

# Understanding HANK: Insights from a PRANK\*

Sushant Acharya<sup>†</sup>

Keshav Dogra<sup>‡</sup>

March 5, 2018

## Abstract

Does market incompleteness radically transform the properties of monetary economies? Using an analytically tractable heterogeneous agent New Keynesian (NK) model, we show that whether incomplete markets resolve ‘policy paradoxes’ in the representative agent NK model (RANK) depends primarily on the cyclical risk, rather than incomplete markets per se. Incomplete markets reduce the effectiveness of forward guidance and multipliers in a liquidity trap only if risk is procyclical. Acyclical or countercyclical risk *amplifies* these puzzles relative to RANK. Cyclical risk also affects determinacy: procyclical risk permits determinacy even under a peg, while countercyclical income risk generates indeterminacy even if the Taylor principle holds. Finally, we uncover a new dimension of monetary-fiscal interaction. Since fiscal policy affects the cyclical risk, it influences the effects of monetary policy even when ‘passive’.

**Keywords:** New Keynesian, incomplete markets, monetary and fiscal policy, determinacy, forward guidance, fiscal multipliers

**JEL codes:** E21, E30, E52, E62, E63

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\*The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York or the Federal Reserve System.

<sup>†</sup>Federal Reserve Bank of New York **Email:** sushant.acharya@ny.frb.org

<sup>‡</sup>Federal Reserve Bank of New York **Email:** keshav.dogra@ny.frb.org

# 1 Introduction

The last few years have seen a surge of interest in heterogeneous agent New Keynesian (HANK) models. Heterogeneity and market incompleteness have been proposed as a means to understand the monetary transmission mechanism (Kaplan et al., 2016), the forward guidance puzzle (McKay et al., 2015), the distributional effects of monetary policy (Gornemann et al., 2016), the efficacy of targeted transfers (Oh and Reis, 2012), automatic stabilizers (McKay and Reis, 2016b) and fiscal stimulus (Hagedorn et al., 2017), among many other topics. These explorations have revealed that the introduction of market incompleteness into the NK model can affect not just the model’s substantive predictions, but also the determinacy properties of equilibrium (Ravn and Sterk, 2017b), (Auclert et al., 2017), which are central to fundamental questions in monetary economics - how is the price level determined, and what kind of policy regime ensures price stability? But the source of the differences between HANK and representative agent New Keynesian (RANK) economies, and the extent to which these differences are a general result rather than a consequence of particular modeling assumptions, remain obscure. This is largely because incomplete market models are generally analytically intractable since the distribution of wealth is an infinite dimensional state variable; thus, most of these studies make use of computational methods. While these papers have highlighted striking differences in the behavior of HANK and RANK economies, the lack of analytical tractability makes it hard to drill down and uncover exactly which features are responsible for these differences.

In this paper we present an analytically tractable HANK model which allows us to pinpoint exactly when, and why, HANKs behave differently from RANKs. We study a standard New Keynesian economy, with the exception that individuals face idiosyncratic, uninsurable shocks to their endowment of labor. Importantly, idiosyncratic income risk varies endogenously with aggregate economic activity, and may be either procyclical or countercyclical. The economy permits closed-form solutions because household utility has constant absolute risk aversion (CARA), rather than constant relative risk aversion (CRRA) as is commonly assumed. This permits linear aggregation and gives us an exact closed form aggregate Euler equation, without having to carry around an infinite-dimensional state variable or impose a degenerate wealth distribution. Since the model aggregates linearly, one can think of it as a Pseudo-Representative Agent New-Keynesian model - in short, a PRANK. Again, our goal is to understand the qualitative differences between HANK and RANK economies and is *not* quantitative in nature.

Our first result is that market incompleteness can alter the determinacy properties of equilibrium, in a way which depends critically on the cyclicity of income risk. RANK models feature indeterminacy under an interest rate peg, or more generally, under interest rate rules which fail to satisfy the Taylor principle. We explain that HANK models can feature determinacy under a peg when income risk is procyclical. In this case, even under a peg, higher future cannot be self-fulfilling, because it would also imply higher income risk, reducing demand via the precautionary savings channel. Whereas procyclical income risk makes indeterminacy less likely, countercyclical risk makes it more likely - if risk is countercyclical, the standard Taylor principle may not even be sufficient to ensure determinacy. In this case, fear of lower output in the future implies higher risk, depressing demand via the precautionary savings channel and generating a self-fulfilling recession. We derive a general, income-risk augmented Taylor principle which depends explicitly on the cyclicity of income risk.

Importantly, the cyclicity of income risk is endogenous. In particular, it depends on the cyclicity of fiscal policy, and on whether redistribution increases or decreases when output is low. This highlights a

new and important dimension of monetary-fiscal interaction, distinct from (but related to) the traditional question concerning whether the fiscal authority adjusts surpluses in order to repay government debt along any hypothetical price path (Leeper, 1991). In HANK economies, what matters is not just the expected path of surpluses, but whether those surpluses are raised in ways that increase or decrease the variance of households’ after-tax income, and whether this depends on the overall level of economic activity.

We next consider how heterogeneity and market incompleteness alter the effects of forward guidance and the size of government spending multipliers at the zero lower bound. In RANK models, announcements of future interest rate cuts are equally, or more, effective than current policy changes in stimulating output and inflation. We find that market incompleteness can reverse this prediction, but only if income risk is strongly procyclical, so the expansionary effect of a promised future boom is offset by an increase in desired precautionary savings, in response to the increased risk generated by the boom. If risk is countercyclical, this prediction is naturally reversed, and incomplete markets worsen the ‘forward guidance puzzle’. Interestingly, HANK models may feature a stronger forward guidance puzzle even if income risk is acyclical or weakly procyclical. Looser monetary policy effectively provides more consumption insurance against income shocks, reducing consumption risk (which is ultimately what matters for precautionary savings) for a given level of income risk, and boosting demand.

RANK models predict that in a liquidity trap, the government spending multiplier is greater than 1 and increasing in the duration of the trap.<sup>1</sup> This is due to the expected inflation channel: when nominal interest rates are constrained due to the zero bound, higher future spending increases expected inflation, lowers real interest rates, and stimulates current spending. If income risk is procyclical, the precautionary savings channel can potentially outweigh the effect of expected inflation in our HANK economy. While future spending lowers real interest rates, it also increases risk, encourages households to save, and moderates the increase in current spending. Consequently, the multiplier can be *less* than 1 and *decreasing* in the duration of the liquidity trap. In contrast, if risk is countercyclical, the precautionary savings and expected inflation channels both work in the same direction, increasing the multiplier.

Like us, some other recent papers such have also made simplifying assumptions in order to solve HANK models analytically in order to better understand the operative channels. For example, recent work by Ravn and Sterk (2017b) and Challe (2017) assume that agents are unable to borrow and the government issues no debt - the so called *zero liquidity limit*. This assumption makes the wealth distribution degenerate, affording analytical tractability. In particular, it allows the authors to study how beliefs about future output affect perceived unemployment risk, the precautionary savings motive, and aggregate demand. Our alternative approach complements these studies in two ways.

First, the models described above incorporate features such as labor market search frictions which are not present in RANK models. We instead make a minimal departure from the RANK framework, incorporating uninsurable income risk, but not labor market frictions, which allows us to more precisely isolate the role of heterogeneity and market incompleteness. In particular, we find that the effect of incompleteness depends crucially on whether income risk is countercyclical (as assumed in the papers described above) or procyclical. This resonates with the results of Werning (2015). He shows that, in an

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<sup>1</sup>This prediction depends on the assumption that the monetary authority targets the zero-output, zero-inflation steady state (using an appropriately specified active Taylor rule) as soon as the underlying shocks abate and the ZLB is no longer binding. We maintain this assumption throughout our analysis. Cochrane (2017b) discusses how fiscal multipliers change when the active Taylor rule assumption is dropped and alternative criteria are used to select among the many bounded rational expectations equilibria consistent with a given path of nominal interest rates.

economy with CRRA utility and zero liquidity, current consumption is more sensitive to future consumption in the presence of countercyclical income risk, implying that consumption is more sensitive to future changes in interest rates than current changes. With acyclical income risk, market incompleteness does not affect the relation between consumption and interest rates.

Second, our analysis complements both [Werning \(2015\)](#) and [Ravn and Sterk \(2017b\)](#) by moving beyond the *zero liquidity limit*. While a useful simplifying assumption, this has the strong implication that income risk passes through one for one to consumption risk. In reality, households can partially insure consumption against income shocks through various mechanisms ([Blundell et al., 2008](#)); thus the pass-through from income to consumption risk is less than one, and importantly, may vary over time. Our approach does not impose zero liquidity, and allows for endogenous, time-varying pass-through of income to consumption risk, an important and as yet understudied component of the precautionary savings channel.<sup>2</sup>

Another difference, relative to [Werning \(2015\)](#), is that we discuss how the cyclicalities of income risk affects determinacy in HANK models, not just the equilibrium response of consumption to interest rates. In this regard our results are related to [Auclert et al. \(2017\)](#), who also analyze how incomplete markets affects determinacy and the economy’s response to increases in government spending, monetary policy shocks and forward guidance.<sup>3</sup> Their analytical results are framed in terms of an infinite dimensional  $\mathbf{M}$  matrix which describes the response of consumption at any date to aggregate output at any other date; for example, they show that determinacy depends on the asymptotic properties of the far-out columns of this matrix. They also present numerical results which generally confirm the results in our closed-form solutions (procyclical risk permits determinacy under a peg, countercyclical risk makes determinacy less likely, and so forth). Our simplifying assumption of CARA utility allows us to analyze determinacy and the economy’s response to shocks in a transparent model permitting closed form solutions.

In older work, [Challe and Ragot \(2011\)](#); [Challe et al. \(2017\)](#) and [Challe et al. \(2017\)](#) make assumptions on preferences, technology and market structure in order to construct analytically tractable *limited heterogeneity equilibria* in which the wealth distribution has finite support. These papers primarily study how the precautionary savings channel can amplify aggregate shocks, which is related to, but distinct from, the themes we address in this paper.

Recent work by [Bilbiie \(2017b\)](#); [Debortoli and Galí \(2017\)](#) presents a TANK (*two agent New Keynesian*) model to shed light on how the responses of HANK models differs from RANK models in response to aggregate shocks. Similarly, [Bilbiie \(2008\)](#) discusses determinacy in a TANK model, [Bilbiie \(2017a\)](#) discusses the effects of forward guidance and other ‘puzzles’, while [Mehrotra \(2017\)](#) compares the effects of transfers and government purchases. As [Debortoli and Galí \(2017\)](#) emphasize, this TANK literature abstracts from precautionary savings (more generally, heterogeneity *within* unconstrained households) in order to study MPC heterogeneity (heterogeneity *between* constrained and unconstrained households). Our approach instead abstracts for the most part from MPC heterogeneity in order to study the precautionary savings channel in detail. In Section 6.1, we introduce heterogeneity into the model and show that this does not qualitatively change our main findings.

