The Side Effects of Safe Asset Creation*

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Abstract

We present a model with incomplete markets in order to understand the costs and benefits of increasing government debt in a low interest rate environment. Higher idiosyncratic risk increases the demand for safe assets and can even lower real interest rates below zero. A fiscal authority can issue more debt to meet this increased demand for safe assets and arrest the decline in real interest rates. While such a policy succeeds in keeping real rates above zero, it comes at a cost as higher real interest rates can lead to permanently lower investment. However, in an environment with nominal rigidities and a zero bound on nominal rates, policymakers may not have a choice. On the one hand, without creating additional safe assets, constrained monetary policy is powerless to combat higher unemployment. On the other hand, creating safe assets can make monetary policy potent again, allowing policymakers to lower unemployment, but such a policy only shifts the malaise elsewhere in the economy where it manifests itself as a permanent investment slump.

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

The past three decades have been marked by a secular decline in real interest rates in the United States and other advanced economies. A growing literature has argued that this decline can be attributed to an increased demand of safe assets, which pushed the natural rate of interest well below zero (Caballero and Farhi (2016)). This in turn caused the zero lower bound to bind in many advanced economies, rendering conventional monetary policy unable to prevent a deep and lasting recession. At the same time, an empirical literature has documented that safe assets such as U.S. Treasury debt enjoy a safety premium, which is responsive to the aggregate supply of debt (Krishnamurthy and Vissing-Jorgensen (2012)). This suggests that by increasing the supply of safe debt, fiscal authorities may be able to reduce the safety premium, satiate the demand for safe assets, and raise the natural rate of interest, allowing monetary policy to regain its potency. Can governments increase the supply of safe assets in this way? And if they can, should they?

In models where Ricardian Equivalence holds, the supply of government debt does not affect equilibrium prices and allocations unless it is assumed to provide nonpecuniary benefits; while such a modelling strategy is convenient, it does not provide satisfactory answers to the questions we are interested in. Thus in order to answer these questions, we present an analytically tractable incomplete markets model with risky capital, safe government debt, and nominal rigidities. The economy has a simple two period overlapping generations (OLG) structure. Young households supply labor inelastically, pay lump sum taxes (which may be negative) and invest in both risky capital and safe government debt. Each old household operates a firm, hiring labor to produce output with its capital. Old households face uninsurable productivity risk. This creates a risk premium between the marginal product of capital and the return on short term government debt.

In order to understand the forces at play, we first study a model without nominal rigidities. An increase in idiosyncratic risk reduces real interest rates, as households attempt to substitute away from risky capital towards safe debt. A sufficiently large increase in risk can push real interest rates below zero. An increase in government debt can offset the decline in real interest rates by satiating the demand for safe assets. However, this comes at the cost of crowding out investment in physical capital. Absent nominal rigidities, this cost is so strong that it is never optimal to prevent real interest rates from falling below zero.

We then introduce downward nominal wage rigidity and a zero lower bound on nominal interest rates. Under conventional monetary policy rules, a (permanently) negative natural rate of interest generates a (permanent) recession, as nominal rates cannot fall sufficiently to restore full employment. This decline in employment in turn causes a persistent investment slump. An increase in the supply of safe assets can raise the natural rate of interest above zero, allowing conventional monetary policy to preserve full employment. As in the model without nominal rigidities, this comes at the cost of crowding out investment. In this sense, the costs of a risk-induced recession may persist even after the economy has returned to full employment, taking the form of sluggish investment and low labor productivity.

Related Literature Our investigation relates to a large recent literature which studies the macroeconomic consequences of the secular decline in safe rates of interest and the supply of safe assets. Most relevant to our work, Caballero and Farhi (2016) study an endowment economy in which safe asset shortages can generate a persistent recession. Relative to their paper, our contribution is to consider the interaction between safe asset shortages and investment in physical capital. While one might have thought that giving households access to a physical storage technology prevents saving gluts from having any adverse effects, this turns out not to be the case when investment in physical capital is risky. Our focus on investment also reveals a new tradeoff. In Caballero and Farhi (2016), increasing government debt can prevent a safety trap without any adverse consequences. In our model, it comes at the cost of crowding out investment in physical capital.

Our approach to modeling safe assets differs from Gorton and Ordonez (2013), for whom a 'safe asset' is an information-insensitive asset. Since individuals are willing to trade information-insensitive assets without fear of adverse selection, such assets circulate widely. The liquidity or moneyness of these assets implies the existence of a convenience yield, so that the pecuniary return on safe assets lies below the yield on comparable securities without these properties. The existence of such a convenience is documented empirically by Krishnamurthy and Vissing-Jorgensen (2012, 2015). One important strand of this literature focuses on the financial stability consequences of low real interest rates, and the role of public debt management in regulating these. Greenwood et al. (2016) and Woodford (2016) ask whether a central bank should increase its supply of short term claims in order to promote financial stability, by reducing the private sector's tendency to engage in socially excessive maturity transformation. Like this literature, our model predicts that government debt trades at a spread below other assets, and this spread varies with the supply of Treasury debt. However, our model attributes this spread entirely to the risk properties of debt - we abstract altogether from liquidity.

In this respect, our paper is related to a growing literature discussing the macroeconomic consequences of the liquidity properties of government debt. A large literature, following Woodford (1990), has studied the role of government debt in relaxing private borrowing constraints. In a model without nominal rigidities, Angeletos et al. (2016) study the optimal provision of government debt when debt provides liquidity services and taxes are distortionary. Their Ramsey planner trades off the benefits of increasing debt and relaxing financial frictions against the cost of raising interest rates, tightening the government's borrowing constraint. We focus on a different role for government debt - providing a safe asset, rather than liquidity services - and a diifferent set of tradeoffs. Our government has access to lump sum taxes, so relaxing the government budget constraint is irrelevant. The cost of issuing more government debt is instead that this crowds out investment; a potential benefit is that it avoids liquidity traps. In this regard, our result is reminiscent of Yared (2013) who shows that while increasing government debt can in principle relax private borrowing constraints, it is not always optimal to do so since this distorts investment decisions.

The role of government debt in providing liquidity services becomes even more important in the presence of nominal rigidities. Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) were among the first to present models in which an exogenous shock to borrowing constraints causes a recession due to the zero lower bound; Guerrieri and Lorenzoni (2011) noted that government debt can in principle completely offset a shock to private borrowing constraints, while Bilbiie et al. (2013) demonstrated a similar result in a Eggertsson and Krugman (2012)-type model. Although it is not central to our narrative, our model allows for the possibility of permanently negative real rates; in that regard, it is more closely related to Eggertsson and Mehrotra (2014). In models without capital where government debt is valued for its liquidity rather than its safety, government debt is generally an extremely powerful tool. Again, in our setting government debt has an additional cost in that it crowds out capital investment.

In an economy with capital and nominal rigidities, Boullot (2016) shows that rational bubbles can ameliorate liquidity traps which are driven by a contraction in private borrowing constraints (as in Eggertsson and Krugman (2012) and Korinek and Simsek (2016)). Depending on whether the zero lower bound binds, rational bubbles (and government debt, which is a perfect substitute for bubbles) may either expand or contract output and the stock of capital. Our focus is on the effect of government debt, rather than bubbles, on the natural rate of interest, and we study the safety channel rather than the liquidity channel. Bacchetta et al. (2016) also study the interaction between government debt and capital in a liquidity trap, albeit in a flexible price economy. Like them, we show that safe assets crowd out capital even in a liquidity trap. Unlike them, we focus on goverment debt's safety properties rather than its liquidity properties, and we study nominal rigidities, giving policymakers a motive for increasing the natural rate of interest which is absent in their flexible price economy. Finally, like us, Auclert and Rognlie (2016) study an incomplete markets model, in which they study the consequences of labor income inequality for aggregate demand. Their results are consistent with our findings: when monetary policy is constrained, public debt issuances are expansionary and crowd in investment. Rather than studying inequality, we instead investigate the role of public debt in moderating increases in *risk premia* driven by idiosyncratic capital income risk.

Our stylized model predicts that an increase in the dispersion of firm-specific productivity causes an increase in the risk premium, which pushes real interest rates below zero while the aggregate marginal product of capital remains high. This is broadly consistent with the empirical literature. Decker et al. (2017) document that the volatility of total factor productivity increased since 1980 for U.S. manufacturing firms. Duarte and Rosa (2015) present evidence from a variety of asset pricing models that the equity risk premium increased significantly between 2000 and 2013. Using the methodology of Gomme et al. (2011), Caballero et al. (2017) document that the real return on productive capital remained flat or even increased over the past three decades, while the return on U.S. Treasuries declined dramatically.

2 Model

Households Time is discrete. At each date t, a cohort of ex-ante identical individuals with measure 1 is born and lives for two periods. Each individual $j \in [0,1]$ has identical preferences which can be described as:

$$\mathbb{U}(c_t^Y, c_{t+1}^O) = (1-\beta)\ln c_t^Y + \beta \mathbb{E}_t \ln c_{t+1}^O$$

where $\beta \in (0, 1/2)$. When young, each household is endowed with one unit of labor which it is willing to supply inelastically and earns a nominal wage W_t per unit. The household also receives a transfer T_t (which could be negative) from the government. Households can invest in two assets: *risky* capital and *safe* government debt. The budget constraints of a household can be written as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} = W_t l_t + P_t T_t \tag{1}$$

$$P_{t+1} c_{t+1}^O(z) = P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} \tag{2}$$

where i_t is the nominal interest rate on government debt and $R_{t+1}^k(z)$ is the return on capital earned by household i at date $t+1$ when it is old, which depends on a random variable z described below. When young, the household must decide in how much to invest in capital without knowing the realization of z in the next period. Notice that a household makes all its decisions when young and just consumes its wealth when old. Lemma 1 below summarizes a households decisions.

