Liquidity Traps, Capital Flows*

Sushant Acharya
FRB New York

Julien Bengui
Université de Montréal

June 2018

Abstract

Motivated by debates surrounding international capital flows during the Great Recession, we conduct a positive and normative analysis of capital flows when a region of the global economy experiences a liquidity trap. Capital flows reduce inefficient output fluctuations in this region by inducing exchange rate movements that reallocate expenditure towards the goods it produces. Restricting capital mobility hampers such an adjustment. From a global perspective, constrained efficiency entails subsidizing capital flows to address an aggregate demand externality associated with exchange rate movements. Absent cooperation, however, dynamic terms-of-trade manipulation motives drive countries to inefficiently restrict capital flows, impeding aggregate demand stabilization.

Keywords: Capital flows, international spillovers, liquidity traps, capital flow management, policy coordination, optimal monetary policy
JEL Classifications: E52, F32, F38, F42

---

*We thank Ozge Akinci, Manuel Amador, Cristina Arellano, Gianluca Benigno, Javier Bianchi, Saroj Bhattarai, Rui Castro, Keshav Dogra, Fernando Duarte, Gauti Eggertsson, Luca Fornaro, Jordi Gali, Jonathan Heathcote, Anton Korinek, Neil Mehrotra, Paolo Pesenti, Gregory Thwaites, Shu Lin Wee, and seminar/conference participants at BIS, BoE, Banque de France, Carleton, CREI, Duke, FRBNY, FRB, Fordham, IMF, Minneapolis Fed, NUS, TSE, UQAM, UNC Chapel Hill, Université de Montréal; 2015 CMSG conference, EUI-CEPR-IMF Conference on Secular Stagnation, Growth and Real Interest Rates, 2015 Minnesota Workshop in Macroeconomic Theory, 2015 Oxford-FRBNY Conference on Monetary Economics, the Norges Bank REFIT Conference, the 2015 RIDGE Workshop in International Macroeconomics, the 2015 RIDGE Workshop on Macroeconomic Crises, the 2015 ESWC, the 2016 IFRMP, the 2016 ITAM-PIER Macro conference and the 2016 SED meeting. Louphou Coulibaly provided superb research assistance. Bengui is grateful to the Fonds de recherche du Québec - Société et culture (FRQSC) for supporting this research under grant 2016-NP-193924. The views expressed in this paper are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of New York or the Federal Reserve System.
Email: Acharya: sushant.acharya@ny.frb.org, Bengui: julien.bengui@umontreal.ca

1 Introduction

Following the global financial crisis of 2007-2008, a large number of advanced economies (including the U.S., the U.K., and the Eurozone) entered a period of anemic economic activity and very low interest rates that resembled a liquidity trap. At the same time, the crisis marked a break in a trend of widening current account imbalances that had been a central feature of the global economy for the previous two decades. After the crisis, on the deficit side, the U.S. experienced an increase its savings rate and a significant reduction in its current account deficit. Meanwhile, on the surplus side, many emerging market economies experienced a surge in capital inflows and a deterioration of their current account position.[1] Some observers and policymakers at the time argued that this incipient unwinding of global imbalances could promote a rebalancing of demand across countries and help facilitate a swifter global recovery (Blanchard, 2009, Blanchard and Milesi-Ferretti, 2009, IMF, 2010). This rebalancing was to crucially rely on the willingness of surplus countries to allow more capital inflows, let their currency appreciate, and thus suffer a loss in external competitiveness (Blanchard and Milesi-Ferretti, 2012). Fearing such prospects, several emerging market countries, likely emboldened by a shift in the stance of multilateral institutions that broke with the Washington consensus view (see Ostry et al., 2010, 2011 and IMF, 2012), adopted forms of capital controls to put the brakes on inflows, with some apparent success (Ahmed and Zlate, 2014).

This narrative raises several questions regarding the foundations and multilateral aspects of capital flow management policies in a liquidity trap. What precise role do capital flows play in global macroeconomic adjustment at the zero lower bound (ZLB)? Do they fulfill this role efficiently, or are capital flow management policies warranted? Are such policies associated with adverse spillover effects? Is coordinating such policies more crucial than in normal times? If so, why? Our goal in this paper is to address these questions.

To this end, we use a general equilibrium two-country model of the world economy in the New Open Economy Macroeconomics tradition, featuring imperfect competition, nominal rigidities, and an explicit zero bound on nominal interest rates. With the global economy's experience of the Great Recession in mind, we interpret Home as the set of advanced economies and Foreign as the set of emerging economies. In line with the recent literature on policy at the zero lower bound, we consider a large unanticipated negative shock to the home discount rate (a negative "demand" shock) that pushes Home, but not Foreign, into a liquidity trap, defined as a situation where the "natural rate" turns negative.[2] Assuming that monetary policy is conducted

[1] This capital inflows surge followed a brief sudden stop in the last quarter of 2008. See Jeanne et al. (2012) for a detailed description of capital flow patterns during this period.

[2] The natural rate is defined as the real interest rate prevailing in an equilibrium with flexible prices and exchange rates, under appropriately specified production subsidies that eliminate monopolistic competition distortions.

optimally, we then compare the global macroeconomic adjustment to the shock under a variety of capital flow regimes.

Three results emerge from our analysis. (i) In a liquidity trap, capital flows help reduce inefficient output fluctuations by decoupling output dynamics from consumption dynamics. Capital flows facilitate this decoupling by generating exchange rate movements that promote expenditure switching in favor of the goods whose provision is the most depressed. (ii) At the ZLB, even a regime of free capital mobility is constrained inefficient. Constrained efficiency calls for subsidizing capital flows, so as to encourage even more decoupling and expenditure switching. Thus, in a liquidity trap, managing capital flows has the potential to increase global welfare. (iii) Despite the desirability of capital account interventions in a liquidity trap, uncoordinated capital flow management policies are not generally warranted. The reason is that dynamic terms-of-trade manipulation incentives partly driving these policies work against macroeconomic stabilization.

To build intuition on the role of capital flows in a liquidity trap, consider the case of a closed economy. When a discount rate shock results in a negative natural rate, monetary policy is constrained by the zero bound. This results in an excessively high real interest rate and output must fall below its efficient level on impact in order to eliminate excess supply of savings.[3] Optimal monetary policy, by committing to keep interest rates at zero past the liquidity trap episode, can engineer a future boom and thereby dissipate excess demand for current savings without as large a fall in output (Eggertsson and Woodford, 2003, Werning, 2012).

In an open economy context, excess savings can be channeled to other economies, which can further limit the initial output drop. The strength of this equilibrating force, however, crucially depends on the degree of capital mobility. Under free capital mobility, the adjustment features large trade imbalances: a more patient Home initially runs a trade surplus and accumulates claims vis-à-vis Foreign. Meanwhile, a negative interest rate differential between Home, for which the ZLB binds, and Foreign, for which it does not, induces a continuous appreciation of the home currency, following a depreciation on impact. This exchange rate response helps redirect expenditure in favor of the Home good early in the liquidity trap, precisely at the time when its provision is the most depressed. In contrast, under closed capital accounts—much like in a closed economy—Home's excess savings cannot be channeled to Foreign. Furthermore, close capital accounts preclude the stabilizing exchange rate movements that occured under free capital mobility. Dissipating excess savings in Home requires home output to fall more on impact. Thus, curtailing capital mobility reduces the potency of the equilibrating force associated with openness.

Does a regime of free capital mobility fulfill the stabilizing role described above efficiently? To answer this question, we formulate a planning problem in which a global planner chooses

[3]This is often referred to as a "demand-driven recession."

a path of taxes or subsidies on capital flows to maximize world welfare. We find that while a regime of free capital mobility is constrained efficient when away from the zero bound, it is constrained *inefficient* when a region of the world economy faces a binding ZLB. In the same way that the real interest rate is excessively high in a closed economy liquidity trap, under free capital mobility the home real exchange rate is also excessively appreciated. The constrained inefficiency of the free capital mobility regime can hence be traced back to an aggregate demand externality resulting from the combination of two factors: output is demand determined and monetary policy is constrained by the zero bound in Home. Atomistic agents do not internalize that their savings decisions lead to adjustments in both inter- and intra-temporal prices. In the presence of nominal rigidities, however, such price adjustments aren't always feasible and quantity adjustments are instead required, resulting in aggregate demand externalities associated with private decisions. Away from the ZLB, optimal monetary policy is able to address this externality. However, at the ZLB it is unable to do so, and capital flow management policy can serve as a useful complement.

We provide a sharp analytical characterization of the constrained efficient capital flow regime, including a closed form expression for the optimal tax wedge on capital flows. During the liquidity trap, this regime entails a subsidy on flows from Home to Foreign and smaller fluctuations in the home output gap.[4] The managed regime also features a steeper exchange rate path, and a more expansionary foreign monetary policy stance during the liquidity trap. Intuitively, capital flow taxes allow exchange rate dynamics to decouple from interest rate differentials, and thereby relax the ZLB constraint in Home without inflicting much harm on Foreign, where monetary policy can adjust.

While our result stands in contrast to the findings of Devereux and Yetman (2014) that capital flow taxes are not desirable in a liquidity trap, the optimal tax formula we derive allows us to reconcile the two views. It shows that a free capital mobility regime is only constrained efficient in knife-edge cases where natural interest rates are equal across countries, the scenario Devereux and Yetman focus on exclusively. Our optimal tax formula also helps distinguish our result from the work of Farhi and Werning (2016), whose general prescription is that optimal financial market taxes should redirect purchasing power toward agents with the highest marginal propensity to consume (MPC) on goods whose provision is relatively more depressed. In fact, our model's prescription entails discouraging spending by home agents at the precise time when the provision of the home good (on which they have a higher MPC) is the most depressed (i.e., early in the liquidity trap). The reason is that such a diversion supports an exchange rate trajectory that induces all agents to redirect expenditure toward the home good at that time. This, in our view, emphasizes the fundamental role of the exchange rate regime in determining

[4]We define the output gap at any date t as the difference between the level of output and its efficient level at the same date. For more details on the path of output under the efficient benchmark, see Section 2.6.

the direction of the inefficiency on capital flows when monetary policy is constrained.

At first glance, our finding that capital does not flow sufficiently in a liquidity trap may seem difficult to reconcile with a recent literature on capital flow management that argues that free capital flows might instead be excessively volatile (see our literature review below). This literature, however, studies capital flow management from the perspective of individual inflow recipient countries, whereas we take a global efficiency standpoint. To illustrate that this distinction is crucial, we also consider a setting where countries manage capital flows non-cooperatively. In this case, we show that the incentives of individual countries to alter capital flows also respond to a desire to manage their dynamic terms-of-trade (dToT), as in Costinot et al. (2014).[5] We show that this dToT manipulation motive leads countries to restrict capital flows and thus conflicts with macroeconomic stabilization in a liquidity trap. Furthermore, for commonly used parameterizations of this model, the dToT manipulation motive can easily dominate the macroeconomic stabilization force in a Nash equilibrium where countries manage their capital account non-cooperatively. In such cases, output gap fluctuations are larger, not only than under the efficient regime, but also than under free capital mobility.[6] This result resonates with the argument in Blanchard and Milesi-Ferretti (2012) that adverse spillover effects of capital controls by recipient countries may be particularly severe in a liquidity trap, and provides a theoretical underpinning for efforts to better coordinate capital flow management policies across countries during such episodes (see IMF, 2011, Ostry et al., 2012).

The rest of the paper is organized as follows. We conclude the introduction with a review of the related literature. We then describe the model in Section 2. Section 3 highlights the role of capital flows at the zero bound, Section 4 analyzes capital flow efficiency, Section 5 studies non-cooperative capital flow management, Section 6 discusses potential extensions, and Section 7 concludes.

Related literature The paper relates to a large body of literature on the conduct of monetary policy in liquidity traps that has developed following the seminal work of Krugman (1998) and Eggertsson and Woodford (2003).[7] In the open economy context, the literature has mainly emphasized spillovers and interdependence of monetary policy across countries (Jeanne, 2009, Haberis and Lipinska, 2012, Cook and Devereux, 2013, Fujiwara et al., 2013, Bodenstein et al.,

[5] This motive arises in every open economy model where countries have some degree of market power over a good they trade. It applies to capital exporters and importers alike, and prevails independently from zero lower bound considerations.

[6] The result that uncoordinated capital flow management policies may lead to a worse outcome than the laissez-faire mirrors the finding of Bengui (2014), who reaches a similar conclusion regarding macroprudential policy in a model of liquidity demand.

[7] See, for instance, Eggertsson and Woodford (2004b,a), Eggertsson (2006, 2010), Christiano et al. (2011), Guerrieri and Lorenzoni (2011), Eggertsson and Krugman (2012), Werning (2012), Correia et al. (2013), and Benigno and Fornaro (2015).

2017).[8] By assuming unitary inter- and intra-temporal elasticities of substitutions, we intentionally abstract from such monetary policy spillovers, and instead focus on the role played by capital mobility in shaping the dynamics of key macro variables in a liquidity trap. Nevertheless, we provide along the way a first analytical characterization of the optimal ZLB exit time in an open economy, extending the closed economy analysis of Werning (2012). Our focus on capital mobility is thus similar to that of Devereux and Yetman (2014), although unlike us they argue that capital controls are not desirable in terms of welfare in a liquidity trap. As mentioned above, we are able to clarify that their result only holds in knife-edge cases where natural interest rates happen to be equal across countries. From an optimal policy perspective, our analysis highlights the role of capital flow taxes/subsidies as an additional tool to overcome the limitations of monetary policy at the ZLB.[9] By analytically characterizing and comparing cooperative and non-cooperative capital flow management regimes, we further uncover a key source of distortion associated with non-cooperativeness and point to the importance of international cooperation during liquidity trap episodes.

Our paper also connects to a wealth of literature on capital flow regulation in emerging market economies. Several recent papers have developed arguments in favor of capital account interventions based on imperfections in financial markets (e.g., Caballero and Krishnamurthy, 2001, Korinek, 2007, 2010, Jeanne and Korinek, 2010, Bianchi, 2011).[10] Others have shown that imperfections in goods markets may also provide a rationale for the use of capital controls. DePaoli and Lipinska (2012) and Costinot et al. (2014) emphasize the role of market power and dynamic terms-of-trade management. Farhi and Werning (2012, 2014) and Schmitt-Grohe and Uribe (2016) stress the role of nominal rigidities. All these papers study optimal capital flow management from the perspective of individual countries. In contrast, we study the desirability of managing capital flows from a global efficiency perspective and highlight how such a regime differs from one where individual countries manage capital flows non-cooperatively.

More generally, our work also speaks to a recent literature on optimal policy interventions in economies with aggregate demand externalities (see, for example, Farhi and Werning, 2012, 2017 and Korinek and Simsek, 2016). While our approach shares several features with this work, our findings stand out from its general message that optimal policy should induce agents with higher MPC on goods that are relatively depressed in some states to tilt their wealth toward these states (Farhi and Werning, 2016).

Finally, the paper also relates to contemporaneous work by Caballero et al. (2015) (CFG),

[8] See also Benigno and Romei (2014) and Fornaro (forthcoming), who study international liquidity traps arising from debt deleveraging episodes.

[9] Korinek (2014) (section 5.2) also briefly analyzes the use of capital flow taxes at the ZLB but does so only from the point of view of a small open economy.

[10] Gabaix and Maggiori (2015) also show that in the presence of financial frictions, capital controls can increase the potency of currency market interventions as a tool to combat exchange rate movements generated by financial turmoil.

Eggertson et al. (2016) (EMSS) and Fornaro and Romei (2018). Like us, these authors study the interplay between international capital flows and liquidity traps. However, the focus of CFG and EMSS is on the steady state analysis of permanent liquidity traps resulting in secular stagnation, while we emphasize transitional dynamics during temporary liquidity trap episodes. With respect to dealing with the multilateral effects of using tools other than monetary policy in a liquidity trap, our papers are complementary: while CFG and EMSS emphasize public debt issuance and fiscal policy, we focus on capital flow management policy and, in particular, on the conflict arising between the dictates of global efficiency and the incentives of individual countries in that regard. Like us, Fornaro and Romei (2018) consider the use of taxes on financial transactions to deal with liquidity traps in an open economy setting, and emphasize the pitfalls of non-cooperative interventions. However, while we contemplate the ex-post use of these policies for stimulatory purposes, they consider them from an ex-ante precautionary standpoint.

2 Model

The world economy consists of two equally sized countries, labeled "Home" and "Foreign."[11] In each country, households consume goods and supply labor, while firms hire labor to produce output. Foreign variables are denoted with asterisks. Following a large body of literature, we adopt the Cole and Obstfeld (1991) parameterization which features unitary inter- and intra-temporal elasticities of substitution. As we shall see, this parameterization eliminates international spillovers from monetary policy and allows us to streamline the role of capital flow regimes. The model is deterministic, and a liquidity trap is generated using a time-varying discount rate for Home.

2.1 Households

Preferences of the representative household in Home are represented by the utility functional

$$\int_0^\infty e^{-\int_0^t (\rho+\zeta_s)ds} \left[\log C_t - \frac{(N_t)^{1+\phi}}{1+\phi} \right] dt,$$

where C_t is consumption, N_t is labor supply, ϕ is the inverse Frisch elasticity of labor supply, ρ is the (steady state) discount rate and ζ_t is a time-varying and country-specific discount rate shifter. Although our model does not feature uncertainty (as of date 0), we will refer to a negative realization of ζ_t as a *negative demand shock*, as such a realization lowers the demand

[11] In the context of the Great Recession, we think of Home as representing the set of *demand deficient* economies and of Foreign as standing for the rest of the world.

for current consumption relative to future consumption (and hence increases the desire to save). C_t is a consumption index defined as

$$C_t \equiv \frac{1}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}} (C_{H,t})^{1-\alpha} (C_{F,t})^{\alpha}$$

where $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$ denotes an index of domestically produced varieties, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$ is an index of foreign produced varieties, and $\alpha \in (0, 0.5]$ is a home bias parameter representing the degree of openness.

Households have access to markets for bonds issued under home and foreign jurisdiction, but they potentially face taxes for investing abroad. Home bonds are denominated in home currency, and foreign bonds are denominated in foreign currency. Since the model does not feature uncertainty, each of the two bonds trivially spans the space of states of nature. The home household's budget constraint is given by

$$\begin{aligned} \dot{D}_{H,t} + \mathcal{E}_t \dot{D}_{F,t} &= i_t D_{H,t} + (i_t^* + \tau_t - \tau_t^*) \mathcal{E}_t D_{F,t} + W_t N_t + T_t + \Pi_t - \int_0^1 P_{H,t}(l) C_{H,t}(l) dl \\ &\quad - \int_0^1 P_{F,t}(l) C_{F,t}(l) dl \end{aligned} \tag{1}$$

where $D_{H,t}$ denotes home currency bond holdings, $D_{F,t}$ denotes foreign currency bond holdings, $\mathcal{E}_t$ is the nominal exchange rate (the price of the foreign currency in terms of the home currency), W_t is the nominal wage, T_t denotes a lump-sum transfer and Π_t denotes the payout of domestic firms. We explicitly allow for taxes and subsidies on capital flows: τ_t is a tax on capital inflows (or a subsidy on capital outflows) in Home, and similarly τ_t^* is a tax on capital inflows (or a subsidy on capital outflows) in Foreign.[12] The proceeds of these taxes are rebated lump sum to domestic households.

Expenditure minimization leads to a home consumer price index (CPI) defined as $P_t \equiv (P_{H,t})^{1-\alpha} (P_{F,t})^{\alpha}$, where $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is Home's producer price index (PPI) and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ is Home's price index of imported goods.[13] The household's demand

[12] A more sophisticated capital flow tax system could feature independent tax rates for inflows and outflows. It would potentially give rise to corner solutions and no-trade equilibria for non-singleton sets of exogenous variables and taxes, thus significantly complicating the analysis. For this reason, we follow the vast majority of the normative literature on capital flow management in assuming that for each country, the tax rate on outflows is constrained to be equal to minus the tax rate on inflows. However, based on the fact that capital flow taxes and nominal interest rates are not perfectly substitutable policy instruments (see Section 2.7), we conjecture that our results would not change substantively under a more sophisticated capital flow tax system.

[13] Similarly, $P_t^* \equiv (P_{F,t}^*)^{1-\alpha} (P_{H,t}^*)^{\alpha}$ is Foreign's CPI, with $P_{F,t}^* \equiv \left[\int_0^1 P_{F,t}^*(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ being Foreign's PPI and $P_{H,t}^* \equiv \left[\int_0^1 P_{H,t}^*(l)^{1-\epsilon} dl\right]^{\frac{1}{1-\epsilon}}$ being Foreign's price index of imported goods.

for a differentiated good l is given by $C_{j,t}(l) = (P_{j,t}(l)/P_{j,t})^{-\epsilon} C_{j,t}$, for $j = H, F$. The law of one price (LOP) implies $P_{j,t}(l) = \mathcal{E}_t P^*_{j,t}(l)$ for $j = H, F$. At the final good level, it implies $P_{j,t} = \mathcal{E}_t P^*_{j,t}$ for $j = H, F$. The terms-of-trade between Home and Foreign are defined as the relative price of the foreign index $S_t \equiv P_{F,t}/P_{H,t} = \mathcal{E}_t P^*_{F,t}/P_{H,t}$, while the real exchange rate is defined as the ratio of CPIs: $Q_t \equiv \mathcal{E}_t P^*_t/P_t$.

The home household chooses consumption, labor supply and bond holdings to maximize utility. His optimal labor supply condition is given by $W_t/P_t = N_t^{\phi} C_t$, and his Euler equations for the home and foreign currency bond holdings are given by

$$\begin{aligned} \frac{\dot{C}_t}{C_t} &= i_t - \pi_t - (\rho + \zeta_t), \\ \frac{\dot{C}_t}{C_t} &= i^*_t + \tau_t - \tau^*_t + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t - (\rho + \zeta_t). \end{aligned} \tag{2}$$

where $\pi_t \equiv \dot{P}_t/P_t$ is home CPI inflation. The combination of these two Euler equations implies a distorted interest parity condition given by

$$i_t = i^*_t + \tau_t - \tau^*_t + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t}. \tag{3}$$

Foreign households are symmetric. Their preferences, constraints and optimality conditions are laid out in online Appendix C.2.

2.2 Firms

Technology Firms in Home and Foreign produce differentiated goods $l \in [0, 1]$ with a linear technology: $Y_t(l) = AN_t(l)$, resp. $Y^*_t(l) = A^* N_t(l)$. Without loss of generality and to streamline the notation, we set the level of productivity in both countries to $A = A^* = 1$.

Price setting We assume that the price of each variety is fully rigid, and normalize this price to 1. As a result, the producer price index (PPI) of both countries in their own currencies are fixed at 1. The CPIs are thus given by $P_t = S_t^{\alpha}$ and $P^*_t = S_t^{-\alpha}$. Furthermore, the real exchange rate is related to the terms-of-trade by $Q_t = S_t^{1-2\alpha}$, and the terms-of-trade coincide with the nominal exchange rate: $S_t = \mathcal{E}_t$. The assumption of fully rigid prices can be regarded as an extreme one, but it has the virtue of significantly improving the analytical tractability of the model and making our results transparent.[14]

[14] Rigid prices rule out PPI inflation or deflation, but do not eliminate the deflation-recession feedback loop that is a key characteristic of liquidity trap episodes. This is because the relevant measure for that mechanism is CPI inflation rather than PPI inflation, and CPI inflation does respond to fluctuations in the nominal exchange rate. We discuss the consequences of relaxing this rigid price assumption in Section 6.