[McKay et al. \(2015\)](#) argued that incomplete markets solve the ‘forward guidance puzzle’, i.e. the fact that in New Keynesian models, announcements of interest rate cuts far in the future are more effective at

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<sup>2</sup>To be clear, this important component of the precautionary savings channel is already implicitly present in models solved using computational methods; the advantage of our approach is that we can observe it analytically.

<sup>3</sup>Indeed, this paper began life as a discussion of [Auclert et al. \(2017\)](#).

stimulating output and inflation than contemporaneous interest rate cuts (Del Negro et al., 2015). (McKay et al., 2017) present a stylized incomplete markets model, again with zero liquidity, in which household consumption is described by a ‘discounted Euler equation’. We also derive a modified Euler equation in our CARA-HANK framework (which does not rely on zero liquidity) and describe the conditions under which forward guidance is less effective than in a RANK model. Importantly though, we find that the model only generates a *discounted* Euler equation and weakens the power of forward guidance if income risk is sufficiently procyclical (as in (McKay et al., 2017)). If instead income risk is countercyclical, the model generates an *explosive* Euler equation and strengthens the power of forward guidance.

The rest of the paper is structured as follows. Section 2 presents the model economy. Section 3 solves the model and discusses the factors affecting the cyclicity of income risk. Section 4 shows how the cyclicity of risk affects determinacy of equilibrium in our HANK economy and derives an income risk-adjusted Taylor principle. Section 5 discusses conditions under which the introduction of incomplete markets solves, or amplifies, two perceived ‘puzzles’ present in the RANK model: the power of forward guidance, and explosive government spending multipliers in a liquidity trap. Section 6 discusses the relative importance of hand-to-mouth agents and the precautionary saving motive in HANK economies, and the pervasive importance of fiscal policy. Section 7 concludes.

## 2 Model

We introduce uninsurable income risk into an otherwise standard New Keynesian model. Households face idiosyncratic income risk and can only save in a nominally riskless bond. The supply side is deliberately kept relatively standard: monopolistically competitive firms combine labor and intermediate inputs to produce differentiated varieties of the output good, and set prices subject to nominal rigidities. For simplicity, we consider an economy with idiosyncratic risk but no aggregate uncertainty.

### 2.1 Households

There is a continuum of households in the economy indexed by  $i \in [0, 1]$ . Households maximize utility

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{\gamma} e^{-\gamma c_t^i} \right\}$$

subject to:

$$P_t c_t^i + \frac{1}{1+i_t} A_{t+1}^i = A_t^i + P_t y_t^i \quad (1)$$

Each household can save only in a risk free nominal bond  $A_{t+1}^i$  which has a price of  $\frac{1}{1+i_t}$  at date  $t$  and pays off 1 in nominal terms at  $t+1$ .  $c_t^i$  is itself an aggregate consumption index defined by:

$$c_t^i = \left[ \int_0^1 c_t^i(k)^{\frac{\theta-1}{\theta}} dk \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

As is standard, the demand for variety  $k$  by household  $i$  can be written as:

$$c_t^i(k) = \left( \frac{P_t(k)}{P_t} \right)^{-\theta} c_t^i \quad (3)$$

Thus, total consumption demand for variety  $k$  can be written as:

$$c_t(k) = \int_0^1 c_t^i(k) di = \left( \frac{P_t(k)}{P_t} \right)^{-\theta} c_t \quad (4)$$

where  $c_t = \int_0^1 c_t^i di$ .

**Income of Household  $i$**   $y_t^i$  denotes the income of household  $i$  in period  $t$  and can be written as:

$$y_t^i = (1 - \tau_t) \omega_t \ell_t^i + D_t^i + T_t \quad (5)$$

The income of each household is made up of three components: (i) real labor income net of taxes  $(1 - \tau_t) \omega_t \ell_t^i$ , (ii) real dividends from the production sector,  $D_t^i$  and (iii) real transfers from the government,  $T_t$ . We discuss each of these subcomponents next.

**Labor Income** Following [Aiyagari \(1994\)](#), we assume that households have a stochastic endowment of labor  $\ell_t^i$  each period which they supply inelastically at the prevailing real wage  $\omega_t$ . In particular, we assume that each period, household  $i$ 's endowment of labor is given by  $\ell_t^i \sim N(\ell, \sigma_{\ell,t}^2)$  where  $\ell$  is the aggregate endowment of labor in this economy. Without loss of generality, we normalize  $\ell = 1$ .  $\tau_t$  denotes the linear tax on labor income. In particular we assume that  $\sigma_{\ell,t}^2$  is given by:

$$\sigma_{\ell,t}^2 = \sigma_{\ell}^2(Y_t) \quad (6)$$

where  $Y_t$  denotes aggregate output. As in [McKay and Reis \(2016a\)](#), this specification allows for cyclical changes in the distributions of earnings risks in line with the empirical evidence documented by [Storesletten et al. \(2004\)](#) and [Guiso et al. \(2014\)](#). To be clear, none of our results depend on the assumption that the variance of labor endowments depends exogenously on economic activity. Even if the variance of endowments  $\sigma_{\ell}^2$  does not vary with economic activity and is constant, the variance of household income will generally still vary with economic activity, as we show in [Section 3.4](#).

**Capital Income** In addition to labor income, each household also receives dividends from the productive sector. Notice that the dividends  $D_t^i$  have an  $i$  superscript, implying that dividends may vary across households. In particular we assume that the distribution of dividends across all households can be expressed as:

$$D_t^i = \bar{D}_t + \delta_t (\ell_{i,t} - 1) \quad (7)$$

As has been highlighted by [Broer et al. \(2016\)](#) and [Werning \(2015\)](#), the distribution of dividends is an important determinant of how an incomplete markets economy responds to various shocks. This convenient specification is fairly general and nests many commonly used cases. For example,  $\delta = 0$  implies that

dividends are distributed equally across all households. Given other things,  $\delta > 0$  implies that households with larger labor income are the recipient of a larger share of dividends. We also allow for the possibility that  $\delta_t$  varies with economic activity:

$$\delta_t = \delta(Y_t) \quad (8)$$

**Net Transfers from the Government** The last source of income is lumpsum transfers net of taxes. We assume the government makes a lump sum transfer  $T_t$  which is the same across all households in each period, and taxes labor income at the rate  $\tau_t$ .

## 2.2 Firms

There is a continuum of monopolistically competitive firms indexed by  $j \in [0, 1]$ . Following [Basu \(1995\)](#); [Woodford \(2003\)](#); [Nakamura and Steinsson \(2010\)](#) and many others, we assume that each firm combines labor and intermediate inputs to produce a differentiated good  $x(j)$  using a constant returns to scale technology:<sup>4</sup>

$$x_t(j) = M_t(j)^\alpha L_t(j)^{1-\alpha} \quad (9)$$

where  $M_t(j)$  is the level of intermediate inputs utilized by the firm producing variety  $j$ .  $M_t(j)$  is itself an aggregate of intermediate inputs defined by:

$$M_t(j) = \left[ \int_0^1 m_t(j, k)^{\frac{\theta-1}{\theta}} dk \right]^{\frac{\theta}{\theta-1}} \quad (10)$$

As is standard, the demand for intermediate input  $k$  by firm  $j$  can be written as:

$$m_t(j, k) = \left( \frac{P_t(k)}{P_t} \right)^{-\theta} M_t(j) \quad (11)$$

Thus, total demand for intermediate input  $k$  can be written as:

$$m_t(k) = \int_0^1 m_t(j, k) dj = \left( \frac{P_t(k)}{P_t} \right)^{-\theta} M_t \quad (12)$$

Firms solve the cost minimization problem

$$\begin{aligned} \min \quad & P_t M_t(j) + W_t L_t(j) \\ \text{s.t.} \quad & M_t(j)^\alpha L_t(j)^{1-\alpha} \geq x_t(j) \end{aligned}$$

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<sup>4</sup>In our setup, since households supply their stochastic endowment of labor inelastically, intermediate inputs are the factor which adjusts in response to demand. This specification of the supply side of the economy is just one possibility among many. Our results are general enough to apply to any specification of the supply side which yields a New Keynesian Phillips curve relationship - with the understanding that different specifications of the supply side may affect whether income risk is procyclical or countercyclical.

yielding

$$\frac{W_t}{P_t} = \frac{1-\alpha}{\alpha} M_t(j), \quad (13)$$

In symmetric equilibrium,  $x_t(j) = x_t$  normalizing the aggregate endowment of labor to 1, we have  $L_t(j) = 1$  and so real wages are given by

$$\omega_t = \frac{W_t}{P_t} = \frac{1-\alpha}{\alpha} x_t^{\frac{1}{\alpha}} \quad (14)$$

Finally, net output is given by

$$Y_t = x_t - M_t = x_t - x_t^{\frac{1}{\alpha}} \quad (15)$$

### 2.3 Nominal Rigidities

Each firm faces a quadratic cost of changing prices following [Rotemberg \(1982\)](#). The pricing decisions of each firm can then be written as:

$$\max_{P_t(k)} \sum_{t=0}^{\infty} \frac{1}{\prod_{s=0}^{t-1} (1+r_s)} \left\{ \left( \frac{P_t(k)}{P_t} - \lambda_t \right) \left( \frac{P_t(k)}{P_t} \right)^{-\theta} - \frac{\Xi}{2} \left( \frac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2 \right\} x_t$$

where  $\lambda_t = \frac{\omega_t^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$  is the real marginal cost faced by firm  $k$ ,  $1+r_t = \frac{1+i_t}{1+\Pi_{t+1}}$  denotes the real interest rate and  $-\frac{\Xi}{2} \left( \frac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2$  denotes the quadratic cost a firm faces if it wants to change its price from last period's level.  $\Xi \geq 0$  is a constant which scales the cost. In equilibrium, the aggregate cost of firms changing prices is given by:

$$C_t = x_t \int_k \frac{\Xi}{2} \left( \frac{P_t(k)}{P_{t-1}(k)} - 1 \right)^2 dk \quad (16)$$

Following [Ascari and Rossi \(2012\)](#) and [Bhandari et al. \(2017a\)](#) we assume that this cost is rebated lumpsum to households along with dividends.<sup>5</sup>

### 2.4 Policy

**Monetary Policy** We assume that the monetary authority sets nominal rates according to some rule:

$$i_t = (1+r)\Pi_t^{\Phi_\pi} \geq 0 \quad (17)$$

where  $(1+r)$  denotes the steady state real interest rate.