Lemma 1 (Saving and Investment Decision)**.** *The household's decisions are described by*

$$c_t^Y = (1-\beta)(\omega_t l_t + T_t) \tag{3}$$

$$k_{t+1} = \beta \eta_t (\omega_t l_t + T_t) \tag{4}$$

$$\frac{1}{R_t} b_{t+1} = \beta(1-\eta_t)(\omega_t l_t + T_t) \tag{5}$$

where $b_t = \dfrac{B_t}{P_t}$ *denotes real debt,* $R_t = \dfrac{(1+i_t)P_t}{P_{t+1}}$ *is the real return on government debt and* η_t*, the portfolio share of risky capital, is implicitly defined by*

$$\mathbb{E}_z \left[\frac{R_{t+1}^k(z) - R_t}{\eta_t R_{t+1}^k(z) + (1-\eta_t) R_t} \right] = 0 \tag{6}$$

Proof. See Appendix A. □

Lemma 1 shows that the young household consumes a fraction $1-\beta$ of its labor income net of transfers when young and invests the rest. Out of the β fraction it chooses to save, the household invests a fraction η_t in risky capital and $1-\eta_t$ in the safe bond. Equation (6) describes the optimal choice of portfolio weights as the solution to a portfolio choice problem seeking to maximize risk-adjusted returns. Using equations (4)-(6), we can express the optimal portfolio share of capital as:[1]

$$\eta_t = \frac{k_{t+1}}{k_{t+1} + b_{t+1}/R_t} = \mathbb{E}_z \left[\frac{R^k_{t+1}(z) k_{t+1}}{R^k_{t+1}(z) k_{t+1} + b_{t+1}} \right] \tag{7}$$

Another interpretation of η_t is that it is the share of capital in the portfolio which maximizes the risk-adjusted return on the portfolio. In order to see this, notice that the denominator of equation (6) is the return on a portfolio comprising of a fraction η of capital and $1-\eta$ of bonds. The portfolio choice function can then be described as

$$\max_{\eta \in [0,1]} \ln \left[\eta R^k(z) + (1-\eta) R_t \right]$$

The first-order necessary condition which characterizes the optimal choice of η to maximize the risk-adjusted return is given by equation (6). The following properties of the optimal portfolio choice are standard.

Lemma 2 (Portfolio Choice)**.** *The optimal portfolio choice satisfies the following:*

1. *The optimal choice of η_t depends negatively on R_t.*
2. *Compare two distributions of the return on capital $R^k(z)$, F and G where G is a mean-preserving spread of F. Then $\eta_F < \eta_G$*

Proof. See Hadar and Seo (1990). □

The Lemma above shows that agents demand a higher level of safe assets (lower η) if bonds are relatively cheap (low $1/R_t$) or when risk is high.

Firms At each date t, each old household operates a firm with a Cobb-Douglas production technology:

$$Y_t(z) = (z_t k_t)^{\alpha} (\ell_t(z))^{1-\alpha}$$

where k_t is the amount of capital that household i invested when young. z is the firm-specific productivity and is i.i.d across all firms with distribution $\ln z \sim N(-\sigma_t^2/2, \sigma_t^2)$. Given its productivity

[1] Notice that the Euler equation for capital can be written as:

$$\frac{1-\beta}{c^Y_t} = \beta \mathbb{E}_z \frac{R^k_{t+1}(z)}{c^O_{t+1}(z)}$$

Notice that $c^O_{t+1}(z) = R^k_{t+1}(z) k_{t+1} + b_{t+1}$. Using the fact that $c^Y_t = \frac{1-\beta}{\beta}(k_{t+1} + b_{t+1}/R_t)$ and multiplying both sides of the Euler equation by k_{t+1} yields the desired expression for η_t.

and capital, the firm hires labor in order to maximize profits:

$$R_t^k(z)k_t := \max_{\ell} (zk_t)^{\alpha} \ell_t^{1-\alpha} - \omega_t \ell_t$$

where ω_t denotes the real wage. Labor demand is given by:

$$\ell_t(z) = \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1}{\alpha}} zk_t \tag{8}$$

and we can write the return to capital as:

$$R_t^k(z) = \alpha \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1-\alpha}{\alpha}} z \tag{9}$$

Government At date t, the government issues non-defaultable nominally safe one period debt B_{t+1} at price $1/(1+i_t)$ and uses the proceeds to repay outstanding debt B_t and to disburse transfers $P_t T_t$ to the young:

$$\frac{1}{1+i_t} B_{t+1} = B_t + P_t T_t \tag{10}$$

The monetary authority sets nominal interest rates i_t according to some rule which we specify later.

Labor Market We begin by assuming that nominal wages can adjust freely to achieve full-employment.[2] Consequently, labor market clearing determines real wages:

$$l_t = 1 \text{ and } \omega_t = (1-\alpha)k_t^{\alpha} \tag{11}$$

Return on capital Given equilibrium wages (11), the return to investing in capital can be written as:

$$R_t^k(z) = \alpha z k_t^{\alpha-1} \tag{12}$$

Goods Market Clearing The aggregate resource constraint of this economy can be written as:

$$c_t^Y + \int_z c_t^O(z) dF_t(z) + k_{t+1} = \int_z (zk_t)^{\alpha} \ell_t(z)^{1-\alpha} dF_t(z) = k_t^{\alpha} \tag{13}$$

where $F_t(z)$ is the cdf of the log-normal distribution defined above. The LHS of the equation above is the sum of total consumption and investment in capital in period t while the RHS is the GDP.

[2] In section 4, we also consider the case in which nominal wages are sticky downwards but flexible upwards.

2.1 Characterizing Equilibrium

Definition 1 (Equilibrium)**.** *Given a sequence $\{B_{t+1}, i_t, T_t\}_{t=0}^{\infty}$ and initial conditions $\{B_0, k_0\}$, an equilibrium is a sequence $\{c_t^Y, c_t^O(z), k_{t+1}, l_t, \ell_t(z), R_t^k(z), P_t, W_t\}_{t=0}^{\infty}$ such that*

1. *$\{c_t^Y, c_t^O(z), k_{t+1}, B_{t+1}\}$ solves the household's problem for each cohort t, given prices $\{i_t, R_t^k(z), P_t, W_t\}$ and transfers $\{T_t\}$*
2. *$\{\ell_t(z), R_t^k(z)\}$ solve the firm's problem at each date t*
3. *the government budget constraints (10), labor market clearing (11) and goods market clearing (13) are satisfied.*

There are two key equations that help us describe the dynamics of the economy. The first of these equations, which we refer to as the aggregate supply of savings equation which can be written as:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta \left[(1-\alpha)k_t^{\alpha} + \frac{b_{t+1}}{R_t} - b_t\right] \tag{14}$$

In order to derive the relationship above, we used the equilibrium expression for labor income (11) and government budget constraint (10) to substitute out for transfers from equation (4). The LHS of (14) denotes the total savings in the economy at date t.

The other equation of interest concerns the demand for capital. In order to derive an expression for this equation, we start by solving for η_t which denotes the portfolio share of capital for young households. Lemma 3 below provides an expression for the optimal portfolio share of capital and describes it's properties.

Lemma 3 (Equilibrium Portfolio Shares)**.** *In equilibrium, the optimal portfolio choices of a young household satisfy:*

1. *The optimal portfolio share of capital can be written as:*

$$\eta_t \quad = \quad \mathbb{E}_z \left[\frac{\alpha z k_{t+1}^{\alpha}}{\alpha z k_{t+1}^{\alpha} + b_{t+1}}\right] \tag{15}$$

2. *The optimal portfolio share of capital is decreasing in σ, i.e. $\frac{\partial \eta_t}{\partial \sigma} < 0$*

Proof. See Appendix B. □

Equation (15) follows immediately from equation (7) with the expression for $R_{t+1}^k(z)$ plugged in. Thus, the equilibrium portfolio share of capital is the same as the expected value of the share of capital income of the old to their total income. Notice that equation (15) shows that the equilibrium portfolio share of capital only depends on capital and bonds only via debt to GDP. That is, we can write $\eta_t = \mathbb{E}_z\left[\alpha z/(\alpha z + \tilde{b}_{t+1})\right]$ where $\tilde{b}_{t+1}$ is defined as $\tilde{b}_{t+1} = b_{t+1}/k_{t+1}^{\alpha}$. In what follows, it will be convenient to work with $\tilde{b}$ instead of b as our measure of fiscal policy.

Throughout our analysis, we will refer to increases in σ as increases in risk. It is important to note that given our specification of $\ln z \sim N(-\sigma^2/2, \sigma^2)$, any increase in σ corresponds to a mean-preserving spread to the distribution of idiosyncratic productivity, leaving the average return on capital (12) unchanged. Lemma 3 above shows that an increase in risk in this sense reduces the equilibrium portfolio share of capital. Finally, using the expression for η_t, the demand for capital can be expressed as:

$$R_t \;=\; \frac{\mathbb{E}_z\left[\dfrac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\dfrac{1}{\alpha z + \tilde{b}_{t+1}}\right]} k_{t+1}^{\alpha-1} \tag{16}$$

In order to get some intuition regarding the demand for capital equation, notice that in equation (16), there is a spread between the return on capital and the safe return:

$$\frac{R_t}{\mathbb{E}_z R_{t+1}^k(z)} = \frac{\mathbb{E}_z\left[\dfrac{z}{\alpha z + \tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\dfrac{1}{\alpha z + \tilde{b}_{t+1}}\right]} < 1$$

where $\mathbb{E}_z R_{t+1}^k(z)$ is the expected return on capital and the expression on the RHS is the safety premium enjoyed by bonds.[3] Interestingly, notice that increasing $\tilde{b}_{t+1}$ reduces the safety premium by satiating the demand for safe assets. Finally, note that there is a declining relationship between capital and the safe real interest rate. Furthermore, Appendix C shows that an increase in σ increases the safety premium, widening the gap between the safe rate and the expected return on capital.

The intersection of the aggregate supply of savings (14) and the demand for capital (16) determines the equilibrium level of investment k_{t+1} and real interest rates R_t given today's capital stock and government debt policy. Using equations (14) and (16), we can describe the evolution of this

[3] Notice that since the spread can be written as:

$$\frac{R_t}{\mathbb{E}_z R_{t+1}^k(z)} = \frac{1}{\alpha}\frac{1 - \mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z + \tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]} = \frac{1}{\alpha}\left[\mathbb{E}_z\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right]$$

Next, from Jensen's inequality, we know that:

$$\frac{\partial\left(\frac{R_t}{\mathbb{R}_{t+1}^k(z)}\right)}{\partial \tilde{b}_{t+1}} = \frac{\mathbb{E}\left[\left(\frac{1}{\alpha z + \tilde{b}_{t+1}}\right)^2\right] - \left(\mathbb{E}\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]\right)^2}{\mathbb{E}\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]\mathbb{E}\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]} > 0$$

economy by the single equation:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) k_t^{\alpha} \tag{17}$$

Notice that the equation above looks a lot like the Solow model where the aggregate savings rate is given by

$$s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) = \frac{\beta(1 - \alpha - \widetilde{b}_t)}{\beta + (1-\beta)\mathbb{E}_t \left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}} \right]^{-1}}$$

Note that the aggregate savings rate is different from the private savings rate which is given by β.

