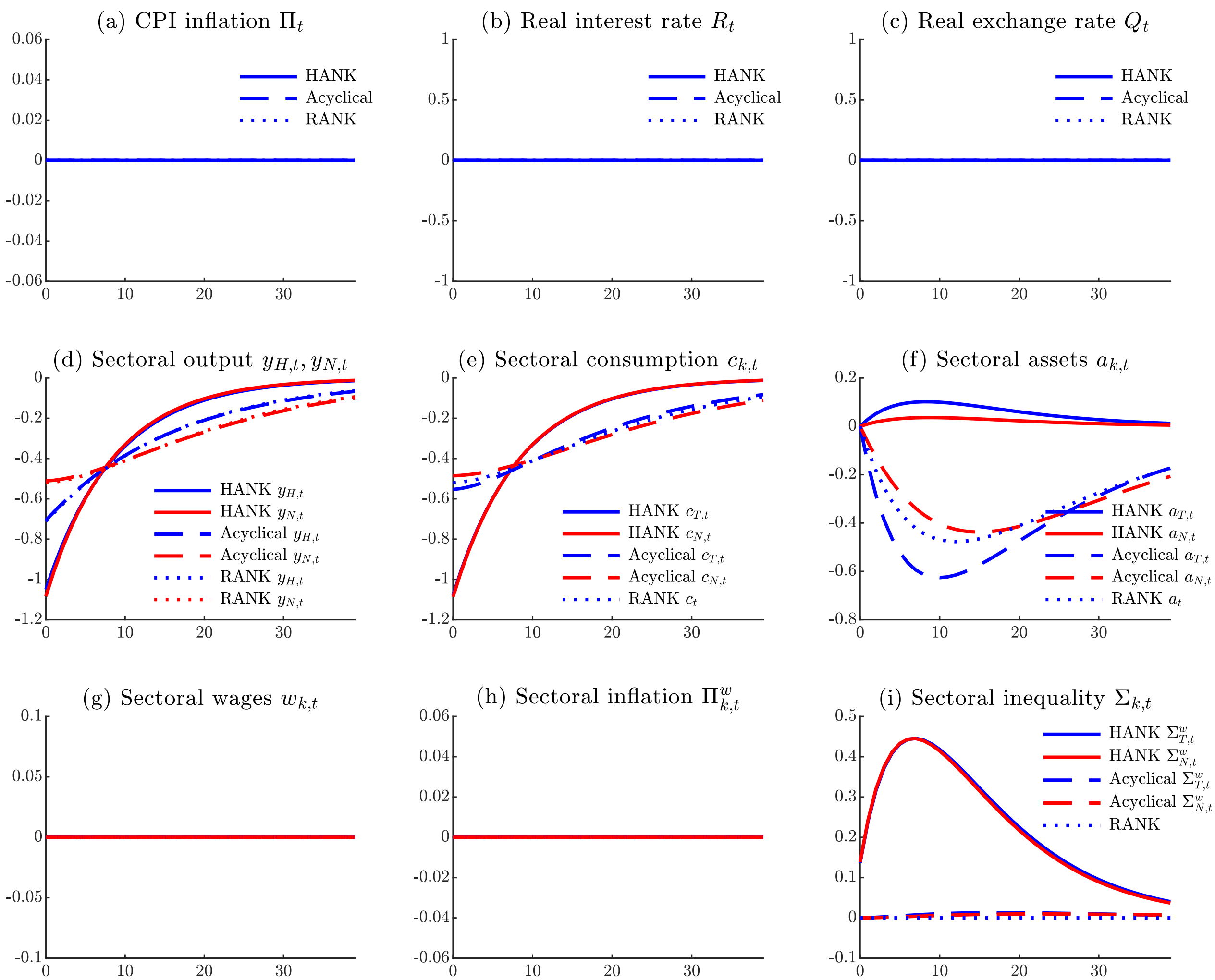

Figure 2: **Isolating intersectoral spillovers: Alternative models under full price rigidity.** In all three models, the slope of the wage Phillips curve is zero (i.e., $\Psi = \infty$ in (21)) and the central bank fully stabilises inflation as well as the nominal and real exchange rates (these are equivalent objectives, given constant internal wages and prices). The solid lines show the response of the HANK model with cyclical risk, the dashed lines the response of the HANK model with acyclical risk, and the dotted lines the response of the comparable RANK benchmark.

subject to

$$c_t^s + \frac{\vartheta}{R_t} a_{t+1}^s = a_t^s + \underbrace{p_{H,t} y_{H,t} + p_{N,t} y_{N,t}}_{\text{national income}}$$

Finally, we compare all models around a steady state with the same (zero) net foreign assets and thus adjust the savings tax accordingly given (35)). Hence, the tax is set to $\tau^\star = e^{\frac{1}{2}(\sigma^y)^2} - 1$ in the acyclical risk model and to $\tau^\star = 0$ in RANK.

To measure the strength of intersectoral spillovers due to cyclical precautionary savings, we must compare the responses of the HANK model with the two benchmark economies (where cyclical precautionary savings are shut down by construction) *holding monetary policy identical across model specifications* — else heterogeneous monetary policy responses across specifications would contaminate the comparison. We thus focus on a special case in which all prices are constant, so that intersectoral spillovers manifest only through quantities. This is achieved by assuming that (i) the slope of the New Keynesian Phillips curves is zero (i.e., $\Psi = \infty$ in (21)) and (ii) monetary policy fully stabilises the nominal exchange rate —or equivalently the real exchange rates or CPI inflation (with constant home-produced tradables and nontradables prices, stabilising one of the three variables directly stabilises the other two).

The outcome of this comparison is shown in Figure 2. Under the assumptions of flat Phillips curves and fixed exchange rates, all inflation rates are constant, and so is the real interest rate. Intersectoral spillovers are evident in Panels (d) and (e), which plot the responses of sectoral output and consumption levels across model specifications. Crucially, the adjustment of output levels is identical across the two benchmarks and much smaller than in the HANK model; this highlights the importance of cyclical precautionary savings as an amplifier of the export contraction. Crucially, moving from the RANK (or acyclical-risk) benchmark to the HANK model produces a *larger relative decline in nontradables output than in tradables output*. This is because, in the two benchmark models, the nontradables sector is partly sheltered by the asymmetric nature of the shock, which hits the tradables sector directly and the nontradables sector only indirectly. In contrast, in HANK, the shock directly propagates to the nontradables sectors via the drop in consumption demand of tradables workers. Ultimately, in equilibrium, the falls in sectoral output and consumption levels in HANK are large and almost symmetric. The importance of intersectoral spillovers is also evident from the rise in inequalities induced by the shock (Panel (i)). Under constant domestic and import prices, the near-symmetric fall in sectoral output levels implies that sectoral inequality rise just as much in the nontradables sector as it does in the tradables sector.

4.5 HANK versus RANK

We conclude this section by comparing, for a given monetary policy regime (either flexible inflation targeting or an exchange-rate peg), how our baseline HANK economy behaves relative to the RANK benchmark —away from constant prices this time. Figures 3 and 4 display this comparision. By construction, the RANK model implies symmetric consumption and asset levels both within and across sectors (Panels (e), (f) and (i)), despite asymmetric sectoral output levels (Panel (d)). More subtly, it also implies symmetric sectoral wages (panel (g)) and wage inflation rates (h) (to first order) *despite the lack of intersectoral workers' mobility*. The reason is that our RANK benchmark implies symmetric marginal rates of substitution between consumption and leisure across sectors, which, combined with (21) and (33), imply symmetric wages and inflation rates.[16] While this extreme outcome follows from linear labor disutility, it highlights that equal consumption mechanically compresses dispersion in MRS across sectors – and ultimately in wages and inflation rates. Finally, to first order, within-sector inequality in RANK does not depart from its steady state value of $\Sigma_k = 1$. This is because (i) there is (trivially) no inequality due to idiosyncratic risk in RANK and (ii) at zero idiosyncratic risk ($\sigma^y = 0$), assets $a_{k,t}$ disappear from equation (31) to first order, leaving no role for between-cohort consumption dispersion due to heterogenous asset holdings. Under both regimes, the real depreciation is stronger in HANK than in RANK, but the gap between the two economies is, in general, larger for the exchange-rate peg. Consistent with Figure 1, the social cost of the peg in terms of sectoral consumption inequality is particularly large (Panel (i)).

[16] Formally, log-linearising (21) around the steady state of the Ramsey plan gives $\pi^w_{k,t} = \Gamma(\widehat{c}_t - \widehat{w}_{k,t}) + \beta\pi^w_{k,t+1}$, for $k = T, N$, where the Phillips curves' slope Γ is positive but small (< 1). Combining this with the log-linear counterpart (33) implies that wage differentials evolve as $\widehat{w}_{T,t} - \widehat{w}_{N,t} = (1-\Gamma)^{-1}(\widehat{w}_{T,t-1} - \widehat{w}_{N,t-1})$, so that $w_{T,t} = w_{N,t}$ in stationary equilibrium.

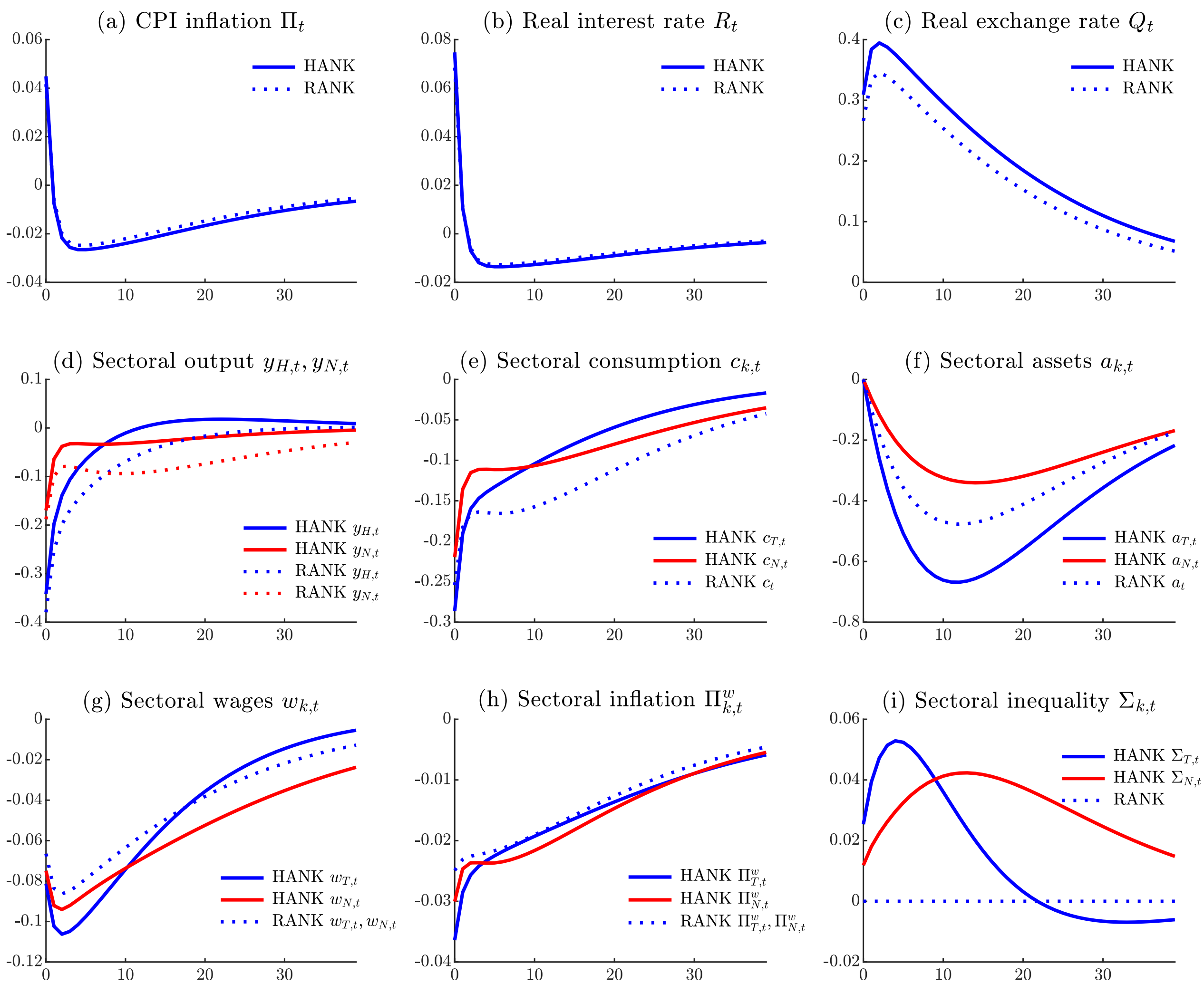

Figure 3: **HANK versus RANK under flexible inflation targeting.** The solid lines show the response of the HANK baseline, and the dashed lines the response of the RANK benchmark. In panels (d) to (i), blue is for the tradables sector and red is for the nontradables sector.

5 Optimal fiscal and monetary policy

Having characterised the dynamic response of the economy under ad hoc policy regimes, we now compare those to the corresponding outcomes under optimal fiscal and monetary policy, assuming that the social planner (i) sets the tax rates optimally to ensure the constrained efficiency of the steady state, and (ii) optimally adjusts the path of real interest rates in response to an export shock – but leaving the tax instruments at their efficient steady-state levels.