**Fiscal Policy** The budget constraint of the fiscal authority can be written as:

$$B_t + P_t G_t + P_t T_t = P_t \tau_t \omega_t + \frac{1}{1+i_t} B_{t+1} \quad (18)$$

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<sup>5</sup>This assumption is made to simplify exposition. Even if we did not rebate this cost to households, it would be zero in a linear approximation of the economy around the zero inflation steady state.



where  $G_t$  denotes government purchases of the final good. We can define real primary surpluses  $\mathcal{S}_t$  as:

$$\mathcal{S}_t = \tau_t \omega_t - T_t - G_t \quad (19)$$

As we will discuss shortly, we allow the rate of labor income taxation  $\tau_t$  to depend in a continuous but otherwise arbitrary fashion on the level of aggregate output in the economy:

$$\tau_t = \tau(Y_t)$$

We assume throughout that lump-sum transfers  $T_t$  adjust as needed to ensure fiscal solvency: fiscal policy is “*passive*” in the sense of [Leeper \(1991\)](#) and this is not an environment where the *fiscal theory of the price level* (FTPL) is at play. We make this assumption to highlight that in the presence of incomplete markets, fiscal policy crucially affects the effects of monetary policy even when it is ‘passive’. Our results identify a new sense in which fiscal policy matters in a way which is logically distinct from the FTPL.

## 2.5 Market Clearing

The aggregate resource constraint implies

$$c_t + G_t = Y_t \quad (20)$$

where  $c_t = \int_0^1 c_t^i di$  denotes aggregate consumption.

## 3 Characterizing General Equilibrium

In this section we characterize equilibrium in our HANK economy. We start by solving the decision problem of each household.

### 3.1 Household decisions

The virtue of assuming CARA utility is that it allows us to characterize the decisions of each household in closed form. The following proposition characterizes each household’s optimal decisions.

**Proposition 1** (Individual decision problem). *Given a sequence of real interest rates, aggregate output and idiosyncratic risk  $\{r_t, y_t, \sigma_{y,t}\}$ ,<sup>6</sup> and initial wealth  $a_{t-1}^i$  each household’s consumption decision can be expressed as:*

$$c_t^i = \chi_t + \mu_t (a_t^i + y_t^i) \quad (21)$$

where  $a_t^i = A_t^i/P_t$  is real net worth at the start of date  $t$ ,  $\mu_t$  is the marginal propensity to consume out of cash-on-hand ( $a^i + y^i$ ) at date  $t$ , and  $\chi_t$  is the common component of consumption across households.  $\chi_t$

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<sup>6</sup>We restrict attention to sequences of interest rates for which there exists a terminal date  $T < \infty$  after which  $r_t > 0$ .

and  $\mu_t$  solve the following recursions:

$$\chi_t [1 + \mu_{t+1} (1 + r_t)] = -\frac{1}{\gamma} \ln \beta (1 + r_t) + \chi_{t+1} + \mu_{t+1} y_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \quad (22)$$

$$\mu_t = \frac{\mu_{t+1} (1 + r_t)}{1 + \mu_{t+1} (1 + r_t)} \quad (23)$$

*Proof.* See Appendix A. □

Equation (21) shows that individual consumption can be decomposed into an aggregate and an idiosyncratic component. The idiosyncratic component of this equation states that each household has marginal propensity to consume (MPC)  $\mu_t$  out of cash-on-hand at any date  $t$  which is common across households. Iterating equation (23) forward reveals that the MPC today depends positively on the future path of real interest rates. If expected future interest rates are high, a household receiving a positive income shock today saves a larger part of that increase. Higher interest rates in the future imply higher interest earning in the future due to this income shock implying that the household is richer in the future, and so increases consumption today by a larger amount than if interest rates were lower. This can be seen clearly in the case of constant real interest rates where  $\mu_t = \frac{r}{1+r}$  is constant across time and is simply the annuity value of a additional dollar of income today.<sup>7</sup>

The aggregate component  $\chi_t$  can be decomposed into 3 terms. To see this, solve (22) forwards to get:

$$\chi_t = \underbrace{\sum_{s=1}^{\infty} Q_{t+s|t} \frac{\mu_t}{\gamma \mu_{t+s}} \ln \left[ \frac{1}{\beta (1 + r_{t+s-1})} \right]}_{\text{impatience}} + \underbrace{\mu_t \sum_{s=1}^{\infty} Q_{t+s|t} y_{t+s}}_{\text{PIH}} - \underbrace{\frac{\gamma \mu_t}{2} \sum_{s=1}^{\infty} Q_{t+s|t} \mu_{t+s} \sigma_{y,t+s}^2}_{\text{precautionary savings}} \quad (24)$$

where  $Q_{t+s|t} = \prod_{k=0}^{s-1} \frac{1}{1+r_{t+k}}$ . The first term in (24) is standard and reflects the effect of impatience and interest rates on savings behavior: if interest rates are higher relative to  $\beta$ , then current consumption is lower as households wish to save more. The second term reflects the permanent income hypothesis: higher expected discounted lifetime income causes a household to increase current consumption.<sup>8</sup> The final term reflects the precautionary savings motive. To the extent that households are risk averse,  $\gamma > 0$ , higher income risk at any date in the future lowers current consumption by increasing the desire of households to save. This third term indicates the effect of uninsurable income risk on aggregate consumption - if households face no idiosyncratic income risk, this term is zero and households are *permanent income consumers*. Note that what matters for the precautionary savings channel is the variance of *consumption*, not income. A given level of income risk  $\sigma_y^2$  depresses current consumption more when the sensitivity of consumption to income, i.e. the MPC  $\mu_t$ , is higher. In our framework, this sensitivity varies over time depending on the path of real interest rates. This effect is absent in popular tractable HANK models which impose the zero liquidity limit, discussed in [Ravn and Sterk \(2017b\)](#); [Werning \(2011\)](#); [McKay et al. \(2017\)](#) among others. In these models, households who are on their Euler equation anticipate that

<sup>7</sup>Caballero (1990) and Weil (1993) solved a generalized version of this decision problem while assuming that the real interest rate was constant.

<sup>8</sup>Recall that the expected future discounted lifetime income a household is common across all households and is the same as the discounted future value of aggregate income or GDP. This follows from our assumption that individual labor endowments are i.i.d..

their consumption will be equal to their income in all future periods; the precautionary savings channel is present, but its strength is not affected by variations in the sensitivity of consumption to income. However, in general HANK models with CRRA preferences do feature this channel if the zero liquidity limit is not imposed. In our CARA framework, it can be studied in an analytically tractable model.

### 3.2 Demand Block

Our setup features no MPC heterogeneity across households, permitting aggregation despite a non-degenerate distribution of wealth. Relative to the existing literature on HANK models, our analysis abstracts from MPC heterogeneity in order to focus on precautionary savings. This also helps clarify the sense in which MPC heterogeneity is, or is not, necessary for heterogeneity to affect aggregate outcomes, as discussed in Section 6.1. The next proposition states the aggregation result formally.

**Proposition 2** (Aggregation). *Since the marginal propensity to consume  $\mu_t$  is the same for all households in any period, the individual consumption function (21) can be aggregated across all households to yield an aggregate consumption function:*

$$c_t = \int_0^1 c_t^i di = \chi_t + \mu_t (a_t + y_t) \quad (25)$$

where  $a_t = \int_0^1 a_t^i di$ .

*Proof.* See Appendix B. □

In general equilibrium, asset and goods markets clear, i.e.  $c_t = Y_t$  and  $a_t = \frac{B_t}{P_t}$ . Plugging these conditions in (25) one can derive the *aggregate IS equation*.<sup>9</sup>

$$Y_t = Y_{t+1} - \frac{\ln \beta (1 + r_t)}{\gamma} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} + G_t - G_{t+1} \quad (26)$$

Notice that if there is no risk, i.e.  $\sigma_y = 0$ , this *exact aggregate Euler equation* looks very much like the linearized IS equation from the 3 equation RANK model (except that this is not linearized). In general though, if  $\sigma_y^2 > 0$ , (26) also features a precautionary savings term which depends on 3 factors: risk aversion  $\gamma$ , the sensitivity of individual consumption to individual income  $\mu_{t+1}$ , and idiosyncratic income risk,  $\sigma_y^2$ . As described above, idiosyncratic risk depends on many factors; most importantly, it can be affected by fiscal policy. More redistribution through higher taxes and transfers (higher  $\tau$  and  $T$ ) reduces the variance of after-tax income and diminishes the precautionary savings effect.

Interestingly, the level of government debt does not directly enter the IS equation. Consider two economies with different levels of initial government debt. In each economy, the lifetime budget constraint of the government must hold without a bubble term,<sup>10</sup> so the economy with higher initial government debt must also have a higher present discounted value of primary surpluses. Suppose this difference is entirely accounted for by a lower path of lumpsum transfers,  $T_t$  in the high debt economy. Then the IS equation

<sup>9</sup>See Appendix B for a derivation.

<sup>10</sup>This is because we only consider positive real interest rates which are less than  $1/\beta$ . Combined with the transversality condition of households, this must be the case.

(26) - and in fact all equilibrium outcomes (as we show next) - will be identical in the two economies. In this sense, *conditional on a path of marginal taxes*  $\tau_t$ , government debt is not a state variable in this economy despite incomplete markets. This is reminiscent of a similar result in [Bhandari et al. \(2017b\)](#).

### 3.3 Phillips Curve

The solution to the pricing problem of a firm described in section 2.3 is given by:

$$\Xi \Pi_t (\Pi_t - 1) = 1 - \theta \left( 1 - x_t^{\frac{1-\alpha}{\alpha}} \right) + \Xi (\Pi_{t+1} - 1) \Pi_{t+1} \left[ \frac{1}{1+r_t} \frac{x_{t+1}}{x_t} \right] \quad (27)$$

where we have imposed a symmetric equilibrium. Aggregate real dividends can then be written as:

$$\bar{D}_t = x_t - \frac{x_t^{\frac{1}{\alpha}}}{\alpha}$$

Note that as  $\Xi \rightarrow \infty$ , equation (27) implies  $\Pi_t = 1$  for all  $t$ . We refer to this case as the “*rigid price benchmark*”. In the “*flexible price benchmark*” ( $\Xi = 0$ ), (27) implies that output is given by:

$$Y^* = \left[ \frac{\theta(1-\alpha) + \alpha}{\theta} \right] \left[ \frac{\alpha(\theta-1)}{\theta} \right]^{\frac{\alpha}{1-\alpha}} \quad (28)$$

In summary, despite being an incomplete markets model, given a path of marginal taxes  $\{\tau_t\}$ , the entire model can be summarized by the following equations which describe the dynamics of only aggregate variables: the IS equation (26), the MPC recursion (23), the Phillips curve (27), the definition of GDP  $Y_t$  (15), the monetary policy rule (17) and finally, the Fisher equation  $\frac{1+i_t}{\Pi_{t+1}} = 1 + r_t$ .