Lemma 4 (Savings Function)**.** *The savings function is decreasing in all its arguments.*

It is straightforward to show that the savings rate is decreasing in both the current period and next period's debt to GDP ratio. Intuitively, higher government debt requires higher taxes on young saves reducing their disposable income and thus, the amount they save. A high government tomorrow, however crowds out investment in physical capital. In equilibrium, a higher $\tilde{b}_{t+1}$ requires that young households hold more bonds in their portfolio, thus reducing the amount they invest in capital. Finally, an increase in risk reduces the aggregate savings rate. In a riskier environment, young savers shift their portfolio away from riskier capital towards safe government debt reducing aggregate savings and hence investment.

2.2 Steady State

In steady state, the aggregate supply of savings (14) can be written as:

$$k^{1-\alpha} = \beta(1-\alpha) - \left[\frac{1-\beta}{R} + \beta\right] \tilde{b} \tag{18}$$

where $\tilde{b} = b/k^{\alpha}$ is the steady state debt to GDP ratio. We use the convention that quantities and prices without time sub-scripts denote their values in steady state. Equation (18) implies that government debt crowds out capital. Intuitively, for a fixed supply of savings, the higher government debt, the less savings remain to finance physical investment. In addition, if $R > 1$ in steady state, a higher debt increases the tax burden on young households, reducing the total supply of savings further. Also note that holding fixed debt to GDP, this equation describes an increasing relationship between steady state k and R. Higher interest rates make the same amount of debt cheaper for young savers, leaving ample funds available to invest in capital. If there is zero government debt outstanding, capital attains its highest attainable steady state level, which is invariant to interest rates. This relationship is captured in the upwards sloping curve in Figure(1).

The upward sloping curves in Figure 2a describes this relationship for a positive level of debt to GDP; the vertical line describes the relationship when debt is zero. Figure 2a describes this relationship for a positive level of debt to GDP; the vertical line describes the relationship when debt is zero.

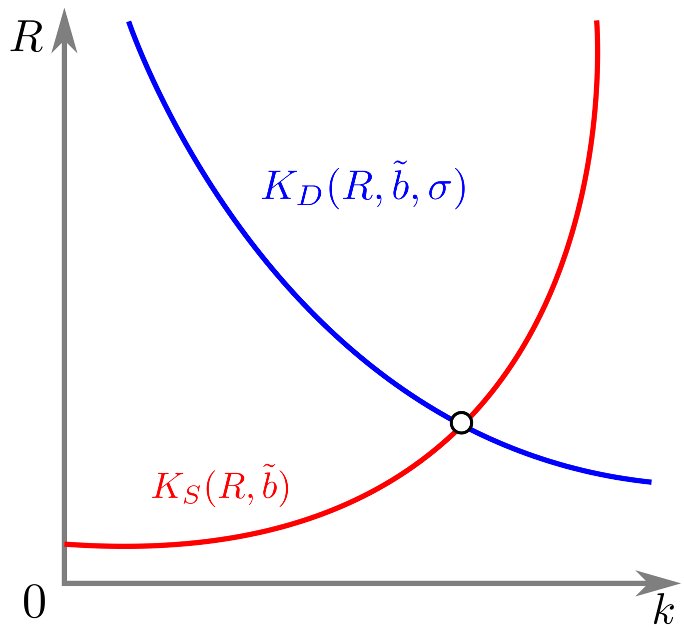


Figure 1. Steady state

On the other hand, the downward sloping curves depicts the demand for capital given by (16) evaluated at steady state $\tilde{b}$. A higher debt to GDP ratio shifts the curve up; an increase in risk shifts the curve down. The intersection of the two curves determines steady state capital and interest rates. The following Lemma solves explicitly for the steady state.

Lemma 5. *Given a steady state debt-to-GDP ratio, $\tilde{b} \in [0, 1-\alpha)$ and steady state risk σ, steady state capital and interest rates are given by:*

$$k\left(\tilde{b}, \sigma\right) = \left[\frac{\beta(1-\alpha-\tilde{b})}{\beta + (1-\beta)\mathbb{E}\left[\frac{\alpha z}{\alpha z+\tilde{b}}\right]^{-1}}\right]^{\frac{1}{1-\alpha}} \tag{19}$$

$$R\left(\tilde{b}, \sigma\right) = \frac{1}{1-\alpha-\tilde{b}}\left[\beta^{-1}\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}}\right]^{-1} - \tilde{b}\right] \tag{20}$$

3 Inspecting the Mechanism

Our analysis of the model is going to be centered around changes in idiosyncratic risk σ and the supply of safe assets. To this end, we start by describing how a permanent increase in risk affects allocations in both the the short run and the long run. We then investigate how changes in the level of government debt can influence the path the economy takes.

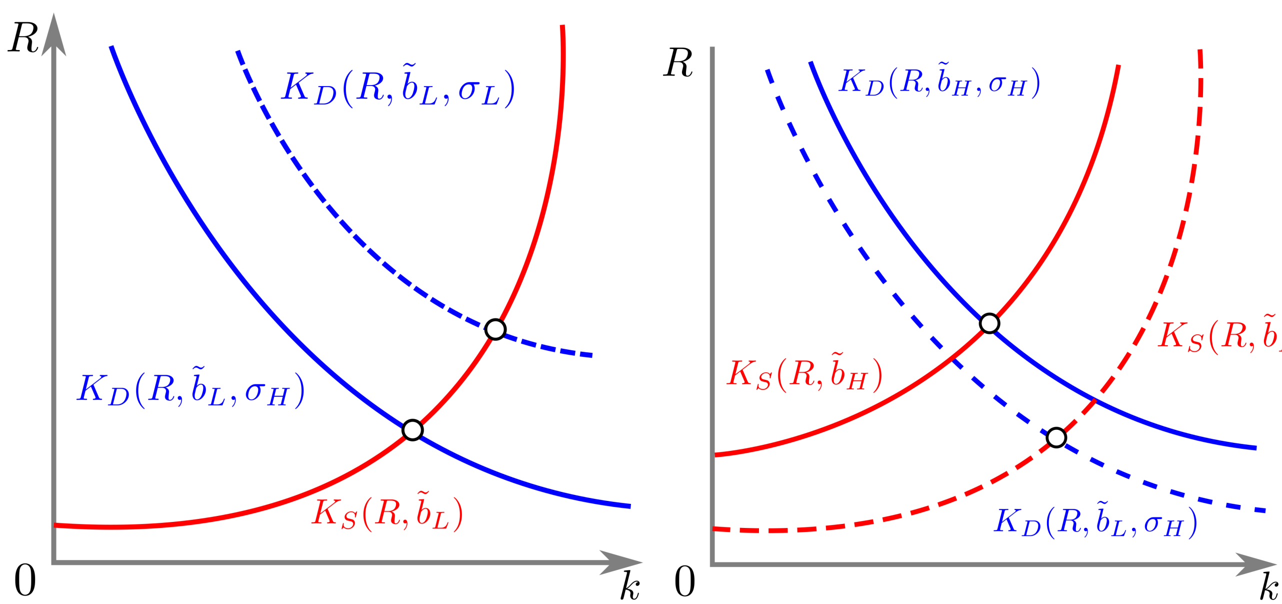
3.1 The effects of an increase in risk

Lemma 6. *For a given level of $\tilde{b}$, the steady state level of capital k and is weakly decreasing in σ while steady state real interest rate R is striclty decreasing in σ.*

The proof of the Lemma is a staightforward application of Jensen's inequality on equations (19) and (20). An increase in risk reduces the demand for capital, as households seek to hold safe bonds rather than risky capital. Given a fixed supply of safe assets, however, the price $1/R$ of these assets must rise in order to equilibrate demand and supply of safe assets. This is the easiest to see in the case where bonds are in zero net supply. In this case, using equation (20), the relationship between the real interest rate and σ can be written as:[4]

$$R = \frac{\alpha}{\beta(1-\alpha)} e^{-\sigma^2}$$

Facing higher prices of safe assets, young savers, who save a fixed fraction β of their total income, have less resources left over to invest in physical capital. Consequently, the aggregate saving rate and the capital stock fall. Figure 2a depicts this graphically. An increase in σ shifts the capital demand schedule to the left while leaving the aggreate supply of savings unchanged, reducing the steady state levels of capital and real interest rates. Importantly, a high enough σ can result in negative real interest rates in steady state, $R < 1$. For example, in the case with $\tilde{b} = 0$, $\sigma > \sqrt{\ln\left[\frac{\alpha}{\beta(1-\alpha)}\right]}$ results in a negative real interest rate.



(a) A permanent increase in σ **(b)** A permanent increase in $\tilde{b}$

Figure 2. Steady States

[4]Recall that if a random variable z is distributed log-normally with the underlying normal distribution $N(-\sigma^2/2, \sigma^2)$, then the mean of z^{-1} is given by e^{σ^2}.

3.2 The effects of an increase in safe assets

Next, we ask whether an increase in the supply of safe assets can mitigate the fall in real interest rates induced by an increase in risk. The following Lemma states that an increase in the debt to GDP ratio always increases interest rates. This crowds out investment, reducing the steady state capital stock.

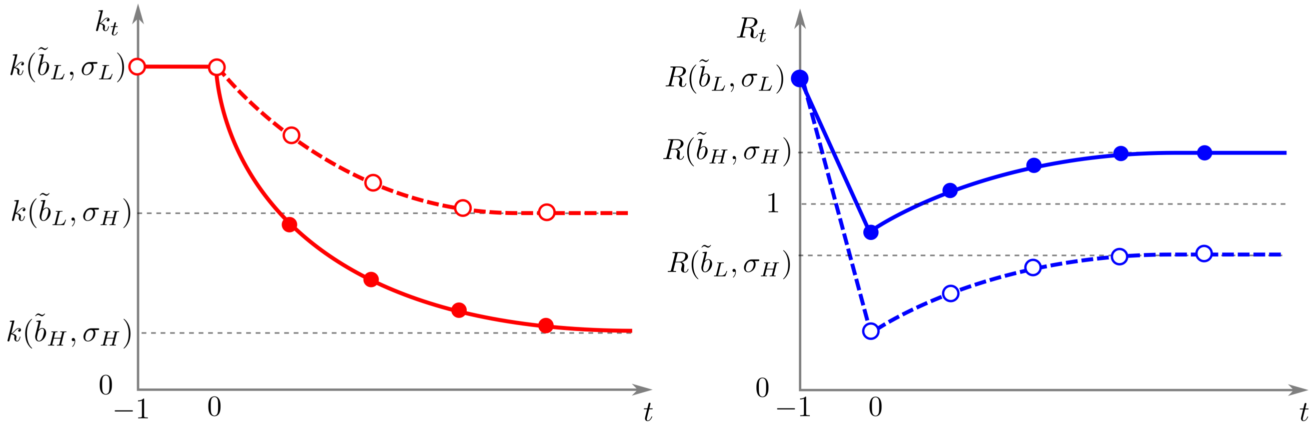
Lemma 7. *The steady state levels of capital k is strictly decreasing in $\tilde{b}$ while the steady state real interest rates R is strictly increasing in $\tilde{b}$.*

Figure 2b depicts the claim of Lemma 7 graphically. An increase in the supply of safe assets satiates the demand for safe assets and reduces the safety premium in equation (16). This makes young households willing to hold more capital for a given real interest rate, shifting the capital demand schedule to the right. However, a higher supply of governmnt debt diverts savings away from capital crowding out investment, shifting the aggregate supply of savings to the left. Overall, capital is unambigously lower and real interest rates are higher in the steady state with a higher supply of safe assets. It follows that in response to any increase in risk, a high enough increase in the debt to GDP ratio can always keep real interest rates positive. However, this comes at the cost of crowding out investment in physical capital.