5.1 Optimal tax rates and interest-rate path

The social planner maximises $\mathbb{W}_0$ in equation (30), taking as constraints all the equilibrium conditions listed in Section 4.1. The Lagrangian function associated with the planning problem is spelt out in Appendix F.1. We note that the optimal policy problem contains five endogenous state variables: beginning-of-period sectoral assets $a_{k,t}$, the sectoral inequality indices $\Sigma_{k,t}$, and the exogenous state $c^*_t/(1+\tau^X_t)$.

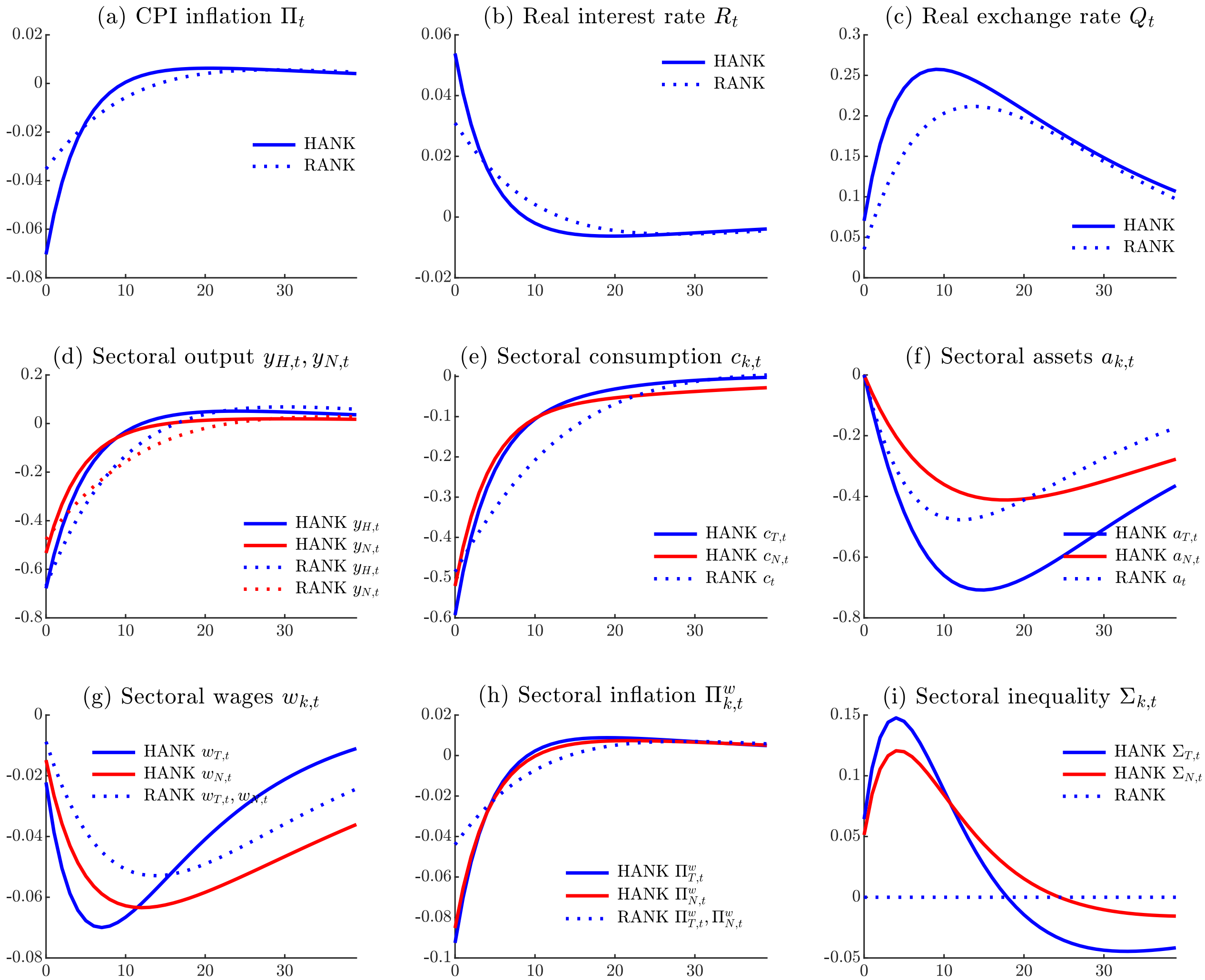

Figure 4: **HANK versus RANK under a nominal exchange-rate peg.** The solid lines show the response of the HANK baseline and the dashed lines the response of the RANK benchmark. In panels (d) to (i), blue is for the tradables sector and red is for the nontradables sector.

Based on the Lagrangian function, we compute the (seventeen) first-order conditions of the planning problem, which are listed in Appendix F.2. Armed with these optimality conditions, we then solve the optimal policy problem in two steps. First, we compute the steady state of the Ramsey plan, imposing that all Lagrange multipliers and endogenous variables are constant in the planner's first-order conditions. This step delivers closed-form expressions for the entire steady state, including the Lagrange multipliers and the optimal tax rates.

Optimal tax rates. As shown in Appendix F.3, the optimal savings and labor-income tax rates satisfy:

$$1 + \tau^{\star} = e^{\frac{(\sigma^y)^2}{2}} \qquad \text{and} \qquad 1 - \tau^w = \mathcal{M}_w \Sigma \left[1 + \frac{(\sigma^y)^2}{(1-\vartheta)(1-\lambda\phi)} \Theta \right] \tag{37}$$

The intuition underlying the optimal tax rates is as follows. First, the optimal savings tax $\tau^{\star}$ is positive whenever labor-earnings risk σ^y is. Intuitively, households have an incentive to accumulate assets for precautionary reasons – see equation (35) –, but positive steady-state sectoral assets generate inefficient between-cohort consumption dispersion – see equation (31) and the discussion that follows. To eliminate this source of welfare loss, the planner must deter asset accumulation until steady-state asset holdings remain at zero despite households' precautionary motive. By equation (35), this is the value of $\tau^{\star}$ in (37).

Second, the optimal labor-income tax τ^w is generically non-zero, due to three forces. First and foremost, even in the absence of any consumption dispersion, the savings tax is equal to $\tau^w = 1 - \mathcal{M}_w < 0$; this is for the usual reason stressed by Erceg et al. (2000) that labor supply must be incentivised to offset the labor market power of wage-setting unions. Next, the mere presence of idiosyncratic risk (implying $\Sigma > 1$), even if it is acyclical ($\Theta = 0$), further lowers the optimal tax rate. This is because consumption dispersion produces a welfare loss that is not internalised by unions, who base their labor-supply decisions on average sectoral consumption. In contrast, the planner understands that sectoral marginal rates of substitution between leisure and consumption are lowered by consumption dispersion. This leads to a greater optimal labor supply than under symmetric consumption, achieved via a lower labor-income tax rate. Last, the optimal labor-income tax rate is affected by the cyclicality of income risk: when risk is countercyclical ($\Theta > 0$), the planner attempts to lower risk by raising output via labor supply; this commands a still lower tax rate – see Acharya et al. (2023) for further discussion.

Optimal interest-rate path. Under optimal payroll, savings, and income tax rates, the steady state of the Ramsey plan is constrained efficient, eliminating any incentive to manipulate the path of the real interest rate to improve welfare absent aggregate shocks. We can then focus on how the export shock alone affects the path of interest rates. We compute the model solution, which includes the optimal sequences of endogenous model variables and Lagrange multipliers, using Schmitt-Grohé and Uribe (2004)'s approach, providing as inputs the planner's first-order conditions (in Appendix F.2) together with our analytically solved steady state (in Appendix F.3). In this step, we make two additional assumptions to ensure stationarity of the Ramsey plan. First, we set the lagged values of the Lagrange multipliers to their steady-state counterparts (rather than zero), as is common in the New Keynesian literature since Woodford (1999). Second, we allow the planner to optimally tax household wealth after the shock has hit but before the path of interest rates is set. As explained in Acharya et al. (2023), this removes the central bank's incentive to use monetary policy for redistribution by exploiting households' unhedged interest-rate exposures, allowing us to focus on monetary policy as an (imperfect) insurer of aggregate and idiosyncratic shocks.

5.2 Optimal monetary policy versus ad hoc policy regimes

Figures 5-6 compare optimal policy and outcomes in your baseline SOE-HANK model with those under the two monetary policy regimes considered above. The first thing to observe is the considerable difference in the extent of monetary policy accommodation between a nominal exchange-rate peg and optimal policy (Figure 5). Indeed, optimal monetary policy commands a persistent fall in real interest rate instead of a rise (Panel (b)), which is manifested by a significant real depreciation on impact (Panel (c)) and spike in consumer price inflation (Panel (a)). Crucially, the optimal monetary policy *considerably mitigates the large hump in within-sector inequality necessary to maintain the peg.* In fact, under the

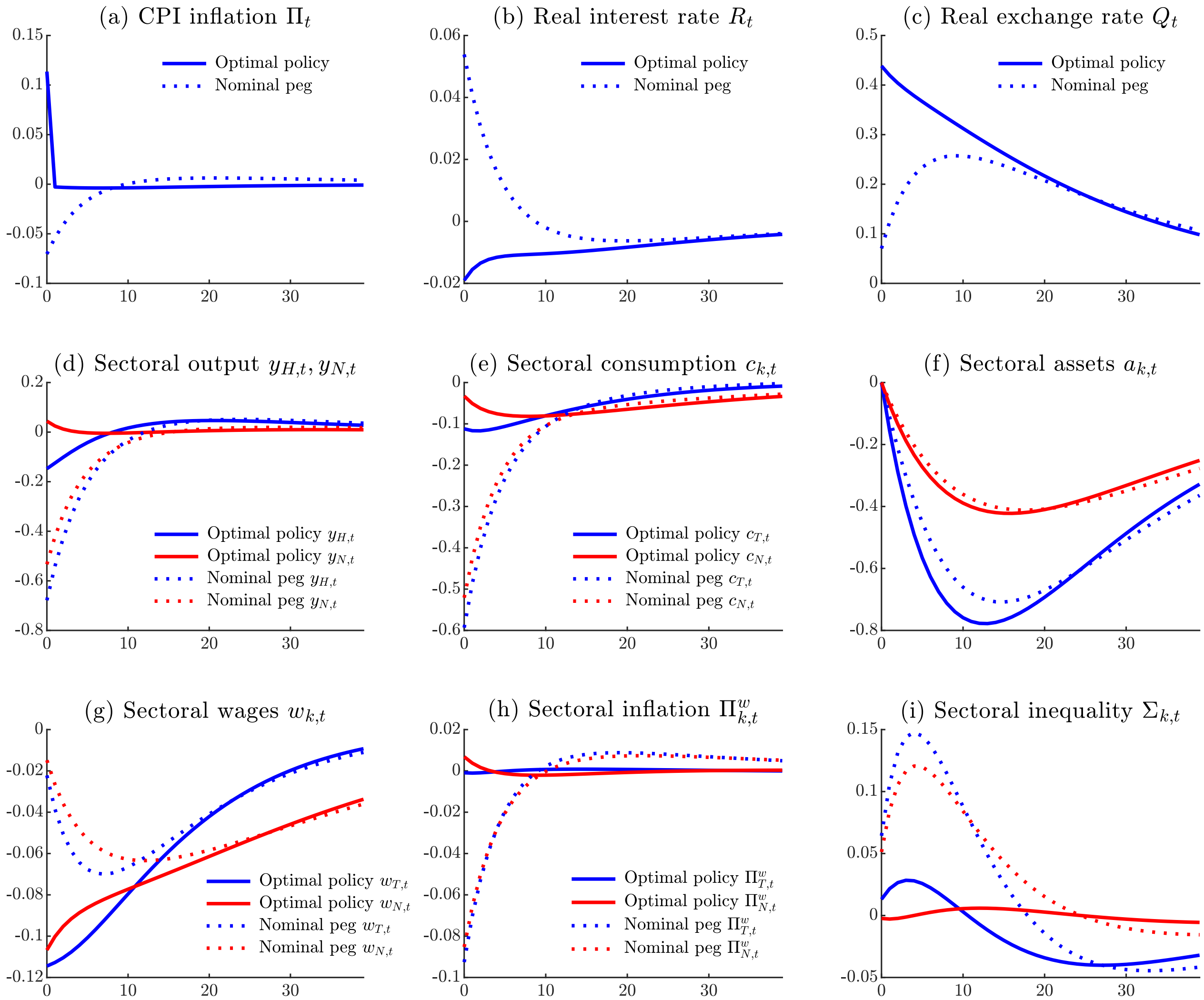

Figure 5: **The inefficiency of a nominal exchange-rate peg.** The solid lines show the economy's response to an export contraction under the optimal monetary policy, while the dashed lines show the same responses under a nominal exchange-rate peg. In panels (d) to (i), blue is for the tradables sector and red is for the nontradables sector.

optimal policy, workers in the nontradables sector are almost perfectly sheltered from any increase in consumption inequality, while within-sector inequalities remain moderate, even in the tradables sector (Panel (i)). This is in sharp contrast to a nominal peg, under which inequalities within both sectors increase substantially and persistently. Importantly, the optimal policy shifts both sectoral consumption levels, but cannot eliminate their differences (Panel (e)). Again, this is because there is only so much an *aggregate* instrument (the policy interest rate) can do against an inherently *asymmetric* shock.