### 3.4 The cyclicity of income risk in General Equilibrium

So far we have not described the properties of individual income risk  $\sigma_{y,t}^2$  which enters the individual household decision problem, and thus the aggregate IS equation (26). We now show that in equilibrium,  $\sigma_{y,t}^2$  depends on aggregate output  $Y_t$  according to some function:

$$\sigma_{y,t}^2 = \sigma^2(Y_t)$$

The structure of  $\sigma^2(Y)$  depends endogenously on many factors - such as the cyclicity of wages, time varying dividend policies and in particular, fiscal policy. For our purposes, however, this rich array of factors affecting the level and cyclicity of income risk can be summarized by  $\sigma^2(Y)$ , as we now explain.

Plugging in the expression for real wages (14) and the dividend function (7) into equation (5) yields the following expression for each household’s income:

$$y_t^i = \left[ \left( 1 - \tau(Y_t) \right) \omega(Y_t) + \delta(Y_t) \right] (\ell_t^i - 1) + y_t \quad (29)$$

where  $y_t$  denotes mean household income and is defined by:<sup>11</sup>

$$y_t = (1 - \tau(Y_t)) \omega(Y_t) + \bar{D}_t + T_t \quad (31)$$

and  $\omega(Y)$  defines the equilibrium real wage consistent with net output being  $Y$ .<sup>12</sup> Thus, in equilibrium, individual income is normally distributed:  $y_{i,t} \sim N(y_t, \sigma^2(Y))$  where

$$\sigma^2(Y) = \left[ (1 - \tau(Y)) \omega(Y)^{1/\alpha} + \delta(Y) \right]^2 \sigma_\ell^2(Y) \quad (32)$$

Thus, household income  $y_t^i$  can be summarized by its mean  $y_t$  and variance  $\sigma^2(Y)$ .

**Determinants of the level of income risk** Equation (32) shows that fiscal policy can affect the level of income risk. Trivially, by setting  $\tau = 1$ , the fiscal authority can totally eliminate labor income risk. In this case, the fiscal authority confiscates each household's labor income and then returns an equal share to each household, thus eliminating any variation in income arising from stochastic endowment shocks. Similarly, higher  $\delta$ , i.e. more unequally distributed dividends increases income risk. Finally, a higher level of endowment/employment risk  $\sigma_\ell^2$  increases income risk to the extent that redistribution through the tax and transfer system is less than perfect ( $\tau < 1$ ) or dividends are unequally distributed ( $\delta > 0$ ).

**Determinants of the cyclical risk of income risk** Define the *cyclical risk of income risk* as  $\frac{d\sigma^2(Y)}{dY}$ . This answers the following question: Supposing all exogenous variables were held fixed, and aggregate income was higher than its steady state level, would the variance of idiosyncratic income be higher or lower?<sup>13</sup> Equation (33) shows that the cyclical risk of income risk, so defined, depends on 4 factors: (i) the cyclical risk of wages  $\omega'(Y)$ ; (ii) the cyclical risk of labor taxes,  $\tau'(Y)$ ; (iii) firms' dividend policy, i.e. whether dividends are more or less unequally distributed in good times compared to bad,  $\delta'(Y)$ ; (iv) the cyclical risk of labor endowment risk  $\frac{d\sigma_\ell^2(Y)}{dY}$ . This last factor can be thought of as unemployment risk which is not explicitly modeled here: if the probability of becoming unemployed is greater in recessions, i.e the probability of drawing a low labor endowment is higher when  $Y$  is low, then  $\frac{d\sigma_\ell^2(Y)}{dY} > 0$ .

$$\frac{d\sigma^2(Y)}{dY} = 2\sigma(Y)\sigma_\ell(Y) \left\{ \underbrace{(1 - \tau(Y)) \omega'(Y)}_{\text{cyclical risk of real wages}} - \underbrace{\tau'(Y) \omega(Y)}_{\text{cyclical risk of taxes}} + \underbrace{\delta'(Y)}_{\text{cyclical risk of dividend risk}} \right\} + \underbrace{\frac{\sigma^2(Y)}{\sigma_\ell^2(Y)} \frac{d\sigma_\ell^2(Y)}{dY}}_{\text{cyclical risk of employment risk}} \quad (33)$$

<sup>11</sup>Note for future reference that mean household income is equal to GDP minus fiscal surplus minus government expenditures:

$$y_t = Y_t - S_t - G_t \quad (30)$$

<sup>12</sup>The function  $\omega(Y)$  solves  $Y = \left(\frac{\alpha}{1-\alpha}\right)^\alpha \omega^\alpha - \left(\frac{\alpha}{1-\alpha}\right) \omega$  with the understanding that we only consider the smaller of the two solutions for  $\omega$ .

<sup>13</sup>In a richer model featuring aggregate shocks, this notion of cyclical risk - which is the relevant one when discussing determinacy and policy puzzles - need not coincide with the definition used in the empirical literature, namely the correlation between income risk and some measure of aggregate economic activity (Storesletten et al. (2004), Guvenen et al. (2014)). In our simple model without aggregate shocks, though, the definitions are essentially equivalent.

**The Importance of Fiscal policy in determining the cyclical risk** Just as the level of taxation affects the level of income risk, the cyclical risk of fiscal policy affects the cyclical risk of income risk, i.e.  $\frac{\sigma^2(Y)}{dY}$ . For example, if the fiscal authority cuts the labor income tax rate in recessions and raises it in booms, i.e.  $\tau'(Y) > 0$ , this would tend to make income risk countercyclical. Conversely, if the government paid lumpsum transfers in recessions financed by proportional taxes, i.e.  $\tau'(Y) < 0$ , this would tend to make income risk more procyclical. As we will see, the cyclical risk of income risk emerges as the central factor determining whether, and how, HANK economies are different from RANK economies. Since fiscal policy itself affects the cyclical risk of income risk, it takes center stage in determining how monetary policy can affect the economy in an economy with incomplete markets. This interplay of fiscal and monetary policy in determining equilibrium outcomes is logically distinct from the well known set of issues dealing with monetary- fiscal coordination discussed by [Leeper \(1991\)](#), [Woodford \(1996\)](#) and [Cochrane \(2017b\)](#) among others. Those issues concern whether the fiscal authority raises sufficient surpluses to remain solvent along any price path. In our environment fiscal policy always ensures solvency and thus is passive. However, *how* the fiscal authority raises these surpluses, and how this varies with economic activity, affects the cyclical risk of income risk in equilibrium and thus the effects of monetary policy and shocks.

Armed with these results, we start by evaluating the key factor which affects determinacy of equilibria in HANK models.

## 4 Determinacy of equilibrium in HANK economies

### 4.1 Equilibrium with an interest rate peg

It is well-known at least since [Sargent and Wallace \(1975\)](#) that under an interest rate peg - a rule which keeps nominal interest rates constant,  $i_t = \bar{i}$  for all  $t$  - a representative agent monetary economy features indeterminacy, i.e. there are multiple bounded equilibria. In flexible price economies, this indeterminacy takes the form of multiple bounded paths of prices consistent with equilibrium. In economies with nominal rigidities, indeterminacy manifests in both nominal and real variables. For example, in the 3 equation NK model, a peg would violate the Taylor principle, permitting multiple bounded paths for inflation and output.<sup>14</sup> In the limiting case where prices are perfectly rigid, a nominal peg fixes the real interest rate and generates indeterminacy which manifests purely in real variables, as prices are fixed.

In contrast, [Auclert et al. \(2017\)](#) numerically find that heterogeneous agent economies can exhibit determinacy even under a constant real interest rate, and this becomes more likely when income risk is procyclical. [Ravn and Sterk \(2017b\)](#) use a different incomplete markets model and argue that even the standard Taylor principle is not sufficient to guarantee determinacy. Our framework explains both results.

We proceed in two steps.<sup>15</sup> First we solve the individual decision problem under the assumption that the real interest rate is constant and equal to its steady state value  $r_t = r > 0$ . Next, we use this solution to characterize general equilibrium under a nominal interest rate peg when firms face an infinite cost  $\Xi$  of changing prices. As in [Werning \(2011\)](#), the extreme assumption of fixed prices allows us to transparently highlight the forces which generate or interfere with determinacy of equilibria. Section 4.3 relaxes this assumption and derives a modified Taylor principle in this general case, showing that our qualitative

<sup>14</sup>Appendix C.1 shows how pegs cause indeterminacy in the standard 3 equation RANK model.

<sup>15</sup>Without loss of generality, in this section we assume that  $G_t = 0$  for all  $t$ .

results do not depend on the assumption of perfectly rigid prices.

**Corollary 1** (Individual decision problem). *With a constant real interest rate  $r_t = r \in (0, \beta^{-1} - 1)$ , each household's consumption decision can be expressed as:*

$$c_t^i = \chi_t + \mu [(1+r)a_{t-1}^i + y_t^i] \quad (34)$$

where  $\mu = \frac{r}{1+r} \in (0, 1)$  is the marginal propensity to consume out of cash-on-hand and  $\chi_t$  is given by:

$$\chi_t = -\frac{\ln[\beta(1+r)]}{\gamma r} + \mu \sum_{s=1}^{\infty} \frac{y_{t+s}}{(1+r)^s} - \frac{\gamma \mu^2}{2} \sum_{s=1}^{\infty} \frac{\sigma_{y,t+1}^2}{(1+r)^s} \quad (35)$$

Equation (22) is a special case of (24) when real interest rates are fixed over time. As before, the three terms reflect impatience, permanent income behavior and precautionary savings respectively. The only difference relative to the characterization in Proposition 1 is that the MPC  $\mu$  is not just the same across all households but also constant across time.

Under an interest rate peg and fixed prices, the IS equation (26) becomes:

$$Y_t = Y_{t+1} - \frac{\ln \beta (1+r)}{\gamma} - \frac{\gamma \mu^2 \sigma_{y,t+1}^2}{2} \quad (36)$$

The benefit of these assumptions is that the entire economy can be described by this single equation. Its determinacy properties depend on the relation between  $\sigma_{y,t+1}^2$  and  $Y_t$ , i.e. the cyclicity of income risk.