3.3 Transitions

Having characterized how steady state interest rates and capital stocks depend on the level of idiosyncratic capital income risk and the quantity of outstanding safe assets, we now describe how permanent, unanticipated shocks to risk affect the transition of interest rates and capital stocks towards their new steady state levels. We also discuss how the level of government debt affects this transition path. We assume that the economy is initially in steady state at date 0 when it is hit by an unaticipated and permanent increase in σ from σ_L to σ_H.

It is convenient to first consider the case in which the government keeps the debt to GDP ratio $\tilde{b}_t = \tilde{b}_L$ constant and positive, adjusting the level of government debt as necessary in response to changes in the capital stock and the level of output. The solid lines in figure 3 illustrate the dynamics of capital and real interest rates under such a policy. Prior to date 0, capital and gross real interest rates are at their steady state levels, $k(\tilde{b}_L, \sigma_L)$, $R(\tilde{b}_L, \sigma_L)$. Following from Lemma 2, the permanent increase in σ decreases the portfolio share of risky capital η, and thus reduces the economy's aggregate savings rate $s(\tilde{b}_L, \tilde{b}_L, \sigma_H)$. With a lower aggregate savings rate, the economy gradually transitions towards a lower steady state capital stock, $k(\tilde{b}_L, \sigma_H)$. Real interest rates fall on impact, undershooting their new steady state level $R(\tilde{b}_L, \sigma_H)$, as the economy initially has a high level of capital (and thus a low marginal product of capital) but also has a high risk premium. As the capital stock gradually declines, both the expected return on risky capital and the safe real interest rate gradually transition to a somewhat higher level, although the real interest rate is still lower than in the low-risk steady state. If the increase in σ is sufficiently large, the long run real interest rate may be negative. Interestingly, even if this is not the case, interest rates may be negative during the transition to the new steady state.



(a) Capital

(b) Gross real interest rate

Figure 3. Dashed lines denote equilibrium without an increase in safe assets, $\tilde{b}_t = \tilde{b}_L \forall t$. Bold lines denote equilibrium with an increase in safe assets, $\tilde{b}_t = \tilde{b}_H$ for $t \geq 1$.

An increase in the supply of safe assets can mitigate the decline in real interest rates. The dashed lines in figure 3 show dynamics of capital and interest rates under a policy which sets $\tilde{b}_t = \tilde{b}_H > \tilde{b}_L$ for $t \geq 1$. The right panel shows that under this policy, real interest rates fall by a smaller amount on impact, and converge to a higher steady state level, $R_{ss}(\tilde{b}_H, \sigma_H)$, than they would in the absence of a change in the debt to GDP ratio. However, this comes at the cost of a sharper decline in the capital stock, as shown in the left panel. In equilibrium, investors must be induced to accommodate an increased supply of safe assets by alocating a larger share of their portfolio to these assets, and a smaller share to risky capital. This immediately reduces the aggregate savings rate. In addition, starting at date 1, young savers must pay higher taxes to finance the increase in debt, further reducing aggregate savings.

3.4 Welfare

Our analysis above showed that an increase in risk can reduce equilibrium real interest rates, even pushing them below zero. We also showed that a sufficiently large increase in the supply of safe assets can mitigate this decline, keeping real interest rates above zero in the new steady state with lower capital. Just because policy can do this, however, does not mean that it should. As we show below, a planner who wishes to maximize steady state welfare would choose not to create additional safe assets in order to counter the negative real interest rates.

We consider a constrained social planner who seeks to maximize steady state welfare, subject to the implementability constraint (19):

$$\max_{k,\tilde{b}} \quad (1-\beta)\ln\left[(1-\alpha-\tilde{b})k^{\alpha}-k\right]+\beta\mathbb{E}_z\ln\left[(\alpha z+\tilde{b})k^{\alpha}\right] \tag{21}$$

subject to

$$k \;=\; \left[\frac{\beta(1-\alpha-\tilde{b})}{\beta+(1-\beta)\mathbb{E}\left[\frac{\alpha z}{\alpha z+\tilde{b}}\right]^{-1}}\right]^{\frac{1}{1-\alpha}} \tag{22}$$

In order to understand why the planner chooses not to increase the supply of safe assets in response to a increase in risk, it is important to understand the trade-off he faces. In equilibrium, an increase in government debt is essentially a forced transfer from the young to the old.[5] If the old accumulate financial claims to consume a greater share of GDP, the young must consume a smaller share. Whether it is desirable to redistribute from the young to the old depends on the planner's preferences for redistribution, and also the amount of risk faced by the old. The higher the level of risk, the higher the expected marginal utility of the average old individual, and thus, higher the benefit from increasing safe assets. In addition to these direct effects, safe asset production also has an indirect effect, crowding out physical capital investment. This harms both young households, who earn lower wages, and old households, who earn less capital income.

Even when risk is high enough to make real interest rates negative, it is not optimal to produce any safe assets. In this sense, negative real interest rates are not a signal of a *shortage of safe assets* per se. In the absence of crowding out, the planner would like to create just enough safe assets that the real interest rate is zero, as the following Lemma states.

Lemma 8. *Consider the unconstrained planner's problem in which the planner maximizes* (21), *ignoring constraint* (22)*. The solution to this unconstrained problem is unique, with either $R \geq 1$ and $\tilde{b}=0$, or $R=1$ and $\tilde{b}>0$.*

Since safe asset creation crowds out physical capital investment, however, in the constrained problem it is optimal to curtail safe asset production, which means that the real interest rate can be negative in the optimal steady state. It is important to note that our results are not driven by dynamic inefficiency: our economy is always dynamically efficient.

Of course, for a sufficiently high level of risk, increasing government debt does increase social welfare but even in this case, it is never desirable to produce enough safe assets that the real interest rate becomes positive. The proposition below summarizes the optimal response of the planner depending on the level of riskiness in the economy.

[5] Recall that we have assumed that debt is financed by lump sum taxes on the old. If in addition there were lump sum taxes on the old, our results would be unchanged, provided that b_t is redefined to be government debt net of taxes on the old. Allowing the government to levy taxes conditional on the realization of an old household's idiosyncratic productivity would (trivially) change our results.

Proposition 1. *There exist $\underline{\sigma}$, $\overline{\sigma}$ such that the solution to the constrained problem has the following properties:*

1. *If risk is low enough, i.e. $\sigma \leq \underline{\sigma}$, then the optimal choices of the planner satisfy $\widetilde{b} = 0$, $R \geq 1$.*

2. *If risk is in the intermediate range $\sigma \in (\underline{\sigma}, \overline{\sigma}]$, the planner still does not choose to create safe assets and his optimal choices satisfy $\widetilde{b} = 0$, $R < 1$.*

3. *If risk is very high, $\sigma > \overline{\sigma}$, then the planner would optimally choose to create some safe assets but still not enough such that real interest rates are positive. In this case, the optimal choices satisfy $\widetilde{b} > 0$, $R < 1$.*

It is also important to realize that the result is not driven by our assumption that the planner maximizes steady state welfare. It is possible to show that irrespective of the Pareto weights the planner puts on the welfare of young and old households respectively, it is never optimal to prevent real interest rates from becoming negative.

In one sense, these results are not surprising. In the absence of nominal rigidities, negative real interest rates are of no particular significance. However, what these results do highlight is that the potential for safe assets to crowd out investment in physical capital is particularly strong in this world.

4 Nominal rigidities

We have seen that in an economy without nominal distortions, even if increases in risk cause interest rates to become negative, it is not necessarily desirable for fiscal policy to reverse this. However, one important motivation for raising the equilibrium real interest rate which we have not discussed so far is that the zero lower bound on nominal rates may prevent the monetary authority from lowering real rates sufficiently to preserve full employment. We now introduce nominal rigidities, by assuming that nominal wages are sticky downwards but flexible upwards. In the spirit of Eggertsson and Mehrotra (2014) and Schmitt-Grohé and Uribe (2016), we assume that workers are unwilling to work for wages below a wage norm given by:

$$\ln \widetilde{W}_t = (1 - \gamma) \ln W_{t-1} + \gamma \ln P_t \omega_t^* \tag{23}$$

where $\omega_t^* := (1 - \alpha) k_t^\alpha$ is the real wage that delivers full employment at date t. With $\gamma = 0$, nominal wages are rigid downwards; with $\gamma = 1$, wages are fully flexible. Formally, we assume that the actual nominal wage is given by:

$$W_t = \max \left\{ \widetilde{W}_t, P_t \omega_t^* \right\} \tag{24}$$

Using equations (23) and (24), the evolution of real wages can be described by the following equation:

$$\frac{W_t}{P_t} = \max\left\{ \left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\gamma} (\Pi_t)^{\gamma-1} (\omega_t^*)^{\gamma}, \omega_t^* \right\} \tag{25}$$

where $\Pi_t = P_t/P_{t-1}$ denotes the gross rate of inflation. When real wages W_t/P_t are higher than ω_t^*, aggregate labor demand is less than the endowment of labor resulting in unemployment: $\int_0^1 \ell_{i,t} di < 1$. Whenever there is unemployment, we assume that households are proportionally rationed, so that each young household supplies the same amount of labor implying that labor supply by each young household is given by $l_t = \int_z \ell_t(z) dF_t(z)$.