Next, one observe from Figure 6 that within-sector inequalities undergo significantly larger fluctuations under flexible CPI targeting than under the optimal monetary policy. Relative to the former, the latter achieves a significant buffering of output, consumption, and Home-driven inflation, at the cost of strong imported inflation. This is ultimately beneficial to within-sector inequality, which rises much less under the optimal policy than under inflation targeting. Note that the path of assets in panel (f) and the difference between sectoral consumption levels in panel (e) would not be majorly affected by a switch

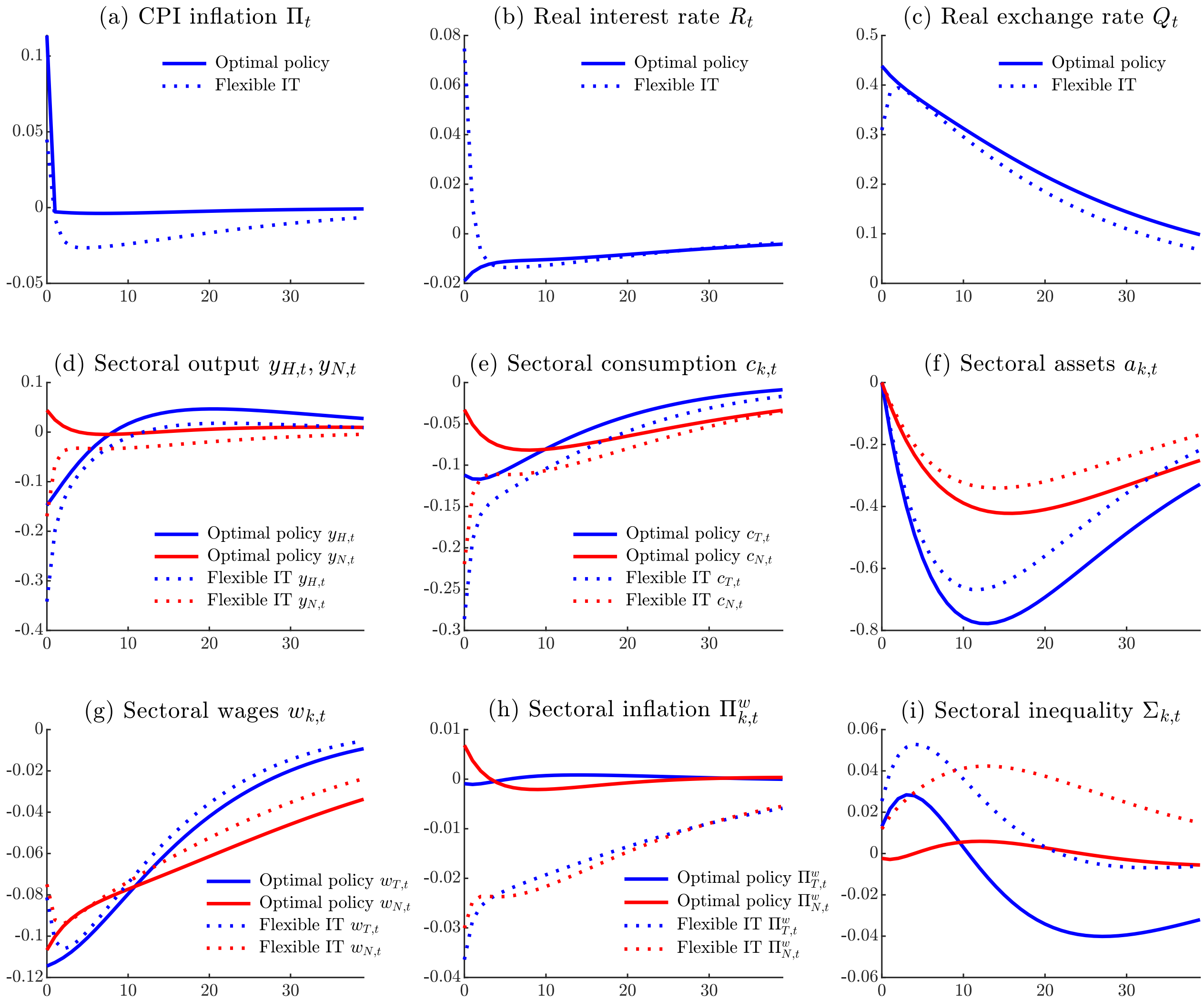

Figure 6: **The inefficiency of flexible inflation targeting.** The solid lines show the economy's response to an export contraction under the optimal monetary policy, while the dashed lines show the same responses under flexible inflation targeting. In panels (d) to (i), blue is for the tradables sector and red is for the nontradables sector.

from inflation targeting to optimal monetary policy. This means (by equations (31)-(32)) that the primary driver of the rise in consumption inequality under inflation targeting is within-cohort/sector dispersion due to countercyclical risk, rather than between-cohort dispersion due to asset accumulation or between-sector dispersion due to mean consumption differences.

6 Concluding remarks

We have examined the desirability and effectiveness of alternative monetary policy regimes for a small, open, heterogeneous-agent economy hit by an asymmetric shock — namely, a fall in export demand that directly contracts the tradables sector, spilling over to the rest of the economy and eventually increasing income dispersion across the population. The crucial feature of the model is that the shock affects consumption inequality, which a benevolent social planner would want to stabilize at some constrained-efficient level. The main takeaway from our analysis is that exchange-rate flexibility is particularly valuable for such an economy. First, exchange rate flexibility mitigates the contraction of the tradables sector as well as the intersectoral precautionary-saving spillovers that contaminate the nontradables sector. Second, and by way of consequence, it mutes down the inefficient increase in consumption inequality that follows the export shock. From this perspective, a nominal exchange-rate peg is socially costly and unambiguously dominated by flexible inflation targeting. Yet, we stress that even inflation targeting may not provide sufficient exchange-rate flexibility. In fact, the optimal policy response is to completely let go of domestic consumer price stability at the time the shock hits. While following the optimal policy has limited ability to correct between-sector consumption inequalities, it efficiently limits the rise in within-sector inequality when idiosyncratic income risk is countercyclical.

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Appendix

A Asset accumulation

A.1 Sectoral tax rebates

The first term in the RHS of (22) is straightforward. The second term in the RHS comes from the uniform rebate of the payroll tax charged to all tradables firms. Indeed, the payroll tax rate is τ^T, so every unit of tradable output yields $\tau^T w_{T,t}$ in tax revenue. Since $m\varphi n_{T,t}$ goods are produced, the total payroll tax revenue is $m\varphi\tau^T w_{T,t} n_{T,t}$. It is uniformly rebated to the mass m of households in the economy so that each household (wherever they work) gets $\varphi\tau^T w_{T,t} n_{T,t}$. Finally, the last term in the RHS of (22) is obtained by summing up all the cohort-specific tax revenues from the tax:

$$\begin{aligned}
\mathbb{T}_{k,t} &= \underbrace{\underbrace{(1-\vartheta)}_{\text{size of cohort } s=t} \times \underbrace{\tau^\star \frac{\vartheta}{R_t} a^t_{k,t+1}}_{\text{mean tax collection}}}_{\text{tax collection from cohort } s=t} + \underbrace{\underbrace{(1-\vartheta)\,\vartheta}_{\text{size of cohort } s=t-1} \times \underbrace{\tau^\star \frac{\vartheta}{R_t} a^{t-1}_{k,t+1}}_{\text{mean tax collection}}}_{\text{tax collection from cohort } t-1} + \ldots \\
&= \frac{\tau^\star}{R_t} \underbrace{\left[(1-\vartheta)\times 0 + (1-\vartheta)\,\vartheta a^t_{k,t+1} + (1-\vartheta)\,\vartheta^2 a^{t-1}_{k,t+1} + \ldots\right]}_{=a_{k,t+1}} \qquad (\text{since } a^{t+1}_{k,t+1} = 0)
\end{aligned}$$

A.2 From individual to sectoral assets

From equation (12), conditional on survival, the individual wealth of a household of cohort s working in sector k evolves as follows:

$$a^s_{k,t}(i) = \frac{R_{t-1}}{\vartheta\,(1+\tau^\star)} \left\{a^s_{k,t-1}(i) + (1-\tau^w) w_{k,t-1} n_{k,t-1} e^s_{k,t-1}(i) + T_{k,t-1} - c^s_{k,t-1}(i)\right\}$$

Aggregating over individuals of the same cohort-sector, we get:

$$a^s_{k,t} = \frac{R_{t-1}}{\vartheta\,(1+\tau^\star)} \left\{a^s_{k,t-1} + (1-\tau^w) w_{k,t-1} n_{k,t-1} + T_{k,t-1} - c^s_{k,t-1}\right\}$$

Aggregating over cohorts in the same sector, we get, for beginning-of-period total assets (before any tax and transfers are collected or rebated):

$$
\begin{aligned}
a_{k,t} &= (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}a_{k,t}^{s} \\
&= (1-\vartheta)\,\vartheta\left\{a_{k,t}^{t-1}+\vartheta a_{k,t}^{t-2}+\ldots\right\} \qquad (\text{since } a_{k,t}^{t}=0) \\
&= (1-\vartheta)\,\vartheta\left\{\frac{R_{t-1}}{\vartheta\,(1+\tau^{\star})}\left\{a_{k,t-1}^{t-1}+(1-\tau^{w})w_{k,t-1}n_{k,t-1}+T_{k,t-1}-c_{k,t-1}^{t-1}\right\}\right. \\
&\qquad\left.+\,\vartheta\frac{R_{t-1}}{\vartheta\,(1+\tau^{\star})}\left\{a_{k,t-2}^{t-1}+(1-\tau^{w})w_{k,t-1}n_{k,t-1}+T_{k,t-1}-c_{k,t-2}^{t-1}\right\}\right\}+\ldots \\
&= \frac{R_{t-1}}{1+\tau^{\star}}\left\{\underbrace{(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}a_{k,t-1}^{s}}_{=a_{k,t-1}}+\underbrace{(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}}_{=1}\left((1-\tau^{w})w_{k,t-1}n_{k,t-1}+T_{k,t-1}\right)\right. \\
&\qquad\left.\underbrace{-\,(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}c_{k,t-1}^{s}}_{=c_{k,t-1}}\right\}
\end{aligned}
$$

This gives equation (19). We can then substitute the sectoral tax rebates (22) into (19) to further specify sectoral asset acumulation equations. For the *tradables* sector we get:

$$
\begin{aligned}
a_{T,t} &= \frac{R_{t-1}}{1+\tau^{\star}}\left\{a_{T,t-1}+(1-\tau^{w})w_{T,t-1}n_{T,t-1}+\underbrace{\varphi\tau^{T}w_{T,t-1}n_{T,t-1}+\tau^{w}w_{T,t-1}n_{T,t-1}+\frac{\tau^{\star}a_{T,t}}{R_{t-1}}}_{=T_{T,t-1}}-c_{T,t-1}\right\} \\
&= \frac{R_{t-1}}{1+\tau^{\star}}\left\{a_{T,t-1}+\left(1+\varphi\tau^{T}\right)w_{T,t-1}n_{T,t-1}-c_{T,t-1}\right\}+\frac{\tau^{\star}a_{T,t}}{1+\tau^{\star}} \\
&= R_{t-1}\left\{a_{T,t-1}+\left(\frac{1+\varphi\tau^{T}}{\varphi\,(1+\tau^{T})}\right)p_{H,t-1}y_{H,t-1}-c_{T,t-1}\right\}
\end{aligned} \tag{38}
$$

For the *nontradables* sector we get:

$$
\begin{aligned}
a_{N,t} &= \frac{R_{t-1}}{1+\tau^{\star}}\left\{a_{N,t-1}+(1-\tau^{w})w_{N,t-1}n_{N,t-1}+\underbrace{\varphi\tau^{T}w_{T,t-1}n_{T,t-1}+\tau^{w}w_{N,t-1}n_{N,t-1}+\frac{\tau^{\star}a_{N,t}}{R_{t-1}}}_{=T_{N,t-1}}-c_{N,t-1}\right\} \\
&= \frac{R_{t-1}}{1+\tau^{\star}}\left\{a_{N,t-1}+w_{N,t-1}n_{N,t-1}+\varphi\tau^{T}w_{T,t-1}n_{T,t-1}-c_{N,t-1}\right\}+\frac{\tau^{\star}a_{N,t}}{1+\tau^{\star}} \\
&= R_{t-1}\left\{a_{N,t-1}+\frac{p_{N,t-1}y_{N,t-1}}{1-\varphi}+\left(\frac{\tau^{T}}{1+\tau^{T}}\right)p_{H,t-1}y_{H,t-1}-c_{N,t-1}\right\}
\end{aligned} \tag{39}
$$

A.3 From sectoral assets to economy-wide assets

Finally, aggregating the asset accumulation equation across sectors using (38)-(39), we obtain the economywide asset holdings, i.e., the Home economy's net foreign assets:

$$
\begin{aligned}
a_t &= \varphi a_{T,t} + (1-\varphi)\, a_{N,t} \\
&= \varphi \left\{ R_{t-1} \left\{ a_{T,t-1} + \left(\frac{1+\varphi\tau^T}{\varphi\,(1+\tau^T)} \right) p_{H,t-1} y_{H,t-1} - c_{T,t-1} \right\} \right\} \\
&\qquad + (1-\varphi) \left\{ R_{t-1} \left\{ a_{N,t-1} + \frac{p_{N,t-1} y_{N,t-1}}{1-\varphi} + \left(\frac{\tau^T}{1+\tau^T} \right) p_{H,t-1} y_{H,t-1} - c_{N,t-1} \right\} \right\} \\
&= R_{t-1} \left\{ a_{t-1} + \left(\frac{1+\varphi\tau^T}{1+\tau^T} \right) p_{H,t-1} y_{H,t-1} + p_{N,t-1} y_{N,t-1} + \left(\frac{(1-\varphi)\,\tau^T}{1+\tau^T} \right) p_{H,t-1} y_{H,t-1} - c_{t-1} \right\} \\
&= R_{t-1} (a_{t-1} + \underbrace{p_{H,t-1} y_{H,t-1} + p_{N,t-1} y_{N,t-1}}_{\text{Home economy's real income}} - c_{t-1})
\end{aligned}
$$

where $c_t = \varphi c_{T,t} + (1-\varphi) c_{N,t}$ is aggregate consumption.