2.3 Government

Each country's government transfers lump-sum the proceeds from capital flow taxes to the domestic household. The home transfer is thus given by

$$T_t = \tau_t D^*_{H,t} - \tau_t \mathcal{E}_t D_{F,t}.$$

2.4 Equilibrium

International "risk"-sharing Combining the home and foreign households' Euler equations for the home bond, it is possible to derive an international consumption smoothing condition relating the ratio of marginal utility in both countries to the real exchange rate[15]

$$C_t = \Theta_t C^*_t Q_t, \tag{4}$$

where $\Theta_t \equiv \Theta_0 \exp\left[\int_0^t \left(\zeta^*_s - \zeta_s + \tau_s - \tau^*_s\right) ds\right]$. Θ_0 is a constant related to initial relative wealth positions. Absent preference shocks and capital flow taxes, (4) would indicate a constant ratio of marginal utilities out of nominal income in both countries. Given our logarithmic utility assumption, this would translate into the ratio of home expenditure to foreign expenditure being equal to a constant (i.e., $P_t C_t = \mathcal{E}_t P^*_t C^*_t$, as $Q_t = \mathcal{E}_t P^*_t / P_t$). Preference shocks and capital flow taxes, however, make this expenditure ratio *time-varying*, following a law of motion given by

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta^*_t - \zeta_t + \tau_t - \tau^*_t \tag{5}$$

Under free capital mobility (i.e., $\tau_t - \tau^*_t = 0$), a scenario where $\zeta^*_t - \zeta_t > 0$ features a relatively more patient home household who experiences a shrinking trade balance and sees its expenditure ratio rise over time: $\dot{\Theta}_t / \Theta_t > 0$. By lending to the foreign household, the home household is able to postpone consumption to when it values it relatively more. In this context, the imposition of a (mild) tax on capital inflows by Foreign (i.e., $\tau^*_t > 0$) discourages capital flows, leading to smoother trade imbalances and a smoother expenditure ratio (i.e., it makes $\dot{\Theta}_t / \Theta_t$ less positive).

Market clearing In equilibrium, bond markets, goods markets and labor markets all have to clear. Market clearing requires $D_{H,t} + D^*_{H,t} = 0$ for the home currency bond, and $D_{F,t} + D^*_{F,t} = 0$

[15] In models featuring uncertainty and complete markets, this condition is often labeled as an international risk-sharing condition. We therefore refer to this condition accordingly, even though risk is absent from our model. (4) is a (potentially distorted) version of what is commonly referred to as the Backus-Smith condition (see Kollmann, 1991 and Backus and Smith, 1993) in which Θ_t would represent a Pareto weight in a planning problem. A detailed derivation of (4) is provided in online Appendix C.3.

for the foreign currency bond.[16] Equilibrium in the market for each good l in Home implies that aggregate home output, defined as $Y_t \equiv \left[\int_0^1 Y_t(l)^{\frac{\epsilon-1}{\epsilon}}\, dl\right]^{\frac{\epsilon}{\epsilon-1}}$, is given by[17]

$$Y_t = (1-\alpha)\, S_t^{\alpha} C_t + \alpha S_t^{\alpha} Q_t C_t^*. \tag{6}$$

Similarly, market clearing for each good in Foreign requires foreign aggregate output, defined as $Y_t^* \equiv \left[\int_0^1 Y_t^*(l)^{\frac{\epsilon-1}{\epsilon}}\, dl\right]^{\frac{\epsilon}{\epsilon-1}}$, to be given by

$$Y_t^* = (1-\alpha)\, S_t^{-\alpha} C_t^* + \alpha S_t^{-\alpha} Q_t^{-1} C_t. \tag{7}$$

Finally, for aggregate employment defined as $N_t \equiv \int_0^1 N_t(l)dl$ and $N_t^* \equiv \int_0^1 N_t^*(l)dl$, equilibrium in the home and foreign labor markets require $N_t = Y_t$ and $N_t^* = Y_t^*$.[18]

The above equilibrium conditions can be combined in a way that greatly simplifies the structure of the optimal policy problems we consider in the next sections. Combining the home and foreign aggregate market clearing conditions (6) and (7) with the international "risk"-sharing condition (4) and the equation linking the real exchange rate to the terms-of-trade, $Q_t = S_t^{1-2\alpha}$, yields expressions for home and foreign aggregate consumption:

$$\begin{aligned} C_t &= Y_t^{1-\alpha} (Y_t^*)^{\alpha}\, \Theta_t^{\alpha} \left(\alpha\Theta_t^{-1} + 1 - \alpha\right)^{-(1-\alpha)} \left(\alpha\Theta_t + 1 - \alpha\right)^{-\alpha}, && (8)\\ C_t^* &= Y_t^{\alpha} (Y_t^*)^{1-\alpha}\, \Theta_t^{-\alpha} \left(\alpha\Theta_t^{-1} + 1 - \alpha\right)^{-\alpha} \left(\alpha\Theta_t + 1 - \alpha\right)^{-(1-\alpha)}, && (9) \end{aligned}$$

Differentiating these equations with respect to time, and substituting the home Euler equation (2), its foreign analogue, as well as the law of motion for the expenditure ratio (5) yields the IS curves

$$\begin{aligned} \frac{\dot{Y}_t}{Y_t} &= i_t - (\rho + \zeta_t) - \frac{\alpha}{(1-\alpha)\,\Theta_t + \alpha} \left(\zeta_t^* - \zeta_t + \tau_t - \tau_t^*\right), && (10)\\ \frac{\dot{Y}_t^*}{Y_t^*} &= i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta}{(1-\alpha) + \alpha\Theta_t} \left(\zeta_t^* - \zeta_t + \tau_t - \tau_t^*\right). && (11) \end{aligned}$$

Lastly, substituting the various equilibrium conditions into the home agent's budget constraint (1) yields an intertemporal budget constraint written as function of the path of the expenditure ratio

$$b_0 = \alpha \int_0^{\infty} e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} \left(\Theta_t - 1\right) dt, \tag{12}$$

[16]Given the redundancy of one of the two bonds, bond portfolios are indeterminate in equilibrium. However, net foreign asset positions as well as prices and allocations are determinate.

[17]For a detailed derivation of the home and foreign aggregate goods market clearing conditions, see online Appendix C.4.

[18]Price dispersion inefficiencies are absent due to our rigid price assumption.

where b_0 is Home's initial net foreign assets expressed in terms of the foreign agent's marginal utility.[19]

Equations (8)-(12) summarize the optimal decisions of private agents and can therefore be regarded as implementability conditions. These equations, along with a description of monetary and capital flow management policy, constitute an equilibrium.

2.5 Demand shock episode

Our analysis concerns a liquidity trap episode. To this end, we assume that just before date 0, the world economy is in a symmetric steady state where both countries have zero net foreign asset positions. Next, as is standard practice in the literature, we generate a liquidity trap via a large unanticipated temporary demand shock which we model as a transitory decrease (from date 0 to T) in the rate of time preference of home households. Formally, the rate of time preference at Home is $\rho + \zeta_t$, where ζ_t is given by

$$\zeta_t = \begin{cases} -\overline{\zeta} & \text{for } t \in [0, T), \\ 0 & \text{for } t \geq T, \end{cases}$$

for $\bar{\zeta} > 0$. The foreign economy is not hit by a demand shock directly and thus $\zeta_t^* = 0 \; \forall t \geq 0$. As our analysis of Section 3 will make clear, for large enough $\bar{\zeta}$, replicating the efficient allocation will require negative nominal interest rates in one or both economies up till date T. Therefore, a monetary authority constrained by the ZLB will fail to achieve the efficient allocation, which will result in a situation akin to a liquidity trap.[20] In the rest of the paper, we refer to the period between dates 0 and T as the *demand shock episode* in the Home economy.

2.6 Socially optimal allocation

A natural way to assess the desirability of decentralized outcomes is to compare these with a socially optimal allocation, which we label *first-best*. The first-best allocation maximizes a symmetrically weighted average of home and foreign agents' utilities subject to worldwide resource constraints and can be described as:[21]

$$N_t^{\text{fb}} = Y_t^{\text{fb}} = \left[\alpha \left(\Xi_t\right)^{-1} + 1 - \alpha\right]^{\frac{1}{1+\phi}}, \qquad \text{and} \qquad N_t^{*\text{fb}} = Y_t^{*\text{fb}} = \left[\alpha \Xi_t + 1 - \alpha\right]^{\frac{1}{1+\phi}}, \tag{13}$$

[19] See online Appendix C.5 for a detailed derivation of this constraint and its analogue for Foreign.

[20] We refer the reader to Section 3 for a formal definition of a liquidity trap in our model.

[21] See Appendix A.1 for a formal description of the planning problem yielding this allocation. The problem is written for an arbitrary Pareto weight but our analysis focuses on a symmetric weight.

and

$$C_t^{\rm fb} = \Xi_t^{\alpha}\left[\left(N_t^{\rm fb}\right)^{1-\alpha}\left(N_t^{*\rm fb}\right)^{\alpha}\right]^{-\phi}, \qquad \text{and} \qquad C_t^{*\rm fb} = \Xi_t^{-\alpha}\left[\left(N_t^{\rm fb}\right)^{\alpha}\left(N_t^{*\rm fb}\right)^{1-\alpha}\right]^{-\phi}, \tag{14}$$

where Ξ_t is a time-varying Pareto weight that denotes the relative weight that the social planner assigns to Home at date t. We assume that, at date 0, the planner weighs the discounted lifetime utility in both economies equally, which for the demand shock episode described in Section 2.5, results in a Pareto weight path given by $\Xi_t \equiv \Xi e^{\min\{T,t\}\bar{\zeta}}$ for $\Xi = (\bar{\zeta}-\rho)/(\bar{\zeta}e^{(\bar{\zeta}-\rho)T}-\rho) < 1$.[22] In other words, the weight assigned to home agents is initially below one, grows during the demand shock episode (reaching one at $\widetilde{T} \equiv -\ln \Xi/\bar{\zeta} < T$) and settles above one from T onwards.

Equations (13)-(14) show that the social planner assigns high employment and low consumption to Home (resp. low employment and high consumption to Foreign) when Ξ_t is low and, accordingly, low employment and high consumption to Home (resp. high employment and low consumption to Foreign) when Ξ_t is high. The paths of these variables are depicted graphically in the panels (a) and (b) of Figure 1.

The path of the planner's shadow values also offer a useful benchmark against which to contrast decentralized outcomes. The *shadow terms-of-trade*, defined as the ratio of the planner's shadow values of the foreign good to the home good, is given by

$$\vartheta_t = \frac{Y_t^{\rm fb}}{Y_t^{*\rm fb}} \times \frac{\alpha\Xi_t + 1 - \alpha}{\alpha + (1-\alpha)\,\Xi_t}. \tag{15}$$

The first term of this expression reflects relative scarcity considerations (i.e., supply factors), while the second term accounts for preference asymmetries between the two goods (i.e., demand factors). Both elements work in favor of a higher relative valuation of the foreign good initially, and a lower valuation of it later on, as displayed in panel (c) of Figure 1.[23]

In the rest of the paper, we refer to the deviation of actual output from the first-best level of output as the *output gap*. Our analysis revolves around the costs imposed by the ZLB under alternative capital flow regimes. These costs can be summarized by three wedges between the decentralized and first-best allocations: the Home labor wedge, $\omega_t \equiv -\ln(MRS_t/MPL_t)$, the Foreign labor wedge, $\omega_t^* \equiv -\ln(MRS_t^*/MPL_t^*)$, and the international "risk-sharing" wedge (which we abbreviate as the international wedge), $\varpi_t \equiv \ln(\Theta_t) - \ln(\Xi_t)$, where $MRS_t \equiv N_t^{\phi}C_t$ and $MPL_t \equiv S_t^{-\alpha}$.[24] The following lemma relates the labor wedges to home output, foreign

[22] Ξ is the Pareto weight assigned by the planner to Home at date 0. Due to differences in discounting, the weight giving both countries equal importance, which we refer to as the *symmetric weight*, is given by $\Xi = \int_0^\infty e^{-\int_0^s(\rho+\zeta_s^*)ds}dt / \int_0^\infty e^{-\int_0^s(\rho+\zeta_s)ds}dt$.

[23] This is true in the presence of home bias. Without home bias (i.e., when $\alpha = 0.5$), preferences for consumption goods are symmetric and the second term is always equal to 1. In this case, shadow terms-of-trade movements only reflects the relative scarcity of the two goods.

[24] Similarly, $MRS_t^* \equiv (N_t^*)^{\phi}C_t^*$ and $MPL_t^* \equiv S_t^{\alpha}$. By definitions, the three wedges are zero in the first-best.

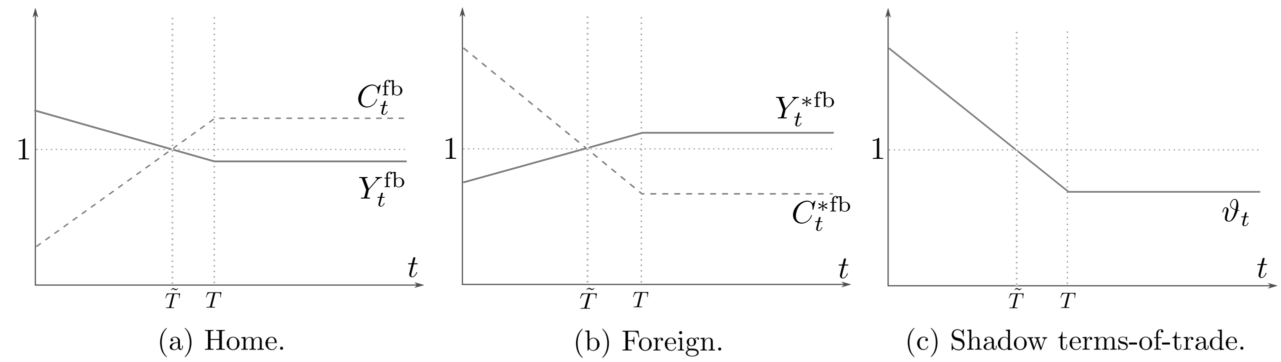


Figure 1: First-best.

output and the expenditure ratio Θ_t.[25]

Lemma 1 (Labor wedges)**.** *In equilibrium, the labor wedges are given by*

$$\omega_t = -\ln\left(\frac{(Y_t)^{1+\phi}}{\alpha\Theta_t^{-1}+1-\alpha}\right), \qquad \omega_t^* = -\ln\left(\frac{(Y_t^*)^{1+\phi}}{\alpha\Theta_t+1-\alpha}\right). \tag{16}$$

Proof. See Appendix B.1. □

2.7 Effects of interest rate policy vs. capital flow tax policy

Before we turn to a formal analysis of a liquidity trap episode, it helps to take a step back and contrast the effect of capital flow tax policy with that of interest rate policy on consumption and output in both Home and Foreign. Recall that our rigid price assumption implies that CPI inflation in both countries is solely driven by exchange rate appreciation/depreciation ($\pi_t = \alpha\dot{\mathcal{E}}_t/\mathcal{E}_t$ and $\pi_t^* = -\alpha\dot{\mathcal{E}}_t/\mathcal{E}_t$). Using the distorted parity condition (3), the home household's Euler equations (2) and its counterpart for the foreign household can thus be expressed as:

$$\frac{\dot{C}_t}{C_t} = (1-\alpha)\, i_t + \alpha i_t^* - (\rho+\zeta_t) + \alpha\,(\tau_t - \tau_t^*)\,, \tag{17}$$

$$\frac{\dot{C}_t^*}{C_t^*} = \alpha i_t + (1-\alpha)\, i_t^* - (\rho+\zeta_t^*) - \alpha\,(\tau_t - \tau_t^*)\,. \tag{18}$$

These Euler equations reveal how the various policy instruments we consider affect consumption growth, and thus household expenditure, in both countries, as we discuss next.

Effect of interest rate policy (17) and (18) show that all else constant, a lower home or foreign interest rate stimulates current consumption in both countries (lower i_t or i_t^* imply lower

[25] We will occasionally refer to a period with a positive labor wedge as a recession and to a period with a negative labor wedge as a boom.

growth rates of consumption $\dot{C}_t/C_t$ and $\dot{C}_t^*/C_t^*$, and thus higher current C_t and C_t^*). This is commonly referred to as the *expenditure changing* effect of monetary policy: lower interest rates induce households in both countries to demand more consumption today.[26]

However, as is well known, in an open-economy context interest rate policy affects economic activity not only by changing the level of expenditure, but also by influencing its allocation across goods produced in different countries. A lower home interest rate depreciates the home currency, leading to *expenditure switching* in favor of home goods and away from foreign goods, consistently with the demand equations (6)-(7). Thus, while the expenditure changing effect of a lower home interest rate is expansionary for both Home and Foreign, the expenditure switching effect is expansionary for Home but contractionary for Foreign. Consequently, the cumulative effect is necessarily expansionary for Home but could be either expansionary (if the expenditure changing effect dominates) or contractionary (if the expenditure switching effect dominates) for Foreign. Under the assumption of unitary inter- and intra-temporal elasticities of substitution, these two forces exactly cancel out and the two economies are "insular" in the sense that the path of foreign output does not depend on home monetary policy (and vice versa, see Corsetti and Pesenti, 2001, Benigno and Benigno, 2003), as indicated by the IS curves (10)-(11).

Effect of capital flow tax policy Like monetary policy, capital flow tax policy influences economic activity through both expenditure changing and expenditure switching effects. Keeping other policy instruments constant, a higher tax on inflows by Home (higher τ_t) requires a higher rate of home exchange rate appreciation going forward by the distorted interest parity condition (3), which induces a lower home CPI inflation and a higher foreign CPI inflation. Thus, it raises the real interest rate faced by home households while lowering the real rate faced by foreign households, which leads to lower home consumption and higher foreign consumption today. Capital flow taxes hence have an asymmetric expenditure changing effect in both countries, consistently with the Euler equations (17)-(18). Naturally, the exchange rate movements associated with capital flow taxes also have expenditure switching effects: the depreciation (on impact) of the home currency brought about by a higher tax on inflows by Home switches expenditure in favor of home goods and away from foreign goods. A key observation from the IS curves (10)-(11) is that under our retained assumptions of unitary inter- and intra-temporal elasticities, this expenditure switching effect actually dominates the expenditure changing ef-

[26]For a given foreign nominal interest rate and for given capital flow taxes, a cut in the home nominal rate lowers the home real rate one for one through a direct effect, but raises it by α through an indirect effect stemming from a drop in home CPI inflation arising from the induced higher rate of home exchange rate appreciation. Hence, the $1-\alpha$ coefficient in front of i_t in equation (17). On the other hand, a cut in the foreign nominal rate lowers the home real rate by α through an indirect effect stemming from a rise in home CPI inflation arising from the induced lower rate of home exchange rate appreciation. Hence, the α coefficient in front of i_t^* in equation (17). The logic is analogous for the coefficients in front of i_t and i_t^* in the foreign household's Euler equation (18).

fect, so that a higher tax on inflows by Home is expansionary for Home, but contractionary for Foreign.

To sum up, capital flow management policy works differently from monetary policy in at least two key respects in our model. First, capital flow taxes have opposite effects on consumption and output. This contrasts with monetary policy, which moves consumption and output in the same direction at home, and does not move output at all abroad. Second, capital flow taxes have opposite effects in both countries (notably on output, but also on consumption). This contrasts with monetary policy, which moves home and foreign consumption in the same direction, and moves output at home but not abroad. This discussion highlights that capital flow taxes cannot be used as a perfect substitute for monetary policy, should the latter be constrained by the ZLB.

The above discussion also helps distinguish the mechanism by which capital flow taxes operate in our flexible exchange rate framework from the one at work in the fixed exchange rate model of Farhi and Werning (2012) and Farhi and Werning (2016) (Section 5.2). Were exchange rates fixed in our model, the expenditure switching effect discussed above would be totally absent, and capital flow taxes would only work through the expenditure changing channel. Keeping the foreign nominal rate constant (but crucially, not the home nominal rate this time), a higher tax on inflows by Home would necessarily lead to a higher home nominal rate through the distorted interest parity condition (3).[27] This would immediately mean a higher home real interest rate and would thus lower home consumption and thereby also home output.[28] Hence, under a fixed exchange rate, the argument for capital flow taxes does not primarily rely on expenditure switching forces (as it does in our model) but rather on the scope to regain monetary autonomy based on expenditure changing forces, in line with the Mundellian trilemma argument.

3 Positive analysis

In order to characterize decentralized outcomes, we need to specify how policy is conducted. Since our interest lies in assessing the performance of alternative capital flow management regimes, we find it convenient to assume that monetary policy is always set optimally by a global monetary authority.[29] Under this assumption, our goal in this section is to shed light on the role played by capital flows in a liquidity trap, by comparing the world economy's adjustment to the demand shock episode described in Section 2.5 under two stylized capital flow regimes:

[27] Note that in this case, the exchange rate would not adjust given the fixed exchange rate regime.

[28] Foreign output would also drop, but given home bias, not by as much as home output. And foreign consumption would be left unchanged (since the foreign nominal and real rates would be unchanged).

[29] This amounts to assuming that it is set cooperatively, and allows us to abstract from any inefficiencies arising from non-cooperative or other suboptimal monetary policy setting. Note that cooperative and non-cooperative monetary policy outcomes differ despite our adopted Cole-Obstfeld parametrization due to level (i.e., steady state) effects related to countries' market power that cannot be undone via labor subsidies since firms do not set prices.

free capital mobility and closed capital accounts.

3.1 Optimal monetary policy with free capital mobility

We assume that a benevolent global monetary authority operates under commitment and specifies the path of nominal interest rates i_t for Home and i_t^* for Foreign in order to maximize a symmetrically weighted sum of welfare in both countries.[30] Importantly, the monetary authority is constrained to set non-negative nominal interest rates in each country at all times.[31] While Appendix A.2 describes and characterizes the optimal monetary policy problem for any capital flow regime, in what follows, we describe the optimal policy problem under a regime of free capital mobility. The problem is given by

$$\max_{i_t \geq 0, i_t^* \geq 0, C_t, C_t^*, Y_t, Y_t^*} \int_0^\infty e^{-\int_0^t \left(\rho + \zeta_h^*\right) dh} \left\{ \Xi_t \left[\ln C_t - \frac{\left(Y_t\right)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{\left(Y_t^*\right)^{1+\phi}}{1+\phi} \right] \right\} dt$$

subject to the implementability constraints (8), (9), (10) and (11) with $\tau_t - \tau_t^* = 0$ and thus, $\Theta_t = \Xi_t$ for all t. The following lemma sets the stage by characterizing the optimal policy away from (or absent) the ZLB.

Lemma 2 (Unconstrained optimal monetary policy)**.** *Absent the ZLB, optimal monetary policy implements the first-best allocation by choosing an initial exchange rate of $\mathcal{E}_0 = \vartheta_0$ and an interest rate path given by:*

$$\mathcal{I}_t = \rho + \frac{(1-\alpha)\,\Xi_t + \frac{\alpha}{1+\phi}}{(1-\alpha)\,\Xi_t + \alpha} \zeta_t \qquad \textit{and} \qquad \mathcal{I}_t^* = \rho + \frac{\alpha\phi}{1+\phi} \frac{\Xi_t}{\alpha \Xi_t + (1-\alpha)} \zeta_t, \tag{19}$$

implying $sign(\mathcal{I}_t^* - \mathcal{I}_t) = -sign(\zeta_t)$ and an exchange rate path of $\mathcal{E}_t = \vartheta_t$.