Regardless of whether there is unemployment or not, since firms are always on their labor demand curve in equilibrium, the actual real wage satisfies $W_t/P_t = (1-\alpha)k_t^{\alpha} l_t^{-\alpha}$. Then using equations (25) and the definition of the market clearing real wage ($\omega_t^* = (1-\alpha)k_t^{\alpha}$), we can derive a relation between changes in employment and the rate of inflation:

$$l_t = \min\left\{ \left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma} l_{t-1}^{1-\gamma} \Pi_t^{\frac{1-\gamma}{\alpha}}, 1 \right\} \tag{26}$$

Equation (26) can be interpreted as a *wage Phillips curve* or the aggregate supply relationship and reveals that the labor market can be in one of two regimes. When last period's nominal wage lies below the wage that would clear markets today, and full employment requires nominal wages today to rise, wages jump to their market clearing level and and there is full employment, $l_t = 1$. However, when last period's wage lies above today's market clearing wage, and full employment requires wages to fall, the wage norm binds not allowing nominal wages to adjust fully. Wages only partially fall towards their market clearing level, resulting in unemployment. In this unemployment regime, employment will be higher, all else equal, if employment was higher in the previous period (as this signals that wages were not too high and do not have too far to fall); if capital is higher today than last period (as this means that the market clearing wage is higher today than last period); or, most importantly, if inflation is higher in the current period.

Finally, in order to close the model with nominal rigidities, we need to specify a monetary policy rule. We assume that monetary policy sets nominal interest rates according to the following inflation targeting rule subject to the zero lower bound (ZLB):

$$1 + i_t = R_t \Pi_{t+1} = \max\left\{ R_t^* \Pi_t^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_y}, 1 \right\} \tag{27}$$

where

$$R_t^* = \frac{\mathbb{E}\left[\dfrac{\alpha z}{\alpha z + \widetilde{b}_{t+1}}\right]}{\mathbb{E}\left[\dfrac{1}{\alpha z + \widetilde{b}_{t+1}}\right]} s\left(\tilde{b}_t, \tilde{b}_{t+1}, \sigma\right)^{\alpha-1} k_t^{\alpha(\alpha-1)} \tag{28}$$

R_t^* is the real interest rate that would prevail in the economy without wage rigidities with the same level of capital. Similarly, $Y_t^* = k_t^\alpha$ is the level of output in the corresponding economy with no nominal rigidities. As is standard, we assume that monetary policy is *active* which requires that $\phi_\pi > 1$.[6]

The remaining equations governing the dynamics of the economy with nominal rigidities are similar to those in flexible wage economy with the exception that the economy might not always be at full-employment. Thus, the aggregate supply of saving is given by:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[(1-\alpha)k_t^\alpha l_t^{1-\alpha} + \frac{b_{t+1}}{R_t} - b_t\right] \tag{29}$$

Notice that the aggregate supply of savings relation (14), described in the flexible wage economy, is the same as (29) evaluated at full employment $l_t = 1$. Equation (29) shows that unemployment today ($l_t < 1$) reduces the labor income of the young, reducing their savings and therefore investment in capital and demand for bonds. In a similar fashion, the optimal portfolio decisions of savers can be written as:

$$\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^\alpha l_{t+1}^{1-\alpha}}{\alpha z k_{t+1}^\alpha l_{t+1}^{1-\alpha} + b_{t+1}}\right] \tag{30}$$

As before the equilibrium portfolio share of capital depends on the expected ratio of capital income to total income of the old. Unemployment reduces the marginal product of capital and thus increases the equilibrium portfolio share of safe assets (reduces η_t) for a given k_{t+1} and b_{t+1}.

As before, we can use the expression (30) to derive the demand for capital:

$$\frac{R_t}{\mathbb{E}_z R_{t+1}^k(z)} \quad = \quad \frac{\mathbb{E}_z\left[\dfrac{z}{\alpha z + \widetilde{b}_{t+1} l_{t+1}^{\alpha-1}}\right]}{\mathbb{E}_z\left[\dfrac{1}{\alpha z + \widetilde{b}_{t+1} l_{t+1}^{\alpha-1}}\right]} \tag{31}$$

where the average marginal product of capital is now $\mathbb{E}_z R_{t+1}^k(z) = \alpha k_t^{\alpha-1} l_t^{1-\alpha}$. Notice that the demand for capital in the flexible wage economy (16) is simply equation (31) evaluated at $l_{t+1} = 1$. The possibility of unemployment at date $t+1$ affects the demand for capital in two ways. First,

[6] We allow for $\phi_y \geq 0$.

a value of l_{t+1} below full employment lowers the average marginal product of capital, reducing the demand for capital for a given R_t. However, a lower l_{t+1} also increases the portfolio share of safe assets, narrowing the spread between the safe rate on bonds and the risky return on physical capital. Intuitively, the consumption of household contains a risky component (capital income) and a safe component (bonds). Thus, the risk premium that young households demand depends on the share of risky to total income when old; when risky income is a large share of total income, the covariance of the return on capital and consumption (when old) will be large. A higher unemployment in the future lowers the risky share of income, leaving old households less exposed to risk, causing them to demand a lower risk premium.

Overall, the dynamics of the economy with nominal rigidities are described by equations (26)-(31).

4.1 How risk leads to stagnation

In Section 3, we have already seen that a permanent increase in risk can lower the equilibrium stock of capital permanently, causing a modest decline in output. This increase in risk also pushes down the safe real interest rate, potentially below zero. In the absence of nominal rigidities, negative real interest rates are not a cause for concern; while an increase in the supply of safe assets can return the real rate to positive territory, there is no reason to do so. However, in the presence of nominal rigidities and a lower bound on nominal rates, permanently negative real interest rates may be incompatible with full employment as we now describe. In such a situation, with monetary policy constrained, an increase in safe assets may be unavoidable if full employment is to be restored.

Our experiment is the same as in Section 3. At date 0, there is a permanent unanticipated increase in σ from σ_L to $\sigma_H > \sigma_L$. As before, we assume that σ_L, σ_H are such that the associated real interest rates in the flexible-wage steady states are, respectively, positive and negative, given the supply of safe assets $\tilde{b}_L$. For now, we assume that fiscal policy keeps $\tilde{b}$ constant. Recall that $\tilde{b}_t = b_t/k_t^\alpha$ is the ratio of debt to the the level of GDP which would prevail under full-employment.[7]

At date 0, following Lemma 2 the increase in risk makes young savers want to reallocate their portfolios away from increasingly risky capital, towards safe government debt. Given that supply of safe assets, $\tilde{b}$ is fixed, this excess demand for bonds necessitates a fall in the real return on bonds in order to equilibrate the bond market. In the absence of inflation, this requires reduction in the nominal interest rate. However, the zero lower bound (ZLB) on nominal interest rates prevents such an adjustment. Consequently, the real return on holding safe assets is *too high* thus, lowering the demand for investment in capital and thus, the price of the final output (which is the price of both consumption and capital). With sticky nominal wages, the fall in price is only partially met resulting in higher real wages and consequently lower labor demand by firms. Lower labor demand

[7]Notice that the definition of $\tilde{b}$ in the model with nominal rigidities is slightly different than in Section 3 because of the possibility that the economy might not be at full employment. Thus, in the case with nominal rigidities, $\tilde{b}$ might be smaller than the ratio of debt to actual GDP.

by firms results in unemployment of labor services in the economy. Overall, total output and labor income decline making both young and old households poorer. The young households, now poorer, have a lower desire to save and hence reduce their demand for bonds, thus equilibrating the bond market but also reduces investment in capital. This is only the beginning of a *risk-induced recession*.

It gets worse. In the subsequent period, the aggregate capital stock in the economy is lower, thus reducing the marginal product of labor and hence demand for labor. In addition, nominal wages are not free to adjust downwards and continue to stay above their market clearing level. This further reduces the demand for labor and results in persistent unemployment. Since nominal wages are slow to adjust, young households also anticipate that real wages will remain above the level consistent with full employment in the future. Thus, anticipating a lower marginal product of capital and hence a lower average return on capital, young households have even less incentive to invest in capital rather than the attractive alternative of safe government debt. Furthermore, with risk permanently high, young households continue to demand more safe assets. Given that fiscal policy holds $\tilde{b}$ fixed, this creates an excess demand for bonds which requires negative real rates for the bond market to clear. Again the ZLB binding prevents this adjustment and causes a further decline in investment. Unemployment remains high in subsequent periods; the excess demand for safe assets is permanent, and so the economy needs permanently lower income in order to equate the demand and supply of safe assets.

The dashed lines in figure 4 illustrate dynamics in the absence of an increase in safe assets. The upper left panel (Figure 4a) depicts the evolution of the capital stock. The increase in risk, and the associated fall in employment, cause a permanent decline in the aggregate saving rate. This causes capital to gradually decline to a new, lower steady state level. The top right panel (Figure 4b) shows the trajectory of the real interest rate. Real rates fall on impact, as the spread between the safe rate and the expected marginal product of capital increases. As the capital stock gradually declines, the expected marginal product of capital rises while the spread remains wider, leading the real rate to increase to its new steady state level. The bottom left panel (Figure 4c) displays the behavior of employment, which falls immediately to its steady state level following the increase in risk. Finally, the bottom right panel (Figure 4d) shows the dynamics of inflation. The collapse in demand at date zero causes a large fall in prices, pushing up real wages and creating unemployment. Inflation then recovers somewhat before gradually declining to its new steady state level. Intuitively, this economy requires relatively lower interest rates early on in the transition to a new steady state, as the capital stock remains high and the marginal product of capital remains low. With the nominal rate of interest stuck at zero, the only way to create a temporarily low real rate is to have a relatively high rate of inflation. As the capital stock gradually declines, the real interest rate rises somewhat, and the rate of inflation falls further.