B Optimal labor supply

There is a continuum of unions $j \in [0,1]$ in each sector, each setting wages on behalf of their members. Each worker i in sector k supplies labor hours to a particular union j, while each union hires a fully representative sample of workers of that sector. Each union j demands the same number of hours $n_{k,t}(j)$ from all its members and then aggregates their effective labor supplies to produce its "output", namely a differentiated labor type $n_{k,t}(j)$. This differentiated labor type is ultimately sold to competitive labor packers who re-aggregate the $n_{k,t}(j)$ into a single composite labor input $n_{k,t}$ sold to sector-k firms according to the production function

$$
n_{k,t} = \left(\int_j n_{k,t}(j)^{\frac{\varkappa-1}{\varkappa}} \mathrm{d}j \right)^{\frac{\varkappa}{\varkappa-1}}, \ \varkappa > 1
$$

Let $W_{k,t}(j)$ be the nominal cost of labor type j sold to labor packers and $W_{k,t}$ the average nominal wage level in the sector. The demand for labor input j by labor packers is given by

$$
n_{k,t}(j) = n_{k,t} \left(\frac{W_{k,t}(j)}{W_{k,t}} \right)^{-\varkappa} = n_{k,t} \left(\frac{w_{k,t}(j)}{w_{k,t}} \right)^{-\varkappa}
$$

Union j of sector k sets nominal wages $W_{k,t}(j)$ (or equivalently labor demand $n_{k,t}(j)$) to maximise

$$
\mathcal{O}_t(j) = \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_{k,t}\left(j\right)\right) - \kappa n_{k,t}(j) - \frac{\Psi}{2} \left(\ln \frac{W_{k,t}(j)}{W_{k,t-1}(j)} \right)^2 \right\}
$$

where $c_{k,t}(j)$ is the average consumption of its members, which by equation (19) is given by:

$$
c_{k,t}\left(j\right) = a_{k,t}\left(j\right) + \left(1-\tau^w\right) w_{k,t}\left(j\right) n_{k,t}\left(j\right) + T_{k,t} - \frac{1+\tau^\star}{R_{t-1}} a_{k,t+1}\left(j\right)
$$

Accordingly, we rewrite $\mathcal{O}_t(j)$ as

$$
\begin{aligned}
&\sum_{t=0}^{\infty} \beta_t^t \Bigg\{ u\left(a_{k,t}(j) + (1-\tau^w)\, w_{k,t}(j)\, n_{k,t}(j) + T_{k,t} - \frac{1+\tau^\star}{R_{t-1}} a_{k,t+1}(j)\right) \\
&\qquad\qquad\qquad\qquad - \kappa n_{k,t}(j) - \frac{\Psi}{2}\left(\ln \frac{W_{k,t}(j)}{W_{k,t-1}(j)}\right)^2 \Bigg\} \\
&\quad = \sum_{t=0}^{\infty} \beta_t^t \Bigg\{ u\left((1-\tau^w)\, w_{k,t} n_{k,t} \left(\frac{w_{k,t}(j)}{w_{k,t}}\right)^{1-\varkappa} + a_{k,t}(j) + T_{k,j} - \frac{1+\tau^\star}{R_{t-1}} a_{k,t+1}(j)\right) \\
&\qquad\qquad - \kappa\left(n_{k,t}(j)\left(\frac{w_{k,t}(j)}{w_{k,t}}\right)^{-\varkappa}\right) - \frac{\Psi}{2}\left(\ln \Pi_t^w + \ln \frac{w_{k,t}(j)}{w_{k,t}} - \ln\left(\frac{w_{k,t-1}(j)}{w_{k,t-1}}\right)\right)^2 \Bigg\}
\end{aligned}
$$

The first-order condition with respect to $w_{k,t}(j)/w_{k,t}$ gives:

$$
\begin{aligned}
&u'\left(c_{k,t}(j)\right)(1-\tau^w) w_{k,t} n_{k,t} (1-\varkappa)\left[\frac{w_{k,t}(j)}{w_{k,t}}\right]^{-\varkappa} + \varkappa\kappa n_{k,t}\left[\frac{w_{k,t}(j)}{w_{k,t}}\right]^{-\varkappa-1} \\
&\qquad - \Psi\left(\ln \Pi_{k,t}^w + \ln \frac{w_{k,t}(j)}{w_{k,t}} - \ln\left(\frac{w_{k,t-1}(j)}{w_{k,t-1}}\right)\right)\frac{w_{k,t}}{w_{k,t}(j)} \\
&\qquad\qquad + \beta\Psi\left(\ln \Pi_{k,t+1}^w + \ln \frac{w_{k,t+1}(j)}{w_{k,t+1}} - \ln\left(\frac{w_{k,t}(j)}{w_{k,t}}\right)\right)\frac{w_{k,t}}{w_{k,t}(j)} = 0
\end{aligned}
$$

Imposing symmetry across unions in the same sector ($c_{k,t}(j) = c_{k,t}, n_{k,t}(j) = n_{k,t}, w_{k,t}(j) = w_{k,t}$) and rearranging gives (21), where $\mathcal{M}_w = \frac{\varkappa}{\varkappa-1} > 1$.

C Sectoral Euler equations

We conjecture (and then verify) that, under the CARA-Normal assumptions, the policy rule for consumption is linear in assets and human wealth. More specifically, we conjecture that household (i,s,k) consumes

$$
c_{k,t}^s(i) = c_{k,t} + \mu_t\left(a_{k,t}^s(i) - a_{k,t}\right) + \sigma_{k,t}^c \xi_{k,t}^s(i), \tag{40}
$$

where $a_{k,t}^s(i)$ is the asset wealth of the household, $a_{k,t}$ the total assets held by households in the same sector, and $\sigma_{k,t}^c$ and $\xi_{k,t}^s(i)$ are defined in equations (20) and (13), respectively.

Next, flip and lead equation (12) to write individual wealth accumulation (for a survivor) as follows:

$$
a_{k,t+1}^s(i) = \frac{R_t}{\vartheta(1+\tau^\star)}\left\{a_{k,t}^s(i) + (1-\tau^w)\, w_{k,t} n_{k,t} e_{k,t}^s(i) + T_{k,t} - c_{k,t}^s(i)\right\}
$$

Using equations (13), (14) and (19), we can rearrange the latter equation as follows:

$$
\begin{aligned}
a^s_{k,t+1}(i) &= \frac{R_t}{\vartheta\left(1+\tau^\star\right)}\left\{a^s_{k,t}(i) + \left(1-\tau^w\right) w_{k,t} n_{k,t}\left[1+\sigma_{k,t}\xi^s_{k,t}(i)\right] + T_{k,t} - c^s_{k,t}(i)\right\} \\
&= \frac{R_t}{\vartheta\left(1+\tau^\star\right)}\left\{a^s_{k,t}(i) + \underbrace{\left(1-\tau^w\right) w_{k,t} n_{k,t}\sigma_{k,t}}_{=\sigma^y_{k,t}}\xi^s_{k,t}(i) + \underbrace{w_{k,t} n_{k,t} + T_{k,t}}_{=(1+\tau^\star)\frac{a_{k,t+1}}{R_t}+c_{k,t}-a_{k,t}} - c^s_{k,t}(i)\right\} \\
&= \frac{R_t}{\vartheta\left(1+\tau^\star\right)}\left\{\left(a^s_{k,t}(i) - a_{k,t}\right) + \sigma^y_{k,t}\xi^s_{k,t}(i) - \underbrace{\left(c^s_{k,t}(i) - c_{k,t}\right)}_{=\mu_t\left(a^s_{k,t}(i)-a_{k,t}\right)+\sigma^c_{k,t}\xi^s_{k,t}(i)}\right\} + \frac{a_{k,t+1}}{\vartheta}
\end{aligned}
$$

to eventually get:

$$
a^s_{k,t+1}(i) - a_{k,t+1} = \frac{R_t}{\vartheta\left(1+\tau^\star\right)}\left\{\left(1-\mu_t\right)\left(a^s_{k,t}(i) - a_{k,t}\right) + \left(\sigma^y_{k,t} - \sigma^c_{k,t}\right)\xi^s_{k,t}(i)\right\} + \left(\frac{1-\vartheta}{\vartheta}\right) a_{k,t+1}
$$

Substituting the latter equation into the consumption rule (40) and using (13) gives:

$$
\begin{aligned}
c^s_{k,t+1}(i) = c_{k,t+1} + \mu_{t+1}&\left[\frac{R_t}{\vartheta\left(1+\tau^\star\right)}[(1-\mu_t)(a^s_{k,t}(i) - a_{k,t}) + (\sigma^y_{k,t} - \sigma^c_{k,t})\xi^s_{k,t}(i)] + \left(\frac{1-\vartheta}{\vartheta}\right) a_{k,t+1}\right] \\
&+ \sigma^c_{k,t+1}\xi^s_t(i) + \sigma^c_{k,t+1}\upsilon^s_{t+1}(i),
\end{aligned}
$$

from which we can infer the conditional moments of individual consumption:

$$
\begin{aligned}
\mathbb{E}_t[c^s_{k,t+1}(i)] = c_{k,t+1} + \mu_{t+1}&\left[\frac{R_t}{\vartheta}[(1-\mu_t)(a^s_{k,t}(i) - a_{k,t}) + (\sigma^y_{k,t} - \sigma^c_{k,t})\xi^s_{k,t}(i)] + \left(\frac{1-\vartheta}{\vartheta}\right) a_{k,t+1}\right] \\
&+ \sigma^c_{k,t+1}\xi^s_{k,t}(i) \qquad (41) \\
\mathbb{V}_t[c^s_{k,t+1}(i)] = (\sigma^c_{k,t+1})^2& \qquad (42)
\end{aligned}
$$

Now, since consumption is conditionally normally distributed, we can log the individual Euler equation (16) and write:

$$
c^s_{k,t}(i) = -\ln\left(\frac{\beta R_t}{1+\tau^\star}\right) + \mathbb{E}_t[c^s_{k,t+1}(i)] - \frac{1}{2}\mathbb{V}_t[c^s_{k,t+1}(i)] \qquad (43)
$$

Substituting the consumption policy function (40) and the moments (41)-(42) into (43) gives:

$$
\begin{aligned}
c_{k,t} + \mu_t\left(a^s_{k,t}(i) - a_{k,t}\right) + \sigma^c_{k,t}\xi^s_{k,t}(i) &= -\ln\left(\frac{\beta R_t}{1+\tau^\star}\right) - \frac{1}{2}(\sigma^c_{k,t+1})^2 + \sigma^c_{k,t+1}\xi^s_{k,t}(i) \\
+ c_{k,t+1} + \mu_{t+1}&\left[\frac{R_t}{\vartheta\left(1+\tau^\star\right)}[(1-\mu_t)(a^s_{k,t}(i) - a_{k,t}) + (\sigma^y_{k,t} - \sigma^c_{k,t})\xi^s_{k,t}(i)] + \left(\frac{1-\vartheta}{\vartheta}\right) a_{k,t+1}\right]
\end{aligned}
$$

Matching the coefficients in $a^s_{k,t}(i) - a_{k,t}$ and rearranging gives (18), while matching the coefficients in $\xi^s_{k,t}$ and rearranging gives (20). Using (20)-(18) to eliminate the terms in $a^s_{k,t}(i) - a_{k,t}$ and $\xi^s_{k,t}$ gives (17).

D Relative prices, real exchange rate, and CPI inflation

D.1 Size of tradables sector

We compute the value of φ such that, in steady state, $w_T = w_N = w$. First, we note from equation (21) that, in the zero-inflation steady state, common steady-state wages imply common steady state sectoral consumption $c_k = c$ since

$$\underbrace{(1-\tau^w)\,w}_{\text{post-tax wage}} = \underbrace{\mathcal{M}_w}_{\text{wage mkup}} \times \underbrace{\kappa e^{c_k}}_{\text{MRS}}, \qquad k = T, N$$

By equations (19) and (22), this also implies common sectoral labor supply $n_k = n$. Next, in world-symmetric steady state where $c = c^*$, the steady state counterpart of the market clearing conditions (26)-(27) become:

$$\varphi n = \frac{\phi}{p_H} c \qquad \text{and} \qquad (1-\varphi)\,n = \frac{1-\phi}{p_N} c,$$

where $p_H = w/(1-\lambda)$ (by equations (10) and (28)) and $p_N = w$. Dividing one market-clearing condition by the other substituting out prices and solving for φ gives (34).