## 4.2 Determinacy and the cyclicity of income risk

**Steady State** We concentrate on the steady state in which output is equal to its flexible price steady state level given by  $Y^*$  defined in (28).<sup>16</sup> Since the fixed price benchmark economy is summarized by one dynamic equation (36), checking for determinacy simply entails evaluating the eigenvalue associated with that equation. Linearizing equation (36) around  $Y^*$  yields:

$$\hat{Y}_t = \Theta \hat{Y}_{t+1} \quad (37)$$

where

$$\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\sigma^2(Y^*)}{dY}$$

Since  $\hat{Y}_t$  is a non-predetermined variable, determinacy requires that  $|\Theta| < 1$ . If this condition is satisfied, the only bounded sequence  $\{\hat{Y}_t\}_{t=0}^{\infty}$  which satisfies (37) is  $\hat{Y}_t = 0$  for all  $t$ . If instead  $|\Theta| \geq 1$ , there are

<sup>16</sup>It is immediate from equation (36) that steady state of this economy must satisfy:

$$\frac{(1+r)^2 \ln \left[ \frac{1}{\beta(1+r)} \right]}{\gamma^2 r^2} = \frac{1}{2} \sigma^2(Y)$$

The LHS is a decreasing function of  $r$ . In other words, a high level of idiosyncratic risk must be accompanied by a lower level of real interest rates in order to clear the savings market. It is straightforward to see that this equation has a unique solution for  $r$  which is positive since the LHS is monotonically decreasing in  $r$  and asymptotes to infinity as  $r \rightarrow 0$ . Another advantage of our CARA economy is that we can establish uniqueness of steady states. Toda (2017) shows that in a similar economy with CARA utility, there may exist multiple steady state interest rates if the labor endowment process is not i.i.d.



infinitely many bounded sequences  $\{\hat{Y}_t\}_{t=0}^{\infty}$  which satisfy (37). To construct such a sequence, take any initial condition  $\hat{Y}_0$ ; we have  $\hat{Y}_t = \Theta^{-t}\hat{Y}_0$  which is bounded and solves (37).

Next, we show that the cyclicity of income risk is the key factor which governs determinacy under a peg in our HANK economy. There are 3 cases to consider:

**Acyclical Income Risk** Acyclical income risk implies that  $\frac{d\sigma^2(Y^*)}{dY} = 0$ . Then  $\Theta = 1$ , and (37) is simply:

$$\hat{Y}_t = \hat{Y}_{t+1}$$

Any constant sequence  $\{\hat{Y}_t\}$  satisfies this equation. This is the standard result, familiar from the RANK model, that a peg generates indeterminacy. Since we consider an economy with fixed prices (for now), this indeterminacy only manifests in real variables.<sup>17</sup> In particular, a special case of acyclical risk is zero risk ( $\sigma^2(y) = 0$ ), in which case our economy is isomorphic to a RANK economy. However, even if idiosyncratic risk is present in the economy but is acyclical, it lowers the equilibrium real interest rate but does not affect determinacy. Thus both the RANK economy and a HANK economy with acyclical risk can feature self-fulfilling, permanent, bounded deviations from steady state under an interest rate peg. Heterogeneity and incomplete markets, per se, need not alter the determinacy properties of an economy.

**Procyclical Income Risk** Procyclical income risk implies that  $\frac{d\sigma^2(Y^*)}{dY} > 0$ , i.e. agents face higher income risk when aggregate economic activity is high. We just saw that if  $\frac{d\sigma^2(Y^*)}{dY} = 0$ , then  $\Theta = 1$ . Now introduce a small amount of procyclical income risk such that  $\Theta \in (0, 1)$ . Then the only bounded solution of (37) is  $\hat{Y}_t = 0$  for all  $t$ : the economy features determinacy even with a peg. Auclert et al. (2017) find similar results using numerical methods in a HANK economy with CRRA preferences. The intuition for this result is as follows: Suppose that at date 0, households conjecture that output at date 1 is going to be higher than steady state, i.e.  $\hat{Y}_1 > 0$ . This belief about higher output at date 1 affects aggregate demand at date 0 in two ways. The first is associated with the permanent income channel. Anticipating higher income at date 1, agents demand more consumption at date 0. This increased date 0 demand raises income at date 0, further raising consumption at date 0. Overall, via the permanent income channel, an increase in  $\hat{Y}_1$  would increase  $\hat{Y}_0$  one-for-one. However, there is also a second effect. Since income risk is procyclical, higher output at date 1 also increases the idiosyncratic risk agents face at date 1. This tends to reduce date 0 consumption, and thus output, via the precautionary savings channel. Overall, an increase in  $\hat{Y}_1$  tends to increase date 0 output less than one-for-one.

How, then, can the expectation of higher date 1 output be sustained? It must be that households foresee even higher output at date 2,  $\hat{Y}_2 > \hat{Y}_1 > \hat{Y}_0$ . But this in turn requires that period 3 output is higher still, and so on. Thus, any equilibrium in which  $\hat{Y}_0 > 0$  must be unbounded. Procyclical income risk is a stabilizing force in this economy. If agents anticipate good times ahead, they also expect higher risk in the future which dampens their demand for consumption. Conversely, if agents anticipate a recession in the future, they know that this will be accompanied by a reduction in risk, which dampens the fall in consumption. Thus, any bounded self-fulfilling booms or recessions are not possible in this economy even though the interest rate fails to adjust.

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<sup>17</sup>See Appendix C.1 for the determinacy properties of a RANK economy under an interest rate peg.



Notice that the case with procyclical income risk also yields a “discounted Euler equation”:

$$\hat{Y}_t = \Theta \hat{Y}_{t+1} - \gamma^{-1} \hat{r}_t \text{ with } \Theta \in (0, 1)$$

while the standard 3 equation RANK model yields an undiscounted Euler equation<sup>18</sup>

$$\hat{Y}_t = \hat{Y}_{t+1} - \gamma^{-1} \hat{r}_t$$

McKay et al. (2015) and McKay et al. (2017) argue that the RANK model feature an unrealistically strong dependence of current consumption on interest rates far in the future while incomplete market HANK economies can dampen this dependence. McKay et al. (2015) find numerically that outcomes in their HANK economy look similar to outcomes in the 3 equation model RANK model if the standard Euler equation is replaced with a “discounted Euler equation”. McKay et al. (2017) derive such an equation analytically in a stylized incomplete markets model with a degenerate wealth distribution.

Our derivation of a similar result clarifies two points. First, it suggests that procyclical income risk - rather than market incompleteness per se - is responsible for generating a discounted Euler equation. Indeed, the model in McKay et al. (2017) features strongly procyclical income risk: “low productivity households receive a constant transfer from the government while high productivity households receive all cyclical wages and dividends, minus the acyclical transfers. ” As a result, the income gap between high and low productivity households (equivalently the variance of individual income) is highest in booms and lowest in recessions.<sup>19</sup> Second, our HANK economy allows us to highlight not just that pro-cyclical income risk generates a discounted Euler equation, but also that this, in turn, generates determinacy under a peg.

**Countercyclical Income Risk** Finally we consider the case of countercyclical income risk. Countercyclical income risk implies that  $\frac{d\sigma^2(Y^*)}{dY} < 0$ , i.e. agents face higher income risk when aggregate economic activity is low, pushing  $\Theta$  above 1. Under a peg, an economy with countercyclical risk features “more” indeterminacy than one with acyclical risk, in the precise sense that there are multiple paths  $\{\hat{Y}_t\}$  which converge to the same steady state  $Y^*$ . In contrast, the acyclical income risk economy features multiple bounded paths but only one of these paths converges to  $Y^*$  - namely the one that starts there.

One natural force which generates countercyclical income risk is unemployment, as highlighted by Challe and Ragot (2016), Challe et al. (2017), Ravn and Sterk (2017a) and Ravn and Sterk (2017b) among others. Lower expected future output depresses aggregate demand today, not just because agents feel poorer, but also because they face a higher risk of becoming unemployed. While our setup does not feature unemployment, one can think of countercyclical income risk as an increase in the probability of drawing low labor endowments (i.e. an increase in the variance of labor endowment) in recessions. Regardless of its source, the mechanism through which countercyclical income risk strengthens indeterminacy is as follows. Suppose that at date 0, households contemplate a lower output than steady state at date 1,  $\hat{Y}_1 < 0$ . This directly depresses consumption via the permanent income channel as agents expect to be poorer in the future; on its own, this would tend to make date 0 output fall one-for-one with date 1 output. In addition,

<sup>18</sup>See Appendix C for details. Here  $\hat{r}_t$  denotes the log-deviation of real interest rates  $1 + r_t$  from their steady state level. Under fixed prices and a peg,  $\hat{r}_t = 0$ .

<sup>19</sup>In a similar vein, Werning (2015) argues that procyclical income risk rather than incomplete markets per se explains why forward guidance is less effective in McKay et al. (2015).

however, agents understand that lower date 1 output implies higher idiosyncratic risk at date 1. This further lowers consumption demand at date 0 via the precautionary savings channel. Overall,  $\hat{Y}_0$  falls more than one-for-one with date  $\hat{Y}_1$ . Thus, a large fall in  $\hat{Y}_0$  can be sustained by a smaller fall in date 1 output which can in turn be sustained by an even smaller fall in date 2 output. In this way, under a interest rate peg, the economy can feature self-fulfilling episodes of low output and high risk.<sup>20</sup>

The result that countercyclical income risk can make indeterminacy more likely resonates with the findings of [Ravn and Sterk \(2017b\)](#) who argue that even if monetary policy satisfies the Taylor principle, equilibrium in a HANK model may be indeterminate. Our analysis makes clear that this result is not a property of HANK models in general - indeed we have just seen that procyclical income risk makes indeterminacy less likely in such models. Instead, indeterminacy is more likely in HANK models only when income risk being countercyclical. Indeed, in the benchmark model of [Ravn and Sterk \(2017b\)](#), both employed and unemployed workers have constant acyclical income while the probability of becoming unemployed increases in recessions, generating counter-cyclical income risk.

Interestingly, while the feedback between unemployment risks and precautionary savings might be a powerful channel amplifying economic fluctuations, our results suggest that this feedback loop makes current consumption even more sensitive to future income and interest rate changes in economies with countercyclical income risk, relative to RANK models. Rather than a discounted Euler equation, counter-cyclical risk generates an “explosive Euler equation” potentially amplifying policy paradoxes such as the “forward guidance Puzzle” which are present in the standard RANK model, as we show in Section 5.1.<sup>21</sup>

### 4.3 An income-risk augmented Taylor Principle

The previous section made the extreme assumption that prices were fixed, preventing us from asking whether the Taylor principle - nominal rates should respond more than one-for one to inflation - is sufficient to ensure determinacy in HANK models. Recent work by [Ravn and Sterk \(2017b\)](#) have shown that this might not be the case. Conversely, [Auclert et al. \(2017\)](#) find that in HANK models, the Taylor Principle may not even be necessary for determinacy. In particular, they numerically find that when income risk is more procyclical, a violation of the standard Taylor principle need not imply indeterminacy. We now relax the assumption of fixed prices in order to explore what our model has to say about these questions.