In the case illustrated in Figure 4, inflation remains negative and the real return on the safe asset remains positive throughout the transition. This need not always be the case. If the economy is hit with a large enough shock, the real return on bonds may actually be negative at date zero,

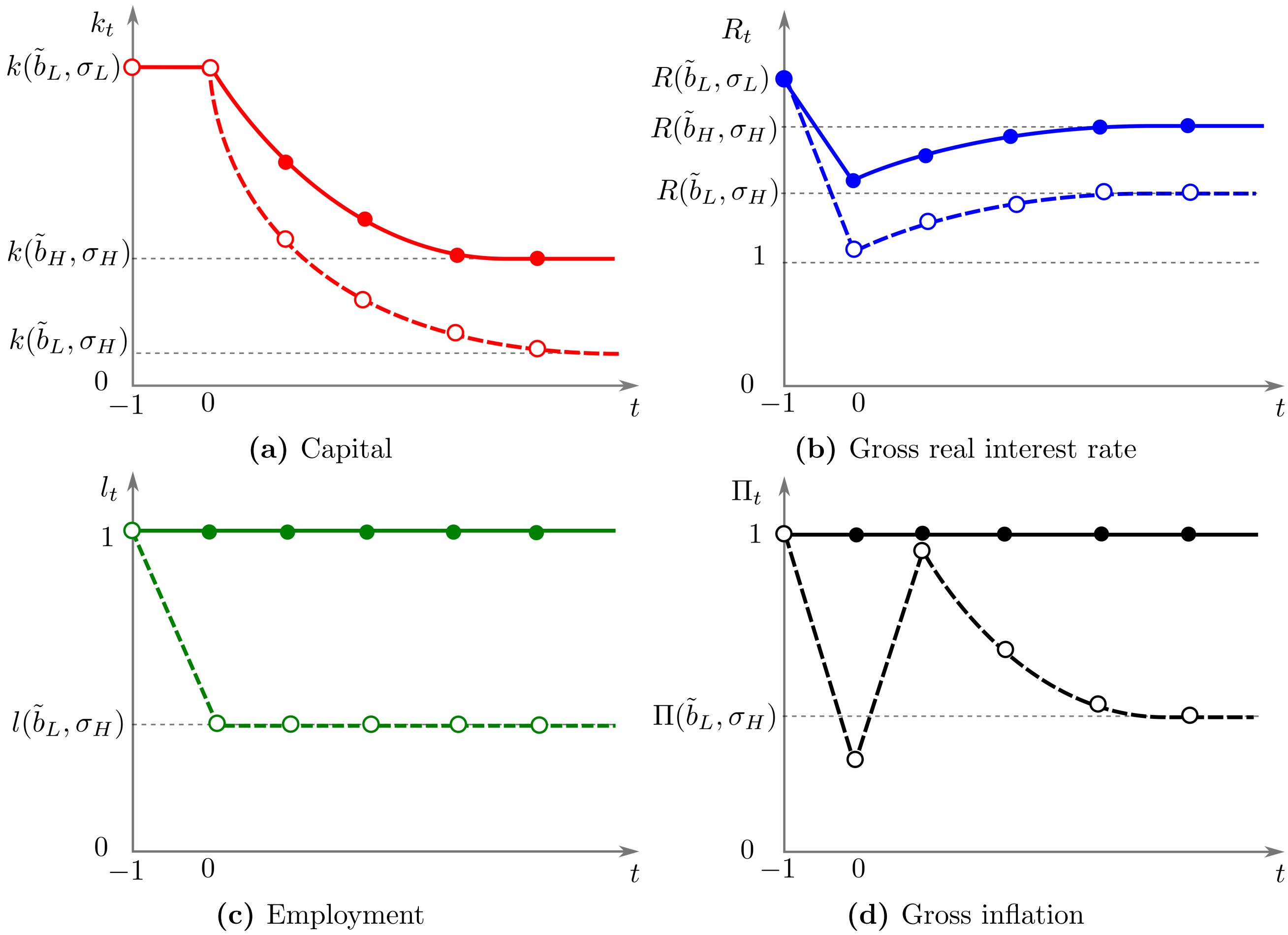


Figure 4. Dashed lines denote equilibrium without an increase in safe assets, $\widetilde{b}_t = \widetilde{b}_L \forall t$. Bold lines denote equilibrium with an increase in safe assets, $\widetilde{b}_t = \widetilde{b}_H$ for $t \geq 1$.

as the economy's capital stock is far above its new steady state level. This in turn requires positive inflation in the short run, even though the economy will eventually arrive at a deflationary steady state. Even though the monetary policy rule we have specified rules out positive inflation in the long run, it may be consistent with keeping rates at zero even in the presence of short run inflation. Recall that the nominal rate of interest depends on R_t^*, the real interest rate that would prevail in the absence of nominal rigidities, and on the shortfall of output relative to its flexible price level. Both these terms will be negative early in the transition process. Whether the monetary authority is willing to keep rates at zero will depend on the weight it attaches to inflation (ϕ_π) relative to the weights on R_t^* and the output gap. If the weight on inflation is too high, the monetary authority might be unwilling to keep rates at zero early on in the transition. In this case, no equilibrium exists given te configuration of fiscal amnd monetary policy that we have specified. The economy desperately requires at least a few periods of negative real rates to smooth the transition to the new steady state, since capital is high in the short run, depressing interest rates even beyond the effect of the increase in risk. A monetary rule which is unable to accommodate temporarily negative real interest rates cannot even engineer a transition to a steady state with inflation and unemployment.

Instead, employment spirals towards zero eventually leaving the government unable to meet its fiscal obligations: Either fiscal or monetary policy must adjust. Even if the monetary rule does allow for temporarily positive inflation, however, all is not well. Instead of imploding, it slumps into a steady state characterized by permanently higher unemployment, low investment and deflation.

To understand how these dire conditions can linger permanently, it is useful to revisit our analysis of the steady states in Section 3. We start by analyzing the differences in the supply side of the two economies. Unlike the flexible wage economy, which was always at full employment, in the presence of nominal wage rigidities, the labor market can be in one of two regimes. Notice that in steady state, equation (26) can be written as:

$$l = \min\left\{\Pi^{\frac{1-\gamma}{\alpha\gamma}}, 1\right\} \tag{32}$$

When inflation is nonnegative ($\Pi \geq 1$), wages are effectively flexible, and we have full employment, $l = 1$. When there is deflation ($\Pi < 1$), in order for real wages to be constant in steady state, nominal wages must be falling. Wages cannot be at their market clearing level, since the nominal wage that would have cleared markets in the last period no longer clears markets today, now that prices have fallen. In fact, equation (32) defines an increasing relationship between inflation and employment in this regime, which can be thought of as a long-run Phillips curve. The degree of wage flexibility γ determines the slope of the Phillips curve in the unemployment regime. When $\gamma = 1$ (perfect flexibility), the Phillips curve is vertical at full employment. At the other extreme, when $\gamma = 0$ (perfect downward nominal wage rigidity), the Phillips curve is inverse-L shaped and is horizontal at zero inflation ($\Pi = 1$). Thus, with $\gamma < 1$, in the deflation regime, the inflation rate affects real allocations in the long run.

Recall that we neglected to specify a monetary policy rule in the model with no nominal rigidities as the classical dichotomy holds in that case. However, as we have just seen, in the presence of nominal rigidities, the classical dichotomy might be violated even in the long run. Thus, the choice of monetary policy affects real allocations. Evaluating the monetary policy rule (27) in steady state, we have:

$$R = \max\left\{R^*\Pi^{\phi_\pi - 1} l^{(1-\alpha)\phi_y}, \Pi^{-1}\right\}$$

where R^* refers to the real interest rate that would prevail in the corresponding economy with no nominal rigidities as defined in equation (28), evaluated at steady state. Like the long-run Phillips curve, this rule defines two steady state regimes, owing to the lower bound on nominal interest rates. When steady state inflation is sufficiently large, the lower bound does not bind, and the gross real interest rate equals $R^*\Pi^{\phi_\pi - 1}$. When inflation is sufficiently low, the lower bound binds, and given a zero nominal rate, the Fisher equation defines the real interest rate as the inverse of the inflation rate, $R = \Pi^{-1}$. For ease of exposition, we consider the limiting case of $\phi_\pi \to \infty$ in the following discussion.[8] In the limiting case, the two regimes of the Phillips curve and the monetary policy rule necessarily coincide in steady state: we either have $l = 1$ or $i = 0$ and $R = \Pi^{-1}$. Combining the monetary policy rule and the Phillips curve, we have the following set-valued map *'LM'* (Labor markets and Monetary policy) relation:

$$R = \begin{cases} l^{-\frac{\alpha\gamma}{1-\gamma}} & \text{if } l < 1 \\ r & \text{for any } r \geq 1 \text{ if } l = 1 \end{cases} \tag{33}$$

The red curve in the top panels of Figure 5 depicts this LM relation. The LM curve is vertical at full employment. At full employment, inflation must be nonnegative, given the Phillips curve; thus, given our monetary policy rule, it must be zero. Zero inflation is possible provided that real interest rates are positive, so that the nominal interest rate does not violate the zero lower bound. When there is less than full employment, the LM curve is backward bending. The Phillips curve tells us that steady state unemployment must be accompanied by deflation. The monetary policy rule only tolerates deflation when it is forced to do so by the zero lower bound. Consequently, the ZLB must bind in this region. Thus lower employment generates more deflation, which, given a fixed nominal rate, implies a higher real interest rate.

The remaining ingredient to complete the characterization of steady state is the young households' investment and savings decisions. Evaluating (29) and (31) at steady state, we have

$$k^{1-\alpha} = \beta(1-\alpha)l^{1-\alpha} - \left[\beta + \frac{1-\beta}{R}\right]\tilde{b} \tag{34}$$

$$R = \frac{\mathbb{E}\left[\dfrac{\alpha z}{\alpha z + \tilde{b}l^{\alpha-1}}\right]}{\mathbb{E}\left[\dfrac{1}{\alpha z + \tilde{b}l^{\alpha-1}}\right]}\left(\frac{k}{l}\right)^{\alpha-1} \tag{35}$$

We can solve explicitly to get k and R as a function of steady state employment l. In fact, these are the same functions defined in Lemma 4:

$$k = k\left(\tilde{b}l^{\alpha-1}, \sigma\right) l \tag{36}$$

$$R = R\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{37}$$

Recall from Lemma 5 that $k(\cdot, \sigma)$ is decreasing in its first argument, and $R(\cdot, \sigma)$ is increasing in its first argument. Consequently, (36) defines an increasing relationship between the capital stock and employment, depicted in the dashed black line on the lower panel of Figure 5. A higher rate of employment raises labor income, increasing savings and the steady state capital stock. Similarly,

[8] Qualitatively, the analysis of steady states does not depend on the magnitude of ϕ_π. Recall that we saw earlier that for a high enough ϕ_π there may be no equilibrium path by which the economy can converge to such a steady state.

(37) defines a decreasing relationship between the interest rate and employment, depicted in the dashed blue curve on the upper panels of Figure 5. A higher rate of employment increases the steady state capital stock and investment. In order to induce households to save more in the form of risky capital, rather than in safe government debt, the rate of return on safe debt (that is, the real interest rate) must fall. In this sense, the two blue curves shows the relation between employment and the rate of interest which must be maintained to make saving equal to investment in steady state. It is therefore convenient to refer to it as an IS curve, following Hicks (1937). The dashed-blue curve denotes the IS curve when risk is relatively low while the solid-blue curve denotes the same relationship with a high σ.