D.2 Foreign prices and real exchange rate

In the limit as $m \to 0$, we have $\alpha^* \to 0$, while Foreign variables (both real and nominal) become unresponsive to domestic variables – hence unindexed by t. Equation (6) thus becomes $1 = (p_N^*)^{1-\phi}\,(p_F^*)^{\phi}$. Moreover, symmetric wages w^* across sectors in the Foreign economy, together with symmetric payroll taxes across countries (so that $1 + \tau^{T*} = 1/(1-\lambda)$) and the fact that prices equal marginal cost (i.e., $p_T^* = (1+\tau^{T*})w^*$ and $p_N^* = w^*$) give:

$$p_N^* = w^* = (1-\lambda)^{\phi}, \qquad p_F^* = (1-\lambda)^{\phi-1}$$

Now, in the Home economy, as $m \to 0$ so that $\alpha \to \lambda$, equation (3) becomes

$$1 = p_{N,t}^{1-\phi} p_{H,t}^{\phi(1-\lambda)} p_{F,t}^{\phi\lambda} \tag{44}$$

Under Producer Currency Pricing, the price of foreign tradables in terms of Home consumption can be expressed as:

$$p_{F,t} = \frac{\mathcal{E}_t P_F^*}{P_t} = \frac{\mathcal{E}_t P^*}{P_t}\frac{P_F^*}{P^*} = Q_t p_F^* = (1-\lambda)^{\phi-1} Q_t, \tag{45}$$

In world-symmetric steady-state where $Q = 1$, we have $p_F = p_F^* = (1-\lambda)^{\phi-1}$, and eventually $p_N = p_N^* = (1-\lambda)^{\phi}$ and $p_H^* = p_H = (1-\lambda)^{\phi-1}$. Moreover, substituting (45) as well as $p_{N,t}$ (in (9)), $p_{T,t}$ (in (10)) and p_F^* into equation (44) and rearranging gives (29).

D.3 CPI inflation

Finally, we compute CPI inflation (which enters the Taylor rule) as follows. In the closed-economy limit where $\alpha \to \lambda$, the CPI is $P_t = P_{N,t}^{1-\phi} P_{H,t}^{\phi(1-\lambda)} P_{F,t}^{\phi\lambda}$, from which we compute CPI inflation as follows:

$$\begin{aligned}
\Pi_t &= \left(\frac{P_{N,t}}{P_{N,t-1}}\right)^{1-\phi} \left(\frac{P_{H,t}}{P_{H,t-1}}\right)^{\phi(1-\lambda)} \left(\frac{P_{F,t}}{P_{F,t-1}}\right)^{\phi\lambda} \\
&= \left(\frac{W_{N,t}}{W_{N,t-1}}\right)^{1-\phi} \left(\frac{W_{T,t}}{W_{T,t-1}}\right)^{\phi(1-\lambda)} \left(\frac{\mathcal{E}_t P_F^*}{\mathcal{E}_{t-1} P_F^*}\right)^{\phi\lambda} \\
&= \left(\Pi_{N,t}^w\right)^{1-\phi} \left(\Pi_{T,t}\right)^{\phi(1-\lambda)} \left(\frac{\mathcal{E}_t P_F^*}{P_t}\right)^{\phi\lambda} \left(\frac{P_{t-1}}{\mathcal{E}_{t-1} P_F^*}\right)^{\phi\lambda} \left(\frac{P_t}{P_{t-1}}\right)^{\phi\lambda} \\
&= \left(\Pi_{N,t}^w\right)^{1-\phi} \left(\Pi_{T,t}\right)^{\phi(1-\lambda)} \left(Q_t/Q_{t-1}\right)^{\phi\lambda} \Pi_t^{\phi\lambda} \\
&= \left(\Pi_{N,t}^w\right)^{\frac{1-\phi}{1-\lambda\phi}} \left(\Pi_{T,t}^w\right)^{\frac{\phi(1-\lambda)}{1-\lambda\phi}} \left(Q_t/Q_{t-1}\right)^{\frac{\phi\lambda}{1-\lambda\phi}}
\end{aligned}$$

E Social welfare function

E.1 Individual wealth and consumption

We first need to compute the dynamics of individual wealth and consumption. Subtracting the sectoral Euler equation (17) from the individual-level Euler equation (43) yields:

$$c_{k,t}^s(i) - c_{k,t} = \mathbb{E}_t\left[c_{k,t+1}^s(i) - c_{k,t+1}\right] - \left(\frac{1-\vartheta}{\vartheta}\right) \mu_{t+1} a_{k,t+1}$$

Next, substituting the consumption function (40) on both side of the latter equation and rearranging, we obtain the dynamics of individual wealth (as a deviation from sectoral wealth):

$$\underbrace{a_{k,t+1}^s(i) - a_{k,t+1}}_{\text{current wealth deviation}} = \underbrace{\left(\frac{1-\vartheta}{\vartheta}\right) a_{k,t+1}}_{\text{aging drift}} + \underbrace{\frac{\mu_t}{\mu_{t+1}}\left(a_{k,t}^s(i) - a_{k,t}\right)}_{\text{past wealth deviation}} + \underbrace{\frac{1}{\mu_{t+1}}\left(\sigma_{k,t}^c - \sigma_{k,t+1}^c\right)\xi_{k,t}^s(i)}_{\text{idiosyncratic shock}}$$

Multiplying both sides by μ_{t+1}, adding $\sigma_{k,t+1}^c \xi_{k,t+1}^s(i)$, using (13) and (40) and rearranging, we obtain the dynamics of individual consumptino (as a deviation from sectoral consumption):

$$\underbrace{c_{k,t}^s(i) - c_{k,t}}_{\text{current consumption deviation}} = \underbrace{\left(\frac{1-\vartheta}{\vartheta}\right) \mu_t a_{k,t}}_{\text{aging drift}} + \underbrace{c_{k,t-1}^s(i) - c_{k,t-1}}_{\text{past consumption deviation}} + \underbrace{\sigma_{k,t}^c \upsilon_{k,t}^s(i)}_{\text{consumption innovation}} \tag{46}$$

Aggregating the latter over individual cohorts s, we find the following recursions for cohort-level moments:

$$c_{k,t}^s - c_{k,t} = c_{k,t-1}^s - c_{k,t-1} + \left(\frac{1-\vartheta}{\vartheta}\right) \mu_t a_{k,t} \quad \text{and} \quad \sigma_k^c(s,t)^2 = \sigma_k^c(s,t-1)^2 + \left(\sigma_{k,t}^c\right)^2 \tag{47}$$

In (47), $c_{k,t}^s - c_{k,t}$ is the deviation of average cohort-s consumption $c_{k,t}^s$ from sectoral consumption $c_{k,t}$, while $\sigma_k^c(s,t)^2$ denotes the time-t cross-sectional variance of consumption within cohort s.

E.2 Flow utility to the planner

We compute the planner's flow utility by aggregating the utility flows of all households in the economy, leveraging the CARA-Normal structure to keep this aggregation tractable. First, since $u(c) = -e^{-c}$, the time-t flow utility of household i from cohort $s \leq t$ can be written as

$$\begin{aligned}
\mathcal{V}_t(s,t,i) &= u\left(c^s_{k,t}(i)\right) - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2 \\
&= u\left(c_{k,t}\right) e^{-\left(c^s_{k,t} - c_{k,t}\right)} e^{-\left(c^s_{k,t}(i) - c^s_{k,t}\right)} - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2,
\end{aligned}$$

where $c^s_{k,t} - c_{k,t}$ and $c^s_{k,t}(i) - c^s_{k,t}$ respectively capture between and within-cohort consumption dispersion. Aggregating over all households i of the same cohort $s \leq t$ gives

$$\begin{aligned}
\mathcal{V}_t(s,t) &= \int \mathcal{V}_t(s,t,i)di \\
&= u\left(c_{k,t}\right) e^{-\left(c^s_{k,t} - c_{k,t}\right)} \underbrace{\int e^{-\left(c^s_{k,t}(i) - c^s_{k,t}\right)} di}_{=[\sigma^c_{k,t}(s,t)]^2} - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2
\end{aligned}$$

Finally, aggregating overall all cohort $s \leq t$ alive at time t and working in sector k, we get:

$$\begin{aligned}
\mathbb{U}_{k,t} &= \sum_{s=-\infty}^{t} (1-\vartheta)\vartheta^{t-s}\mathcal{V}_t(s,t) \\
&= u\left(c_{k,t}\right) \underbrace{(1-\vartheta)\sum_{j=0}^{\infty} \vartheta^j e^{-\left(c^{t-j}_{k,t} - c_{k,t}\right)} e^{\frac{1}{2}[\sigma^c_k(t-j,t)]^2}}_{\equiv \Sigma_{k,t}} - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2 \\
&= u\left(c_{k,t}\right) \Sigma_{k,t} - \kappa n_{k,t} - \frac{\Psi}{2}\left(\ln \Pi^w_{k,t}\right)^2
\end{aligned}$$

where $\Sigma_{k,t}$ is an index capturing the dispersion in marginal utility of consumption between households working in the same sector, whether this dispersion comes from within- or between-cohort heterogeneity. The total time-t flow utility to the planner is $\mathbb{U}_t = \varphi \mathbb{U}_{T,t} + (1-\varphi)\mathbb{U}_{N,t}$.

E.3 Dynamics of utility-dispersion index

The social cost of inequality is captured by the sectoral indices $\Sigma_{k,t}$, $k = T, N$. Because inequalities are persistent, these indices are state variables that enter the planning problem. The law of motion of $\Sigma_{k,t}$

is computed as follows:

$$
\begin{aligned}
\Sigma_{k,t} &= (1-\vartheta)\sum_{j=0}^{\infty}\vartheta^k e^{-[c_{k,t}^{t-j}-c_{k,t}]}e^{\frac{1}{2}[\sigma_c^c(t-k,t)]^2} \\
&= (1-\vartheta)e^{\mu_t a_{k,t}}e^{(\sigma_{k,t}^c)^2} + \vartheta(1-\vartheta)\left\{e^{-[c_{k,t}^{t-1}-c_{k,t}]}e^{\frac{1}{2}[\sigma_k^c(t-1,t)]^2} + \vartheta e^{-[c_{k,t}^{t-2}-c_{k,t}]}e^{\frac{1}{2}[\sigma_k^c(t-2,t)]^2}\dots\right\} \\
&= (1-\vartheta)e^{\mu_t a_{k,t}}e^{(\sigma_{k,t}^c)^2} + \vartheta(1-\vartheta)\Big\{e^{-\left[c_{k,t-1}^{t-1}-c_{k,t-1}+\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}\right]}e^{\frac{1}{2}[[\sigma_{k,}^c(t-1,t-1)]^2+(\sigma_{k,t}^c)^2]} \\
&\qquad + \vartheta e^{-\left[c_{k,t-1}^{t-2}-c_{k,t-1}+\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}\right]}e^{\frac{1}{2}[[\sigma_k^c(t-2,t-1)]^2+(\sigma_{k,t}^c)^2]}\dots\Big\} \\
&= (1-\vartheta)e^{\mu_t a_{k,t}}e^{(\sigma_{k,t}^c)^2} \\
&\quad + \vartheta(1-\vartheta)e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}}e^{\frac{1}{2}(\sigma_{k,t}^c)^2}\left\{e^{-\left[c_{k,t-1}^{t-1}-c_{k,t-1}\right]}e^{\frac{1}{2}[\sigma_k^c(t-1,t-1)^2]} + \vartheta e^{-\left[c_{k,t-1}^{t-2}-c_{k,t-1}\right]}e^{\frac{1}{2}[\sigma_k^c(t-2,t-1)^2}\dots\right\} \\
&= (1-\vartheta)e^{\mu_t a_{k,t}}e^{\frac{1}{2}(\sigma_{k,t}^c)^2} + \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}}e^{\frac{1}{2}(\sigma_{k,t}^c)^2}\Sigma_{k,t-1} \\
&= e^{\frac{1}{2}(\sigma_{k,t}^c)^2}\left[(1-\vartheta)e^{\mu_t a_{k,t}} + \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{k,t}}\Sigma_{k,t-1}\right]
\end{aligned}
$$

The two $\Sigma_{k,t}$ can then be aggregated as follows. First, use the expression above for $\mathbb{U}_{k,t}$ rewrite the total flow utility in (30) as

$$
\begin{aligned}
\varphi\mathbb{U}_{T,t} + (1-\varphi)\mathbb{U}_{N,t} = \underbrace{\varphi\left(-e^{-c_{T,t}}\right)\Sigma_{T,t} + (1-\varphi)\left(-e^{-c_{N,t}}\right)\Sigma_{N,t}}_{=-e^{-c_t}\Sigma_t} \\
- \varphi\left[\kappa n_{T,t} + \frac{\Psi}{2}\left(\ln\Pi_{T,t}^w\right)^2\right] - (1-\varphi)\left[\kappa n_{N,t} + \frac{\Psi}{2}\left(\ln\Pi_{N,t}^w\right)^2\right]
\end{aligned}
\tag{48}
$$

where

$$
\begin{aligned}
\Sigma_t &= \varphi\frac{e^{-c_{T,t}}}{e^{-c_t}}\Sigma_{T,t} + (1-\varphi)\frac{e^{-c_{N,t}}}{e^{-c_t}}\Sigma_{N,t} \\
&= \varphi e^{-(1-\varphi)\left(c_{T,t}-c_{N,t}\right)}\Sigma_{T,t} + (1-\varphi)e^{\varphi\left(c_{T,t}-c_{N,t}\right)}\Sigma_{N,t}
\end{aligned}
\tag{49}
$$

since $c_t = \varphi c_{T,t} + (1-\varphi)c_{N,t}$.