Since our model can be solved in closed form, we can derive analytical conditions under which determinacy is achieved and what kind of monetary policy rules ensure determinacy. We derive a new Taylor Principle which crucially depends on the cyclicity of income risk. We refer to this as the *income-risk augmented Taylor Principle*. The intuition from the fixed price analysis holds more generally - procyclical income risk makes indeterminacy less likely, so a smaller Taylor rule coefficient below 1 suffices for determinacy, while countercyclical risk makes indeterminacy more likely, so a larger coefficient above 1 is required. Before we present the augmented Taylor principle, it is useful to make the following assumption.

**Assumption 1.** *Income risk is not too countercyclical.*

$$\Theta \in (0, \bar{\Theta})$$

<sup>20</sup>Equally, since our model is symmetric, it can feature self-fulfilling booms characterized by low risk.

<sup>21</sup>[Bilbiie \(2017a\)](#) finds a similar result in a TANK model in which the precautionary savings motive is absent - modifications of the RANK model which increase amplification also worsen the forward guidance puzzle.

where  $\bar{\Theta}$  is defined in Appendix D and is greater than 1 for  $\sigma_y^2$  sufficiently small.

In the general case of sticky but not perfectly rigid prices, linearizing around the zero inflation steady state yields the following 4-equation model:<sup>22</sup>

$$\hat{Y}_t = \Theta \hat{Y}_{t+1} - \frac{1}{\gamma} (i_t - \pi_{t+1}) - \Lambda \hat{\mu}_{t+1} \quad (38)$$

$$\hat{\mu}_t = \tilde{\beta} \hat{\mu}_{t+1} + \tilde{\beta} (i_t - \pi_{t+1}) \quad (39)$$

$$\hat{\pi}_t = \tilde{\beta} \pi_{t+1} + \kappa \hat{Y}_t \quad (40)$$

$$i_t = \Phi_\pi \pi_t \quad (41)$$

where we define  $\tilde{\beta} = \frac{1}{1+r}$  as the inverse of the steady state real interest rate and  $\Lambda = \gamma \mu^2 \sigma_y^2$ . The last two equations are familiar from the canonical RANK model: (40) is the standard linearized Phillips curve and finally (41) denotes the interest rate rule where  $i_t$  denotes the log deviation of  $1 + i_t$  from steady state while,  $\pi_t$  denotes the log deviation of inflation  $\Pi_t$  from steady state  $\Pi = 1$ . The difference between HANK and RANK is concentrated in the aggregate demand block, represented by the first two equations.

(38) is a dynamic IS equation which relates aggregate output today to output tomorrow. This equation is the analog of (37) in the general case where prices are not fixed (so inflation can move) and the nominal interest rate is not pegged (and allowed to move). The first term on the RHS is the same as in (37): whereas in the standard RANK model we would have  $\Theta = 1$ , here we can have  $\Theta < 1$  for pro-cyclical income risk and  $\Theta > 1$  for countercyclical income risk, as was described in the previous section. The second term on the RHS of (38) denotes the standard intertemporal channel - higher real interest rates induce households to seek a steeper path of consumption.

The third term on the RHS of (38) is new, and arises from interactions between the precautionary savings motive and variations in real interest rates. Recall that  $\hat{\mu}_t$  denotes the log-deviation of households' (common) MPC. When the MPC is high, individual consumption is more responsive to individual income, and so a given level of income risk translates into more volatile consumption, and thus a stronger precautionary savings motive. Thus when  $\hat{\mu}_{t+1}$  is high, households seek to reduce consumption today relative to tomorrow.  $\hat{\mu}_{t+1}$  in turn depends on the whole future path of interest rates as shown in (39). In this sense, the  $-\Lambda \hat{\mu}_{t+1}$  term in (38) and equation (39) represent a novel channel of monetary policy: tighter monetary policy increases the sensitivity of individual consumption to individual income shocks, raising consumption risk and reducing demand via the *precautionary savings* channel. This is in addition to the standard *intertemporal substitution* channel of monetary policy, represented by the second term in equation (38). If  $\sigma(Y) = 0$ ,  $\Theta = 1$ ,  $\Lambda = 0$ , and this economy reduces to the standard 3-equation RANK model.

The following Proposition describes local determinacy in this economy.

**Proposition 3** (An income-risk augmented Taylor Principle). *Consider the nominal interest rule:*

$$i_t = \Phi_\pi \pi_t$$

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<sup>22</sup> Appendix C derives the linearized model.

The following condition is necessary and sufficient for equilibrium to be locally determinate:

$$\Phi_\pi > 1 + \frac{\gamma}{\kappa} \left[ \frac{(1 - \tilde{\beta})^2}{(1 - \tilde{\beta}) + \gamma \tilde{\beta} \Lambda} \right] \underbrace{(\Theta - 1)}_{\substack{\text{cyclical} \\ \text{of} \\ \text{income} \\ \text{risk}}} \quad (42)$$

*Proof.* See Appendix D. □

If income risk is acyclical,  $\Theta = 1$ , then determinacy requires  $\Phi_\pi > 1$ , as in the RANK model. Thus the introduction of incomplete markets does not necessarily change the determinacy properties of equilibrium. Away from the acyclical risk benchmark, procyclical income risk tends to make determinacy more likely, while countercyclical risk makes it less likely, as we found in the fixed price limit. More precisely, if income risk is procyclical ( $\Theta < 1$ ), determinacy obtains even if the standard Taylor principle is violated and  $\Phi_\pi < 1$  - unlike in the 3-equation RANK model, raising nominal rates more than one for one with inflation is not *necessary* to ensure determinacy. A HANK economy with procyclical income risk contains a powerful additional stabilizing force: a higher path of output implies higher risk, which reduces demand and prevents the rise in output from being self-fulfilling.<sup>23</sup> Again, our results are consistent with Auclert et al. (2017) who find numerically that in a HANK economy with CRRA preferences, determinacy can be ensured with a lower Taylor rule coefficient when income risk is sufficiently procyclical.

Conversely, if income risk is countercyclical ( $\Theta > 1$ ), the standard Taylor principle  $\Phi_\pi > 1$  is not even *sufficient* for determinacy, unlike in the RANK model. Countercyclical risk creates an additional destabilizing force: lower output implies higher risk, reducing demand and allowing the fall in output to become self-fulfilling. Monetary policy must respond more aggressively to prevent such self-fulfilling fluctuations. This is consistent with the results of Ravn and Sterk (2017b), who show that a HANK economy with countercyclical risk arising from labor market frictions can experience local indeterminacy even when the standard Taylor principle holds. Our results emphasize that this is not a general property of HANK economies, but depends critically on the countercyclicality of income risk.

Finally, all else equal, a higher  $\Lambda$  weakens the extent to which pro- or counter-cyclical income risk warrants a deviation from the classic Taylor principle. A higher  $\Lambda$  makes monetary policy more powerful: smaller changes in interest rates have a larger effect on aggregate output, operating not just through the intertemporal channel but also by changing the passthrough from income to consumption shocks and affecting desired precautionary savings.<sup>24</sup>

## 5 Some RANK policy puzzles

Recent work has argued that RANK models make unrealistic predictions about the depth of recessions and deflation during liquidity trap episodes, the size of government spending multipliers at the zero lower bound, and the effects of forward guidance. These perceived shortcomings have often been explained

<sup>23</sup>This argument hinges on the fact that output is partially demand-determined in this economy with nominal rigidities. In the flexible price limit,  $\kappa \rightarrow \infty$ , we recover the standard Taylor principle  $\Phi_\pi > 1$ , whatever the cyclicalty of income risk.

<sup>24</sup>As mentioned elsewhere, this channel would be absent if we had considered an economy with zero liquidity, in which case the passthrough from income to consumption shocks is always equal to 1 and  $\hat{\mu}_t = 0$ .

by the notion that the intertemporal substitution channel is ‘too strong’ in the RANK model, in which the representative agent is essentially a permanent income consumer, and households are on their Euler equation at all point in time. This diagnosis suggests that moving towards a HANK model might reverse the three ‘unrealistic’ predictions described above. Our analytically tractable framework allows us to shed some light on how, if at all, market incompleteness might actually affect these predictions.

Turning first to forward guidance, the RANK model predicts that an interest rate cut far in the future has a greater effect than an interest rate cut today. [McKay et al. \(2015\)](#) argued that market incompleteness softens this prediction, generating behavior which can be approximated in terms of a ‘discounted Euler equation’, in which output today moves less than one for one with output in the far future. [Werning \(2015\)](#) conjectured that this result might be sensitive to the assumption of procyclical income risk. In our setting, we confirm both [McKay et al. \(2015\)](#)’s result and [Werning \(2015\)](#)’s conjecture. Introducing acyclical income risk into our New Keynesian model would actually amplify the forward guidance puzzle, making far future interest rate cuts even more effective in stimulating demand today; countercyclical income risk would make forward guidance more effective yet. However, procyclical income risk indeed creates a discounted Euler equation in our setting, and reduces the effectiveness of forward guidance.

Similarly, when we turn to the predictions of the NK model regarding government spending multipliers in a liquidity trap, procyclical income risk tends to reduce the perceived ‘puzzles’, while countercyclical risk amplifies them. The 3-equation RANK model predicts multipliers substantially above 1 at the ZLB, which are larger the longer the ZLB episode is expected to persist. Procyclical income risk reduces both the size of the multiplier, and its dependence on the duration of the liquidity trap. Countercyclical risk, however, increases both the size of the multiplier and its dependence on the length of the liquidity trap.