Proof. See Appendix B.2. □

Intuitively, optimal policy responds to the demand shock episode described in Section 2.5 by lowering the nominal interest rates in both countries, but more so in Home. Owing to the interest parity condition, the resulting interest rate differential is accompanied by a continuous appreciation of the home currency during the episode ($\dot{\mathcal{E}}_t / \mathcal{E}_t < 0$ for $0 \leq t < T$), following a depreciation on impact (i.e., at $t = 0$, $\mathcal{E}_0$ jumps up from 1). The resulting terms-of-trade path coincides with that of the shadow terms-of-trade in the first-best: home goods are relatively cheaper initially (from 0 to $\widetilde{T}$) and then more expensive (from $\widetilde{T}$ onwards). Since output is

[30] The symmetric Pareto weight is given explicitly in Section 2.6 (see in particular Footnote 22).

[31] We are agnostic about whether such ZLB constraints exist because the monetary authority is truly unable to set negative rates or because it has imposed such a constraint on itself voluntarily.

demand determined, this relative price path naturally allows the monetary authority to achieve the first-best allocation.

Home's trade balance is given in terms of the home good by

$$TB_t = \alpha(1-\Theta_t)C_t^* \mathcal{E}_t^{1-\alpha}. \tag{20}$$

For $\Theta_t = \Xi_t$, it is positive initially (from 0 to $\widetilde{T}$), and then negative (from $\widetilde{T}$ onwards). After the shock has dissipated (i.e., after T), Home runs a permanent trade deficit financed by the foreign assets accumulated during the demand shock episode. Hence, trade imbalances and capital flows are a key part of the adjustment allowing the first-best to be achieved. They allow temporarily more patient home agents to reduce both consumption and leisure simultaneously, and catch up later with accordingly higher consumption and leisure.

However, the unconstrained policy described in Lemma 2 is not always implementable. That is the case when one of the interest rate expressions in (19) results in a negative nominal rate. We refer to such a situation as a *liquidity trap* in the country in question. For the rest of the paper, we focus on a situation where the demand shock is large enough to make Home experience a liquidity trap, yet small enough not to make Foreign experience one. We thus make the following assumption:

Assumption 1 (Liquidity trap in Home only)**.** *The demand shock size $\bar{\zeta}$ satisfies:*[32]

$$\rho + \frac{\alpha\rho}{(1-\alpha)\left(\bar{\zeta}-\rho\right)} e^{\bar{\zeta}T}\left[\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho\right] < \bar{\zeta} < \rho + \frac{(1-\alpha)\,\rho}{\alpha\left(\bar{\zeta}-\rho\right)} e^{-\bar{\zeta}T}\left[\bar{\zeta}e^{\left(\bar{\zeta}-\rho\right)T}-\rho\right]. \tag{21}$$

Under these circumstances, the optimal policy is described by the following lemma.

Lemma 3 (Optimal monetary policy at the ZLB)**.** *Consider a demand shock scenario for which Assumption 1 holds. Then the optimal policy is characterized as follows:*

1. *In Home, the ZLB binds, the interest rate path is described by $i_t = 0$ for $t \in [0, \widehat{T})$ and $i_t = \mathcal{I}_t = \rho$ for $t \geq \widehat{T}$, while the output path and optimal ZLB exit time $\widehat{T} > T$ are jointly determined by:*

$$0 = \int_0^{\widehat{T}} e^{-\int_0^t (\rho+\zeta_s)ds}\left[(Y_t^{fb})^{1+\phi} - (Y_t)^{1+\phi}\right]dt \tag{22}$$

and the differential equation (10) with terminal condition $Y_{\widehat{T}} = Y_{\widehat{T}}^{fb}$. Furthermore, the output gap is negative on impact: $Y_0 < Y_0^{fb}$.

[32]Notice that in the limiting case of extreme home bias ($\alpha \to 0$), this condition trivially reduces to $\rho < \bar{\zeta}$, i.e., the condition under which the natural rate becomes negative in the closed economy. Thus, a small α is enough to ensure that condition (21) is satisfied if $\rho < \bar{\zeta}$. More generally, a necessary condition for the parameter set satisfying condition (21) to be non-empty is $\bar{\zeta}T < \ln\left(\frac{1-\alpha}{\alpha}\right)$ (remember that home bias requires $\alpha < 1/2$). Loosely speaking, for a given duration of the liquidity trap T the shock $\bar{\zeta}$ cannot be too large, or equivalently, for a given shock size $\bar{\zeta}$, the duration of the trap T cannot be too long.

2. *In Foreign, the ZLB does not bind, the interest rate path is described by* $i_t^* = \mathcal{I}_t^*$*, and output is at its first best level* $Y_t^* = Y_t^{*fb}$ *at all time.*

3. *The initial exchange rate is given by* $\mathcal{E}_0 = \frac{Y_0}{Y_0^{*fb}} \times \frac{\alpha\Xi_0+1-\alpha}{(1-\alpha)\Xi_0+\alpha} < \vartheta_0$.

Proof. See Appendix B.3. □

Some aspects of the optimal policy outcome common to closed economy frameworks are worth mentioning. First, it is optimal for the monetary authority to make a commitment to keep the home interest rate at zero even after the demand shock scenario has ended at date T. This commitment, known to be a feature of optimal monetary policy at the ZLB and often referred to as *forward guidance*, generates a boom in demand after the end of the liquidity trap. This future boom in turn dampens the initial decline of output via the intertemporal channel. Second, the ZLB exit time $\widehat{T}$ is precisely chosen so as to minimize (weighted) average deviations from the first-best output path. We will refer to the period from 0 to T as phase I of the liquidity trap, and to the period from T to $\widehat{T}$ as phase II. The ZLB therefore implies that home output falls short of its first-best level on impact, grows continuously during phase I, overshooting its first-best level late during that phase, before reverting back to it by the end of phase II, as shown in panel (a) of Figure 2 (dark solid line).[33]

Owing to our unitary elasticities assumptions, each country's interest rate is set only with regard to its own output path. Our model therefore abstracts from the monetary policy interdependence and spillovers that have been the focus of most of the open economy literature on the ZLB (e.g., Haberis and Lipinska, 2012 and Fujiwara et al., 2013). This feature delivers a sharp characterization of exchange rate dynamics at the ZLB. Under a regime of free capital mobility, the exchange rate path is tightly linked to interest rate differentials through the interest parity condition. Relative to what would prevail absent the ZLB, the differential is smaller during phase I and larger during phase II. As a result, the home currency appreciates too slowly during phase I and too fast during phase II. Lemma 3 indicates that as a result, the home exchange rate does not depreciate sufficiently on impact. This is shown in the right panel of Figure 2 (dark solid line).

The above characterization indicates that early in phase I, in addition to being too expensive relative to the future home good (a notion familiar from the closed economy analysis), the current home good is also too expensive relative to the current foreign good. Thus, in analogy with the mechanism by which output has to drop on impact to make consumers content with their savings choice when facing an excessively high interest rate in a liquidity trap, here home output has to drop relative to foreign output to make consumers content with their intra-temporal expenditure allocation decision when facing an excessively appreciated home currency.

[33] This characterization of optimal policy is reminiscent of earlier results in the closed economy ZLB literature (e.g., Eggertsson and Woodford, 2003 and Werning, 2012).

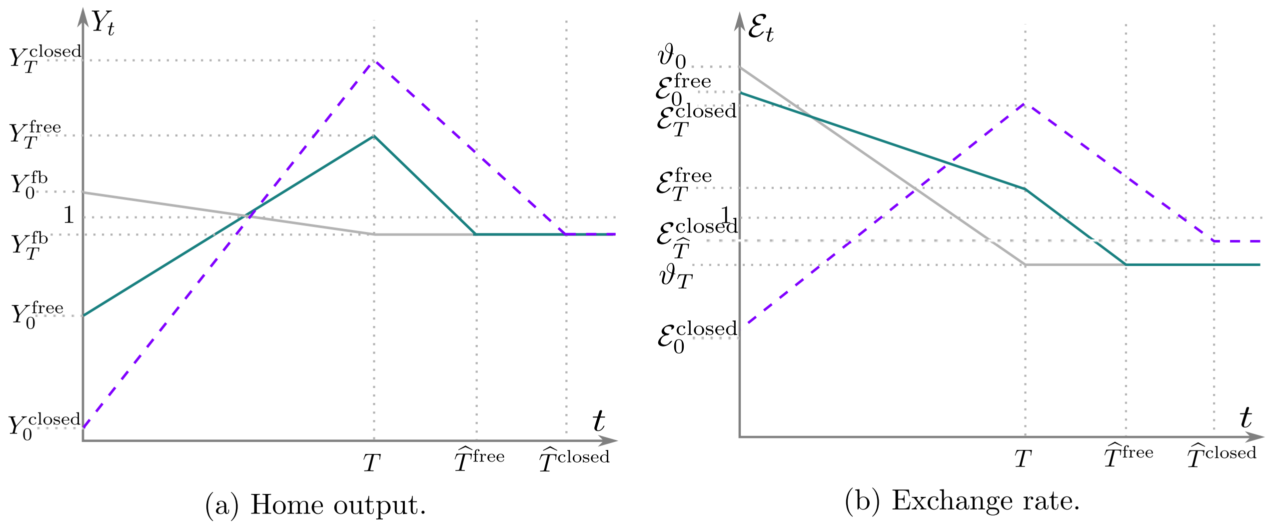


Figure 2: Home output and exchange rate paths under ZLB with free capital mobility (solid dark), ZLB with closed capital accounts (dashed dark) and unconstrained policy with free capital mobility (solid light).

Regarding consumption dynamics, it is easy to establish that home consumption falls on impact, tilts up during phase I and tilts down during phase II. Foreign consumption, on the other hand, jumps up on impact, before tilting down during phase I and II. The paths of the main model variables are shown for illustration purposes in Figure 3 (dark solid line).[34]

The distortions caused by the ZLB constraint can be summarized by the three wedges defined in Section 2.6. Under a regime of free capital mobility, a binding ZLB constraint in Home translates into an opening of the home labor wedge, but not of the foreign labor wedge and the international wedges. More precisely, the home labor wedge path mirrors the output gap path. It jumps up on impact, decreases during phase I, and increases during phase II. This corroborates the narrative that the ZLB causes a recession-boom cycle in Home, and suggests that this cycle is the key source of efficiency losses relative to the unconstrained policy outcome when capital flows freely across countries.

3.2 Capital flows at the ZLB

To shed light on the role played by international capital flows in a liquidity trap, we conduct the experiment of shutting down capital accounts and contrast the resulting allocations and prices to

[34] The parametrization used to generate the figure relies on standard values from the literature. We set the discount rate to $\rho = 0.04$, the openness parameter to $\alpha = 0.2$, and the inverse Frisch elasticity of labor supply to $\phi = 3$. For parameters pertaining to our demand shock trap scenario, we follow Werning (2012). The duration of the shock is set to $T = 2$ years, and the size of the demand shock is set to $\bar{\zeta} = 2\rho$. In a closed economy benchmark, such a shock size would result in a natural real interest rate of -4% for the duration of the liquidity trap. These parameter values satisfy Assumption 1. Unless noted otherwise, they are used for all our subsequent figures.

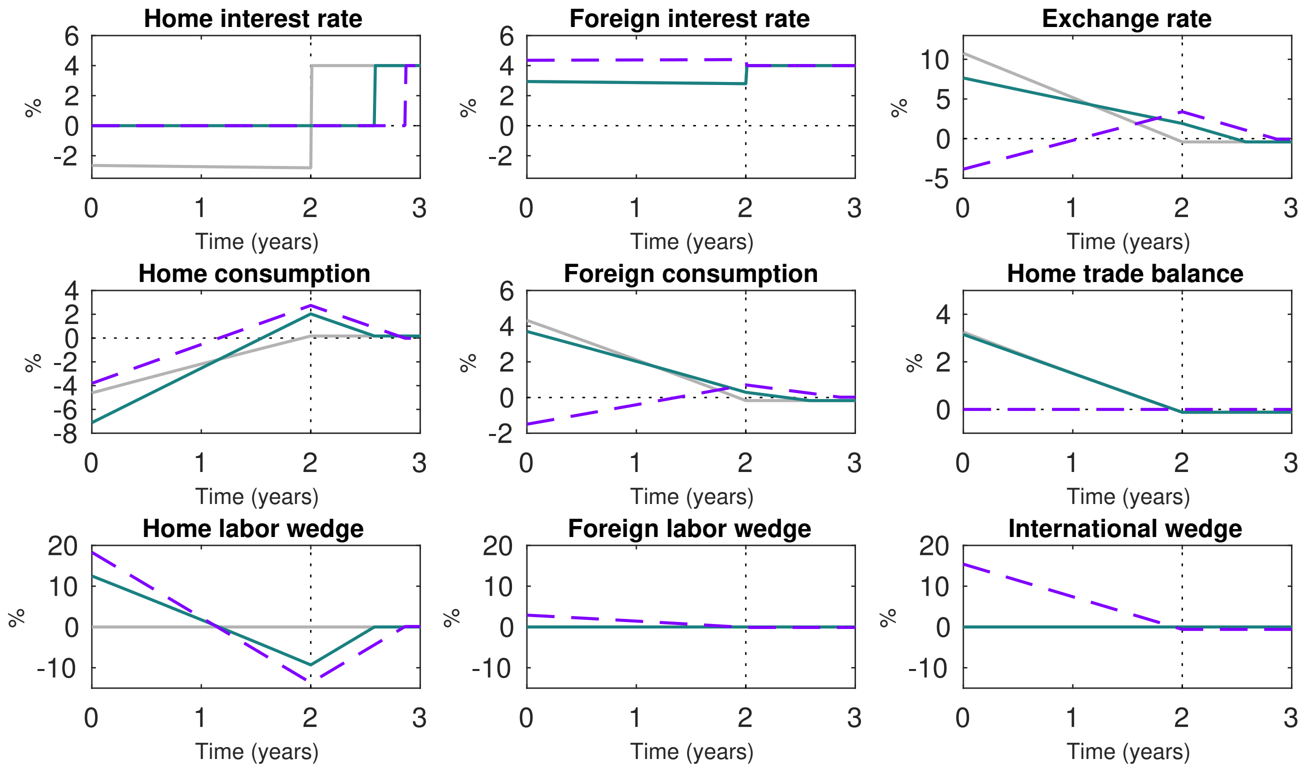


Figure 3: Variable paths under ZLB with free capital mobility (solid dark), ZLB with closed capital accounts (dashed dark) and unconstrained policy with free capital mobility (solid light).

those obtained under a regime of free capital mobility. In doing so, we allow for intra-temporal trade but require it to be balanced period by period. The price implementation of shutting down international capital flows entails setting a tax wedge of $\tau_t - \tau_t^* = \zeta_t$.[35] This implies that the expenditure ratio is fixed at 1. The optimal monetary policy problem under closed capital accounts is thus isomorphic to the case of free capital mobility, but with $\tau_t - \tau_t^* = \zeta_t$ and $\Theta_t = 1$ $\forall t \geq 0$. In these circumstances, the IS equations (10) and (11) are given by their closed economy counterparts and the unconstrained interest rate expressions in (19) become[36]

$$\mathcal{I}_t^{\text{closed}} = \rho + \left(1 + \frac{\alpha}{1+\phi}\frac{\Xi_t^{-1}}{1-\alpha+\alpha\Xi_t^{-1}}\right)\zeta_t \qquad \text{and} \qquad \mathcal{I}_t^{*\text{closed}} = \rho - \frac{\alpha}{1+\phi}\frac{\Xi_t}{\alpha\Xi_t+1-\alpha}\zeta_t. \quad (23)$$

The following lemma adapts Lemma 3's description of optimal monetary policy at the ZLB to the case of closed capital accounts.

Lemma 4 (Optimal monetary policy at the ZLB with closed capital accounts)**.** *Consider a liquidity trap scenario for which Assumption 1 holds. Then the optimal policy outcome is isomorphic to that of Lemma 3, with the following modifications: (i) the foreign interest rate is given in (23), and (ii) the initial exchange rate is given by* $\mathcal{E}_0^{closed} = Y_0^{closed}/Y_0^{*fb}$.

[35] More generally, it entails setting $\tau_t - \tau_t^* = \zeta_t - \zeta_t^*$ in order to achieve $\dot{\Theta}_t/\Theta_t = 0$ according to (5).
[36] See Appendix A.2 for an analysis of optimal monetary policy for an arbitrary capital flow regime.

Proof. The proof of Lemma 3 applies. □

The unconstrained interest rate expressions in (23) indicate that in our demand shock scenario, the following inequalities hold for $t \in [0, T)$: $\mathcal{I}_t^{\text{closed}} < \mathcal{I}_t^{\text{free}} < 0$ and $\mathcal{I}_t^{*\text{closed}} > \mathcal{I}_t^{*\text{free}} > 0$. In other words, closing capital accounts makes Home experience a deeper liquidity trap, while pushing Foreign further away from experiencing one. We will now argue that closing capital accounts hampers the adjustment process in a number of additional dimensions.

Proposition 1 (Capital flows in a liquidity trap)**.** *Relative to the free capital mobility regime, a regime of closed capital accounts results in*

1. *a further delay of the optimal ZLB exit time ($\widehat{T}^{closed} > \widehat{T}^{free}$),*
2. *a more variable path of home output and output gap.*

Proof. See Appendix B.5. □

This comparison is illustrated in Figure 2, where the free capital mobility and closed capital accounts regime are respectively represented by a dark solid line and a dark dashed line. The proposition indicates that the adjustment process happening in Home takes longer *and* features larger inefficient output fluctuations when capital flows are constrained. We interpret this as evidence that capital flows play a fundamentally smoothing role in a liquidity trap. In what follows, we describe the paths of other key macro variables under closed capital accounts to shed light on the mechanisms behind this result.

Our first observation is that under a closed capital account, exchange rate movements lose their stabilizing role during the liquidity trap; rather than facilitating expenditure switching in favor of the home good early on, they hamper it. Despite a positive interest rate differential between Foreign and Home – in fact, a larger one than under free capital mobility – the now distorted interest parity condition does not require the home currency to continuously appreciate during phase I. Instead, it continuously depreciates ($\dot{\mathcal{E}}_t/\mathcal{E}_t = -\mathcal{I}_t^{*\text{closed}} + \tau_t^* - \tau_t > 0$ for $t \in [0, T)$), and may even appreciate (rather than depreciate) on impact if the adverse shock is severe enough.[37] Hence, the exchange rate "moves the wrong way" from the perspective of stabilizing expenditure reallocation during the liquidity trap. Under free capital mobility, a continuous appreciation of the home currency during phase I contributed to encourage expenditure switching in favor of the home good in the early stage of the trap, precisely at the time when the home good was the most under-provided. Without capital flows, in contrast, a continuous depreciation of the home currency during phase I diverts expenditure away from the home good when its provision is the most depressed.

[37] To establish the depreciation during phase I, note that for $t \in [0, T)$, $\zeta_t = -\bar{\zeta}$ so that $-\mathcal{I}_t^{*\text{closed}} + \tau_t^* - \tau_t = \frac{\frac{\phi}{1+\phi}\alpha\Xi_t + (1-\alpha)}{\alpha\Xi_t + (1-\alpha)}\bar{\zeta} - \rho > \frac{(1-\alpha)}{\alpha\Xi_t + (1-\alpha)}\bar{\zeta} - \rho > 0$, where the last inequality follows from Assumption 1.

Our second observation is that the absence of capital flows prevents the decoupling of consumption from output which is characteristic of the first-best allocation. Under free capital mobility, trade surpluses in the early stage of phase I allowed Home to satisfy its desire to shift consumption forward, while letting current demand for its good be supported by foreign consumers. This channel is unavailable when capital is not able to flow. Instead of experiencing an initial consumption boom, Foreign experiences a consumption bust on impact.[38] The contrast in the exchange rate paths between both regimes can be further interpreted from the perspective of the trade balance. Under free capital mobility, Foreign was initially running a trade deficit. Under closed capital accounts, for trade to be balanced, the relative price of home goods must rise enough to reduce Foreign's incentives to import them, which in turn requires a more appreciated home currency.

The paths of the main model variables are contrasted with their free capital mobility counterparts in Figure 3 (dark dashed line). The figure shows an opening of the foreign labor wedge and of the international wedge, along with a more variable home labor wedge than under the free capital mobility regime.

4 Efficient capital flows

The positive analysis of the preceding section emphasized the stabilizing role played by capital mobility in a scenario where a region of the world economy experiences a liquidity trap. Its concluding paragraph hinted at the additional distortions caused by impediments to cross-border capital flows in such an episode. In this section, we adopt a normative perspective and analyze the constrained efficiency properties of the free capital mobility regime.

4.1 Constrained planning problem

We frame this constrained efficiency question by formulating a Ramsey planning problem. We endow the global planner with the ability to tax or subsidize international financial transactions, in addition to its ability to set monetary policy under the zero bound constraint. The planning problem is given by

$$\max_{\{i_t \geq 0, i_t^* \geq 0, \tau_t, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t \left(\rho+\zeta_h^*\right)dh} \left\{ \Xi_t \left[\ln C_t - \frac{\left(Y_t\right)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{\left(Y_t^*\right)^{1+\phi}}{1+\phi} \right] \right\} dt$$

[38]The behavior of home consumption on impact relative to the free capital flow regime depends on two counteracting forces. On the one hand, the lack of savings opportunity pushes consumption up on impact. On the other hand, the fact that Home output is more depressed pushes consumption down.

subject to (5), (8), (9), (10) and (11), with Θ_t only allowed to jump at $t = 0$. τ_t is the tax on capital inflows into (or subsidy on outflows out of) Home. Without loss of generality, we can assume that the planner sets the foreign capital flow tax τ_t^* to zero.[39] (5) and (8)-(11) are implementability conditions, while $i_t \geq 0$ and $i_t^* \geq 0$ are zero bound constraints.

4.2 Characterization of efficient regime

Framing the efficiency question via the above Ramsey problem has several advantages. First, we can evaluate the constrained efficiency of the free capital mobility regime by asking a very simple question, namely: *Is the planner's optimal choice characterized by $\tau_t = 0$, $\forall t$?* Second, anticipating a negative answer to our first question, we can learn about the direction of the inefficiency by analyzing the sign of the optimal capital flow tax. The following lemma provides a characterization of the efficient capital flow regime.

Lemma 5 (Targeting rule in efficient capital flow regime)**.** *The constrained efficient capital flow regime is characterized by the targeting rule*

$$1 - e^{-\omega_t - \varpi_t} = \Xi_t \left(1 - e^{-\omega_t^* + \varpi_t}\right). \tag{24}$$

Proof. See part of Appendix A.3 leading up to equation (A.19). □

As is standard with targeting rules in New Keynesian models (see, e.g., Woodford, 2003, Gali, 2015), this rule does not directly describe what optimal policy should be, but rather what it should target. It indicates that the planner aims for a balance between the distortions experienced by Home (left-hand-side of (24)) and the ones experienced by Foreign (right-hand-side of (24)), with a weight reflecting the time-varying Pareto weight Ξ_t. Absent (or away from) the ZLB, all three wedges, i.e., the home labor wedge ω_t, the foreign labor wedge ω_t^* and the international wedge ϖ_t, are zero. As a result, intervening in international financial markets is undesirable in that case as $\tau_t = \dot{\varpi}_t = 0$ and the free capital mobility regime is constrained efficient.[40] This is no longer true when the ZLB binds for at least one country. When the ZLB binds in Home but not in Foreign, free capital mobility is constrained inefficient, for it would imply an opening of the home labor wedge only, a contradiction with the targeting rule (24).[41]

[39] We follow the literature in normative open-economy macroeconomics in assuming that the planner has access to a date 0 transfer across the two countries. This assumption allows us to drop the country resource constraint (12) from the planning problem and makes the tax differential $\tau_t - \tau_t^*$ (rather the individual taxes τ_t, τ_t^*) the only relevant instrument. Normalizing $\tau_t^* = 0$ is thus without loss of generality.