F Computation of optimal policy

F.1 Lagrangian function

The planner maximises (30) subject to all the constraints listed in Section 4.1. Before formulating the Lagrangian function, we make three substitutions for convenience. First, we replace the $n_{k,t}$s by $y_{H,t}/\varphi$ and $y_{N,t}/(1-\varphi)$, as is implied by (8). Second, we replace the prices $p_{H,t}$ and $p_{N,t}$ by $w_{T,t}/(1-\lambda)$ and $w_{N,t}$ using (9)-(10). Last, we replace φ (the relative size of the tradables sector) in the sectoral asset accumulation equations (38)-(39) by its value in (34). We end up with a Lagrangian function involving 16 sequences of constraints for 33 sequences of unknowns, namely 16 Lagrange multipliers and the following 17 endogenous variables:

$$
\{c_{T,t}, c_{N,t}, y_{Ht}, y_{Nt}, \Pi_{T,t}^w, \Pi_{N,t}^w, R_t, a_{T,t}, a_{N,t}, Q_t, w_{T,t}, w_{N,t}, \mu_t, \sigma_{T,t}^c, \sigma_{N,t}^c, \Sigma_{T,t}, \Sigma_{N,t}\}_{t=0}^{\infty}
$$

The 17 planner's FOCs will provide the missing equations. The Lagrangian function is as follows:

$$\mathcal{L} =$$

$$\sum_{t=0}^{\infty}\beta^t\left[\varphi\left\{-e^{-c_{T,t}}\Sigma_{T,t}-\kappa\frac{y_{H,t}}{\varphi}-\frac{\Psi}{2}\left(\ln\Pi^w_{T,t}\right)^2\right\}+(1-\varphi)\left\{-e^{-c_{N,t}}\Sigma_{N,t}-\kappa\frac{y_{N,t}}{1-\varphi}-\frac{\Psi}{2}\left(\ln\Pi^w_{N,t}\right)^2\right\}\right]$$

$$+\sum_{t=0}^{\infty}\beta^t M^T_{1,t}\varphi\left\{-c_{T,t}-\ln\beta-\ln R_t+\ln(1+\tau^\star)+c_{T,t+1}+\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{T,t+1}-\frac{\left(\sigma^c_{T,t+1}\right)^2}{2}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{1,t}(1-\varphi)\left\{-c_{N,t}-\ln\beta-\ln R_t+\ln(1+\tau^\star)+c_{N,t+1}+\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{N,t+1}-\frac{\left(\sigma^c_{N,t+1}\right)^2}{2}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^T_{2,t}\varphi\left\{-\frac{a_{T,t+1}}{R_t}+a_{T,t}+\frac{w_{T,t}y_{H,t}}{\phi(1-\lambda)}-c_{T,t}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{2,t}(1-\varphi)\left\{-\frac{a_{N,t+1}}{R_t}+a_{N,t}+\frac{1-\lambda\phi}{1-\phi}w_{N,t}y_{N,t}+\frac{\lambda}{1-\lambda}w_{T,t}y_{H,t}-c_{N,t}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M_{3,t}\left\{-\ln R_t+\ln R^*+\ln Q_{t+1}-\ln Q_t\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^H_{4,t}\left\{-y_{H,t}+\frac{1-\lambda}{w_{T,t}}\left\{\phi(1-\lambda)\left[(1-\varphi)c_{N,t}+\varphi c_{T,t}\right]+\lambda\phi\frac{Q_t c^*_t}{1+\tau^X_t}\right\}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{4,t}\left\{-y_{Nt}+\frac{1-\phi}{w_{N,t}}\left[(1-\varphi)c_{N,t}+\varphi c_{T,t}\right]\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^T_{5,t}\varphi\left\{-\ln\Pi^w_{T,t}+\frac{y_{H,t}}{\Psi(\mathcal{M}_w-1)\varphi}\left(\kappa\mathcal{M}_w-(1-\tau^w)w_{T,t}e^{-c_{T,t}}\right)+\beta\ln\Pi^w_{T,t+1}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{5,t}(1-\varphi)\left\{-\ln\Pi^w_{N,t}+\frac{y_{N,t}}{\Psi(\mathcal{M}_w-1)(1-\varphi)}\left(\kappa\mathcal{M}_w-(1-\tau^w)w_{N,t}e^{-c_{N,t}}\right)+\beta\ln\Pi^w_{N,t+1}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M_{6,t}\left\{-\ln\Pi^w_{T,t}+\ln\Pi^w_{N,t}+\ln w_{T,t}+\ln w_{N,t-1}-\ln w_{T,t-1}-\ln w_{N,t}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M_{7,t}\left\{\frac{1-\phi}{\phi\lambda}\ln w_{N,t}+\frac{1-\lambda}{\lambda}\ln w_{T,t}-\left(\lambda^{-1}-\phi\right)\ln(1-\lambda)+\ln Q_t\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^T_{8,t}\varphi\left\{\frac{1}{2}(\sigma^c_{T,t})^2+\ln\left[(1-\vartheta)e^{\mu_t a_{T,t}}+\vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{T,t}}\Sigma_{T,t-1}\right]-\ln\Sigma_{T,t}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{8,t}(1-\varphi)\left\{\frac{1}{2}(\sigma^c_{N,t})^2+\ln\left[(1-\vartheta)e^{\mu_t a_{N,t}}+\vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{N,t}}\Sigma_{NT,t-1}\right]-\ln\Sigma_{N,t}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M_{9,t}\left\{\mu_t^{-1}-1-\frac{\vartheta(1+\tau^\star)}{R_t}\mu_{t+1}^{-1}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^T_{10,t}\varphi\left\{\sigma^c_{T,t}-\mu_t\sigma^y\exp\left\{-\Theta\left(\frac{y_{H,t}}{y_H}-1\right)\right\}-(1-\mu_t)\sigma^c_{T,t+1}\right\}$$

$$+\sum_{t=0}^{\infty}\beta^t M^N_{10,t}(1-\varphi)\left\{\sigma^c_{N,t}-\mu_t\sigma^y\exp\left\{-\Theta\left(\frac{y_{N,t}}{y_N}-1\right)\right\}-(1-\mu_t)\sigma^c_{N,t+1}\right\}$$

F.2 First-order conditions

In this section, we derive the first-order conditions associated with all the endogenous sequences summarised above, and also evaluate those conditions in the steady state (which will eventually allow us to derive analytical expressions for all steady-state Lagrange multipliers and optimal tax rates). In this second step, we exploit the steady-state condition of Section 4.2 and also conjecture (and verify later on) the following relationships between the steady-state Lagrange multipliers:

$$M_1^T = M_1^N, \quad M_2^T = M_2^N, \quad M_5^T = M_5^N = M_6 = 0, \quad M_{10}^T = M_{10}^N$$

F.2.1 FOCs with respect to $c_{T,t}$ and $c_{N,t}$

The FOC w.r.t. $c_{T,t}$ is

$$0 = e^{-c_{T,t}}\Sigma_{T,t} - M_{1,t}^T + \beta^{-1}M_{1,t-1}^T - M_{2,t}^T + M_{4,t}^H\frac{(1-\lambda)^2}{w_{T,t}}\phi + M_{4,t}^N\frac{1-\phi}{w_{N,t}} + M_{5,t}^T\frac{y_{H,t}\left(1-\tau^w\right)w_{T,t}}{\Psi\left(\mathcal{M}_w-1\right)\varphi}e^{-c_{T,t}}$$

and, evaluated at steady state using $M_5^T = 0$:

$$0 = e^{-c}\Sigma + M_1^T\left(\beta^{-1}-1\right) - M_2^T + M_4^H\frac{(1-\lambda)^2}{w}\phi + M_4^N\frac{1-\phi}{w}$$

The FOC w.r.t. $c_{N,t}$ is

$$\begin{aligned}0 = e^{-c_{N,t}}\Sigma_{N,t} - M_{1,t}^N + \beta^{-1}M_{1,t-1}^N - M_{2,t}^N + M_{4,t}^H\frac{(1-\lambda)^2}{w_{T,t}}\phi \\ + M_{4,t}^N\frac{1-\phi}{w_{N,t}} + M_{5,t}^N\frac{y_{N,t}\left(1-\tau^w\right)w_{N,t}}{\Psi(\mathcal{M}_w-1)\left(1-\varphi\right)}e^{-c_{N,t}},\end{aligned}$$

and, evaluated at steady state using $M_5^N = 0$:

$$0 = e^{-c}\Sigma + M_1^N\left(\beta^{-1}-1\right) - M_2^N + M_4^H\frac{(1-\lambda)^2}{w}\phi + M_4^N\frac{1-\phi}{w}$$

F.2.2 FOCs with respect to $y_{H,t}$ and $y_{N,t}$

The FOC w.r.t. $y_{H,t}$ is:

$$\begin{aligned}\kappa = M_{2,t}^T\frac{w_{T,t}}{1-\lambda\phi} + M_{2,t}^N\frac{1-\phi}{1-\lambda\phi}\frac{\lambda w_{T,t}}{1-\lambda} - M_{4,t}^H \\ + M_{5,t}^T\frac{1}{\Psi(\mathcal{M}_w-1)}\left(\kappa\mathcal{M}_w - \left(1-\tau^w\right)w_{T,t}e^{-c_{T,t}}\right) + M_{10,t}^T\frac{\varphi\mu_t\sigma^y}{y_H}\Theta e^{-\Theta\left(\frac{y_{H,t}}{y_H}-1\right)},\end{aligned}$$

and, evaluated at steady state and using $M_5^T = 0$ and $M_2^T = M_2^N$:

$$\kappa = \frac{M_2^T w}{1-\lambda} - M_4^H + M_{10}^T\frac{\mu\sigma^y}{n}\Theta$$

The FOC w.r.t. $y_{N,t}$ is:

$$\kappa = M_{2,t}^N w_{N,t} - M_{4,t}^N + M_{5,t}^N \frac{1}{\Psi(\mathcal{M}_w - 1)} \left(\kappa \mathcal{M}_w - (1-\tau^w) w_{N,t} e^{-c_{N,t}} \right) + M_{10,t}^N \left(\frac{1-\varphi}{y_N} \right) \mu_t \sigma^y \Theta e^{-\Theta\left(\frac{y_{N,t}}{y_N}-1\right)}$$

and, evaluated at steady state using $M_5^N = 0$ and $M_2^T = M_2^N$:

$$\kappa = M_2^T w - M_4^N + M_{10}^N \frac{\mu \sigma^y}{n} \Theta$$

F.2.3 FOCs with respect to Q_t

The FOC w.r.t. Q_t is:

$$0 = -M_{3,t} + \beta^{-1} M_{3,t-1} + M_{4,t}^H \frac{1-\lambda}{w_{T,t}} \lambda \phi \frac{Q_t c_t^*}{1+\tau_t^X} + M_{7,t}$$

and, evaluated at steady state:

$$0 = \left(\beta^{-1} - 1\right) M_3 + M_4^H \lambda \left(\frac{1-\lambda}{w} \phi c \right),$$

We note that $\frac{1-\lambda}{w} \phi c = y_H$ and rewrite the latter expression as:

$$M_7 = -\left(\beta^{-1} - 1\right) M_3 - M_4^H \lambda y_H$$

F.2.4 FOCs with respect to $w_{T,t}$ and $w_{T,t}$

The FOC w.r.t. $w_{T,t}$ is:

$$\begin{aligned} 0 = M_{2,t}^T \frac{w_{T,t} y_{H,t}}{1-\lambda\phi} + M_{2,t}^N \frac{(1-\phi)\lambda w_{T,t} y_{H,t}}{(1-\lambda\phi)(1-\lambda)} - M_{4,t}^H y_{H,t} \\ - M_{5,t}^T \frac{y_{H,t}(1-\tau^w) w_{T,t}}{\Psi(\mathcal{M}_w - 1)} e^{-c_{T,t}} + M_{6,t} - \beta M_{6,t+1} + M_{7,t} \frac{1-\lambda}{\lambda} \end{aligned}$$

and, evaluated at steady state using $M_2^N = M_2^T$, $M_5^T = M_6 = 0$ and the FOC w.r.t. Q_t to eliminate M_7:

$$\begin{aligned} 0 &= M_2^T \frac{w y_H}{1-\lambda\phi} + M_2^N \frac{(1-\phi)\lambda w y_H}{(1-\lambda\phi)(1-\lambda)} - M_4^H y_{H,t} - \frac{1-\lambda}{\lambda} \left[\left(\beta^{-1} - 1\right) M_3 + M_4^H \lambda y_H \right] \\ &= M_2^T w \left[\frac{1}{1-\lambda\phi} + \frac{(1-\phi)\lambda}{(1-\lambda\phi)(1-\lambda)} \right] - (2-\lambda) M_4^H - \frac{1-\lambda}{\lambda y_H} \left(\beta^{-1} - 1\right) M_3 \\ &= \frac{M_2^T w}{1-\lambda} - (2-\lambda) M_4^H - \frac{1-\lambda}{\lambda y_H} \left(\beta^{-1} - 1\right) M_3 \end{aligned}$$