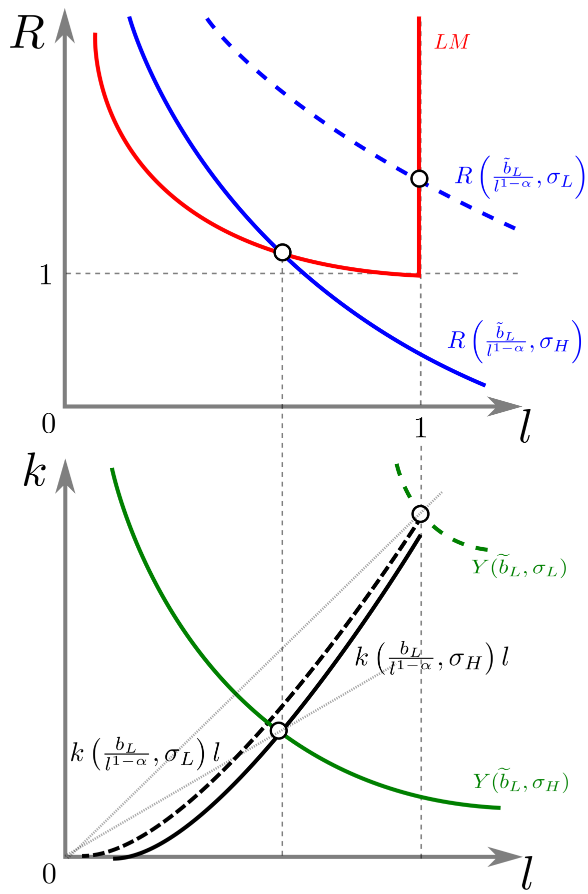


Figure 5. A permanent increase in σ keeping $\tilde{b}$ fixed

The intersection of the IS and LM curves determines the steady state real interest rate and level of employment. The dashed line in Figure 5 (with $\sigma = \sigma_L$) intersects the full employment portion of the LM curve at a positive real interest rate. The lower panel shows that this high rate of employment generates a high steady state capital stock. The green curves on the lower panel indicate isoquants of the aggregate production function, $Y = k^\alpha l^{1-\alpha}$. With a high rate of employment and a high capital stock, the level of output is relatively high in this low risk steady state, shown by the dashed-green higher isoquant.

The solid lines in Figure 5 shows the case in which risk has risen to a higher level σ_H. This shifts the IS curve to the left, as savers shift their portfolios away from increasingly risky capital towards safe debt, so that it would take a lower return on bonds to push them back towards capital. Indeed, the IS curve intersects the dashed vertical full employment line at a negative real interest rate, indicating that this economy requires negative rates to sustain full employment. Given the constraints on monetary policy, however, the LM curve does not permit negative real rates. Instead, the zero lower bound binds, and the IS and LM intersect at a level of employment below 1. This unemployment in turn generates persistent deflation, raising real interest rates with the nominal rate stuck at zero. The economy enters a stagnant steady state. The stagnant level of employment has consequences for the steady state capital stock and rate of investment. Lower employment implies lower income for young savers, and less investment, implying a smaller steady state capital stock, as the solid black line in the lower panel in Figure 5 indicates. With a decline in both capital and employment, the economy falls to a much lower level of output, indicated by a lower isoquant in the bottom panel. In particular, the increase in risk is associated with a lower capital labor ratio as indicated by the diagonal gray lines passing through the the origin in the lower panel.

At this point, it is important to note a difference from standard New Keynesian models of liquidity traps. In standard New Keynesian models with no physical storage technology such as capital, a large enough increased supply of savings pushes the economy to the ZLB. With real interest rates constrained by the ZLB to be *too high* output is the variable which must adjust to equilibrate the supply and demand of savings. However, if a physical storage technology such as capital was available, the higher desire of households to save can be accommodated by an increase in investment - leading to a boom instead of a recession. These scenarios are different from our liquidity trap because the shock we consider - an increase in risk - does not lead to an overall increase in the the level of desired savings relative to consumption. Instead, it shifts the desired composition of savings by increasing the demand for safe assets relative to capital. Thus, even though physical storage technology is available capital is available, an increase in the demand for *safe* assets can cause a recession, and in fact a permanent investment slump.

It is also worth noting that in our model, stagnation is accompanied by a *higher* expected marginal product of capital, since the capital-labor ratio falls. Gomme et al. (2015) present evidence that while the return on government debt has remained at a low level following the financial crisis, the real return on productive capital has rebounded, with the after-tax return on business capital at its highest level over the past three decades. They interpret this as evidence against the variants of the secular stagnation hypothesis which emphasize a scarcity of investment opportunities. It is, however, entirely consistent with our risk-based view of stagnation. Higher risk may deter investment even though the *average* return on capital remains high. Furthermore, through the lens of our model, an increase in σ would be consistent with a decline in the safe rate and a larger risk premium, as documented empirically by Duarte and Rosa (2015) and more recently Caballero et al. (2017).

4.2 How safe assets can preserve full employment

The previous section showed that in an economy with nominal rigidities, an increase in risk can lead to undesirable outcomes such as permanently high unemployment and low investment. The essential problem is that when the return on capital is very risky, the only way to sustain investment in capital is to have the real return on safe assets fall below zero. The ZLB prevents this adjustment leading to lower permanently lower economic activity.

This raises the question as to whether there is a way out of this conundrum? We have already seen that in an economy without nominal wage rigidities, an increase in government debt can offset an increase in idiosyncratic risk and keep real interest rates positive. In the absence of nominal rigidities, there is no reason to be concerned about real interest rates falling below a particular threshold. However, we have just seen that a negative natural rate R^* causes problem in the presence of nominal rigidities. It is therefore natural to ask whether issuing government debt can redress this situation by raising R^*.

The answer is a qualified yes. Suppose that in response to the increase in idiosyncratic income risk, the fiscal authority raises $\widetilde{b}$ from $\widetilde{b}_L$ to a higher level $\widetilde{b}_H$. The solid lines in Figure 4 describe the transition following an increase in the supply of safe assets starting at date 1. As the top right panel shows (Figure 4b), this increase in the supply of safe assets accommodates increased demand, equilibrating asset markets without the need for a fall in incomes. Real rates fall somewhat on impact, as the economy suddenly finds itself with a high level of capital relative to the new steady state. However, as capital gradually falls towards the new steady state (as shown in the top left panel), real interest rates rise again to their new steady state level. Figure 6a shows that the higher level of safe assets $\tilde{b}_H$ raises the natural rate R^* above 0 (solid curve in Figure 6b) unshackling monetary policy from the ZLB, allowing it to restore full employment (see equation (33)). The solid line is the path of nominal interest rates with $\tilde{b}_H$ while the dashed line depicts the path with a lower level. Notice that with a higher $\tilde{b}_H$, the nominal rates are not constrained by the ZLB while monetary policy is constrained by the ZLB in the case with $\tilde{b}_L$. The bottom-left panel of figure 4c shows that with a higher supply of safe assets $l_t = 1$ throughout (solid green line) while it is below full employment with a lower supply of safe assets $\tilde{b}_L$.

However, it is important to note that, as in the flexible wage economy, an increase in safe assets still crowds out investment in physical capital - in the precise sense that investment is lower than it would be if there were no increase in safe assets *and if wages were fully flexible*. But in the presence of nominal rigidities, increasing safe assets ‘crowds in’ investment, in that investment is higher than it would be if there were no increase in safe assets, given that nominal wages are not fully flexible. Producing safe assets reduces the capital-labor ratio, thus reducing real wages and labor productivity. But this is more than compensated by an increase in employment.

A permanently higher level of safe assets $\tilde{b}_H$ does more than just smooth transitions; it also affects long run outcomes inducing a steady state characterized by full employment. The dashed

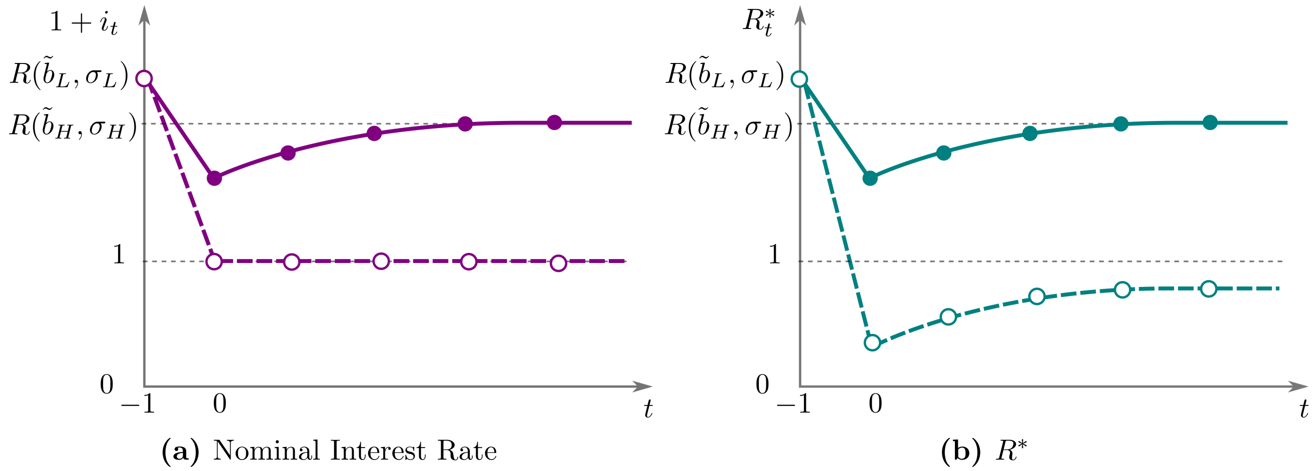


Figure 6. Dashed lines denote equilibrium without an increase in safe assets, $\widetilde{b}_t = \widetilde{b}_L \forall t$. Bold lines denote equilibrium with an increase in safe assets, $\widetilde{b}_t = \widetilde{b}_H$ for $t \geq 1$.

lines in Figure 7 depict the steady state without the increase in debt; the solid lines depict steady state with higher debt. Higher debt satiates investors' demand for safe assets, reducing the risk premium and raising the real interest rate. The IS curve shifts up and to the right. A sufficiently large increase in $\widetilde{b}$ restores full employment and zero inflation.

However, as we have already seen, this is not costless. As the lower panel of Figure 7 shows, the increase in safe assets shifts down the steady state capital stock for any level of employment. While an increase in safe assets raises real interest rates and restores full employment, it crowds out investment in physical capital, in the precise sense that investment is lower than it would be if there were no increase in safe assets *and if wages were fully flexible*. Thus, the creation of additional safe assets ultimately leaves the capital stock and investment lower before the increase in risk. With a lower level of capital, output remains lower than its pre-crisis level even though full employment has been restored, as the isoquants show. Indeed the new full employment equilibrium has a lower capital-labor ratio, and thus lower real wages and labor productivity, not just relative to the low risk steady state but also the low debt, high risk, high unemployment steady state (see the grey lines in the lower panel which pass through the origin). In this sense, our model suggests that a risk-induced recession can continue to depress output, wages and productivity even when fiscal policy has restored full employment.