[40] It actually turns out to be simply efficient (i.e., without the "constrained" qualification), since the first-best allocation is achieved in this case.

[41] This logic generically holds in the case where the ZLB binds in both countries. We briefly discuss this case at the end of this section.

This result, together with a characterization of the constrained efficient regime, constitutes our main normative contribution.

To gain insights into the logic of the optimal policy intervention, it is useful to resort a first-order Taylor approximation of some key equations around the symmetric steady state.[42] First, linearizing the targeting rule (24) yields

$$\omega_t + \varpi_t = \omega_t^* - \varpi_t. \tag{25}$$

Hence, faced with asymmetric labor wedges across countries, the planner wants to distort the allocation of consumption in favor of the country with the smallest labor wedge, i.e., experiencing the least severe recession (or the largest boom). Second, linearizing the labor wedge expressions in (16) delivers

$$\omega_t = -\alpha\varpi_t - (1+\phi)\,\tilde{y}_t, \qquad \text{and} \qquad \omega_t^* = \alpha\varpi_t - (1+\phi)\,\tilde{y}_t^* \tag{26}$$

for the output gaps $\tilde{y}_t \equiv \hat{Y}_t - \hat{Y}_t^{fb}$, $\tilde{y}_t^* \equiv \hat{Y}_t^* - \hat{Y}_t^{*fb}$, where $\hat{Y}_t \equiv \ln(Y_t/Y)$, $\hat{Y}_t^* \equiv \ln(Y_t^*/Y^*)$, $\hat{Y}_t^{fb} \equiv -\alpha\hat{\Xi}_t/(1+\phi)$, $\hat{Y}_t^{*fb} \equiv \alpha\hat{\Xi}_t/(1+\phi)$ and $\hat{\Xi}_t \equiv \ln \Xi_t$.[43] In a scenario of interest where the ZLB does not bind in Foreign, the foreign output gap is zero at all times, $\tilde{y}_t^* = 0$, and the foreign labor wedge is therefore proportional to the international wedge: $\omega_t^* = \alpha\varpi_t$. Taking a first-order Taylor approximation of the IS curves (10)-(11) and expressing these in gaps yields

$$\dot{\tilde{y}}_t = i_t - r_t^n - \alpha(\tau_t - \tau_t^*), \qquad \text{and} \qquad \dot{\tilde{y}}_t^* = i_t^* - r_t^{*n} + \alpha(\tau_t - \tau_t^*) \tag{27}$$

where $r_t^n \equiv \rho + \left(1 - \alpha + \frac{\alpha}{1+\phi}\right)\zeta_t$ and $r_t^{*n} \equiv \rho + \left(\alpha - \frac{\alpha}{1+\phi}\right)\zeta_t$ are the home and foreign natural real interest rate, respectively. Differentiating the two equations in (26) as well as the targeting rule (25) with respect to time, and combining them with the IS equations in (27) (when Home is at the ZLB but Foreign is not) yields:

$$\tau_t - \tau_t^* = -\Psi r_t^n, \qquad i_t^* = r_t^{n*} + \alpha\Psi r_t^n, \qquad \dot{e}_t = -r_t^{n*} + (1-\alpha)\,\Psi r_t^n, \qquad \text{and} \qquad \dot{\tilde{y}}_t = -\left(1 - \alpha\Psi\right) r_t^n \tag{28}$$

where $e_t \equiv \ln(\mathcal{E}_t)$, for $\Psi \equiv (1+\phi)/[2 + \alpha\,(\phi - 1)] > 0$.[44] This compares with $\tau_t - \tau_t^* = 0$,

[42]This symmetric steady state is described in Appendix A.3.1.

[43]Thus, absent the international wedge, the labor wedges are simply proportional to the negative output gaps, as in the standard closed economy model (coinciding with the limit where $\alpha \to 0$). However, for given output gaps, a positive international wedge is associated with a smaller (i.e., more negative) home labor wedge and a larger (i.e., more positive) foreign labor wedge. The intuition is that for given output levels, a larger international wedge translates into higher home consumption, and therefore lower home marginal utility and a higher home marginal rate of substitution (MRS) of consumption for leisure, thus reducing the labor wedge. Likewise, in Foreign, a larger international wedge translates into lower consumption, higher marginal utility, a lower MRS, and thus a higher labor wedge.

[44]Note that $\Psi \lesseqgtr 1$ and $\partial\Psi/\partial\alpha \gtreqless 0$ for $\phi \lesseqgtr 1$. Furthermore, $0 < \alpha\Psi < 1$.

$i_t^* = r_t^{n*}$, $\dot{e}_t = -r_t^{n*}$ and $\dot{\tilde{y}}_t = -r_t^n$ in the free capital mobility regime. Meanwhile, in either regime, home monetary policy is set such that the home output gap averages out to zero over time: (22) (which applies irrespective of the capital flow regime) is given in linearized form by $\int_0^{\widehat{T}} e^{-\rho t}\tilde{y}_t dt = 0$. (25) and (26) then show that in the constrained efficient regime, the home labor wedge, foreign labor wedge and international wedge must also average out to zero over time. Given the these properties, one can think of the degree of distortions associated with a regime as being related to the slope of these variables: smoother/less variable gaps or wedges (as represented by smaller growth rates in absolute values) indicate smaller distortions.[45] With this in mind, we can summarize our main normative results in the following proposition.

Proposition 2 (Constrained efficient capital flow regime)**.** *The free capital mobility regime is constrained efficient if and only if the ZLB constraints never bind. Furthermore, when the ZLB binds in Home but not in Foreign from 0 to T, up to a first-order, the constrained efficient regime compares with the free capital mobility regime as follows:*

1. *Capital flows out of Home are subsidized in phase I and taxed in phase II.*
2. *The home output gap is smoother, while the foreign output gap is still zero.*
3. *The home labor wedge is smoothed out at the expense of the foreign labor wedge and international wedge, which both open.*
4. *The home exchange rate appreciates at a faster rate in phase I and at a slower rate in phase II.*
5. *Monetary policy in Foreign is more expansionary in phase I and less expansionary in phase II.*

Proof. See argument in text. □

Point 1. reflects the expression for τ_t in (28) according to which the optimal tax wedge on flows from Home to Foreign is proportional to the negative of the home natural rate during the time spent by Home at the ZLB. Since the home interest rate is zero at the ZLB, $-r_t^n$ represents the gap between the home interest rate and its ideal level. This suggests that the capital flow tax is used as a substitute for deficient home monetary policy, but only to the extent that the policy deviates from its unconstrained target.[46] Since $r_t^n < 0$ during phase I and $r_t^n = \rho > 0$ during phase II, the optimal policy consists in subsidizing flows out of Home during phase I, and taxing

[45] This representation admittedly abstracts from the fact that the ZLB exit time differs across capital flow regimes.

[46] "Unconstrained" here refers to the absence of a ZLB constraint.

such flows during phase II.[47] The intervention yields a smoother real adjustment in Home, as evidenced by a reduction in the growth rates of the home output gap and home labor wedges (points 2. and 3.).[48] This improvement is achieved at the expense of a mild destabilization of the foreign labor wedge and international wedge. Thus, from an optimal taxation perspective, the planner's intervention in the efficient capital flow regime can be seen as reflecting wedge management: it is desirable to strike a balance between fluctuations in the model's three wedges so as to satisfy (25).

How does the described path of the capital flow tax result in a smoother adjustment in Home? In line with our discussion of Section 2.7, the intuition has to do with the tilting of the exchange rate path induced by the optimal tax (points 4.). Through the distorted interest parity condition, a positive τ_t increases the required rate of appreciation of the home currency in phase I, while a negative τ_t decreases it in phase II. This tilting of the exchange rate path stabilizes demand for the home good, by switching expenditure in its favor in the early stage of phase I, and at its expense in the late stage of phase I and in phase II. Accordingly, and since expenditure switching occurs vis-à-vis the foreign good, the capital flow tax is contractionary in Foreign when it is expansionary in Home and vice-versa (see IS curves in (27)). And as the ZLB on the foreign nominal rate is not binding, foreign monetary policy is optimally adjusted to a more expansionary (contractionary) stance in phase I (II) so as to align foreign output with its first-best level. Thus, in analogy to the manner in which delaying exit from the ZLB in a closed economy allows borrowing monetary policy room from the future, constrained efficient capital flow management can be interpreted as enabling a transfer of monetary policy room across regions.

It is worth noting that although a full relaxation of the home ZLB constraint can always be engineered through a high enough tax on capital inflows into Home, such a policy is not desirable.[49] Intuitively, capital flow taxes can relax the home ZLB constraint, but only at the cost of distorting the international allocation of consumption. Monetary policy, in contrast, stimulates aggregate demand without distorting the international wedge. Our discussion of Section 2.7 hinted at the fact that capital flow management is not a perfect substitute for monetary policy. Our normative analysis of this section further shows that it *should not* be used to fully relax constraints on monetary policy.

[47]For $t \geq \hat{T}$, expression (28) does not hold any more, since the home output gap is back to zero. As a result, and consistently with the targeting rule (25), all wedges are zero and so is the optimal capital flow tax.

[48]The home labor wedge growth rate is given by $\dot{\omega}_t = \frac{2(1-\alpha)+\alpha}{2(1-\alpha)+\alpha(1+\phi)}\left(1+\phi\right) r_t^n$ in the efficient regime, while it was given by $\dot{\omega}_t = \left(1+\phi\right) r_t^n$ in the free capital mobility regime.

[49]The fact that a high enough capital inflow tax can fully relax the home ZLB constraint is apparent from the home New Keynesian IS curve in (27). The fact that this is not desirable follows from the observation that imposing zero output gaps (a consequence of fully relaxed ZLB constraints) in the labor wedge expressions in (26) and substituting these into the targeting rule (25) leads to a zero international wedge, which in turn require zero capital flow taxes (a contradiction).

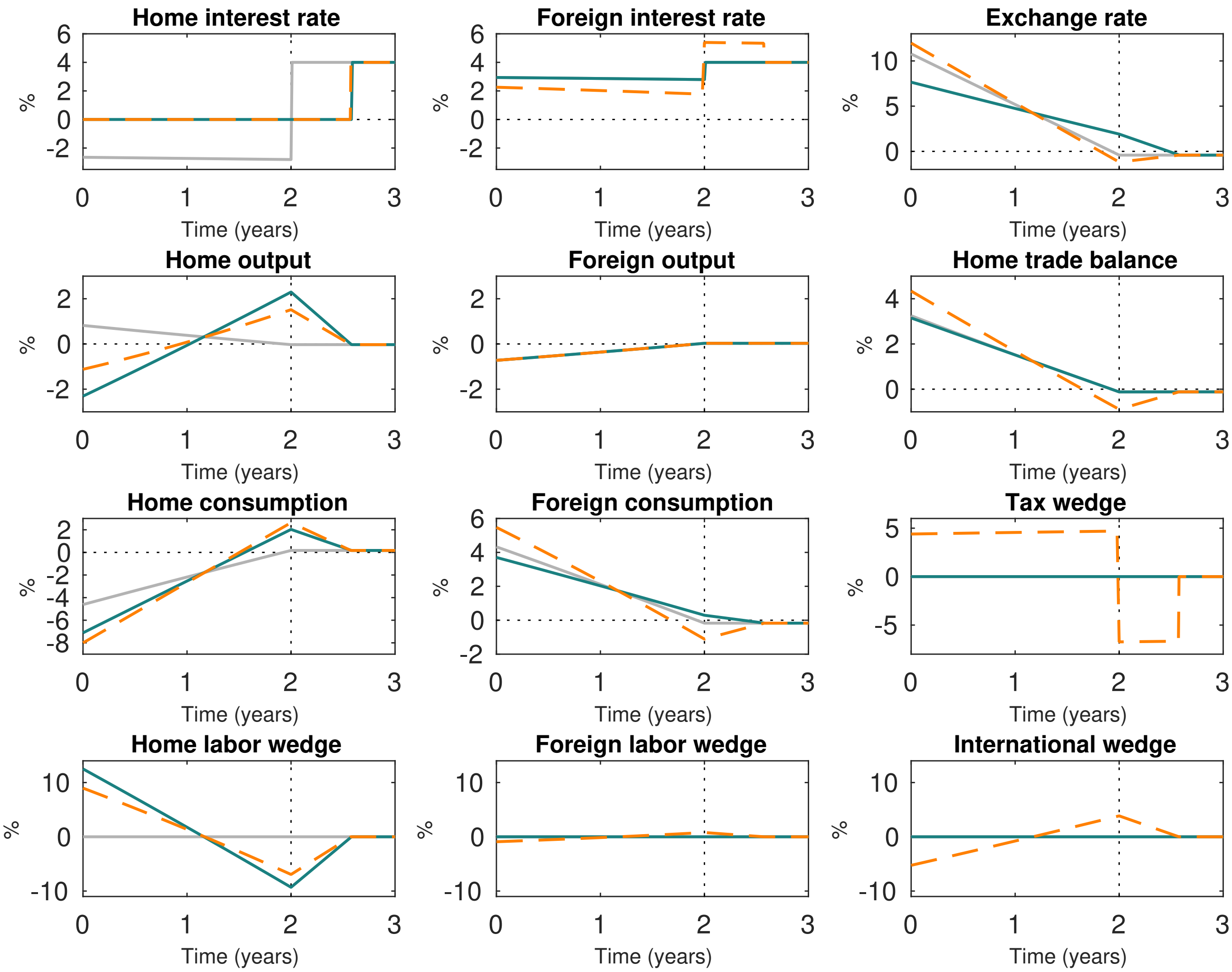


Figure 4: Variable paths under ZLB with free capital mobility (solid dark), ZLB with efficient regime (dashed) and unconstrained policy (solid light).

This stabilizing effect of efficient capital flow management is depicted in Figure 4, which contrasts the paths of key variables in the efficient regime with their counterparts in the free capital mobility regime and unconstrained benchmark.[50] It is evident that the capital account intervention results in a steeper exchange rate (and hence terms-of-trade) path, a smoother home output gap, more pronounced consumption fluctuations and larger current account imbalances. Accordingly, the home labor wedge is stabilized at the expense of an opening of the foreign labor wedge and the international wedge.

The underlying rationale for the constrained inefficiency of the free capital mobility regime is an aggregate demand externality generically present in economies with nominal rigidities where constraints on monetary policy make the socially optimal allocation unattainable (see Blanchard and Kiyotaki, 1987 for an early discussion focusing on pricing decisions, and more recently,

[50]The picture is produced using an exact non-linear solution to the planning problem, using the parametrization described in Footnote 34.

Farhi and Werning, 2016 for a general treatment regarding financial choices). With the nominal rigidities present in our model, prices fail to fulfill their allocative role, and the fall in demand associated with agents' increased desire to save pushes the economy into a recession. The monetary authority internalizes these effects, attempts to nullify them by affecting intertemporal prices, and is successful at correcting the externality with monetary policy alone absent (or away from) the ZLB. But at the ZLB, it cannot lower the nominal rate sufficiently, and distorting international savings decision leads to exchange rate movements that help curtail the severity of the bust-booms cycle in Home.

While our constrained inefficiency result might appear to fall under the umbrella of the general theory put forward by Farhi and Werning (2016), the multiple-currency model structure called for by our particular application makes the mechanics of the intervention, and thus our contribution, distinct. Rather than aiming to simply direct purchasing power toward agents with the highest marginal propensity to consume (MPC) on relatively more depressed goods, the optimal intervention in our model is guided by a desire to switch expenditure toward more depressed goods by manipulating the only flexible component of relative prices, namely the exchange rate. In fact, the resulting policy prescription is at odds with Farhi and Werning's general principle: while home agents have a higher MPC on the home good, our model's prescription entails discouraging spending by these agents at the precise time when this good is relatively more depressed (early in phase I). The reason is that such a diversion supports an exchange rate trajectory that induces all agents to redirect expenditure toward the home good at that time. This underlines the relevance of the exchange rate regime for the direction of the desirable intervention. Were the constraint on monetary policy arising from a peg rather than a ZLB, this expenditure switching channel would be absent, and Farhi and Werning's general principle would apply (see Farhi and Werning, 2012 and our discussion of Section 2.7).

A further benefit of our two-country model structure is that it naturally lends itself to an investigation of the coordination problem inherent to capital flow policies in a liquidity trap, an issue to which we turn in Section 5.

Global liquidity trap While our focus is on a scenario where only a region of the world economy experiences a liquidity trap, it is worth noting that our normative results carry over to an alternative global liquidity trap scenario (i.e., where the ZLB binds in both countries). Indeed, combining the labor wedge expressions in (26) with the IS curves in (27), under the assumption that $i_t = i_t^* = 0$, yields[51]

$$\tau_t - \tau_t^* = -\dot{e}_t = -\Psi^{g}\left(r_t^n - r_t^{n*}\right), \quad \dot{\tilde{y}}_t = -\left(1-\alpha\Psi^{g}\right)r_t^n - \alpha\Psi^{g} r_t^{n*}, \quad \dot{\tilde{y}}_t^* = -\alpha\Psi^{g} r_t^n - \left(1-\alpha\Psi^{g}\right)r_t^{n*}. \tag{29}$$

where the g superscript stands for "global" liquidity trap, and $\Psi^{g} \equiv (1+\phi)/[2(1+\alpha\phi)]$. This contrasts with $\dot{e}_t = 0$, $\dot{\tilde{y}}_t = -r_t^n$ and $\dot{\tilde{y}}_t^* = -r_t^{n*}$ under free capital mobility. Thus, when the ZLB binds everywhere, our result translates into one indicating that it is optimal to subsidize outflows out of the country with the lowest natural interest rate. Devereux and Yetman (2014) argue that imposing capital controls necessarily reduce welfare during a liquidity trap, using an environment where due to an absence of home-bias and preference shocks that do not affect the disutility from labor supply, natural interest rates are by construction equal across countries at all times. Our optimal tax expression in (29) reveals the knife-edge nature of their result by showing that the free capital mobility regime is only constrained efficient in non-generic cases where natural rates happen to be equal in Home and Foreign.

5 Capital flow management and currency wars

Given the constrained inefficiency of a free capital mobility regime established in Section 4, it is natural to ask whether the constrained efficient outcome can also be achieved in a decentralized (i.e., non-cooperative) setting where each country sets its own capital flow taxes independently. Our objective in this section is to tackle this question.

5.1 Game between planners

In order to focus on the potential coordination problem pertaining to capital flow management, we still delegate monetary policy decisions to a global monetary authority but let national planners in Home and Foreign set capital flow taxes optimally.[52] The global planner sets monetary policy optimally for all future dates and chooses a date 0 transfer b_0 from Foreign to Home to maximize global welfare. The home planner chooses a path for home capital flow taxes to maximize home welfare, and the foreign planner chooses a path for foreign capital flow taxes to maximize foreign welfare. The three planners choose their actions simultaneously at date 0.

[51] These expressions hold at any instant where the ZLB binds in both countries. However, as the optimal ZLB exit times does generally not coincide for Home and Foreign, after the first country has exited the ZLB but before the second ones has, there is a time interval where expressions analogous to those in (28) will hold.

[52] We retain the assumption of cooperative monetary policy so as to study the implications of non-cooperativeness of capital flow management policies in a transparent fashion. In an earlier version (Acharya and Bengui, 2015), we obtained qualitatively similar results and predictions under the assumption of non-cooperative monetary policy.

The problem of the global planner is given by

$$\max_{\{i_t\geq 0, i_t^*\geq 0, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}, b_0} \int_0^\infty e^{-\int_0^t (\rho+\zeta_h^*)dh} \left\{ \Xi_t \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] \right\} dt$$

subject to (5), (8), (9), (10), (11) and (12); the problem of the home planner is given by

$$\max_{\{\tau_t, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t (\rho+\zeta_h)dh} \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] dt$$

subject to (5), (8), (9), (10), (11) and (12); and the problem of the foreign planner is given by

$$\max_{\{\tau_t^*, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t (\rho+\zeta_h^*)dh} \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] dt$$

subject to (5), (8), (9), (10), (11) and (12).

The three planning problems are analyzed formally in Appendix A.4. In the next two sections, we discuss the motivations faced by national planners and explain how these shape the macroeconomic adjustment in a liquidity trap.

5.2 Best responses

Monetary policy is set by the global planner following the same principles as in Sections 3 and 4. The global planner aims to replicate the first-best output paths in both countries, and, whenever the interest rates necessary to achieve that goal violate the ZLB, uses forward guidance to minimize average output gaps over time. The new element of the policy setting analyzed in this section is that the path of the expenditure ratio Θ_t, whose growth rate is taken as given by the global monetary authority, is determined by the interaction of the capital flow taxes set by the home and foreign planners. We thus turn to the forces driving optimal capital flow management. The following lemma characterizes the national planners' choices.

Lemma 6 (Targeting rules in non-cooperative capital flow regime)**.** *When capital flow taxes are set non-cooperatively, the home and foreign planners' choices are characterized by the targeting rules*

$$\Gamma_H \Theta_0 e^{\int_0^t \tau_s ds} = 1 + \Xi_t^{-1} e^{-\omega_t - \varpi_t}, \tag{30}$$

$$\Gamma_F \Theta_0^{-1} e^{\int_0^t \tau_s^* ds} = 1 + \Xi_t e^{-\omega_t^* + \varpi_t}, \tag{31}$$

where Γ_H and Γ_F denote the planners' multipliers on their own country's lifetime budget constraints.

Proof. See Appendix B.4. $\square$

In stark contrast with their efficient regime counterpart (24), these targeting rules indicate that non-zero taxes are desirable for the national planners even when wedges are zero. In order to gain further insights into the logic of the national planners' optimal policy, it is again useful to resort a first-order approximation of some key equations around the symmetric steady state.[53] Linearizing the home targeting rule (30) and then taking the time derivative yields:

$$\tau_t = -\frac{1}{2}\dot{\omega}_t - \frac{1}{2}\frac{\dot{\Theta}_t}{\Theta_t}. \tag{32}$$

This relation embeds the two types of incentives faced by the home planner when setting its tax on capital flows. The first term on the right-hand side of (32) represents a macroeconomic stabilization motive: when the home labor wedge is contracting over time ($\dot{\omega}_t < 0$), such as in phase I when the output gap is growing ($\dot{\tilde{y}}_t > 0$), smoothing it requires engineering a more depreciated home currency, which is achieved by encouraging outflows ($\tau_t > 0$). The second term represents a dynamic terms-of-trade (henceforth, dToT) manipulation motive entailing that it is optimal for the home planner to smooth out the expenditure ratio Θ_t. This second motive is related to Costinot et al. (2014)'s result that when a country's trade balance grows or shrinks, managing the capital account provides a subtle way of extracting rents from foreigners by exerting market power differentially across time periods. In the context of our model and scenario, from the trade balance expression (20), $\dot{\Theta}_t > 0$ during phase I indicates that Home's trade surplus (measured in marginal utility terms) is shrinking over time. When this is the case, taxing capital outflows induces the home consumer to front load consumption and thereby contributes to smooth surpluses over time. This intertemporal reallocation of exports implies that Home sells at higher prices during high export periods and at lower prices during low export periods. The home national planner internalizes that such a dynamic manipulation of the terms-of-trade hence brings a net utility benefit to the home economy. Equation (32) makes clear that the two motives just described conflict during a liquidity trap.