The FOC w.r.t. $w_{N,t}$ is:

$$0 = M_{2,t}^N w_{N,t} y_{N,t} - M_{4,t}^N y_{N,t} - M_{5,t}^N \frac{y_{N,t}(1-\tau^w) w_{N,t}}{\Psi(\mathcal{M}_w - 1)} e^{-c_{N,t}} - M_{6t} + \beta M_{6t+1} + M_{7,t} \frac{1-\phi}{\phi\lambda}$$

and, evaluated at steady state using $M_2^N = M_2^T$, $M_5^N = M_6 = 0$, and again the FOC w.r.t. Q_t:

$$
\begin{aligned}
0 &= M_{2,t}^N w y_N - M_{4,t}^N y_{NT} - \frac{1-\phi}{\phi\lambda}\left\{\left(\beta^{-1}-1\right)M_3 + M_4^H \lambda y_H\right\} \\
&= M_{2,t}^T w - M_{4,t}^N - M_4^H \frac{1-\phi}{\phi}\frac{y_H}{y_N} - \frac{1-\phi}{\phi\lambda y_{NT}}\left(\beta^{-1}-1\right)M_3 \\
&= M_{2,t}^T w - M_{4,t}^N - M_4^H (1-\lambda) - \frac{1-\phi}{\phi\lambda y_{NT}}\left(\beta^{-1}-1\right)M_3
\end{aligned}
$$

F.2.5 FOCs with respect to R_t

The FOC w.r.t. R_t gives:

$$
0 = -\varphi M_{1,t}^T - (1-\varphi)M_{1,t}^N + \varphi M_{2,t}^T \frac{a_{T,t+1}}{R_t} + (1-\varphi)M_{2,t}^N \frac{a_{N,t+1}}{R_t} - M_{3,t} + M_{9,t}\frac{\vartheta\left(1+\tau^\star\right)}{\mu_{t+1}R_t}
$$

and, evaluated at steady state using $M_1^T = M_1^N$ and $M_2^T = M_2^N$:

$$
0 = -M_1^T + \frac{M_2^T}{R}\left(\varphi a_T + (1-\varphi)a_N\right) - M_3 + M_9 \frac{\vartheta\left(1+\tau^\star\right)}{\mu R}
$$

F.2.6 FOCs with respect to $a_{T,t+1}$ and $a_{N,t+1}$

The FOC w.r.t. $a_{T,t+1}$ gives:

$$
\begin{aligned}
0 = M_{1,t}^T \left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1} - M_{2,t}^T \frac{1}{R_t} + M_{2,t+1}^T \beta \\
+ \beta(1-\vartheta)\mu_{t+1} M_{8,t+1}^T \left\{e^{\mu_{t+1}a_{T,t+1}} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{T,t+1}}\Sigma_{T,t}\right\}\frac{e^{\frac{1}{2}\left(\sigma_{T,t+1}^c\right)^2}}{\Sigma_{t+1}^T},
\end{aligned}
$$

that is, evaluated at steady state:

$$
0 = M_1^T \left(\frac{1-\vartheta}{\vartheta}\right)\mu + \beta(1-\vartheta)\mu M_8^T \left\{e^{\mu a} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a}\Sigma\right\}\frac{e^{\frac{1}{2}(\sigma^y)^2}}{\Sigma}
$$

The FOC w.r.t. $a_{N,t+1}$ gives:

$$
\begin{aligned}
0 = M_{1,t}^N \left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1} - M_{2,t}^N \frac{1}{R_t} + M_{2,t+1}^N \beta \\
+ M_{8,t+1}^N \beta(1-\vartheta)\mu_{t+1} \left\{e^{\mu_{t+1}a_{N,t+1}} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{N,t+1}}\Sigma_{N,t}\right\}\frac{e^{\frac{1}{2}\left(\sigma_{N,t+1}^c\right)^2}}{\Sigma_{t+1}^N},
\end{aligned}
$$

that is, evaluated at steady state using $M_1^N = M_1^T$:

$$
0 = M_1^T \left(\frac{1-\vartheta}{\vartheta}\right)\mu + \beta(1-\vartheta)\mu M_8^N \left\{e^{\mu a} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a}\Sigma\right\}\frac{e^{\frac{1}{2}(\sigma^y)^2}}{\Sigma}
$$

F.2.7 FOCs with respect to $\Pi^w_{T,t}$ and $\Pi^w_{N,t}$

The FOC w.r.t. $\Pi^w_{T,t}$ gives

$$0 = \varphi\Psi \frac{1}{\Pi^w_{T,t}} \ln \Pi^w_{T,,t} - M^T_{5,t}\varphi \frac{1}{\Pi^w_{T,t}} + M^T_{5,t-1}\varphi \frac{1}{\Pi^w_{T,t}} - M_{6,t}\frac{1}{\Pi^w_{T,t}},$$

and thus, evaluated at steady state using $M^T_5 = M_6 = 0$, we get $\Pi^w_T = 1$.

Similarly, the FOC w.r.t $\Pi^w_{N,t}$ gives

$$0 = (1-\varphi)\Psi \frac{1}{\Pi^w_{N,t}} \ln \Pi^w_{N,t} - M^N_{5,t}(1-\varphi) \frac{1}{\Pi^w_{N,t}} + M^N_{5,t-1}(1-\varphi) \frac{1}{\Pi^w_{N,t}} + M_{6,t}\frac{1}{\Pi^w_{N,t}},$$

and thus, evaluated at steady state using $M^N_5 = M_6 = 0$, so that we get $\Pi^w_N = 1$.

F.2.8 FOCs with respect to $\mu_{T,t}$

The FOC w.r.t. μ_t is

$$\begin{aligned}
0 = {} & \beta^{-1}M^T_{1,t-1}\varphi\left(\frac{1-\vartheta}{\vartheta}\right)a_{T,t} + M^T_8\varphi\left(e^{\mu_t a_{T,t}} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{T,t}}\Sigma_{T,t-1}\right)\frac{e^{\frac{1}{2}(\sigma^c_{T,t})^2}}{\Sigma_{T,t}}(1-\vartheta)\,a_{T,t} \\
& + M^N_{8,t}(1-\varphi)\left(e^{\mu_t a_{N,t}} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_{N,t}}\Sigma_{N,t-1}\right)\frac{e^{\frac{1}{2}(\sigma^c_{N,t})^2}}{\Sigma_{N,t}}(1-\vartheta)a_{N,t} - M_{9,t}\frac{1}{\mu_t^2} + \beta^{-1}M_{9,t-1}\frac{\vartheta\,(1+\tau^\star)}{R_t\mu_t^2} \\
& + M^T_{10,t}\varphi\left[-\sigma^y\exp\left\{-\Theta\left(\frac{y_{H,t}}{y_H}-1\right)\right\}+\sigma^c_{T,t+1}\right] + M^N_{10,t}\varphi\left[-\sigma^y\exp\left\{-\Theta\left(\frac{y_{N,t}}{y_N}-1\right)\right\}+\sigma^c_{N,t+1}\right],
\end{aligned}$$

or, evaluated at steady state:

$$\begin{aligned}
0 = {} & \beta^{-1}M^T_1\varphi\left(\frac{1-\vartheta}{\vartheta}\right)a + M^T_8\varphi\left(e^{\mu a} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a}\Sigma\right)\frac{e^{\frac{1}{2}(\sigma^y)^2}}{\Sigma}(1-\vartheta)\,a \\
& + M^N_{8,t}(1-\varphi)\left(e^{\mu a} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a}\Sigma\right)\frac{e^{\frac{1}{2}(\sigma^y)^2}}{\Sigma}(1-\vartheta)a - \frac{M_9}{\mu^2}\left[1-\vartheta\,(1+\tau^\star)\right]
\end{aligned}$$

F.2.9 FOCs with respect to $\sigma^c_{T,t}$ and $\sigma^c_{N,t}$

The FOC w.r.t. $\sigma^c_{T,t}$ is:

$$0 = -\beta^{-1}M^T_{1,t-1}\sigma^c_{T,t} + M^T_{8,t}\sigma^c_{T,t} + M^T_{10,t} - \beta^{-1}M^T_{10,t-1}\,(1-\mu_t)\,,$$

so that we have, in steady state:

$$0 = \left(\beta M^T_8 - M^T_1\right)\sigma^y - M^T_{10}\,(1-\mu-\beta)$$

Symmetrically, the FOC w.r.t $\sigma^c_{N,t}$ gives:

$$0 = -\beta^{-1}M^N_{1,t-1}\sigma^c_{N,t} + M^N_{8,t}\sigma^c_{N,t} + M^N_{10,t} - \beta^{-1}M^N_{10,t-1}\,(1-\mu_t)$$

and, in steady state and using the facts that $M_1^T = M_1^N$ and $M_{10}^T = M_{10}^N$:

$$0 = \left(\beta M_8^N - M_1^T\right)\sigma^y - M_{10}^T\left(1 - \mu - \beta\right)$$

F.2.10 FOCs with respect to $\Sigma_{T,t}$ and $\Sigma_{N,t}$

Finally, the FOC w.r.t. $\Sigma_{T,t}$ gives:

$$0 = -e^{-c_{T,t}} - M_{8,t}^T \frac{1}{\Sigma_{T,t}} + M_{8,t+1}^T \beta \frac{e^{\frac{1}{2}\left(\sigma_{T,t+1}^c\right)^2}}{\Sigma_{t+1}^T} \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1} a_{T,t+1}},$$

that is, in steady state:

$$0 = -e^{-c}\Sigma + M_8^T\left[\beta\vartheta e^{\frac{1}{2}\sigma_c^2} e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a} - 1\right]$$

On the other hand, the FOC w.r.t. $\Sigma_{N,t}$ is:

$$0 = -e^{-c_{T,t}} - M_{8,t}^N \frac{1}{\Sigma_{N,t}} + M_{8,t+1}^N \beta \frac{e^{\frac{1}{2}\left(\sigma_{N,t+1}^c\right)^2}}{\Sigma_{t+1}^N} \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1} a_{N,t+1}},$$

and therefore, in steady state:

$$0 = -e^{-c}\Sigma + M_8^N\left[\beta\vartheta e^{\frac{1}{2}\left(\sigma^y\right)^2} e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu a} - 1\right]$$

Note that the latter two steady-state relationships imply that

$$M_8^N = M_8^T = -\frac{e^{-c}\Sigma}{\mu} = -\frac{e^{-c}}{\mu}\left(\frac{1-\vartheta}{e^{-\frac{\left(\sigma^y\right)^2}{2}} - \vartheta}\right)$$

F.3 Steady state of the Ramsey plan

The steady state of the Ramsey plan is the solution to a nonlinear system involving 33 equations (16 constraints and 17 FOCs) for 33 unknowns (the 16 Lagrange multipliers and 17 endogenous variables of the model). Tedious calculations show the following steady-state multipliers and endogenous variables to solve this nonlinear system (which can easily be checked by plugging back the solution into the above steady-state FOCs). In what follows, we set the labor disutility parameter κ to

$$\kappa = \frac{\left(1 - \tau^w\right) w}{\mathcal{M}_w} e^{-1},$$

which, by equation (21), normalises steady-state per capita consumption to $c = 1$. This normalisation implies equality of the coefficients of relative and absolute risk aversion (respectively CRRA and CARA) for the median household, so that the elasticity of intertemporal substitution (the inverse of the CRRA coefficient) is equal to 1.

Steady-state model variables. The steady-state values of the endogenous variables associated with the solution are:

$$
\begin{aligned}
&Q = 1, \quad w = p_N = (1-\lambda)^{\phi}, \quad p_H = (1-\lambda)^{\phi-1} \\
&c_k = c = 1, \quad a_k = a = 0, \quad R = \beta^{-1}, \quad \mu = 1 - \vartheta\beta e^{\frac{1}{2}(\sigma^y)^2} \\
&\Pi_T^w = \Pi_N^w = 1, \quad y_H = \phi(1-\lambda)^{1-\phi}, \quad y_N = \frac{1-\phi}{(1-\lambda)^{\phi}} \\
&\sigma_k^c = \sigma^y, \quad \Sigma_k = \Sigma = \frac{1-\vartheta}{e^{-\frac{(\sigma^y)^2}{2}} - \vartheta}
\end{aligned}
$$

From the above relations, we also get

$$
n = \frac{y_H}{\varphi} = \frac{1-\lambda\phi}{(1-\lambda)^{\phi}}
$$

Steady-state Lagrange multipliers. The steady-state values of the Lagrange multipliers associated with the solution are:

$$
\begin{aligned}
M_1^T &= M_1^N = -\frac{\beta\vartheta\tau^{\star}e^{-1}\Sigma}{\mu(1-\vartheta)} \\
M_2^T &= M_2^N = \left[2 - \lambda - \frac{\vartheta(1-\beta)\tau^{\star}}{\mu\lambda\phi(1-\vartheta)}\right] e^{-1}\Sigma \\
M_3 &= \frac{\beta\vartheta\tau^{\star}e^{-1}\Sigma}{\mu(1-\vartheta)} \\
M_4^H &= \left[1 - \frac{(1-\beta)\vartheta\tau^{\star}}{\mu(1-\vartheta)\lambda\phi}\right](1-\lambda)^{\phi-1} e^{-1}\Sigma \\
M_4^N &= \left[1 - \lambda - \frac{(1-\beta)\vartheta\tau^{\star}}{\mu(1-\vartheta)\lambda\phi}\right](1-\lambda)^{\phi} e^{-1}\Sigma \\
M_5^T &= M_5^N = M_6 = 0 \\
M_7 &= -\phi\lambda e^{-1}\Sigma \\
M_8^T &= M_8^N = -\frac{e^{-1}\Sigma}{\mu} \\
M_9 &= 0 \\
M_{10}^T &= M_{10}^N = \frac{\sigma^y e^{-1}\Sigma}{(1-\vartheta)\mu}
\end{aligned}
$$

Optimal tax rates. The associated tax rates are in equation (37). If τ^w were different, costly steady-state wage inflation (by (21)) would follow. If $\tau^{\star}$ were different, positive steady state assets and hence costly between-cohort consumption dispersion would follow (by (35) and (31)). Both possibilities would be inconsistent with the constrained-efficient steady state computed above.