As we saw in the model without nominal rigidities, an economy with a high level of nominal risk *needs* negative real interest rates in order to sustain a high level of investment. In the presence of nominal rigidities and a monetary policy rule that targets zero inflation whenever unconstrained by the zero lower bound, negative rates are simply not a possibility. Instead, an economy with a negative natural rate experiences a recession, as monetary policy loses its potency at the zero lower bound. Issuing public debt satiates the demand for safe assets and raises the natural rate of interest, relaxing the ZLB constraint and rendering monetary policy potent again. But this does not change the fact that a risky economy **requires negative real rates** in order to sustain high

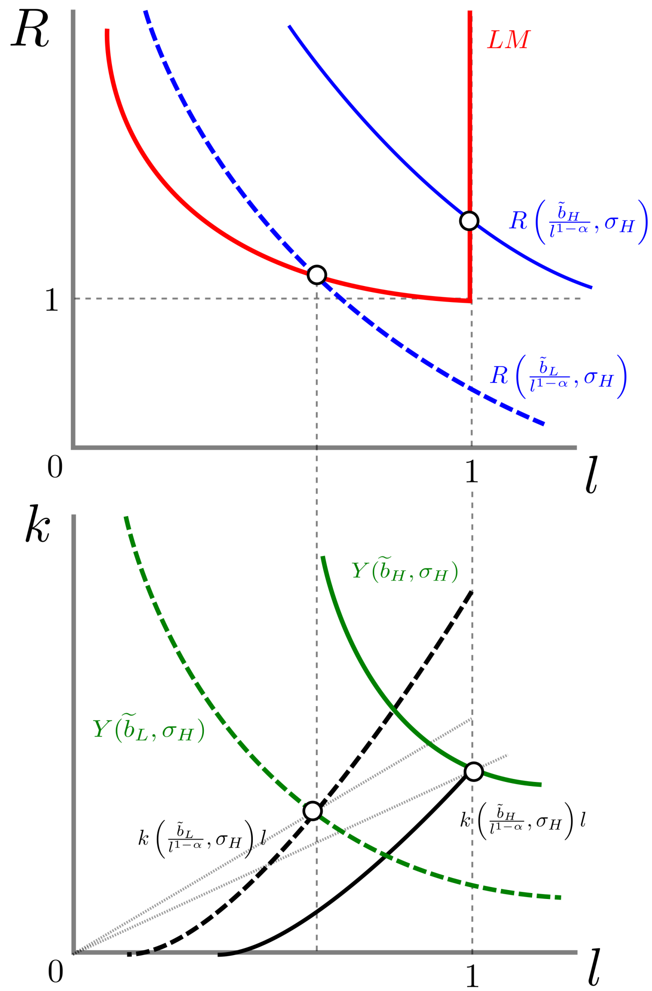


Figure 7. A permanent increase in $\tilde{b}$ in a high σ environment

investment. The same increase in debt which restores full employment crowds out investment in physical capital.

From the perspective of our model, rather than increasing government debt, a better policy would be to engineer negative real interest rates, either through negative nominal rates or a permanently higher inflation target. Negative real rates would permit both full employment and high investment. Our model is not the place for a full cost-benefit analysis of such alternative policies, since it abstracts both from the costs associated with a higher steady state rate of inflation (Coibion et al. (2016)), and the potential consequences of persistently low rates for financial stability (Greenwood et al. (2016)). If these costs are large, risk-induced secular stagnation may present problems which cannot be solved by either monetary or fiscal policy.[9] Nonetheless, such policies are worth considering, because safe asset creation is no panacea.

This provides a new perspective on an argument made by economists such as Paul Krugman and Brad DeLong during the Great Recession. Their argument was that when governments can

[9]Another possibility, within the lens of our model, would be to produce safe assets but keep investment at its efficient level by other means, for example with an appropriately designed subsidy.

borrow at negative real interest rates while unemployment is high, they should obviously do so. Higher deficits reduce unemployment in the short run; moreover, negative real rates make it an exceptionally good time for the government to borrow (since the private sector is effectively paying them to take its money). In our model, this argument is correct as far as it goes: increasing deficits prevents unemployment, and actually results in higher investment. This is preferable to the alternative of tight fiscal policy and maintaining the monetary policy rule. However, a better alternative to both these options would be to keep a low level of public debt, but allow for a higher inflation target. Such a policy would engineer negative real interest rates, permitting a high level of investment without the need for safe assets.

5 Conclusion

We presented a model in which the natural rate of interest is affected both by idiosyncratic risk and by fiscal policy. By increasing the supply of safe assets, the government can prevent an increase in risk from driving real interest rates below zero, allowing monetary policy to operate effectively rather than being constrained by the zero lower bound on nominal rates. However, our analysis uncovers that negative real interest rates do not necessarily indicate a *shortage* of safe assets. While it is possible for fiscal policy to keep interest rates above zero, this is generally not desirable in the absence of nominal rigidities, because increasing debt crowds out investment in physical capital. In the presence of nominal rigidities and a lower bound on nominal interest rates, a negative natural rate of interest (along with a low inflation target) can cause persistent investment and employment slumps. An increase in government debt can prevent this, relaxing the zero bound constraint and allowing monetary policy to restore full employment. However, the reprieve comes at the cost of a decline in investment and the steady state capital stock. The return to full employment merely disguises the deeper problem - that the economy requires negative interest rates in order to operate at potential - which manifests itself in the form of sluggish investment and productivity growth. In this sense, the cost of a risk-induced recession may linger even once the economy has returned to full employment. Rather than increasing government debt, it may be preferable to engineer negative real interest rates, either through negative nominal rates or a higher inflation target; such policies sustain a high level of investment while preventing unemployment.

A full empirical evaluation of this theory lies beyond the scope of this paper. Nevertheless, the scenario we have described is in some respects disturbingly similar to the experience of the U.S. and other advanced economies during the recovery from the Great Recession.

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Appendix

A Household's Optimal Choices

Using equations (1)-(2), the objective function of the households can be written as:

$$\max_{k_{t+1},b_{t+1}} (1-\beta)\ln\left[\omega_t l_t + T_t - \frac{1}{R_t}b_{t+1} - k_{t+1}\right] + \beta\mathbb{E}_z \ln\left[R^k_{t+1}(z)k_{t+1} + b_{t+1}\right]$$

where $\omega_t = \frac{W_t}{P_t}$ and $b_{t+1} = \frac{B_{t+1}}{P_{t+1}}$ and $R_t = \frac{\Pi_t}{1+i_t}$. The first order conditions w.r.t k_{t+1} and b_{t+1} can be written as:

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta \mathbb{E}_z \left[\frac{R^k_{t+1}}{R^k_{t+1}(z) k_{t+1} + b_{t+1}} \right] \tag{38}$$

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta \mathbb{E}_z \left[\frac{R_t}{R^k_{t+1}(z) k_{t+1} + b_{t+1}} \right] \tag{39}$$

Next multiply equation (38) by k_{t+1}, (39) by $\frac{b_{t+1}}{R_t}$ and add them up:

$$\frac{k_{t+1} + \frac{b_{t+1}}{R_t}}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \frac{\beta}{1-\beta} \tag{40}$$

which can be rearranged to yield:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta \left[\omega_t l_t + T_t \right] \tag{41}$$

i.e. the young household save a fraction β of its labor income net of transfers. Using the budget constraint, it is straightforward to see that

$$c^Y_{t+1} = (1-\beta) \left[\omega_t l_t + T_t \right]$$

Using these equations, we can re-write the objective as:

$$\max_{\eta_t} (1-\beta) \ln \Big[(1-\beta)(\omega_t l_t + T_t) \Big] + \beta \ln \Big[(\omega_t l_t + T_t) \Big] + \beta \mathbb{E}_z \ln \Big[\eta_t R^k_{t+1}(z) + (1-\eta_t) R_t \Big]$$

where we define the portfolio share of capital as η_t as $\frac{k_{t+1}}{k_{t+1} + \frac{b_{t+1}}{R_t}}$. The optimal choice of η_t can then be written as:

$$\mathbb{E}_z \ln \left[\frac{R^k_{t+1}(z) - R_t}{\eta_t R^k_{t+1}(z) + (1-\eta_t) R_t} \right] = 0$$

B Proof of Lemma 3

Multiplying (6) by $(\eta_t - 1)$ and rearranging, we have

$$\mathbb{E}_z \left[\frac{R^k_{t+1}}{\eta_t R^k_{t+1}(z) + (1-\eta_t) R_t} \right] = 1$$

Using the expression for $R^k(z)$ in equation (12) and multiplying both sides by η_t we can rewrite the equation above as:

$$\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^{\alpha}}{\alpha z k_{t+1}^{\alpha} + \frac{1-\eta_t}{\eta_t} R_t}\right] \tag{42}$$

Next, using equations (4) and (5) we know that:

$$\frac{1-\eta_t}{\eta_t} \quad = \quad \frac{b_{t+1}}{R_t k_{t+1}} = \frac{\tilde{b}_{t+1}}{R_t} k_{t+1}^{\alpha-1} \tag{43}$$

where we used the definition of $\tilde{b}$ to go from the first to the second equality. Plugging in this expression into (42) and simplifying, we get:

$$\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]$$

In order to see that η_t is decreasing in σ, consider instead, the expression for $1-\eta_t$:

$$1-\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z + \tilde{b}_{t+1}}\right]$$

which is clearly increasing in σ because of Jensen's inequality.

C Deriving an Expression for the Real Interest Rate

In order to derive the expression for the real interest rate, substitute the expression (43) for η_t into equation (43) and rearrange:

$$R_t \quad = \quad \tilde{b}_{t+1} \frac{\eta_t}{1-\eta_t} k_{t+1}^{\alpha-1} = \frac{\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]} k_{t+1}^{\alpha-1}$$

How does R_t change with σ? Notice that we can also write the expression for R_t as:

$$R_t = \left(\mathbb{E}_t\left[\frac{1}{\alpha z + \tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right) k_t^{\alpha-1}$$

Then since $\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1}$ is increasing in σ (from Jensen's inequality), the whole expression is decreasing and thus, $\frac{\partial R_t}{\partial \sigma} < 0$.