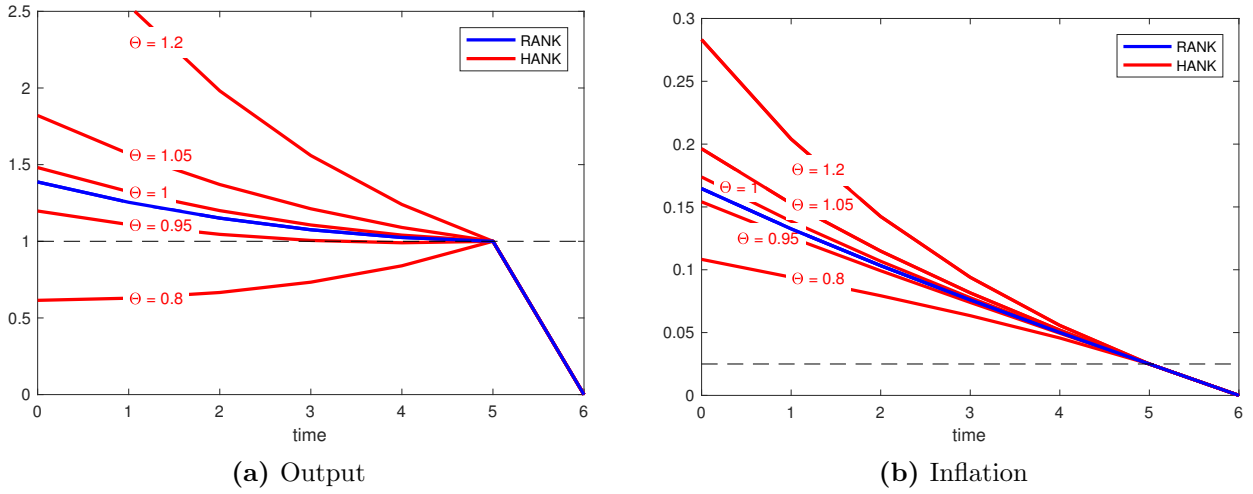
## 5.1 Forward guidance

As in [Del Negro et al. \(2015\)](#) and [McKay et al. \(2015\)](#), the effect of forward guidance in the RANK model is best illustrated with a simple experiment. Suppose the monetary authority announces at date  $t$  a temporary decline in the short-term nominal interest rate at date  $t + k$ :  $i_{t+k} = -\varepsilon < 0$ ,  $i_{t+s} = 0$  for all  $s \neq k$ . How does the effect of this shock on date  $t$  output depend on the horizon of forward guidance  $k$ ? In the RANK version of our economy  $\Theta = 1$ ,  $\Lambda = 0$ , and so iterating the IS equation forward yields

$$\hat{Y}_t = -\gamma^{-1} \sum_{k=0}^{\infty} (i_{t+k} - \pi_{t+k+1})$$

Under rigid prices ( $\kappa = 0$ ),  $\pi_{t+k} = 0$ , and nominal and real rates move by the same amount. In this case, whatever the horizon of forward guidance  $k$ , output and consumption increase by  $\varepsilon$  at date  $t$  and remain at this level until  $t + k$  - announcements about far future interest rates are *equally* as effective as contemporaneous changes in interest rates. Under sticky (not rigid) prices ( $\kappa > 0$ ), announcements about far future interest rates are even *more* effective than contemporaneous changes. Inflation can also be written as the present discounted value of future output gaps:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \tilde{\beta}^k \hat{Y}_{t+k}$$



**Figure 1.** Response of output and inflation to a unit drop in nominal interest rates 5 periods in the future. Blue line indicates RANK economy; red lines indicate HANK economies with lower lines corresponding to lower values of  $\Theta$ .

A larger  $k$  (more distant changes in policy) implies output will be high for longer, creating a larger increase in inflation, which in turn reduces real interest rates and stimulates output further. Note that in the RANK model, all of these effects can be understood in terms of the so called *intertemporal* channel of monetary policy. Lower real interest rates - caused both by a commitment to lower nominal rates and the resulting higher expected inflation - induce households to desire a declining path of consumption. With output at some date in the future fixed at its steady state level (implicitly by an active Taylor rule after date  $t + k$ ) a declining growth rate of consumption implies a higher level of consumption and hence output today.

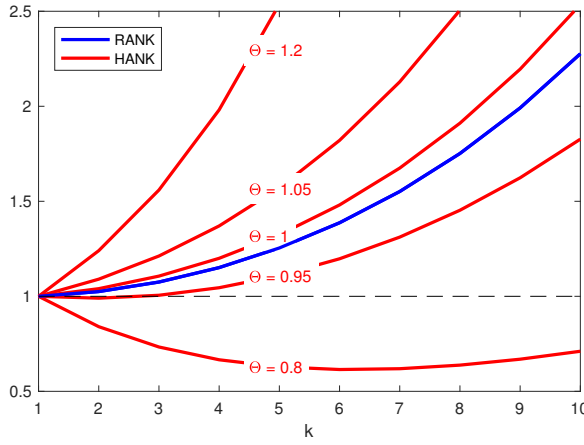
The blue line in Figure 1a shows the response of output to the announcement of a unit cut in nominal interest rates at date  $t + 5$ .<sup>25</sup> As can be seen in Figure 1a the response of output remains positive until the announced change in policy is enacted at date 5, with the largest effect on the day of announcement - date 0. Figure 1b shows that inflation behaves similarly, jumping at date 0 in anticipation of a sustained period of higher output and then gradually declining. Figure 2 shows how the impact effect of a future one-time cut in interest rates depends on the horizon of the policy change. Again the blue curve describes outcomes in a RANK economy. Announced future policy changes are more effective than contemporaneous policy changes. This phenomenon has been described as the *forward guidance puzzle* (Del Negro et al., 2015).

Now consider the same experiment in our HANK economy. Again, start with the case of rigid prices. Iterating the IS equation forward now yields

$$\hat{Y}_t = -\gamma^{-1} \sum_{k=0}^{\infty} \Theta^k i_{t+k} - \Lambda \sum_{k=0}^{\infty} \Theta^k \sum_{s=1}^{\infty} \tilde{\beta}^s i_{t+k+s} \quad (43)$$

where we have used the fact that  $\hat{\mu}_{t+k} = \sum_{s=0}^{\infty} \tilde{\beta}^{s+1} i_{t+k+s}$  with fixed prices. Consequently, one can express

<sup>25</sup>Henceforth, whenever we plot graphs, we will use the following parameters. We set the coefficient of absolute risk aversion  $\gamma = 1$ , discount factor  $\beta = 0.98$ , the slope of the Phillips curve  $\kappa = 0.025$ . In addition, we set the steady state level of idiosyncratic risk to  $\sigma_y^2 = 100$ . While this does not matter in the RANK economy, this level of steady state risk generates a steady state real interest rate of  $r = \tilde{\beta}^{-1} - 1 = 0.0126$  in the HANK economy. This exercise is not quantitative in nature and is only for illustration purposes.



**Figure 2.** Impact change in output as a function of horizon of forward guidance  $k$ . Monetary policy cuts nominal interest rates only at date  $t + k$

the sensitivity of output at date  $t$  to interest rate changes at date  $t + k$  as:<sup>26</sup>

$$\frac{d\hat{Y}_t}{di_{t+k}} = -\gamma^{-1}\Theta^k - \Lambda \sum_{s=1}^k \tilde{\beta}^s \Theta^{k-s} \quad (44)$$

Suppose first that idiosyncratic income risk is countercyclical, so  $\Theta > 1$ . With  $\Theta > 1$ ,  $\Theta^k$  is increasing in  $k$  and so is  $\sum_{s=1}^k \tilde{\beta}^s \Theta^{k-s}$ . Thus, announcements in the far future are *more* effective in stimulating demand than contemporaneous changes in policy even with fixed prices:

$$\left| \frac{d\hat{Y}_t}{di_{t+k+1}} \right| > \left| \frac{d\hat{Y}_t}{di_{t+k}} \right|, \forall k \geq 0$$

The *intertemporal channel* is operative in both the HANK and RANK economies. Lower interest rates in the future induce a declining path of consumption and an increase in consumption (and hence output) on impact, in both economies. However, the FGP is more severe in this HANK economy than in the RANK economy for two additional novel reasons which are associated with the *precautionary savings channel*. The first reason is that higher output in the future reduces idiosyncratic income risk faced by households. When households at date  $t + k - 1$  anticipate higher income and hence less risk at date  $t + k$ , they increase spending more than one for one, leading to a larger boom at date  $t + k - 1$ . This in turn leads to an even larger boom at date  $t + k - 2$ , and so forth. The second reason is that lower real interest rates make consumption less responsive to current changes in income, i.e.  $\hat{\mu} < 0$ . Thus, for a given path of income risk, consumption risk - which ultimately matters for precautionary savings - is lower, further boosting spending today. Overall, the forward guidance puzzle can be substantially amplified relative to the RANK

<sup>26</sup>In the case where  $\tilde{\beta} \neq \Theta$ , this can be written as

$$\frac{d\hat{Y}_t}{di_{t+k}} = -\gamma^{-1}\Theta^k - \beta\Lambda \frac{\Theta^k - \tilde{\beta}^k}{\Theta - \tilde{\beta}}$$

In the knife-edge case where  $\tilde{\beta} = \Theta$  the formula simply becomes:

$$\frac{d\hat{Y}_t}{di_{t+k}} = -(\Lambda k + \gamma^{-1})\Theta^k$$



economy even though markets are incomplete if income risk is countercyclical.

It is straightforward to see that if income risk was acyclical  $\Theta = 1$  while the first effect just described would be absent, the second effect would still be operative. Indeed with  $\Theta = 1$ , we can express  $\left| \frac{d\hat{Y}_t}{d\hat{i}_{t+k}} \right|$  as:

$$\left| \frac{d\hat{Y}_t}{d\hat{i}_{t+k}} \right| = \gamma^{-1} + \beta\Lambda \frac{1 - \tilde{\beta}^k}{1 - \tilde{\beta}}$$

which is strictly increasing in  $k$ . While forward guidance does not affect income risk - which does not depend on the level of aggregate economic activity in this case - it does reduce the sensitivity of consumption to income by lowering  $\mu_t$  and thus, boosts consumption by lowering desired precautionary savings.

Thus procyclical income risk ( $\Theta < 1$ ) is essential if incomplete markets are to resolve the forward guidance puzzle. This generates a discounted Euler equation as in [McKay et al. \(2017\)](#), and the response of date  $t$  output to a unit reduction in interest rates at  $t+k$  has two components, as can be seen from (44). The first term  $\gamma^{-1}\Theta^k$  is decreasing in  $k$ , which tends to make announcements about future interest rate cuts *less* effective in stimulating demand than contemporaneous cuts. This is not because households are not forward-looking, or because they anticipate being borrowing constrained in the future; our households are infinitely lived and unconstrained. Instead, it is because idiosyncratic income risk is procyclical. At date  $t+k-1$ , households anticipate that the cut in the policy rate at date  $t+k$  will increase output and average income, but they also expect this to generate an increase in idiosyncratic income risk. Consequently, while higher average income would induce them to increase consumption one for one, the increase in risk tends to reduce their consumption response, so spending at date  $t+k-1$  increases less than one for one. By the same logic, at date  $t+k-2$ , households increase spending less than one for one in response to a smaller expected increase in income at date  $t+k-1$ , and so forth.