G RANK benchmark

In the RANK benchmark, there is no uninsured risk, and all labor earnings (from either sector) are pooled at the cohort level. However, the Blanchard-Yaari demographic structure is maintained to keep net foreign assets stationary, and there is still imperfect labor mobility of workers across sectors.

Sections 2.1, 2.3, 2.4, 3.1 and 3.2 are unchanged relative to the baseline HANK model: only the "Home household block" is modified. We adjust taxes accordingly: since there is no idiosyncratic risk, equation (37) implies that zero assets (hence zero between-cohort dispersion) is achieved at $\tau^{\star} = 0$, while the labor-income tax only corrects for labor market power, i.e., $\tau^{w} = 1 - \mathcal{M}_{w}$.

G.1 Home households

The Representative Agent of cohort s solves

$$\max \mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left\{ -e^{-c_t^s} - \kappa\left(\varphi n_{T,t} + (1-\varphi)\, n_{N,t}\right) - \frac{\Psi}{2}\left[\varphi\left(\ln \Pi_{T,t}^{w}\right)^2 + (1-\varphi)\left(\ln \Pi_{N,t}^{w}\right)^2\right] \right\}$$

subject to, upon survival:

$$\begin{aligned} c_t^s + \frac{\vartheta}{R_t} a_{t+1}^s &= a_t^s + \underbrace{(1-\tau^w)\left[\varphi w_{T,t} n_{T,t} + (1-\varphi)\, w_{N,t} n_{N,t}\right]}_{\text{total disposable income}} \\ &\quad + \underbrace{\varphi \tau^T w_{T,t} n_{T,t}}_{\text{rebate of payroll tax}} + \underbrace{\tau^w \left[\varphi w_{T,t} n_{T,t} + (1-\varphi)\, w_{N,t} n_{N,t}\right]}_{\text{rebate of income tax}} \\ &= a_t^s + \underbrace{(1+\tau^T) w_{T,t}}_{p_{H,t}} \underbrace{\varphi n_{T,t}}_{y_{H,t}} + \underbrace{w_{N,t}}_{p_{N,t}} \underbrace{(1-\varphi)\, n_{N,t}}_{y_{N,t}} \end{aligned}$$

Solving the household problem by going through the same steps as in Section C yields the following log-Euler condition:

$$c_t = c_{t+1} - \ln(\beta R_t) + \left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1} a_{t+1},$$

where

$$\mu_t^{-1} = 1 + \frac{\vartheta}{R_t}\mu_{t+1}^{-1}$$

Finally, aggregating cohort-level assets a_{t+1}^s as in Section A and the consumption dispersion index Σ_t as in Section E, we obtain:

$$a_{t+1} = R_t \left\{ a_t + \frac{w_{T,t}}{1-\lambda} y_{H,t} + w_{N,t} y_{N,t} - c_t \right\}$$

and

$$\Sigma_t = (1-\vartheta)\, e^{\mu_t a_t} + \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_t} \Sigma_{t-1}$$

The latter expression makes it clear that only between-cohort heterogeneity can produce socially costly consumption dispersion – neither idiosyncratic risk nor asymmetric sectoral exposure to the aggregate export shock can.

G.2 Optimal policy problem

G.2.1 Lagrangian function

Aggregating the flow utilities to the planner across sectors as in Section E, we find the total utility flow to be the same as in (48)-(49), except that now $c_{T,t} = c_{N,t} = 0$ and $\Sigma_{T,t} = \Sigma_{N,t} = \Sigma_t$. Accordingly, we can write the Lagrangian function as

$$
\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \left\{ -e^{-c_t}\Sigma_t - \kappa y_{H,t} - \kappa y_{N,t} - \frac{\Psi}{2}\left[\varphi\left(\ln \Pi^w_{T,t}\right)^2 + (1-\varphi)\left(\ln \Pi^w_{N,t}\right)^2\right]\right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{1,t} \left\{ -c_t - \ln\beta - \ln R_t + c_{t+1} + \left(\frac{1-\vartheta}{\vartheta}\right)\mu_{t+1}a_{t+1} \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{2,t} \left\{ -\frac{a_{t+1}}{R_t} + a_t + \frac{w_{T,t}}{1-\lambda} y_{H,t} + w_{N,t} y_{N\$,t} - c_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{3,t} \left\{ -\ln R_t + \ln R^* + \ln Q_{t+1} - \ln Q_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}^H_{4,t} \left\{ -y_{H,t} + \frac{1-\lambda}{w_{T,t}} \left\{ \phi(1-\lambda) c_t + \lambda\phi \frac{Q_t c^*_t}{1+\tau^X_t} \right\} \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}^N_{4,t} \left\{ -y_{N,t} + \frac{1-\phi}{w_{N,t}} c_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}^T_{5,t} \varphi \left\{ -\ln \Pi^w_{T,t} + \frac{\mathcal{M}_w y_{H,t}}{\Psi(\mathcal{M}_w - 1)\varphi}\left(\kappa - w_{T,t} e^{-c_t}\right) + \beta \ln \Pi^w_{T,t} \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}^N_{5,t} (1-\varphi) \left\{ -\ln \Pi^w_{N,t} + \frac{\mathcal{M}_w y_{N,t}}{\Psi(\mathcal{M}_w - 1)(1-\varphi)}\left(\kappa - w_{N,t} e^{-c_t}\right) + \beta \ln \Pi^w_{N,t} \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{6,t} \left\{ -\ln \Pi^w_{T,t} + \ln \Pi^w_{N,t} + \ln w_{T,t} + \ln w_{N,t-1} - \ln w_{T,t-1} - \ln w_{N,t} \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{7,t} \left\{ \frac{1-\phi}{\phi\lambda} \ln w_{N,t} + \frac{1-\lambda}{\lambda} \ln w_{T,t} - \left(\lambda^{-1} - \phi\right) \ln(1-\lambda) + \ln Q_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{8,t} \left\{ \ln\left[(1-\vartheta)e^{\mu_t a_t} + \vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right)\mu_t a_t}\Sigma_{t-1}\right] - \ln \Sigma_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t \widetilde{M}_{9,t} \left\{ \mu_t^{-1} - 1 - \frac{\vartheta}{R_t}\mu^{-1}_{t+1} \right\}
\end{aligned}
$$

G.2.2 First-order conditions

The first-order conditions are as follows:

- FOC w.r.t. c_t:

$$
0 = e^{-c_t}\Sigma_t - \widetilde{M}_{1,t} + \beta^{-1}\widetilde{M}_{1,t-1} - \widetilde{M}_{2,t} + \widetilde{M}^H_{4,t}\frac{(1-\lambda)^2}{w_{T,t}}\phi + \widetilde{M}^N_{4,t}\frac{1-\phi}{w_{N,t}} + \widetilde{M}^T_{5,t}\frac{\mathcal{M}_w y_{H,t}}{\Psi(\mathcal{M}_w - 1)\varphi} w_{T,t} e^{-c_t}
$$

- FOC w.r.t. $y_{H,t}$:

$$\kappa = \widetilde{M}_2^T \frac{w_{T,t}}{1-\lambda} - \widetilde{M}_{4,t}^H + \widetilde{M}_{5,t}^T \frac{\mathcal{M}_w}{\Psi\left(\mathcal{M}_w - 1\right)} \left(\kappa - w_{T,t} e^{-c_t}\right)$$

- FOC w.r.t. $y_{N,t}$:

$$\kappa = \widetilde{M}_2^N w_{N,t} - \widetilde{M}_{4,t}^N + \widetilde{M}_{5,t}^N \frac{\mathcal{M}_w}{\Psi\left(\mathcal{M}_w - 1\right)} \left(\kappa - w_{N,t} e^{-c_t}\right)$$

- FOC w.r.t. Q_t:

$$0 = -\widetilde{M}_{3,t} + \beta^{-1} \widetilde{M}_{3,t-1} + \widetilde{M}_{4,t}^H \frac{1-\lambda}{w_{T,t}} \lambda \phi \frac{Q_t c_t^*}{1+\tau_t^X} + \widetilde{M}_{7,t}$$

- FOC w.r.t. $w_{T,t}$:

$$0 = \widetilde{M}_{2,t} \frac{1}{1-\lambda} y_{H,t} \widetilde{w}_{T,t} - \widetilde{M}_{4,t}^H y_{H,t} - \widetilde{M}_{5,t}^T \frac{\mathcal{M}_w y_{H,t}}{\Psi\left(\mathcal{M}_w - 1\right)} w_{T,t} e^{-c_t} + \widetilde{M}_{6,t} - \beta \widetilde{M}_{6,t+1} + \widetilde{M}_{7,t} \frac{1-\lambda}{\lambda}$$

- FOC w.r.t. $w_{N,t}$:

$$0 = \widetilde{M}_{2,t} y_{N,t} w_{N,t} - \widetilde{M}_{4,t}^N y_{N,t} - \widetilde{M}_{5,t}^N \frac{\mathcal{M}_w y_{N,t}}{\Psi\left(\mathcal{M}_w - 1\right)} w_{N,t} e^{-c_t} - \widetilde{M}_{6,t} + \beta \widetilde{M}_{6,t+1} + \widetilde{M}_{7,t} \frac{1-\phi}{\phi \lambda}$$

- FOC w.r.t. R_t:

$$0 = -\widetilde{M}_{1,t} + \widetilde{M}_{2,t} \frac{a_{t+1}}{R_t} - \widetilde{M}_{3,t}$$

- FOC w.r.t. a_{t+1}

$$0 = \widetilde{M}_{1,t} \left(\frac{1-\vartheta}{\vartheta}\right) \mu_{t+1} - \widetilde{M}_{2,t} \frac{1}{R_t} + \beta \widetilde{M}_{2,t+1} + \beta \widetilde{M}_{8,t+1}^T (1-\vartheta) \mu_{t+1} \frac{e^{\mu_{t+1} a_{t+1}} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right) \mu_{t+1} a_{t+1}} \Sigma_t}{\Sigma_{t+1}}$$

- FOCs w.r.t. $\Pi_{T,t}^w$ and $\Pi_{N,t}^w$:

$$0 = \varphi \Psi \frac{1}{\Pi_{T,t}^w} \ln \Pi_{T,t}^w - M_{5,t}^T \varphi \frac{1}{\Pi_{T,t}^w} + M_{5,t-1}^T \varphi \frac{1}{\Pi_{T,t}^w} - M_{6,t} \frac{1}{\Pi_{T,t}^w}$$

$$0 = (1-\varphi) \Psi \frac{1}{\Pi_{N,t}^w} \ln \Pi_{N,t}^w - M_{5,t}^N (1-\varphi) \frac{1}{\Pi_{N,t}^w} + M_{5,t-1}^N (1-\varphi) \frac{1}{\Pi_{N,t}^w} + M_{6,t} \frac{1}{\Pi_{N,t}^w}$$

- FOC w.r.t. μ_t:

$$0 = \beta^{-1} \widetilde{M}_{1,t-1} \left(\frac{1-\vartheta}{\vartheta}\right) a_t + + \widetilde{M}_{8,t} \left(1-\vartheta\right) \left(\frac{e^{\mu_t a_t} - e^{-\left(\frac{1-\vartheta}{\vartheta}\right) \mu_t a_t} \Sigma_{t-1}}{\Sigma_t}\right) a_t - \widetilde{M}_{9,t} \frac{1}{\mu_t^2} + \widetilde{M}_{9,t-1} \beta^{-1} \frac{\vartheta}{R_t \mu_t^2}$$

- FOC w.r.t. Σ_t:

$$0 = -e^{-c_t} - \widetilde{M}_{8,t} \frac{1}{\Sigma_t} + \widetilde{M}_{8,t+1} \beta \frac{\vartheta e^{-\left(\frac{1-\vartheta}{\vartheta}\right) \mu_{t+1} a_{t+1}}}{\Sigma_{t+1}}$$

G.3 Steady state of the Ramsey plan

Just as we did with the HANK model, evaluate all the planner's FOC at steady state, again setting κ such that steady state consumption $c = 1$. For the endogenous variables of the model, we get:

$$
\begin{aligned}
&Q = 1, \quad w = p_N = (1-\lambda)^{\phi}, \quad p_H = (1-\lambda)^{\phi-1} \\
&c = 1, \quad a = 0, \quad R = \beta^{-1}, \quad \mu = 1 - \vartheta\beta, \quad \Sigma = 1 \\
&\Pi^w_T = \Pi^w_N = 1, \quad y_H = \phi\,(1-\lambda)^{1-\phi}, \quad y_N = \frac{1-\phi}{(1-\lambda)^{\phi}},
\end{aligned}
$$

and for the Lagrange multipliers we get:

$$
\begin{aligned}
\widetilde{M}_1 &= \widetilde{M}_3 = \widetilde{M}^T_5 = \widetilde{M}^N_5 = \widetilde{M}_6 = \widetilde{M}_9 = 0 \\
\widetilde{M}_2 &= (2-\lambda)\, e^{-1} \\
\widetilde{M}^H_4 &= (1-\lambda)^{\phi-1}\, e^{-1} \\
\widetilde{M}^N_4 &= (1-\lambda)^{\phi+1}\, e^{-1} \\
\widetilde{M}_7 &= -\phi\lambda e^{-1} \\
\widetilde{M}_8 &= -\frac{e^{-1}}{\mu}
\end{aligned}
$$