Using the linearized equilibrium labor wedge expressions in (26), the linearized IS curves in (27), and the law of motion for the expenditure ratio (5), relation (32) and its foreign counterpart lead to linearized best responses describing a (home or foreign) planner's optimal choice as a function of the other two planners' choices and exogenous variables

$$\tau_t = \frac{1+\phi}{3+\phi\alpha}\left(i_t - r_t^n\right) + \frac{\zeta_t}{3+\phi\alpha} + \frac{1+\phi\alpha}{3+\phi\alpha}\tau_t^* \tag{33}$$

$$\tau_t^* = \frac{1+\phi}{3+\phi\alpha}\left(i_t^* - r_t^{n*}\right) - \frac{\zeta_t}{3+\phi\alpha} + \frac{1+\phi\alpha}{3+\phi\alpha}\tau_t \tag{34}$$

[53] This symmetric steady state is described in Appendix A.3.1.

In both best response functions, the first term reflects macroeconomic stabilization motives, the second one reflects dToT manipulation motives, and the third one reflects a combination of both.[54]

Next, we look at the interplay between the macroeconomic stabilization and dToT manipulation motives in the Nash equilibrium of the game played by the three planners.

5.3 Nash equilibrium

In the absence of the zero bound, it is straightforward to establish that from 0 to T the global planner sets interest rates so as to perfectly stabilize the output gap in both countries, while the home planner taxes outflows and the foreign planner taxes inflows.[55] Consistent with our discussion of relation (32) in the preceding section, this suggests that independently from ZLB considerations, uncoordinated capital flow management hinders intertemporal trade and capital flows across countries. The intuition is that demand shocks lead to trade imbalances, which both countries face incentives to reduce based on dToT manipulation considerations.[56] In other words, the path of the expenditure ratio Θ_t is "smoother" than that of the Pareto weight Ξ_t, so that the international wedge opens. Hence, in the absence of the ZLB, despite monetary policy being able to successfully stabilize aggregate demand so as to implement the first-best level of output everywhere, the first-best allocation is not achieved. Relative to this first-best allocation, the path of the terms-of-trade $S_t = \left(Y_t^{\text{fb}}/Y_t^{*\text{fb}}\right)\left(1-\alpha+\alpha\Theta_t\right)/\left[\left(1-\alpha\right)\Theta_t+\alpha\right]$ is excessively smooth and consumption decouples insufficiently from output.

When monetary policy is constrained by the ZLB, it is unable to stabilize aggregate demand. In this case, the insufficient adjustment in the terms-of-trade brought about by non-cooperative capital flow management distorts not just relative consumption but also output, resulting in even more pervasive efficiency losses. In our case of interest where the ZLB binds in Home but not in Foreign, interest rates are given by $i_t = 0$ and $i_t^* = r_t^{n*} - \alpha(\tau_t - \tau_t^*)$ and the global monetary authority is unable to implement the first-best output path in Home.[57] Substituting

[54]Notice that the third term implies that the capital flow taxes are strategic complements. A higher tax on inflows by Foreign pushes capital to flow into Home, leading to a more appreciated home currency. As a result of both the macroeconomic stabilization motive and the dToT manipulation motive, the home planner responds by adjusting its capital flow tax upwards. The same logic applies to the foreign planner.

[55]See equations (A.26) and (A.27) for our statement regarding output levels. From the IS equations in (27), the interest rates are given by $i_t = r_t^n + \alpha(\tau_t - \tau_t^*)$ and $i_t^* = r_t^{n*} - \alpha(\tau_t - \tau_t^*)$. Substituting these expressions into the best responses (33) and (34) leads to equilibrium interest rates of $i_t = r_t^n + \alpha\zeta_t/(2-\alpha)$ and $i_t^* = r_t^{*n} - \alpha\zeta_t/(2-\alpha)$, and equilibrium capital flow taxes of $\tau_t = \zeta_t/[2(2-\alpha)]$ and $\tau_t^* = -\zeta_t/[2(2-\alpha)]$, and thus to an equilibrium tax wedge of $\tau_t - \tau_t^* = \zeta_t/(2-\alpha)$.

[56]Initially, Home wants to reduce its trade surpluses to exert monopoly power, while Foreign wants to reduce its trade deficit to exert monopsony power. Current account positions flip later on, but regardless of the sign of imbalances, it is attractive for both countries to restrict capital flows.

[57]Our analysis of non-cooperative capital flow management can easily be extended to accommodate situations where the ZLB binds in both countries.

the interest rate expressions into the best responses (33) and (34) yields

$$\begin{aligned}
\tau_t - \tau_t^* &= -(1-\Phi)\Psi r_t^n + \Phi\zeta_t, & i_t^* &= r_t^{n*} + \alpha\left[(1-\Phi)\Psi r_t^n - \alpha\Phi\zeta_t\right], \\
\dot{e}_t &= -r_t^{n*} + (1-\alpha)\left[(1-\Phi)\Psi r_t^n - \Phi\zeta_t\right], & \dot{\tilde{y}}_t &= -\left[1-(1-\Phi)\alpha\Psi\right] r_t^n - \Phi\zeta_t.
\end{aligned} \tag{35}$$

for $\Phi \equiv 2/[4+\alpha(\phi-1)]$.[58] The Nash equilibrium tax wedge expression in (35) is a weighted average of an aggregate demand stabilization term, already present in the efficient regime expression in (28), and a new, conflicting, dToT manipulation term. Home monetary policy is again set such that the home output gap averages out to zero over time (i.e. $\int_0^{\widehat{T}} e^{-\rho t}\tilde{y}_t dt = 0$).[59] The main properties of the Nash regime is summarized in the following proposition.

Proposition 3 (Non-cooperative capital flow regime)**.** *When the ZLB binds in Home but not in Foreign from 0 to T, up to a first-order, the non-cooperative regime compares with the constrained efficient regime as follows:*

1. *Capital flows out of Home are less subsidized in phase I and less taxed in phase II.*
2. *The home output gap is less smooth.*
3. *The home exchange rate appreciates at a slower rate in phase I and at a faster rate in phase II.*

Moreover, in phase I, relative to the free capital mobility regime, capital flows out of Home may even be taxed (rather than subsidized), the home output gap may be less smooth, and the home exchange rate may appreciate at a slower rate.

Proof. See argument in the text above. □

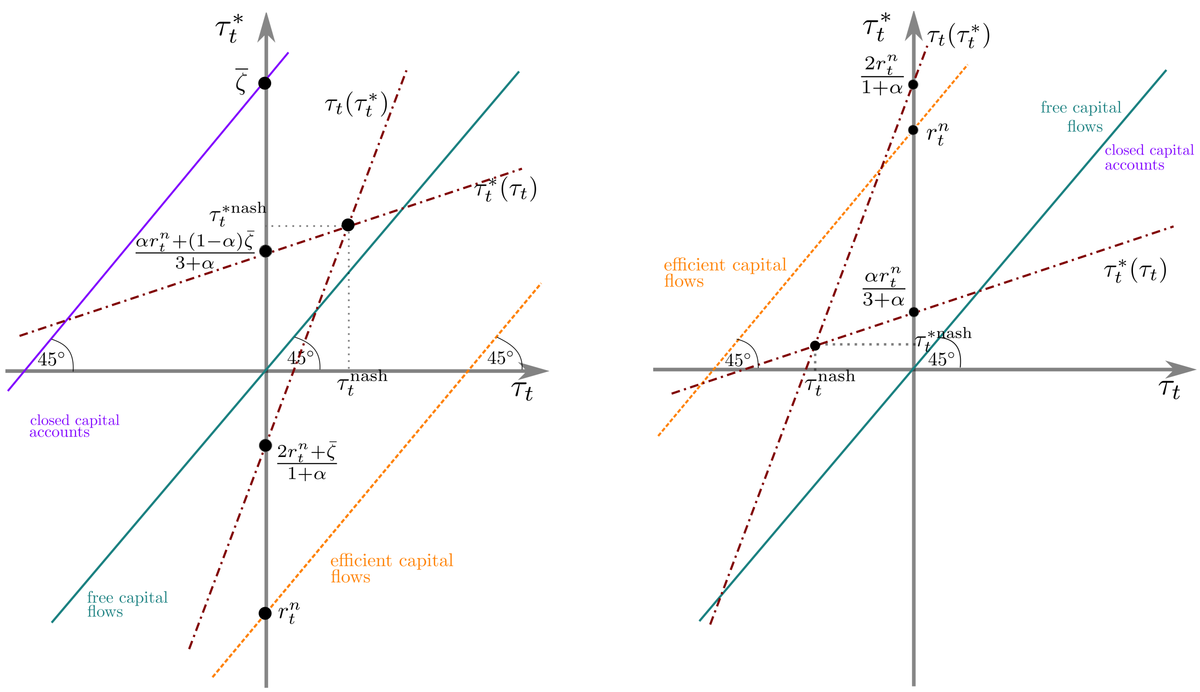
These results show that the idea put forward in the context of the best responses that dToT management motives conflict with macroeconomic stabilization finds its way to the Nash equilibrium. Notably, they indicate that the tax wedge in the non-cooperative regime falls short of its efficient value and may even take the "wrong" sign during phase I of the liquidity trap.

The determination of the Nash equilibrium is illustrated for the special case of a unit Frisch elasticity ($\phi = 1$, implying $\Psi = 1$ and $\Phi = 1/2$) in the (τ_t, τ_t^*) space in Figure 5. The free capital mobility regime, corresponding to a zero tax wedge, is represented by the straight line $\tau_t^* = \tau_t$. Points to the South-East of this line represent regimes associated with net subsidies on flows from Home to Foreign, while points to the North-West of this line represent regimes associated with net taxes on such flows. The closed capital account regime, corresponding to a

[58]Note that $0 < \Phi < 1$, $\partial\Phi/\partial\phi < 0$ and $\partial\Phi/\partial\alpha \gtrless 0$ for $\phi \lessgtr 1$. Moreover, $\lim_{\phi\to\infty}\Phi = 0$.

[59]Note however that unlike in the constrained efficient and free capital mobility regime, the home labor wedge, foreign labor wedge, and international wedge do not necessarily also average out to zero over time

tax wedge of ζ_t, is represented by the straight line $\tau_t^* = -\zeta_t + \tau_t$. The constrained efficient regime, corresponding to a tax wedge of $-r_t^n$, is represented by the straight line $\tau_t^* = r_t^n + \tau_t$. Finally, the home and foreign planners' best responses (33) and (34), drawn at the Nash equilibrium interest rates, are represented by the straight lines $\tau_t^* = [(3+\alpha)\,\tau_t + 2r_t^n - \zeta_t] \,/\, (1+\alpha)$ and $\tau_t^* = [(1+\alpha)\,\tau_t + \alpha r_t^n - (1+\alpha)\,\zeta_t] \,/\, (3+\alpha)$. The figure shows that the Nash outcome features too small a subsidy to flows from Home to Foreign in phase I, and too small a subsidy to flows from Foreign to Home in phase II. In phase I, it even illustrates a case where flows from Home to Foreign end up being taxed, while efficiency consideration would require them to be subsidized. With unit Frisch elasticity, the sign of the tax wedge is unambiguously negative in phase I of the liquidity trap (unlike in the efficient regime where it was positive). More generally, it is unambiguously negative in phase I when $\phi < \left(\rho + \bar{\zeta}\right) / \left[(1-\alpha)\,\bar{\zeta} - \rho\right]$, i.e., when labor supply is sufficiently elastic. The intuition is that with sufficiently elastic labor supply, labor wedge or output gap fluctuations are not too costly for the home planner, and the dToT manipulation force more easily dominates the macroeconomic stabilization force in equilibrium.[60]



(a) Phase I: $t \in [0, T)$.

(b) Phase II: $t \in [T, \widehat{T})$.

Figure 5: Linearized best-responses and Nash equilibrium when $\phi = 1$.

Figure 6 illustrates the effects of the distortions induced by the non-cooperative regime by plotting the paths of the model's main variables.[61] It is again apparent that relative to the

[60]For the liquidity trap scenario parametrization of Werning (2012), for which $\bar{\zeta} = 2\rho$, the condition becomes $\phi < 3/\,(1-2\alpha)$. This condition is satisfied for most values of the Frisch elasticity and trade openness used in the literature.

[61]The picture is produced using an exact non-linear solution of the game, again using the parametrization

efficient regime, the tax wedge has the wrong sign during phase I, and is too small during phase II. As a consequence, the exchange rate path is smoother during phase I than under both the efficient regime *and* the free capital mobility regime. The uncoordinated capital flow management regime thus hampers smooth adjustment, as suggested by the more variable home output gap, home labor wedge, foreign labor wedge, and international wedge paths than under the free capital mobility regime.

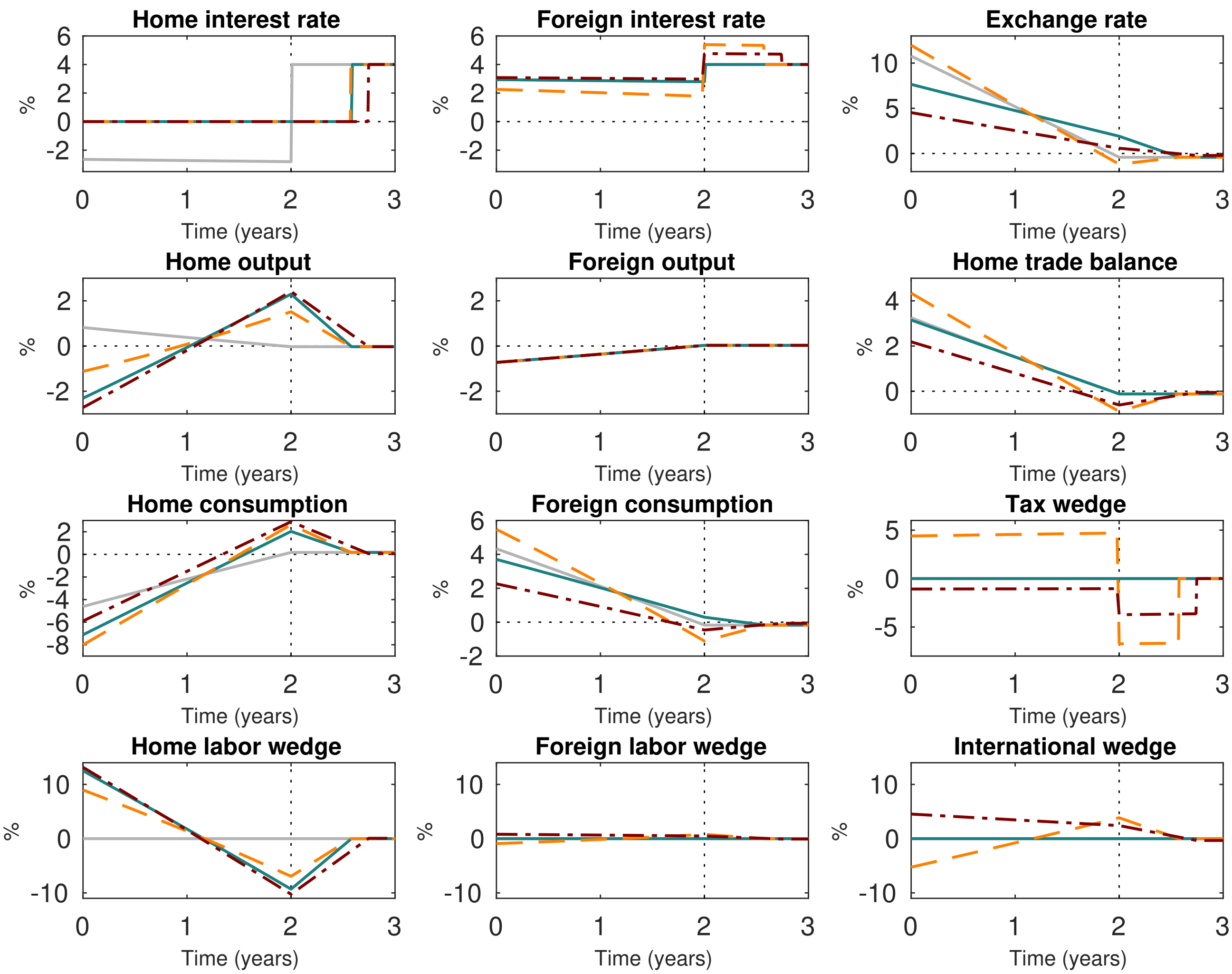


Figure 6: Variable paths under ZLB with free capital mobility (solid dark), ZLB with efficient regime (dashed), ZLB with non-cooperative regime (dashed-dotted), and unconstrained policy (solid light).

Global liquidity trap While our main scenario of interest is one where a single region experiences a liquidity trap, it is easy to extend our analysis to a situation in which the demand shock at Home is large enough to push the entire global economy into a liquidity trap. In this global

described in Footnote 34. See Appendix D for details on the model solution.

liquidity trap case, similar forces are at play - dToT management motives still work counter to aggregate demand stabilization. As in the case where only Home hits the ZLB, the Nash equilibrium tax wedge is a weighted average of an aggregate demand stabilization term already present in the efficient regime expression in (29), and a dToT manipulation term. The resulting non-cooperative equilibrium in addition features Foreign output away from its first-best level.

6 Discussion of potential extensions

Our positive and normative analysis of capital flows in a liquidity trap was conducted in a purposefully stylized general equilibrium model of the world economy. In this section, we briefly discuss the likely robustness of our results with respect to several natural extensions.

Sticky prices For the sake of analytical tractability, we have assumed that prices were fully rigid. A more realistic (and standard) assumption would be one under which prices would instead be sticky. Allowing for sticky prices (e.g., à la Rotemberg, 1982 or Calvo, 1983) would likely not qualitatively change our results, but would substantially reduce tractability. From a positive perspective, increased price stability is known to often be destabilizing in a liquidity trap (see Eggertsson, 2010 and Bhattarai et al., 2014). In our context, sticky rather than rigid prices would allow a stronger deflationary pressure in Home than in Foreign to materialize and thereby create an additional destabilizing force by making the terms-of-trade respond perversely (i.e., shifting expenditure *away* from home goods at the beginning of the liquidity trap).[62] As such, they would not eliminate – and may even instead reinforce – the benefits of the stabilizing exchange rate movements associated with international capital flows. Our main insights can therefore be expected to apply as long as prices are not fully flexible.

Global liquidity trap While the global events constituting our motivation of the paper led us to focus on a scenario where only Home experiences a liquidity trap, the brief analysis in the concluding paragraph of Section 4 showed that our main insights carry through to a situation where the entire world enters a liquidity trap. Pursuing a detailed analysis in this direction would not compromise our framework's tractability and may provide a fruitful avenue for future research.

Pricing currency The mechanism driving our results on the stabilizing role of capital flows in a liquidity trap crucially relies on the expenditure switching effect brought about by exchange rate movements. As is well known, this effect is at work under the standard producer currency

[62] Controlling for the exchange rate, a higher PPI deflation in Home than in Foreign early in the liquidity trap would make the home good cheaper and cheaper over time, relative to the foreign good. Cook and Devereux (2013) refer to this effect as a "perverse response" of the terms-of-trade in a liquidity trap.

pricing (PCP) assumption we have adopted, but would be absent under the alternative assumption of local currency pricing. Given our motivation stemming from the Great Recession and accordingly, our interpretation of Home as the set of advanced economies and of Foreign as the set of emerging economies, an arguably more realistic pricing assumption would be that of *dominant currency pricing* (see Gopinath, 2016 and Casas et al., 2016). Under this paradigm, all internationally traded goods would be priced in Home's currency (i.e., the dominant currency). As a result, expenditure switching would not operate on home consumers but it would still operate on foreign consumers. Under this empirically more plausible pricing currency assumption, capital flows would thus help stabilize aggregate demand in Home by triggering expenditure switching by foreign consumers only (as opposed to by all consumers under PCP). The strength of the forces underlying our mechanism would consequently be somewhat weakened, but our main insights would not change qualitatively.

Non-cooperative monetary policy Throughout the paper, we have assumed that monetary policy was conducted cooperatively. In an earlier version (Acharya and Bengui, 2015), we analyzed the case where monetary policy is instead conducted non-cooperatively and obtained qualitatively similar results and predictions.

7 Conclusion

Using a standard open-economy New Keynesian framework, we argue that when a large region of the world economy experiences a liquidity trap, global capital flows enable a reallocation of demand and expenditure, and are therefore stabilizing. Owing to aggregate demand externalities operating at the zero lower bound, free capital flows are nonetheless constrained inefficient and result in reallocations that are too small. Global efficiency requires larger flows during and after the liquidity trap, to compensate for monetary policy's inability to stimulate aggregate demand in the region where the zero bound on interest rates is binding. Despite pointing to inefficient capital flows in a liquidity trap, our analysis does not support the management of capital flows by individual countries. To the contrary, it suggests that the terms-of-trade management objectives underlying such policies may interfere with aggregate demand stabilization and thus hamper, rather than promote, a smooth global macroeconomic adjustment.

Since the Great Recession, a lot of attention has been devoted to understanding the effects of U.S. monetary policy onto the rest of the world (e.g., Rey, 2013, 2016; Rajan, 2014; Dedola et al., 2017). By purposefully abstracting from such considerations, the present paper transparently makes the case that capital flow management policies, whose adoption and endorsement have experienced a revival in recent years, may also be associated with important international spillover effects. Furthermore, it suggests that these effects may be particularly harmful in low interest rate environments. As a new consensus is emerging on the global acceptability of such policies, an important task for both theoretical and empirical research is to improve our understanding of their cross-border implications.

References

Acharya, Sushant and Julien Bengui, "Liquidity Traps, Capital Flows," Working Papers, CIREQ December 2015.

Ahmed, Shaghil and Andrei Zlate, "Capital flows to emerging market economies: A brave new world," *Journal of International Money and Finance*, 2014, pp. 221–248.

Backus, David K. and Gregor W. Smith, "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, November 1993, *35* (3-4), 297–316.

Bengui, Julien, "Macroprudential Policy Coordination," Mimeo 2014.

Benigno, Gianluca and Luca Fornaro, "Stagnation Traps," Working Paper 2015.

_ **and Pierpaolo Benigno**, "Price Stability in Open Economies," *Review of Economic Studies*, 2003, *70*, 743–764.

Benigno, Pierpaolo and Federica Romei, "Debt deleveraging and the exchange rate," *Journal of International Economics*, 2014, *93* (1), 1–16.

Bhattarai, Saroj, Gauti B. Eggertsson, and Raphael Schoenle, "Is Increased Price Flexibility Stabilizing (Redux)," Working Paper 19886, NBER 2014.

Bianchi, Javier, "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, December 2011, *101* (7), 3400–3426.

Blanchard, Olivier, "Sustaining a Global Recovery," *Finance and Development*, September 2009, pp. 8–12.

_ **and Gian Maria Milesi-Ferretti**, "Global Imbalances: In Midstream?," IMF Staff Position Note December 2009.

_ **and** _ , "(Why) Should Current Account Balances Be Reduced," *IMF Economic Review*, 2012, (1), 139–150.

_ **and Nobuhiro Kiyotaki**, "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, 1987, *77* (4), 647–666.

Bodenstein, Martin, Christopher Erceg, and Luca Guerrieri, "The Effects of Foreign Shocks when U.S. Interest Rates are Zero," *Canadian Journal of Economics*, 2017, *50*, 660–664.

Caballero, Ricardo J. and Arvind Krishnamurthy, "International and Domestic Collateral Constraints in a Model of Emerging Market Crises," *Journal of Monetary Economics*, 2001, *48* (1), 513–548.

__ , Emmanuel Farhi, and Pierre-Olivier Gourinchas, "Global Imbalances and Currency Wars at the ZLB.," Working Paper 21670, National Bureau of Economic Research October 2015.

Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, September 1983, *12* (3), 383–398.

Casas, Camila, Federico J. Diez, Gita Gopinath, and Pierre-Olivier Gourinchas, "Dominant Currency Paradigm," Working Paper 22943, National Bureau of Economic Research December 2016.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When is the Goverment Spending Multiplier Large," *Journal of Political Economy*, 2011, *119* (1), 78–121.

Cole, Harold L. and Maurice Obstfeld, "Commodity trade and international risk sharing : How much do financial markets matter?," *Journal of Monetary Economics*, August 1991, *28* (1), 3–24.

Cook, David and Michael B. Devereux, "Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap," *American Economic Journal: Macroeconomics*, July 2013, *5* (3), 190–228.

Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles, "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, June 2013, *103* (4), 1172–1211.

Corsetti, Giancarlo and Paolo Pesenti, "Welfare and macroeconomic interdependence," *Quarterly Journal of Economics*, March 2001, pp. 421–445.

Costinot, Arnaud, Guido Lorenzoni, and Ivan Werning, "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 2014, *122* (1), 77 – 128.

Dedola, Luca, Giulia Rivolta, and Livio Stracca, "If the Fed Sneezes, who catches a cold?," *Journal of International Economics*, May 2017, *108* (Supplement 1), S23–S41.

DePaoli, Bianca and Anna Lipinska, "Capital controls: a normative analysis," *Proceedings*, 2012, (Nov), 1–36.

Devereux, Michael B. and James Yetman, "Capital Controls, Global Liquidity Traps, and the International Policy Trilemma," *Scandinavian Journal of Economics*, 01 2014, *116* (1), 158–189.

Eggertson, Gauti B., Neil R. Mehrotra, Singh Sanjay R., and Laurence H. Summers, "A Contagious Malady? Open Economy Dimensions of Secular Stagnation," *IMF Economic Review*, 2016, *64* (4).

Eggertsson, Gauti B., "The Deflation Bias and Committing to being Irresponsible," *Journal of Money, Credit and Banking*, 2006, *36*, 283–321.

_ , "What Fiscal Policy Is Effective at Zero Interest Rates?," *NBER Macroeconomic Annual*, 2010, *25*, 59–112.

_ **and Michael Woodford**, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *34* (1), 139–235.

_ **and** _ , "Optimal Monetary and Fiscal Policy in a Liquidity Trap," *NBER International Seminar on Macroeconomics*, 2004, *1*, 75–131.

_ **and** _ , "Policy Options in a Liquidity Trap," *American Economic Review P&P*, 2004, *94* (2), 76–79.

_ **and Paul Krugman**, "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 2012, *127* (3).

Farhi, Emmanuel and Ivan Werning, "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates," NBER Working Paper 2012.

_ **and** _ , "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows," *IMF Economic Review (Special Volume in Honor of Stanley Fischer)*, 2014, *62* (4).

_ **and** _ , "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 2016, *84* (5), 1645–1704.

_ **and** _ , "Fiscal Unions," *American Economic Review*, 2017, *107* (12), 3788–3834.

Fornaro, Luca, "International debt deleveraging," *Journal of the European Economic Association*, forthcoming.

— **and Federica Romei**, "The Paradox of Global Thrift," Discussion Paper DP12894, CEPR 2018.

Fujiwara, Ippei, Tomoyuki Nakajima, Nao Sudo, and Yuki Teranishi, "Global liquidity trap," *Journal of Monetary Economics*, 2013, *60* (8), 936–949.

Gabaix, Xavier and Matteo Maggiori, "International Liquidity and Exchange Rate Dynamics," *Quarterly Journal of Economics*, 2015, pp. 1369–1420.

Gali, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduciton to the New Keynesian Framework* 2015.

Gopinath, Gita, "The International Price System," *Jackson Hole Symposium Proceedings*, 2016.

Guerrieri, Veronica and Guido Lorenzoni, "Credit Crises, Precautionary Savings, and the Liquidity Trap," Working Papers 17583, National Bureau of Economic Research, Inc 2011.

Haberis, Alex and Anna Lipinska, "International policy spillovers at the zero lower bound," Finance and Economics Discussion Series, Board of Governors of the Federal Reserve System (U.S.) 2012.

IMF, *World Economic Outlook: Rebalancing Growth* April 2010.

— , "The Multilateral Aspects of Policies Affecting Capital Flows," Technical Report October 2011.

— , "The Liberalization and Management of Capital Flows: An Institutional View," IMF Staff Paper November 2012.

Jeanne, Olivier, "The global liquidity trap," Working Paper 2009.

— **and Anton Korinek**, "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach," *American Economic Review*, May 2010, *100* (2), 403–07.

— **, Arvind Subramanian, and John Williamson**, *Who needs to open the capital account?* 2012.

Kollmann, Robert, "Essays on International Business Cycles." PhD dissertation, University of Chicago 1991.

Korinek, Anton, "Excessive Dollar Borrowing in Emerging Markets: Balance Sheet Effects and Macroeconomic Externalities," 2007. University of Maryland.

_ , “Regulating Capital Flows to Emerging Markets: An Externality View,” 2010. University of Maryland.

_ , “International Spillovers and Guidelines for Policy Cooperation: A Welfare Theorem for National Economic Policymaking,” Technical Report 2014.

_ **and Alp Simsek**, “Liquidity Trap and Excessive Leverage,” *American Economic Review*, 2016, *106* (3), 699–738.

Krugman, Paul R., “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 1998, *29* (2), 137–206.

Ostry, Jonathan D., Atish R. Ghosh, Karl Habermeier, Luc Laeven, Marcos Chamon, Mahvash S. Qureshi, and Annamaria Kokenyne, “Managing Capital Inflows: What Tools to Use?,” IMF Discussion Note April 2011.

_ , _ , _ , Marcos Chamon, Mahvash S. Qureshi, and Dennis B.S. Reinhardt, “Capital Inflows: The Role of Controls,” IMF Position NoteWorking Paper February 2010.

_ , Atish R. Gosh, and Anton Korinek, “Multilateral Aspects of Managing the Capital Account,” IMF Discussion Note September 2012.

Rajan, Raghuram, “Competitive Monetary Easing: Is It Yesterday Once More?,” Reserve Bank of India 2014.

Rey, Helene, “Dilemma not trilemma: the global cycle and monetary policy independence,” *Proceedings - Economic Policy Symposium - Jackson Hole*, 2013, pp. 1–2.

Rey, Hélène, “International Channels of Transmission of Monetary Policy and the Mundellian Trilemma,” *IMF Economic Review*, May 2016, *64* (1), 6–35.

Rotemberg, Julio J, “Sticky Prices in the United States,” *Journal of Political Economy*, December 1982, *90* (6), 1187–1211.

Schmitt-Grohe, Stephanie and Martin Uribe, “Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment,” *Journal of Political Economy*, 2016, *124* (5), 1466–1514.

Werning, Ivan, “Managing a Liquidity Trap: Monetary and Fiscal Policy,” Working Paper, MIT 2012.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy* 2003.

A Optimal policy appendix

A.1 First-Best Allocation

The socially optimal allocation, which we refer to as *first-best*, is the allocation that solves an unconstrained social planning problem. Imposing symmetric consumption of the differentiated goods produced within a country,[63] the planning problem amounts to a sequence of static problems of the form

$$\max_{C_t, C_t^*, C_{H,t}, C_{F,t}, C_{H,t}^*, C_{F,t}^*, N_t, N_t^*} \Xi_t \left[\ln(C_t) - \frac{(N_t)^{1+\phi}}{1+\phi} \right] + \left[\ln(C_t^*) - \frac{(N_t^*)^{1+\phi}}{1+\phi} \right]$$

subject to the constraints:

$$\begin{aligned}
C_t &= \frac{(C_{H,t})^{(1-\alpha)} (C_{F,t})^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \\
C_t^* &= \frac{\left(C_{H,t}^*\right)^{(1-\alpha)} \left(C_{F,t}^*\right)^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \\
C_{H,t} + C_{H,t}^* &= N_t, \qquad (A.1) \\
C_{F,t} + C_{F,t}^* &= N_t^*, \qquad (A.2)
\end{aligned}$$

where $\Xi_t \equiv \Xi e^{-\int_0^t (\zeta_s - \zeta_s^*) ds}$ is a time-varying Pareto weight assigned by the planner to Home.[64] The first-order conditions of this problem lead to first-best employment/output

$$N_t^{\text{fb}} = Y_t^{\text{fb}} = \left[\alpha \Xi_t^{-1} + 1 - \alpha\right]^{\frac{1}{1+\phi}} \quad \text{and} \quad N_t^{*\text{fb}} = Y_t^{*\text{fb}} = \left[\alpha \Xi_t + 1 - \alpha\right]^{\frac{1}{1+\phi}}, \qquad (A.3)$$

and aggregate consumption

$$C_t^{\text{fb}} = \Xi_t^{\alpha} \left[\left(Y_t^{\text{fb}}\right)^{1-\alpha} \left(Y_t^{*\text{fb}}\right)^{\alpha} \right]^{-\phi} \quad \text{and} \quad C_t^{*\text{fb}} = \Xi_t^{-\alpha} \left[\left(Y_t^{\text{fb}}\right)^{\alpha} \left(Y_t^{*\text{fb}}\right)^{1-\alpha} \right]^{-\phi}. \qquad (A.4)$$

Consumption of home and foreign goods are accordingly given by $C_{H,t}^{\text{fb}} = (1-\alpha)\left(Y_t^{\text{fb}}\right)^{-\phi}$, $C_{F,t}^{\text{fb}} = \alpha \Xi_t \left(Y_t^{*\text{fb}}\right)^{-\phi}$, $C_{F,t}^{*\text{fb}} = (1-\alpha)\left(Y_t^{*\text{fb}}\right)^{-\phi}$ and $C_{H,t}^{*\text{fb}} = \alpha \Xi_t^{-1} \left(Y_t^{\text{fb}}\right)^{-\phi}$.

Furthermore, the multipliers on the home and foreign resource constraints, (A.1) and (A.2), are given by $\vartheta_{H,t} = \Xi_t \left(Y_t^{\text{fb}}\right)^{\phi}$ and $\vartheta_{F,t} = \left(Y_t^{*\text{fb}}\right)^{\phi}$. Accordingly, the planner's *shadow terms of trade* is given by

$$\vartheta_t \equiv \frac{\vartheta_{F,t}}{\vartheta_{H,t}} = \frac{Y_t^{\text{fb}}}{Y_t^{*\text{fb}}} \times \frac{\alpha \Xi_t + 1 - \alpha}{\alpha + (1-\alpha)\Xi_t}. \qquad (A.5)$$

[63] Such a symmetry is trivially optimal, given the assumed preferences and technologies.

[64] Ξ is the Pareto weight assigned by the planner to Home at date 0. Note that due to differences in discounting, the weight giving both countries equal importance, which we refer to as the *symmetric weight*, is given by $\Xi = \int_0^\infty e^{-\int_0^s (\rho + \zeta_s^*) ds} dt / \int_0^\infty e^{-\int_0^s (\rho + \zeta_s) ds} dt$.

A.2 Optimal monetary policy

For given paths of $\Theta_t, \tau_t, \tau_t^*$, the optimal monetary policy problem is an optimal control problem with control variables i_t, i_t^*, and state variables Y_t, Y_t^*:

$$\max_{\{i_t, i_t^*\}} \int_0^\infty e^{-\int_0^t (\rho+\zeta_s^*)ds} \left\{ \ln \left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right\}$$

subject to

$$\frac{\dot{Y}_t}{Y_t} = i_t - (\rho + \zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*), \tag{A.6}$$

$$\frac{\dot{Y}_t^*}{Y_t^*} = i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*), \tag{A.7}$$

$$i_t \geq 0 \tag{A.8}$$

$$i_t^* \geq 0 \tag{A.9}$$

The associated present value Hamiltonian is given by

$$\begin{aligned} \mathcal{H} = {} & e^{-\int_0^t (\rho+\zeta_s^*)ds} \left\{ \ln \left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right\} \\ & + \lambda_t Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t i_t \\ & + \lambda_t^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t^* i_t^*, \end{aligned}$$

where λ_t, λ_t^* are the co-state variables associated with Y_t, Y_t^*, and ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates.

The planner's optimal choice is characterized by the conditions

$$\lambda_t i_t = 0, \qquad i_t \geq 0, \qquad \lambda_t \geq 0, \tag{A.10}$$

$$\lambda_t^* i_t^* = 0, \qquad i_t^* \geq 0, \qquad \lambda_t^* \geq 0, \tag{A.11}$$

the laws of motion for the co-state variables

$$\dot{\lambda}_t = -e^{-\int_0^t (\rho+\zeta_s)ds} \left[\left(Y_t^{\text{fb}} \right)^{1+\phi} - (Y_t)^{1+\phi} \right] \frac{1}{Y_t} - \lambda_t \frac{\dot{Y}_t}{Y_t} \tag{A.12}$$

$$\dot{\lambda}_t^* = -e^{-\int_0^t (\rho+\zeta_s^*)ds} \left[\left(Y_t^{*\text{fb}} \right)^{1+\phi} - (Y_t^*)^{1+\phi} \right] \frac{1}{Y_t^*} - \lambda_t^* \frac{\dot{Y}_t^*}{Y_t^*}, \tag{A.13}$$

initial conditions $\lambda_0 = \lambda_0^* = 0$ for the co-state variables, and transversality conditions $\lim_{t\to\infty} \lambda_t Y_t = 0$ and $\lim_{t\to\infty} \lambda_t^* Y_t^* = 0$.

Integrating (A.12) and (A.13) from 0 to ∞, and using the initial conditions and transversality conditions, yields

$$0 = \int_0^\infty e^{-\int_0^t(\rho+\zeta_s)ds}\left[\left(Y_t^{\text{fb}}\right)^{1+\phi} - (Y_t)^{1+\phi}\right]dt \tag{A.14}$$

$$0 = \int_0^\infty e^{-\int_0^t(\rho+\zeta_s^*)ds}\left[\left(Y_t^{*\text{fb}}\right)^{1+\phi} - (Y_t^*)^{1+\phi}\right]dt, \tag{A.15}$$

The planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \lambda_t, \lambda_t^*, i_t, i_t^*$ consisting of (A.6), (A.7), (A.8), (A.9), (A.10), (A.11), (A.12) and (A.13) with boundary conditions (A.14), (A.15), $\lambda_0 = 0$ and $\lambda_0^* = 0$.

A.2.1 Symmetricy steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = 1$, $\lambda_t = \lambda_t^* = 0$ and $i_t = i_t^* = \rho$. The associated steady-state labor wedges are hence equal to zero: $\omega_t = \omega_t^* = 0$.

A.3 Efficient capital flows

The optimal policy problem is an optimal control problem with control variables i_t, i_t^*, and state variables Y_t, Y_t^*, Θ_t:

$$\max_{\{i_t, i_t^*, \tau_t\}} \int_0^\infty e^{-\int_0^t(\rho+\zeta_s^*)ds}\left\{\ln\left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)}(Y_t^*)^{\alpha\Xi_t+1-\alpha}\right] - \frac{1}{1+\phi}\left[\Xi_t(Y_t)^{1+\phi} + (Y_t^*)^{1+\phi}\right]\right.$$
$$\left. - \ln\left[\Theta_t^{\alpha(1-\Xi_t)}\left(\alpha\Theta_t^{-1}+1-\alpha\right)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)}(\alpha\Theta_t+1-\alpha)^{\alpha\Xi_t+1-\alpha}\right]\right\}dt$$

subject to (A.6)-(A.9) (with $\tau_t^* = 0$) and

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta_t^* - \zeta_t + \tau_t \tag{A.16}$$

The associated present value Hamiltonian is given by

$$\begin{aligned}
\mathcal{H} = {} & e^{-\int_0^t(\rho+\zeta_s^*)ds}\left\{\ln\left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)}(Y_t^*)^{\alpha\Xi_t+1-\alpha}\right] - \frac{1}{1+\phi}\left[\Xi_t(Y_t)^{1+\phi} + (Y_t^*)^{1+\phi}\right]\right. \\
& \left. - \ln\left[\Theta_t^{\alpha(1-\Xi_t)}\left(\alpha\Theta_t^{-1}+1-\alpha\right)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)}(\alpha\Theta_t+1-\alpha)^{\alpha\Xi_t+1-\alpha}\right]\right\} \\
& +\lambda_t Y_t\left[i_t - (\rho+\zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha}(\zeta_t^* - \zeta_t + \tau_t)\right] + \nu_t i_t \\
& +\lambda_t^* Y_t^*\left[i_t^* - (\rho+\zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t+1-\alpha}(\zeta_t^* - \zeta_t + \tau_t)\right] + \nu_t^* i_t^* \\
& +\mu_t\Theta_t(\zeta_t^* - \zeta_t + \tau_t),
\end{aligned}$$

where $\lambda_t, \lambda_t^*, \mu_t$ are the co-state variables associated with Y_t, Y_t^*, Θ_t, and ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates.

The planner's optimal choice is characterized by the complementary slackness conditions (A.10), (A.11), the first-order condition for τ_t

$$\mu_t \Theta_t - \lambda_t Y_t \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} + \lambda_t^* Y_t^* \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} = 0, \tag{A.17}$$

the laws of motion for the co-state variables (A.12), (A.13) and

$$\begin{aligned} \dot{\mu}_t &= -e^{-\int_0^t (\rho + \zeta_s^*) ds} \Theta_t^{-1} \alpha \left[\Xi_t - 1 + \frac{\Xi_t \Theta_t^{-1} \left(Y_t^{\text{fb}}\right)^{1+\phi}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{\Theta_t \left(Y_t^{*\text{fb}}\right)^{1+\phi}}{\alpha \Theta_t + 1 - \alpha} \right] \\ & -\lambda_t Y_t \frac{\alpha (1-\alpha) \Theta_t^{-2}}{\left(\alpha \Theta_t^{-1} + 1 - \alpha\right)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda_t^* Y_t^* \frac{\alpha (1-\alpha)}{(\alpha \Theta_t + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_t \frac{\dot{\Theta}_t}{\Theta_t} \end{aligned} \tag{A.18}$$

non-negativity conditions $\lambda_t \geq 0$, $\lambda_t^* \geq 0$, $\mu_t \geq 0$, initial conditions $\lambda_0 = \lambda_0^* = \mu_0 = 0$ for the co-state variables, and transversality conditions $\lim_{t\to\infty} \lambda_t Y_t = 0$, $\lim_{t\to\infty} \lambda_t^* Y_t^* = 0$, and $\lim_{t\to\infty} \mu_t \Theta_t = 0$.

Differentiating (A.17) with respect to time, substituting the co-state laws of motion (A.12), (A.13) and (A.18), and using the wedge expressions in (16), yields:

$$1 - e^{-\omega_t - \varpi_t} = \Xi_t \left(1 - e^{-\omega_t^* + \varpi_t}\right) \tag{A.19}$$

Differentiating this equation with respect to time yields

$$\begin{aligned} 0 &= e^{-\omega_t - \varpi_t} \left((1+\phi) \frac{\dot{Y}_t}{Y_t} - \frac{1-\alpha}{\alpha \Theta_t^{-1} + 1 - \alpha} \frac{\dot{\Theta}_t}{\Theta_t} \right) - \Xi_t e^{-\omega_t^* + \varpi_t} \left((1+\phi) \frac{\dot{Y}_t^*}{Y_t^*} + \frac{1-\alpha}{\alpha \Theta_t + 1 - \alpha} \frac{\dot{\Theta}_t}{\Theta_t} \right) \\ & + \left(1 + \Xi_t e^{-\omega_t^* + \varpi_t}\right) (\zeta_t^* - \zeta_t) \end{aligned} \tag{A.20}$$

Integrating (A.12), (A.13) and (A.18) from 0 to ∞, and using the initial conditions and transversality conditions, yields (A.14), (A.15) and[65]

$$0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[\Xi_t \frac{\Theta_t^{-1} (Y_t)^{1+\phi}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{\Theta_t (Y_t^*)^{1+\phi}}{\alpha \Theta_t + 1 - \alpha} \right] dt. \tag{A.21}$$

The planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \Theta_t, \lambda_t, \lambda_t^*, i_t, i_t^*$ consisting of (A.6), (A.7) (both with $\tau_t^* = 0$), (A.8), (A.9), (A.10), (A.11), (A.12), (A.13) and (A.20) with boundary conditions (A.14), (A.15), (A.21), $\lambda_0 = 0$ and $\lambda_0^* = 0$. The path of τ_t then follows from (A.16), and that of μ_t solves (A.18) with initial condition $\mu_0 = 0$.

[65] Integration by parts is required to obtain (A.21).

A.3.1 Symmetric steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = \Theta_t = 1$, $\lambda_t = \lambda_t^* = \mu_t = 0$, $i_t = i_t^* = \rho$ and $\tau_t = 0$. The associated steady-state wedges are hence equal to zero: $\omega_t = \omega_t^* = \varpi_t = 0$.

A.4 Noncooperative capital flow management

A.4.1 Global planner's problem

The global planner's problem is an optimal control problem with control variables i_t, i_t^* and b_0 (a date 0 transfer), and state variables Y_t, Y_t^*, Θ_t:

$$\max_{\{i_t, i_t^*\}, b_0} \int_0^\infty e^{-\int_0^t (\rho+\zeta_s^*)ds} \Bigg\{ \ln \left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right]$$
$$- \ln \left[\Theta_t^{\alpha(1-\Xi_t)} \left(\alpha\Theta_t^{-1} + 1 - \alpha\right)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (\alpha\Theta_t + 1 - \alpha)^{\alpha\Xi_t+1-\alpha} \right] \Bigg\}$$

subject to (A.6)-(A.9) and

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta_t^* - \zeta_t + \tau_t - \tau_t^* \tag{A.22}$$
$$b_0 = \alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_s^*-\tau_s^*)ds} (\Theta_t - 1)\, dt \tag{A.23}$$

The associated present value Hamiltonian is given by

$$\begin{aligned} \mathcal{H}_G = \; & e^{-\int_0^t (\rho+\zeta_s^*)ds} \Bigg\{ \ln \left[(Y_t)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \\ & - \ln \left[\Theta_t^{\alpha(1-\Xi_t)} \left(\alpha\Theta_t^{-1} + 1 - \alpha\right)^{\Xi_t\left(\alpha\Xi_t^{-1}+1-\alpha\right)} (\alpha\Theta_t + 1 - \alpha)^{\alpha\Xi_t+1-\alpha} \right] \Bigg\} \\ & + \lambda_{G,t} Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t i_t \\ & + \lambda_{G,t}^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t^* i_t^* \\ & + \mu_{G,t} \Theta_t (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) + \Gamma_G \left[\alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_s^*-\tau_s^*)ds} (\Theta_t - 1)\, dt - b_0 \right], \end{aligned}$$

where $\lambda_{G,t}, \lambda_{G,t}^*, \mu_{G,t}$ are the co-state variables associated with Y_t, Y_t^*, Θ_t; ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates and Γ_G is the multiplier on the home lifetime budget constraint.

The global planner's optimal choice is characterized by the complementary slackness conditions

$$\lambda_{G,t} i_t = 0, \qquad i_t \geq 0, \qquad \lambda_{G,t} \geq 0 \tag{A.24}$$

$$\lambda^*_{G,t} i^*_t = 0, \qquad i^*_t \geq 0, \qquad \lambda^*_{G,t} \geq 0 \tag{A.25}$$

the first-order condition for the transfer $\Gamma_G = 0$, the laws of motion for the co-state variables

$$\dot{\lambda}_{G,t} = -e^{-\int_0^t (\rho+\zeta_s) ds} \left[\left(Y_t^{\text{fb}} \right)^{1+\phi} - (Y_t)^{1+\phi} \right] \frac{1}{Y_t} - \lambda_{G,t} \frac{\dot{Y}_t}{Y_t}, \tag{A.26}$$

$$\dot{\lambda}^*_{G,t} = -e^{-\int_0^t (\rho+\zeta^*_s) ds} \left[\left(Y_t^{*\text{fb}} \right)^{1+\phi} - (Y^*_t)^{1+\phi} \right] \frac{1}{Y^*_t} - \lambda^*_{G,t} \frac{\dot{Y}^*_t}{Y^*_t}, \tag{A.27}$$

$$\begin{aligned} \dot{\mu}_{G,t} = & -e^{-\int_0^t (\rho+\zeta^*_s) ds} \Theta_t^{-1} \alpha \left[\Xi_t - 1 + \frac{\Xi_t \Theta_t^{-1} \left(Y_t^{\text{fb}} \right)^{1+\phi}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{\Theta_t \left(Y_t^{*\text{fb}} \right)^{1+\phi}}{\alpha \Theta_t + 1 - \alpha} \right] \\ & -\lambda_{G,t} Y_t \frac{\alpha (1-\alpha) \Theta_t^{-2}}{\left(\alpha \Theta_t^{-1} + 1 - \alpha \right)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda^*_{G,t} Y^*_t \frac{\alpha (1-\alpha)}{(\alpha \Theta_t + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_{G,t} \frac{\dot{\Theta}_t}{\Theta_t}, \end{aligned} \tag{A.28}$$

non-negativity conditions $\lambda_{G,t} \geq 0$, $\lambda^*_{G,t} \geq 0$, $\mu_{G,t} \geq 0$, initial conditions $\lambda_{G,0} = \lambda^*_{G,0} = \mu_{G,0} = 0$ for the co-state variables, and transversality conditions $\lim_{t\to\infty} \lambda_{G,t} Y_t = 0$, $\lim_{t\to\infty} \lambda^*_t Y^*_{G,t} = 0$, and $\lim_{t\to\infty} \mu_{G,t} \Theta_t = 0$.

Integrating (A.26), (A.27) and (A.28) from 0 to ∞, and using the initial and transversality conditions, yields (A.14), (A.15) and (A.21). For given paths for τ_t, τ^*_t, the global planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y^*_t, \Theta_t, \lambda_{G,t}, \lambda^*_{G,t}, i_t, i^*_t$ consisting of (A.6), (A.7), (A.8), (A.9), (A.22), (A.24), (A.25), (A.26) and (A.27) with boundary conditions (A.14), (A.15), (A.21), $\lambda_{G,0} = 0$ and $\lambda^*_{G,0} = 0$. The optimal transfer is then given by (A.23), and the path of $\mu_{G,t}$ solves (A.28) with initial condition $\mu_{G,0} = 0$.

A.4.2 Home planner's problem

The home planner's problem is an optimal control problem with control variables τ_t, and state variables Y_t, Y^*_t, Θ_t:

$$\max_{\{\tau_t\}} \int_0^\infty e^{-\int_0^t (\rho+\zeta_s) ds} \left\{ \ln \left[(Y_t)^{1-\alpha} (Y^*_t)^\alpha \right] - \frac{(Y_t)^{1+\phi}}{1+\phi} - \ln \left[\Theta_t^{-\alpha} \left(\alpha \Theta_t^{-1} + 1 - \alpha \right)^{1-\alpha} (\alpha \Theta_t + 1 - \alpha)^\alpha \right] \right\}$$

subject to (A.6)-(A.9), (A.22), (A.23).

The associated present value Hamiltonian is given by

$$
\begin{aligned}
\mathcal{H}_H \;=\;& e^{-\int_0^t(\rho+\zeta_s)ds}\left\{\ln\left[(Y_t)^{1-\alpha}(Y_t^*)^{\alpha}\right]-\frac{(Y_t)^{1+\phi}}{1+\phi}-\ln\left[\Theta_t^{-\alpha}\left(\alpha\Theta_t^{-1}+1-\alpha\right)^{1-\alpha}(\alpha\Theta_t+1-\alpha)^{\alpha}\right]\right\}\\
&+\lambda_{H,t}Y_t\left[i_t-(\rho+\zeta_t)-\frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha}(\zeta_t^*-\zeta_t+\tau_t-\tau_t^*)\right]\\
&+\lambda_{H,t}^*Y_t^*\left[i_t^*-(\rho+\zeta_t^*)+\frac{\alpha\Theta_t}{\alpha\Theta_t+1-\alpha}(\zeta_t^*-\zeta_t+\tau_t-\tau_t^*)\right]\\
&+\mu_{H,t}\Theta_t(\zeta_t^*-\zeta_t+\tau_t-\tau_t^*)+\Gamma_H\left[\alpha\int_0^{\infty}e^{-\int_0^t(\rho+\zeta_s^*-\tau_s^*)ds}(\Theta_t-1)\,dt-b_0\right],
\end{aligned}
$$

where $\lambda_{H,t},\lambda_{H,t}^*,\mu_{H,t}$ are the co-state variables associated with Y_t,Y_t^*,Θ_t; Γ_H is home planner's multiplier on the home lifetime budget constraint.

The home planner's optimal choice is characterized by the first-order condition

$$
\mu_{H,t}\Theta_t-\lambda_{H,t}Y_t\frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha}+\lambda_{H,t}^*Y_t^*\frac{\alpha\Theta_t}{\alpha\Theta_t+1-\alpha}=0, \tag{A.29}
$$

the laws of motion of the co-state variables

$$
\begin{aligned}
\dot\lambda_{H,t} &= -e^{-\int_0^t(\rho+\zeta_s)ds}\left[(1-\alpha)-(Y_t)^{1+\phi}\right]\frac{1}{Y_t}-\lambda_{H,t}\frac{\dot Y_t}{Y_t} && \text{(A.30)}\\
\dot\lambda_{H,t}^* &= -e^{-\int_0^t(\rho+\zeta_s)ds}\alpha\frac{1}{Y_t^*}-\lambda_{H,t}^*\frac{\dot Y_t^*}{Y_t^*} && \text{(A.31)}\\
\dot\mu_{H,t} &= -e^{-\int_0^t(\rho+\zeta_s)ds}\left[\frac{\alpha}{\Theta_t}+\frac{(1-\alpha)\,\alpha\Theta_t^{-2}}{\alpha\Theta_t^{-1}+1-\alpha}-\frac{\alpha^2}{\alpha\Theta_t+1-\alpha}\right]+\alpha\Gamma_He^{-\int_0^t(\rho+\zeta_s^*-\tau_s^*)ds}\\
&\quad-\lambda_{H,t}Y_t\frac{\alpha(1-\alpha)\Theta_t^{-2}}{\left(\alpha\Theta_t^{-1}+1-\alpha\right)^2}\frac{\dot\Theta_t}{\Theta_t}-\lambda_{H,t}^*Y_t^*\frac{\alpha(1-\alpha)}{(\alpha\Theta_t+1-\alpha)^2}\frac{\dot\Theta_t}{\Theta_t}-\mu_{H,t}\frac{\dot\Theta_t}{\Theta_t} && \text{(A.32)}
\end{aligned}
$$

non-negativity conditions $\lambda_{H,t}\geq 0$, $\lambda_{H,t}^*\geq 0$, $\mu_{H,t}\geq 0$, initial conditions $\lambda_{H,0}=\lambda_{H,0}^*=0$ for the co-state variables associated with Y_t,Y_t^*, and transversality conditions $\lim_{t\to\infty}\lambda_{H,t}Y_t=0$, $\lim_{t\to\infty}\lambda_t^*Y_{H,t}^*=0$, and $\lim_{t\to\infty}\mu_{H,t}\Theta_t=0$.

Differentiating (A.29) with respect to time, and substituting the co-state laws of motion (A.30), (A.31), (A.32), and using the wedge expressions in (16), yield:

$$
\Gamma_H\Theta_te^{\int_0^t(\zeta_s-\zeta_s^*+\tau_s^*)ds}=1+\Theta_t^{-1}e^{-\omega_t}. \tag{A.33}
$$

Taking natural logarithms and differentiating with respect to time, and re-arranging yields

$$
\left(1+\frac{e^{-\omega_t}}{\Theta_t+e^{-\omega_t}}\times\frac{1-\alpha}{\alpha\Theta_t^{-1}+1-\alpha}\right)\frac{\dot\Theta_t}{\Theta_t}+(\zeta_t-\zeta_t^*+\tau_t^*)=(1+\phi)\frac{e^{-\omega_t}}{\Theta_t+e^{-\omega_t}}\frac{\dot Y_t}{Y_t}. \tag{A.34}
$$

Integrating (A.30) and (A.31) from 0 to ∞, and using the initial and transversality conditions for λ_t,λ_t^* yields (A.14) and (A.15). For given paths for i_t,i_t^*,τ_t^* and a given transfer b_0, the home planner's optimal plan can be described as the particular solution to the system of first-order differential equations in Y_t, Θ_t consisting of (A.6) and (A.34), with boundary conditions (A.14) and (A.23). The path of τ_t then follows from (A.22); that of Y_t^* solves (A.7) with boundary condition (A.15); those of $\lambda_{H,t}, \lambda_{H,t}^*$ solve (A.30), (A.31) with boundary conditions $\lambda_{H,0} = \lambda_{H,0}^* = 0$; and that of $\mu_{H,t}$ follows from (A.29). Finally, the value of Γ_H can then be backed out from (A.32).

A.4.3 Foreign planner's problem

The foreign planner's problem is an optimal control problem with control variables τ_t^*, and state variables Y_t, Y_t^*, Θ_t:

$$\max_{\{\tau_t^*\}} \int_0^\infty e^{-\int_0^t(\rho+\zeta_s^*)ds}\left\{ \ln\left[(Y_t)^\alpha (Y_t^*)^{(1-\alpha)}\right] - \frac{(Y_t^*)^{1+\phi}}{1+\phi} - \ln\left[\Theta_t^\alpha \left(\alpha\Theta_t^{-1}+1-\alpha\right)^\alpha (\alpha\Theta_t+1-\alpha)^{1-\alpha}\right]\right\}$$

subject to (A.6)-(A.9), (A.22), and

$$\tilde{b}_0 = \alpha \int_0^\infty e^{-\int_0^t(\rho+\zeta_h-\tau_h)dh}\left(\Theta_t^{-1}-1\right)dt \tag{A.35}$$

where $\tilde{b}_0$ is the transfer between Foreign and Home set by the global planner, but expressed in terms of date 0 home (rather than foreign) marginal utility.[66]

The associated present value Hamiltonian is given by

$$\begin{aligned}
\mathcal{H}_F \;=\; & e^{-\int_0^t(\rho+\zeta_s^*)ds}\left\{ \ln\left[Y_t^\alpha (Y_t^*)^{(1-\alpha)}\right] - \frac{(Y_t^*)^{1+\phi}}{1+\phi} - \ln\left[\Theta_t^\alpha \left(\alpha\Theta_t^{-1}+1-\alpha\right)^\alpha (\alpha\Theta_t+1-\alpha)^{1-\alpha}\right]\right\} \\
& +\lambda_{F,t}Y_t\left[i_t-(\rho+\zeta_t)-\frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha}(\tau_t-\tau_t^*+\zeta_t^*-\zeta_t)\right] \\
& +\lambda_{F,t}^*Y_t^*\left[i_t^*-(\rho+\zeta_t^*)+\frac{\alpha\Theta_t}{\alpha\Theta_t+1-\alpha}(\tau_t-\tau_t^*+\zeta_t^*-\zeta_t)\right] \\
& +\mu_{F,t}\Theta_t(\zeta_t^*-\zeta_t+\tau_t-\tau_t^*)+\Gamma_F\left[\tilde{b}_0-\alpha\int_0^\infty e^{-\int_0^t(\rho+\zeta_h-\tau_h)dh}\left(\Theta_t^{-1}-1\right)dt\right],
\end{aligned}$$

where $\lambda_{F,t}, \lambda_{F,t}^*, \mu_{F,t}$ are the co-state variables associated with Y_t, Y_t^*, Θ_t; Γ_F is foreign planner's multiplier on the foreign lifetime budget constraint.

The foreign planner's optimal choice is characterized by the first-order condition

$$\mu_{F,t}\Theta_t - \lambda_{F,t}Y_t\frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha} + \lambda_{F,t}^*Y_t^*\frac{\alpha\Theta_t}{\alpha\Theta_t+1-\alpha} = 0, \tag{A.36}$$

[66] It is analytically more convenient to express the foreign country's lifetime budget constraint in terms of home marginal utility. See online Appendix C.5.2 for details.

the laws of motion of the co-state variables

$$\dot{\lambda}_{F,t} = -e^{-\int_0^t(\rho+\zeta_s^*)ds}\alpha\frac{1}{Y_t} - \lambda_{F,t}\frac{\dot{Y}_t}{Y_t} \tag{A.37}$$

$$\dot{\lambda}_{F,t}^* = -e^{-\int_0^t(\rho+\zeta_s^*)ds}\left[(1-\alpha)-(Y_t^*)^{1+\phi}\right]\frac{1}{Y_t^*} - \lambda_{F,t}^*\frac{\dot{Y}_t^*}{Y_t^*} \tag{A.38}$$

$$\begin{aligned}\dot{\mu}_{F,t} = & -e^{-\int_0^t(\rho+\zeta_s^*)ds}\left[-\frac{\alpha}{\Theta_t}+\alpha\frac{\alpha\Theta_t^{-2}}{\alpha\Theta_t^{-1}+1-\alpha}-(1-\alpha)\frac{\alpha}{\alpha\Theta_t+1-\alpha}\right]-\alpha\Gamma_F e^{-\int_0^t(\rho+\zeta_s-\tau_s)ds}\Theta_t^{-2} \\ & -\lambda_{F,t}Y_t\frac{\alpha(1-\alpha)\Theta_t^{-2}}{\left(\alpha\Theta_t^{-1}+1-\alpha\right)^2}\frac{\dot{\Theta}_t}{\Theta_t}-\lambda_{F,t}^*Y_t^*\frac{\alpha(1-\alpha)}{(\alpha\Theta_t+1-\alpha)^2}\frac{\dot{\Theta}_t}{\Theta_t}-\mu_{F,t}\frac{\dot{\Theta}_t}{\Theta_t}\end{aligned} \tag{A.39}$$

non-negativity conditions $\lambda_{F,t} \geq 0$, $\lambda_{F,t}^* \geq 0$, $\mu_{F,t} \geq 0$, initial conditions $\lambda_{F,0} = \lambda_{F,0}^* = 0$ for the co-state variables associated with Y_t, Y_t^*, and transversality conditions $\lim_{t\to\infty}\lambda_{F,t}Y_t = 0$, $\lim_{t\to\infty}\lambda_t^* Y_{F,t}^* = 0$, and $\lim_{t\to\infty}\mu_{F,t}\Theta_t = 0$.

Differentiating (A.36) with respect to time, and substituting the co-state laws of motion (A.37), (A.38), (A.39), and using the wedge expressions in (16), yield:

$$\Gamma_F\Theta_t^{-1}e^{\int_0^t(-\zeta_s+\zeta_s^*+\tau_s)ds} = 1+\Theta_t e^{-\omega_t^*}. \tag{A.40}$$

Taking natural logarithms and differentiating with respect to time, and re-arranging yields

$$-\left(1+\frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}}\times\frac{1-\alpha}{\alpha\Theta_t+1-\alpha}\right)\frac{\dot{\Theta}_t}{\Theta_t}+(\zeta_t^*-\zeta_t+\tau_t) = (1+\phi)\frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}}\frac{\dot{Y}_t^*}{Y_t^*}. \tag{A.41}$$

Integrating (A.30) and (A.31) from 0 to ∞, and using the initial and transversality conditions for λ_t, λ_t^* yields (A.14) and (A.15). For given paths for i_t, i_t^*, τ_t^* and a given transfer $\tilde{b}_0$, the home planner's optimal plan can be described as the particular solution to the system of first-order differential equations in Y_t^*, Θ_t consisting of (A.7) and (A.41), with boundary conditions (A.15) and (A.35). The path of τ_t^* then follows from (A.22); that of Y_t solves (A.6) with boundary condition (A.14); those of $\lambda_{F,t}, \lambda_{F,t}^*$ solve (A.37) and (A.38) with boundary conditions (A.14) and $\lambda_{F,0} = \lambda_{F,0}^* = 0$; and that of $\mu_{F,t}$ follows from (A.36). Finally, the value of Γ_F can then be backed out from (A.39).

A.4.4 Nash equilibrium

A Nash equilibrium of the game is a set of policy actions by the three planners $\{i_t, i_t^*, \tau_t, \tau_t^*\}_{t\geq 0}, b_0$ and associated allocations $\{Y_t, Y_t^*, \Theta_t\}_{t\geq 0}$ such that:

1. Taking $\{\tau_t, \tau_t^*\}_{t\geq 0}$ as given, the actions $\{i_t, i_t^*\}_{t\geq 0}, b_0$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t\geq 0}$ solve the global planner's problem.

2. Taking $\{i_t, i_t^*, \tau_t^*\}_{t\geq 0}$ and b_0 as given, the actions $\{\tau_t\}_{t\geq 0}$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t\geq 0}$ solve the home planner's problem.

3. Taking $\{i_t, i_t^*, \tau_t\}_{t\geq 0}$ and b_0 as given, the actions $\{\tau_t^*\}_{t\geq 0}$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t\geq 0}$ solve the foreign planner's problem.

Combining (A.22), (A.34) and (A.41) to eliminate τ_t and τ_t^* yields

$$\left(1+\frac{e^{-\omega_t}}{\Theta_t+e^{-\omega_t}}\times\frac{1-\alpha}{\alpha\Theta_t^{-1}+1-\alpha}+\frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}}\times\frac{1-\alpha}{\alpha\Theta_t+1-\alpha}\right)\frac{\dot{\Theta}_t}{\Theta_t}$$
$$+\left(\zeta_t^*-\zeta_t\right)+(1+\phi)\left(\frac{e^{-\omega_t}}{\Theta_t+e^{-\omega_t}}\frac{\dot{Y}_t}{Y_t}-\frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}}\frac{\dot{Y}_t^*}{Y_t^*}\right). \tag{A.42}$$

The Nash equilibrium allocations can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \Theta_t, \lambda_{G,t}, \lambda_{G,t}^*, i_t, i_t^*$ consisting of (A.6), (A.7), (A.24), (A.25), (A.26), (A.27) and (A.42), with boundary conditions (A.14), (A.15) and (A.21). The paths of τ_t and τ_t^* the follow from (A.22) and (A.34); the transfer is given by (A.23); and the home and foreign lifetime budget constraint multipliers Γ_H and Γ_F can be backed out from (A.33) and (A.40).

A.4.5 Symmetric steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = \Theta_t = 1$, $\lambda_{G,t} = \lambda_{G,t}^* = 0$, $i_t = i_t^* = \rho$ and $\tau_t = \tau_t^* = 0$. The associated steady-state wedges are hence equal to zero: $\omega_t = \omega_t^* = \varpi_t = 0$, and the multipliers are given by $\Gamma_H = \Gamma_F = 2$.

B Proofs appendix

B.1 Proof of Lemma 1

The home labor wedge is given by

$$\begin{aligned}\omega_t &= -\ln\left(\frac{MRS_t}{MPL_t}\right)=-\ln\left(S_t^{\alpha}\frac{(N_t)^{\phi}C_t}{1}\right)=-\ln\left(S_t^{\alpha}\frac{(Y_t)^{1+\phi}C_t}{Y_t}\right)\\ &= -\ln\left(S_t^{\alpha}\frac{(Y_t)^{1+\phi}C_t}{(1-\alpha)\,S_t^{\alpha}C_t+\alpha S_t^{\alpha}Q_tC_t^*}\right)\\ &= -\ln\left(\frac{(Y_t)^{1+\phi}}{\alpha\Theta_t^{-1}+1-\alpha}\right)\end{aligned}$$

where the second line follows from the home aggregate market clearing condition (6), and the thrid line follows from the international "risk"-sharing condition (4).

The foreign labor wedge is given by

$$\begin{aligned}\omega_t^* &= -\ln\left(\frac{MRS_t^*}{MPL_t^*}\right)=-\ln\left(S_t^{-\alpha}\frac{(N_t^*)^{\phi}C_t^*}{1}\right)=-\ln\left(S_t^{-\alpha}\frac{(Y_t^*)^{1+\phi}C_t^*}{Y_t^*}\right)\\ &= -\ln\left(S_t^{-\alpha}\frac{(Y_t^*)^{1+\phi}C_t^*}{(1-\alpha)\,S_t^{-\alpha}C_t^*+\alpha S_t^{-\alpha}Q_t^{-1}C_t}\right)\\ &= -\ln\left(\frac{(Y_t^*)^{1+\phi}}{\alpha\Theta_t+1-\alpha}\right)\end{aligned}$$

where the second line follows from the foreign aggregate market clearing condition (7), and the thrid line follows from the international "risk"-sharing condition (4).

B.2 Proof of Lemma 2

At a point in time where the home ZLB constraint (A.8) does not bind, the home co-state variable λ_t is zero according to (A.10), home output is at its first-best level $Y_t = Y_t^{\text{fb}}$ according to (A.12), and the home nominal rate, which we refer to as the *unconstrained* home nominal interest rate, is given by

$$i_t = \mathcal{I}_t \equiv \rho + \zeta_t + \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha}\left(\zeta_t^* - \zeta_t + \tau_t - \tau_t^*\right) - \frac{1}{1+\phi}\frac{\alpha\Xi_t^{-1}}{\alpha\Xi_t^{-1} + 1 - \alpha}\left(\zeta_t^* - \zeta_t\right). \tag{B.1}$$

Similarly, at a point in time where the foreign ZLB constraint (A.9) does not bind, the foreign co-state variable λ_t^* is zero according to (A.11), foreign output is at its first-best level $Y_t^* = Y_t^{*\text{fb}}$ according to (A.13), and the foreign nominal rate, which we refer to as the *unconstrained* foreign nominal interest rate, is given by

$$i_t^* = \mathcal{I}_t^* \equiv \rho + \zeta_t^* - \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha}\left(\zeta_t^* - \zeta_t + \tau_t - \tau_t^*\right) + \frac{1}{1+\phi}\frac{\alpha\Xi_t}{\alpha\Xi_t + 1 - \alpha}\left(\zeta_t^* - \zeta_t\right). \tag{B.2}$$

Hence, under a regime of free capital flows where $\tau_t = \tau_t^* = 0$ and (as a result) $\Theta_t = \Xi_t$, the expressions reduce to those in (19) (for $\zeta_t^* = 0$). Similarly, under a regime of closed capital accounts where $\tau_t - \tau_t^* = \zeta_t - \zeta_t^*$ and (as a result) $\Theta_t = 1$, the expressions reduce to those in (23) (for $\zeta_t^* = 0$).

B.3 Proof of Lemma 3

The proof is by construction. First, we observe that the system of first-order differential equations described at the end of Appendix A.2 can be split into two separate systems in Y_t, λ_t, i_t on one hand, and $Y_t^*, \lambda_t^*, i_t^*$ on the other hand. Therefore, one can analyze the solution for these two sets of variables separately.

Starting with the foreign country, it is easy to verify that Assumption 1 implies that the unconstrained foreign interest rate is strictly positive for any t, both under the free capital mobility regime (expression in (19)) and under the closed capital account regime (expression in (23)). It follows that the unconstrained policy is feasible and optimal for the foreign country[67] The conditions (A.11), (A.13), (A.15) and $\lambda_0^* = 0$ are hence satisfied with $i_t^* = \mathcal{I}_t^*$, $Y_t^* = Y_t^{*\text{fb}}$ and $\lambda_t^* = 0$ for all $t \geq 0$.

Turning to the home country, it is easy to verify that Assumption 1 implies that the unconstrained home interest rate is strictly negative for $t \in [0, T)$ but strictly positive for $t \geq T$, both under the free capital mobility regime (expression in (19)) and under the closed capital account regime (expression in (23)). We conjecture that the optimal plan consists in setting $i_t = 0$ for $t \in [0, \widehat{T})$ and $i_t \geq \widehat{T}$ provided

[67]Notice that the planner's objective is additively separable in Y_t and Y_t^*, and that Y_t is independent of i_t^*.

$\widehat{T} > T$ is chosen to satisfy

$$0 = \int_0^{\widehat{T}} e^{-\int_0^t (\rho+\zeta_s)ds} \left[\left(\frac{Y_t^{\text{fb}}}{Y_T^{\text{fb}}} \right)^{1+\phi} - e^{(1+\phi)\int_t^{\widehat{T}} (\rho+\zeta_s)ds} \left(\frac{\alpha\Theta_t^{-1} + 1 - \alpha}{\alpha\Theta_{\widehat{T}}^{-1} + 1 - \alpha} \right)^{1+\phi} \right] dt. \tag{B.3}$$

The associated values of the state and co-state variables are given as follows: $Y_t = Y_t^{\text{fb}}$ and $\lambda_t = 0$ for $t \geq \widehat{T}$; and

$$Y_t = Y_T^{\text{fb}} e^{\int_t^{\widehat{T}} (\rho+\zeta_s)ds} \frac{\alpha\Theta_t^{-1} + 1 - \alpha}{\alpha\Theta_{\hat{T}}^{-1} + 1 - \alpha},$$

and

$$\lambda_t = \frac{1}{Y_t} \int_t^{\widehat{T}} e^{-\int_0^h (\rho+\zeta_s^*)ds} \Xi_h \left[\left(Y_h^{\text{fb}} \right)^{1+\phi} - (Y_h)^{1+\phi} \right] dh$$

for $t \in [0, \widehat{T})$. It is straightforward to verify that this plan (by construction) satisfies all relevant conditions (A.10), (A.12), (A.14) and $\lambda_0 = 0$.

B.4 Proof of Lemma 6

Substituting (A.22) and the definition of the international wedge $\varpi_t \equiv \ln\Theta_t - \ln\Xi_t$ into (A.33) and (A.40) yields (30) and (31), respectively.

B.5 Proof of Proposition 1

Claim 1. Defining the functions

$$\begin{aligned} f_1(z) &\equiv \int_0^T e^{-(\rho-\bar{\zeta})t} \left[\left(\frac{Y_t^{\text{fb}}}{Y_T^{\text{fb}}} \right)^{1+\phi} - \left(\frac{\alpha\Theta_t^{-1} + 1 - \alpha}{\alpha\Theta_{\hat{T}}^{-1} + 1 - \alpha} \right)^{1+\phi} e^{(1+\phi)\int_t^z (\rho+\zeta_s)ds} \right] dt, \\ f_2(z) &\equiv -e^{\bar{\zeta}T} \int_T^z e^{-\rho t} \left[1 - e^{(1+\phi)\rho(z-t)} \right] dt, \end{aligned}$$

condition (B.3) can be written as

$$f_1(\widehat{T}) = f_2(\widehat{T}). \tag{B.4}$$

The functions satisfy $f_1'(z) < 0$, $f_2'(z) > 0$, with $f_1(T) > 0$, $f_2(T) = 0$, $\lim_{z\to-\infty}$ and $\lim_{z\to\infty} f_2(z) = +\infty$.[68] (B.4) therefore has a unique solution $\widehat{T} > T$.

Now, observe that under free capital mobility, $\Theta_t^{-1} > \Theta_T^{-1}$ for $t < T$, and $\Theta_t = \Theta_T$ for $t \geq T$, while under closed capital accounts $\Theta_t = 1$ for all $t \geq 0$. As a result, we have $f_1^{\text{free}}(z) < f_1^{\text{closed}}(z)$ and $f_2^{\text{free}} = f_2^{\text{closed}}$ for $z > T$. It must thus be that $\widehat{T}^{\text{free}} < \widehat{T}^{\text{closed}}$.

Claim 2. We observe that under both capital flow regimes under consideration, the growth rate of home output is given by $-\rho$ for $t \in [T, \widehat{T})$. Since $\ln Y_{\widehat{T}^{\text{closed}}}^{\text{closed}} = \ln Y_{\widehat{T}^{\text{free}}}^{\text{free}} = \ln Y_T^{\text{fb}}$, it must be that

[68] A sufficient condition for $f_1(T) > 0$ is that $\dot{Y}_t/Y_t > 0$ for $t \in [0, T)$. Assumption 1 ensures that this holds true under $i_t = 0$ for both the regime of free capital mobility and the regime of closed capital accounts.

$\ln Y_T^{\text{closed}} > \ln Y_T^{\text{free}} > \ln Y_T^{\text{fb}}$. This in turn implies, in light of the higher growth rate of home output under closed capital accounts $(-\rho + \bar{\zeta} > 0)$ than under free capital mobility $(-\rho + \bar{\zeta}(1-\alpha)/(\alpha\Theta_t^{-1} + 1 - \alpha) > 0)$, that $\ln Y_0^{\text{closed}} < \ln Y_0^{\text{free}} < \ln Y_0^{\text{fb}}$. Claim #2 follows immediately.

C Model appendix (online appendix)

C.1 Home household

Using the price index definitions, the home household's budget constraint (1) can be expressed as

$$\dot{a}_t = i_t a_t + W_t N_t + T_t + \Pi_t - P_t C_t + \left(i_t^* - i_t + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} \right) \mathcal{E}_t D_{F,t}. \tag{C.1}$$

for net foreign assets $a_t \equiv D_{H,t} + \mathcal{E}_t D_{F,t}$. Expenditure minimization requires $C_{H,t}(l) = \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}$, $C_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}\ \forall l$, $C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t$ and $C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t$. The households' optimality conditions for labor supply, home currency bonds and foreign currency bonds are given by

$$\begin{aligned}
\frac{W_t}{P_t} &= N_t^{\phi} C_t, \\
\frac{\dot{C}_t}{C_t} &= i_t - \pi_t - (\rho + \zeta_t), && \text{(C.2)} \\
\frac{\dot{C}_t}{C_t} &= i_t^* + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t - (\rho + \zeta_t). && \text{(C.3)}
\end{aligned}$$

C.2 Foreign household

Preferences of the foreign households are represented by the utility functional

$$\int_0^{\infty} e^{-\int_0^t (\rho + \zeta_h^*) dh} \left[\ln C_t^* - \frac{(N_t^*)^{1+\phi}}{1+\phi} \right] dt$$

with $C_t^* \equiv \left(C_{F,t}^*\right)^{1-\alpha} \left(C_{H,t}^*\right)^{\alpha} / [(1-\alpha)^{1-\alpha} \alpha^{\alpha}]$, $C_{F,t}^* \equiv \left[\int_0^1 C_{F,t}^*(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$ and $C_{H,t}^* \equiv \left[\int_0^1 C_{H,t}^*(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$. Its budget constraint expressed in its own currency is given by

$$\frac{\dot{D}_{H,t}^*}{\mathcal{E}_t} + \dot{D}_{F,t}^* = (i_t + \tau_t^* - \tau_t) \frac{D_{H,t}^*}{\mathcal{E}_t} + i_t^* D_{F,t}^* + W_t^* N_t^* + T_t^* + \Pi_t^* - \int_0^1 P_{H,t}^*(l)\, C_{H,t}^*(l)\, dl - \int_0^1 P_{F,t}^*(l)\, C_{F,t}^*(l)\, dl$$

Defining $a_t^* \equiv D_{H,t}^*/\mathcal{E}_t + D_{F,t}^*$ as the foreign household's net assets in foreign currency terms and making use of the price index definitions, the budget constraint can be expressed as

$$a_t^* = i_t^* a_t^* + W_t^* N_t^* + T_t^* + \Pi_t^* - P_t^* C_t^* + \left(i_t - i_t^* + \tau_t^* - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} \right) \frac{D_{H,t}^*}{\mathcal{E}_t}. \tag{C.4}$$

Expenditure minimization requires $C_{F,t}^*(l) = \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*}\right)^{-\epsilon} C_{F,t}^*$, $C_{H,t}^*(l) = \left(\frac{P_{H,t}^*(l)}{P_{H,t}^*}\right)^{-\epsilon} C_{H,t}^*\ \forall l$, $C_{F,t}^* = (1-\alpha)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-1} C_t^*$ and $C_{H,t}^* = \alpha \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-1} C_t^*$. The households' optimality conditions for labor supply, home currency bonds and foreign currency bonds are given by

$$\begin{aligned}
\frac{W_t^*}{P_t^*} &= (N_t^*)^{\phi} C_t^*, \\
\frac{\dot{C}_t^*}{C_t^*} &= i_t + \tau_t^* - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t^* - (\rho + \zeta_t^*), &\text{(C.5)} \\
\frac{\dot{C}_t^*}{C_t^*} &= i_t^* - \pi_t^* - (\rho + \zeta_t^*). &\text{(C.6)}
\end{aligned}$$

C.3 International "risk"-sharing condition

Subtracting Foreign's Euler equation for the home currency bond (C.5) from Home's Euler equation for the home currency bond (C.2) yields

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*} = \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} + \pi_t^* - \pi_t + \zeta_t^* - \zeta_t,$$

which can be rewritten as

$$\frac{d}{ds}\left[\ln\left(\frac{C_s}{C_s^* Q_s}\right)\right] = \tau_s - \tau_s^* + \zeta_s^* - \zeta_s.$$

Integrating from 0 to t, we obtain the international "risk"-sharing condition (4), or

$$C_t = \Theta_t C_t^* Q_t, \tag{C.7}$$

with $\Theta_t \equiv \Theta_0 \exp\left[\int_0^t (\zeta_s^* - \zeta_s + \tau_s - \tau_s^*)\, ds\right]$.

C.4 Goods market equilibrium

Clearing on the market for variety l in Home requires

$$\begin{aligned}
Y_t(l) &= C_{H,t}(l) + C_{H,t}^*(l) \\
&= \left(\frac{P_{H,t}(l)}{P_{H,t}}\right)^{-\epsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + \alpha\left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-1} C_t^*\right]
\end{aligned}$$

Given price symmetry ($P_{H,t}(l) = P_{H,t}\ \forall l$), substituting this equation into the definition of home aggregate output $Y_t \equiv \left[\int_0^1 Y_t(l)^{\frac{\epsilon-1}{\epsilon}}\, dl\right]^{\frac{\epsilon}{\epsilon-1}}$, we obtain

$$\begin{aligned}
Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + \alpha\left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*}\right)^{-1} C_t^* \\
&= (1-\alpha) S_t^{\alpha} C_t + \alpha S_t^{\alpha} Q_t C_t^* \\
&= \left[(1-\alpha)\Theta_t + \alpha\right] S_t^{\alpha} Q_t C_t^* &\text{(C.8)}
\end{aligned}$$

Similarly, market clearing for foreign aggregate output $Y_t^* \equiv \left[\int_0^1 Y_t^*(l)^{\frac{\epsilon-1}{\epsilon}} dl\right]^{\frac{\epsilon}{\epsilon-1}}$ requires

$$\begin{aligned} Y_t^* &= (1-\alpha)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-1} C_t^* + \alpha\left(\frac{P_{F,t}^*}{P_t/\mathcal{E}_t}\right)^{-1} C_t \\ &= (1-\alpha) S_t^{-\alpha} C_t^* + \alpha S_t^{-\alpha} Q_t^{-1} C_t \\ &= [(1-\alpha)+\alpha\Theta_t] S_t^{-\alpha} C_t^* \end{aligned} \tag{C.9}$$

C.5 Intertemporal budget constraints

C.5.1 Home intertemporal budget constraint

The equilibrium lump-sum rebate T_t in Home is the sum of the negative of the subsidy expenses on outflows and the tax proceeds on inflows:

$$\begin{aligned} T_t &= -\tau_t \mathcal{E}_t D_{F,t} + \tau_t D_{H,t}^* \\ &= -\tau_t a_t + \tau_t \left(D_{H,t} + D_{H,t}^*\right) \\ &= -\tau_t a_t, \end{aligned} \tag{C.10}$$

where the second line follows from the definition of home net foreign assets $a_t = D_{H,t} + \mathcal{E}_t D_{F,t}$, and the third line follows from the market clearing condition for home currency bond $D_{H,t} + D_{H,t}^* = 0$.

Substituting $\Pi_t = P_{H,t}Y_t - W_t N_t$ and (C.10) into the home household' budget constraint (C.1), while recognizing that the Euler equations (C.2)-(C.3) (or, for that matter, (C.5)-(C.6)) imply a distorted interest parity condition $i_t - i_t^* - \tau_t + \tau_t^* - \dot{\mathcal{E}}_t/\mathcal{E}_t = 0$, we obtain Home's resource constraint:

$$\dot{a}_t = (i_t - \tau_t) a_t + P_{H,t} Y_t - P_t C_t.$$

Expressed in terms of the marginal utility of foreign agents (i.e., normalizing by $P_t^* \mathcal{E}_t C_t^*$), noting that $P_t = \mathcal{E}_t^\alpha$ and $P_t^* = \mathcal{E}_t^{-\alpha}$, the resource constraint is given by

$$\dot{b}_t = \left(i_t - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t^* - \frac{\dot{C^*}_t}{C_t^*}\right) b_t + (C_t^*)^{-1}\left(S_t^{-(1-\alpha)} Y_t - Q_t^{-1} C_t\right),$$

for $b_t \equiv a_t/(P_t^* C_t^* \mathcal{E}_t)$. Substituting Foreign's Euler equation for the home currency bond (C.5) yields a current account equation given by

$$\dot{b}_t = (\rho + \zeta^* - \tau_t^*) b_t - (C_t^*)^{-1}\left(Q_t^{-1} C_t - S_t^{-(1-\alpha)} Y_t\right).$$

Integrating from 0 to ∞, while imposing a no-Ponzi game condition, yields the intertemporal budget constraint

$$b_0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (C_t^*)^{-1}\left(Q_t^{-1} C_t - S_t^{-(1-\alpha)} Y_t\right) dt.$$

Using the home good's market clearing condition (C.8) and the international "risk"-sharing condition

(C.7), this constraint can be expressed as (12).

C.5.2 Foreign intertemporal budget constraint

The derivation is similar to that of the home intertemporal budget constraint. The equilibrium lump-sum rebate T_t^* in Foreign is the sum of minus the subsidy expenses on outlflows and the tax proceeds on inflows:

$$\begin{aligned} T_t^* &= -\tau_t^* \frac{D_{H,t}^*}{\mathcal{E}_t} + \tau_t^* D_{F,t} \\ &= -\tau_t^* a_t^* + \tau_t^* \left(D_{F,t}^* + D_{F,t} \right) \\ &= -\tau_t^* a_t^*, \end{aligned} \tag{C.11}$$

where, again, the second line follows from the definition of foreign net assets $a_t^* \equiv D_{H,t}^*/\mathcal{E}_t + D_{F,t}^*$, and the third line follows from the market clearing condition for the foreign currency bond $D_{F,t}^* + D_{F,t} = 0$.

Substituting $\Pi_t^* = P_{F,t}^* Y_t^* - W_t^* N_t^*$ and (C.11) into the foreign household' budget constraint (C.4), while recognizing that the Euler equations (C.2)-(C.3) (or, for that matter, (C.5)-(C.6)) imply a distorted interest parity condition $i_t - i_t^* - \tau_t + \tau_t^* - \dot{\mathcal{E}}_t/\mathcal{E}_t = 0$, we obtain Foreign's resource constraint:

$$\dot{a}_t^* = \left(i_t^* - \tau_t^*\right) a_t^* + P_{F,t}^* Y_t^* - P_t^* C_t^*. \tag{C.12}$$

Expressed in terms of the marginal utility of foreign agents (i.e., normalizing by $P_t^* C_t^*$), noting that $P_t = \mathcal{E}_t^{\alpha}$ and $P_t^* = \mathcal{E}_t^{-\alpha}$, the resource constraint is given by

$$\dot{b}_t^* = \left(i_t^* - \tau_t^* - \pi_t^* - \frac{\dot{C}_t^*}{C_t^*} \right) b_t^* - \left(C_t^*\right)^{-1} \left(C_t^* - S_t^{-\alpha} Y_t^* \right),$$

for $b_t^* \equiv a_t^*/(P_t^* C_t^*)$. Substituting Foreign's Euler equation for the foreign currency bond (C.6) yields a current account equation given by

$$\dot{b}_t^* = \left(\rho + \zeta_t^* - \tau_t^*\right) b_t^* - \left(C_t^*\right)^{-1} \left(C_t^* - S_t^{-\alpha} Y_t^* \right).$$

Integrating from 0 to ∞, while imposing a no-Ponzi game condition, yields the intertemporal budget constraint

$$b_0^* = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} \left(C_t^*\right)^{-1} \left(C_t^* - S_t^{-\alpha} Y_t^* \right) dt.$$

Using the foreign good's market clearing condition (C.9), this constraint can be expressed as (12), with $-b_0^*$ on the left-hand side instead of b_0.

Alternatively, expressing (C.12) in terms of the marginal utility of home agents (i.e., normalizing by $P_t C_t/\mathcal{E}$), and substituting the home Euler equation for the home bond (C.2) and the interest parity condition, the current account equation is given by

$$\dot{\tilde{b}}_t^* = \left(\rho + \zeta_t - \tau_t\right) \tilde{b}_t^* - C_t^{-1} \left(Q_t C_t^* - Y_t^* S_t^{1-\alpha} \right),$$

for $\tilde{b}_t^* \equiv a_t^* \mathcal{E}_t/(P_t C_t)$. Integrating from 0 to ∞, while imposing a no-Ponzi game condition and using the foreign good's market clearing condition (C.9) yields

$$\tilde{b}_0^* = \alpha \int_0^\infty e^{-\int_0^t (\rho+\zeta_s-\tau_s)ds} \left(\Theta_t^{-1} - 1\right) dt.$$

This version of Foreign's lifetime budget constraint does not explicitly feature τ_t^*, and is therefore more convenient to work with in setting up the foreign planner's problem in Appendix A.4.3.

D Computations appendix (online appendix)

This appendix provides information on the numerical (non-linear) solution of the model under the various capital flow regimes we consider. These computations underlie our plots in Figures 3, 4 and 6.

D.1 Free capital mobility and closed capital accounts

In the free capital mobility and closed capital account regimes, the equilibrium paths of all variables are available in closed form conditional on the ZLB exit time in Home.[69] Solving for the equilibrium thus simply requires numerically solving equation (22) (or equivalently (B.3)) for $\widehat{T}$. Under free capital mobility, the path of the expenditure ratio is given by $\Theta_t = \Xi_t \; \forall t \geq 0$, while under closed capital accounts, it is given by $\Theta_t = 1 \; \forall t \geq 0$.

D.2 Efficient capital flow regime

In the efficient capital flow regime, the solution in addition requires solving for the path of the expenditure ratio Θ_t. We therefore proceed in two steps.

First, we conjecture a home ZLB exit time $\widehat{T}$, and conditional on this $\widehat{T}$ we solve for the path of Θ_t satisfying the system of ordinary differential equations (ODEs) defined by (A.6) and (A.20) with boundary conditions $Y_{\widehat{T}} = Y_T^{\text{fb}}$ and (A.21). This amounts to solving the ODE system

$$\begin{aligned}
\dot{\Theta}_t &= \Theta_t \frac{-\left(1+\phi\right) e^{-\omega_t - \varpi_t}\left(\rho+\zeta_t\right) + e^{-\omega_t^* + \varpi_t}\frac{\alpha \Xi_t^2}{\alpha\Xi_t + (1-\alpha)}\zeta_t - \left(1 + \Xi_t e^{-\omega_t^* + \varpi_t}\right)\zeta_t}{e^{-\omega_t - \varpi_t}\left(1 + \frac{\phi\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha}\right) + \frac{(1-\alpha)\Xi_t e^{-\omega_t^* + \varpi_t}}{\alpha\Theta_t + 1 - \alpha}}, \\
\dot{Y}_t &= Y_t \left[-\left(\rho + \zeta_t\right) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha}\frac{\dot{\Theta}_t}{\Theta_t}\right], \qquad \text{(D.1)} \\
\dot{g}_t &= e^{-\rho t}\left(\Xi_t \Theta_t^{-1} e^{-\omega_t} - \Theta_t e^{-\omega_t^*}\right), \qquad \text{(D.2)}
\end{aligned}$$

with boundary conditions

$$\begin{aligned}
0 &= g_0, \qquad \text{(D.3)} \\
0 &= Y_{\widehat{T}} - Y_T^{\text{fb}}, \qquad \text{(D.4)} \\
0 &= g_{\widehat{T}} + \left[\Xi_T \frac{\Theta_{\widehat{T}}^{-1}\left(Y_T^{\text{fb}}\right)^{1+\phi}}{\alpha\Theta_{\widehat{T}}^{-1} + 1 - \alpha} - \frac{\Theta_{\widehat{T}}\left(Y_T^{\text{fb}*}\right)^{1+\phi}}{\alpha\Theta_{\widehat{T}} + 1 - \alpha}\right]\frac{e^{-\rho\widehat{T}}}{\rho}. \qquad \text{(D.5)}
\end{aligned}$$

We solve this system using MATLAB's ODE solver bvp4c.

Second, in an outer loop, we employ a bisection method to find the value of $\widehat{T}$ that solves equation (22) (or equivalently, (B.3)). This requires solving the ODE system described above repeatedly.

[69] Recall that under Assumption 1, the ZLB never binds in Foreign.

D.3 Nash capital flow regime

As in the efficient capital flow regime, solving for the equilibrium in the Nash capital flow regime requires jointly solving for the ZLB exit time $\widehat{T}$ and the path of the expenditure ratio Θ_t. Hence, we again proceed in two steps.

First, we conjecture a ZLB exit time $\widehat{T}$, and conditional on this $\widehat{T}$ we solve for the path of Θ_t satisfying the system of ordinary differential equations (ODEs) defined by (A.6) and (A.42) with boundary conditions $Y_{\widehat{T}} = Y_T^{\mathrm{fb}}$ and (A.21). This amounts to solving the ODE system consisting of

$$\dot{\Theta}_t = \Theta_t \frac{-(1+\phi)\frac{e^{-\omega_t}}{\Theta_t + e^{-\omega_t}}(\rho+\zeta_t) + \frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}}\frac{\alpha\Xi_t}{\alpha\Xi_t+(1-\alpha)}\zeta_t - \zeta_t}{1 + \frac{e^{-\omega_t}}{\Theta_t+e^{-\omega_t}}\left(1 + \frac{\phi\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1}+1-\alpha}\right) + \frac{e^{-\omega_t^*}}{\Theta_t^{-1}+e^{-\omega_t^*}} \times \frac{1-\alpha}{\alpha\Theta_t+1-\alpha}},$$

(D.1) and (D.2) with boundary conditions (D.3), (D.4) and (D.5), which we solve using MATLAB's ODE solver bvp4c.

Second, in an outer loop, we employ a bisection method to find the value of $\widehat{T}$ that solves equation (22) (or equivalently, (B.3)). This requires solving the ODE system described above repeatedly.