Even if forward guidance increases idiosyncratic risk in HANK, reducing its stimulative effect, it also reduces the sensitivity of individual consumption to income (by lowering  $\mu_t$ ), increasing its stimulative effect. Thus, the overall effect on consumption risk and hence precautionary saving is ambiguous. For mildly procyclical income risk ( $\Theta$  close to 1), this second channel may still reduce desired precautionary savings on net, increasing the impact of interest rate cuts far in the future, and leaving the forward guidance puzzle unresolved. However, for sufficiently procyclical income risk  $\Theta$  low enough, desired precautionary savings are reduced in the net, thus reducing the effectiveness of interest rate cuts far in the future.

The cyclicity of income risk remains key even when prices are not perfectly rigid ( $\kappa > 0$ ). In this case, the forward guidance puzzle is more pronounced even in the RANK model due to the expected inflation channel which further reduces real interest rates; this carries over to HANK economies. Thus, even with moderately procyclical income risk, the puzzle may persist in the sense that announced changes have a larger effect on output than contemporaneous changes. Nonetheless, more procyclical income risk (lower  $\Theta$ ) still reduces both the size of the output response to announced policy changes, and the gains (if any) from future announcements. The red lines in [Figure 1a](#) illustrate the paths of output in response to a unit cut in nominal interest rates at date 5, for various values of  $\Theta$ . As can be seen, lower  $\Theta$  reduces the effect of forward guidance on output at all horizons while higher  $\Theta$  increases the effect; [Figure 1b](#) shows that the same is true for inflation.



## 5.2 Fiscal multipliers

The textbook 3-equation RANK model predicts large declines in output and inflation during a liquidity trap, when the natural rate of interest is negative and the nominal rate is constrained by the zero lower bound. Temporary increases in government spending during the liquidity trap have unusually large multipliers (substantially greater than 1) which grow with the duration of the liquidity trap. We now explore whether, and how, these predictions are modified in an incomplete markets model.

With nonzero government purchases and time varying  $\beta_t$ , our linearized Euler equation becomes:<sup>27</sup>

$$\hat{Y}_t = \Theta \hat{Y}_{t+1} - \gamma^{-1} (i_t - \pi_{t+1} - \rho_t) - \Lambda \hat{\mu}_{t+1} + \hat{G}_t - \hat{G}_{t+1} \quad (45)$$

where  $-\rho_t$  denotes the log-deviation of  $\beta_t$ . We consider a scenario in which  $\rho_t = -\bar{\rho} < 0$  for  $t < T$  and  $\rho_t = 0$  for  $t > T$ . Monetary policy is assumed to be constrained by the ZLB until date  $T$ : in log-deviations,  $i_t = -\iota < 0$  for  $t < T$ . Starting at date  $T$ , we assume that monetary policy implements the zero-output gap, zero inflation equilibrium (for example, with an appropriately specified active Taylor rule) so that  $i_t = 0$ . Furthermore, consider a fiscal policy that sets  $\hat{G}_t = g > 0$  for the duration of the liquidity trap ( $0 \leq t < T$ ) and zero thereafter. Then the fiscal multiplier can be expressed as:

$$\frac{\partial \hat{Y}_t}{\partial g} = \begin{cases} \Theta^{T-t-1} & \text{for } 0 \leq t < T \\ 0 & \text{else} \end{cases} \quad (46)$$

Equation (46) in the RANK case  $\Theta = 1$ , implies that the multiplier at each date during the liquidity trap is 1, independent of the duration of the trap  $T$ . Recall that in this special case, we assumed that prices were perfectly rigid implying that the expected inflation channel is not in operation. Incomplete markets, per se, need not change this prediction: when risk is acyclical, also  $\Theta = 1$  and the multiplier is 1 at all horizons, independent of the duration of the trap.

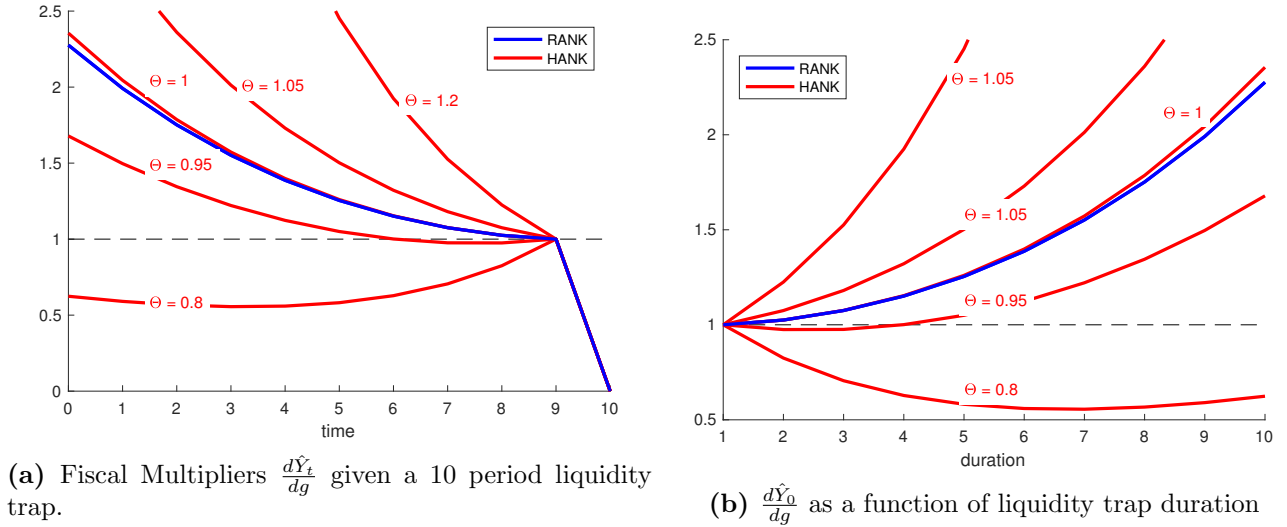
However, pro-cyclical income risk ( $\Theta < 1$ ) reduces the government spending multiplier below 1. Further, in this case, the multiplier is *decreasing* in the duration of the trap  $T - t$ ; equivalently, the multiplier becomes larger as the end of the trap (and end of the increased spending) approaches. Intuitively, when households anticipate higher government spending throughout the duration of the trap, they also expect higher aggregate income; but because idiosyncratic risk is procyclical, this carries with it a higher level of risk faced by each household, inducing them to spend less when real interest rates are fixed.

If on the other hand, risk is countercyclical,  $\Theta > 1$ , the multiplier is greater than 1 at all horizons and is increasing in the duration of the trap. It is important to realize that these large and increasing multipliers are not due to the expected inflation channel (Woodford, 2011; Eggertsson, 2011). Instead, they are due to the precautionary savings channel: higher future government spending increases aggregate output which reduces idiosyncratic risk. Anticipating this, households consume more for a fixed real interest rate.

The intuition broadly carries over to an environment with sticky rather than fixed prices. In addition to the forces mentioned above, the expected inflation channel is now back in play and generally tends to raise fiscal multipliers, especially in more protracted liquidity traps. However, it remains true that procyclical risk tends to dampen the fiscal multiplier and its dependence on the duration of the liquidity trap episode,

<sup>27</sup>See Appendix B for a derivation.

while countercyclical risk amplifies the fiscal multipliers further relative to the RANK model. Figure 3a plots the fiscal multiplier  $\frac{d\hat{Y}}{dg}$  for a liquidity trap that lasts 10 periods. As described above, we consider a fiscal policy which increases fiscal spending for the duration of the liquidity trap by some constant amount  $g > 0$ . Recall that in the fixed price scenario described above, the fiscal multipliers were identical in the RANK economy and the acyclical risk HANK economy. This is not literally true in the case when prices are somewhat flexible even when risk is acyclical. In the RANK economy, higher future government spending stimulates output which stimulates future inflation via the Phillips curve. When nominal interest rates are constrained by the zero lower bound, higher expected inflation reduces real interest rates, encouraging consumption and output. This expected inflation channel is also present in the HANK economy. In fact it is qualitatively strengthened in the presence of idiosyncratic risk. Lower real interest rates lower  $\mu_t$ , the sensitivity of household consumption to income shocks. Even when the volatility of income is fixed, as in the acyclical risk case, this reduces the volatility of household consumption in the future, further stimulating consumption via the precautionary savings channel. For the parameters used to plot Figure 3a this second effect is quantitatively small with the blue line corresponding to the RANK economy lying almost on top of the red line corresponding to the HANK economy with  $\Theta = 1$ . Instead, as in the fixed price case, the most important way in which incomplete markets affect the fiscal multiplier is again, via the cyclical risk of income. As can be seen in Figure 3a, more procyclical income risk (lower  $\Theta$ ) lowers the fiscal multiplier while countercyclical risk amplifies the multiplier relative to the RANK economy.



**Figure 3.** Fiscal Multipliers

Figure 3b illustrates how the duration of the liquidity trap episode affects the impact government spending multiplier  $d\hat{Y}_0/dg$ . In the RANK economy, the multiplier is greater than 1 and is increasing in the duration of the liquidity trap. Relative to the fixed price scenario where this multiplier was constant as a function of duration, here the multiplier is larger in a longer trap entirely because of the expected inflation channel. As can be seen in the figure, procyclical risk reduces the level of the impact multiplier. In fact, when income risk is sufficiently procyclical, the multiplier is decreasing in the duration of the trap. The effect of the expected inflation channel is counteracted by procyclical risk. A long episode of higher government spending financed by higher lumpsum taxes raises output. The longer the duration of the episode, the longer output remains high. This naturally is more inflationary since inflation is just

the sum of net discounted future output. This is the expected inflation channel. At the same time, if risk is procyclical, a longer episode of higher output raises risk more and thus depresses private spending more, *ceteris paribus* (see equation (24)). Thus, procyclical risk can cancel out or even overwhelm the expected inflation channel lowering impact multipliers. If instead, risk is countercyclical, the expected inflation channel and this precautionary channel work in the same direction making the multiplier larger and strongly increasing in the duration of the liquidity trap.

## 6 Some additional thoughts

### 6.1 Adding some hand-to-mouth agents

In the nascent HANK literature, it is sometime suggested that incomplete markets matter to the extent that households are or expect to be ‘off their Euler equation’, because this makes households less forward-looking and weakens the strength of the intertemporal channel. In our model, households are never off their Euler equation, yet the predictions of the model can differ substantially from that of the RANK model, due to the precautionary savings motive. This should not really be surprising in light of the earlier consumption literature. As is well known, in partial equilibrium, precautionary savings arises even for an unconstrained individual if the third derivative of the period utility function is positive, as it is in our CARA economy (Leland, 1968), (Sandmo, 1970).<sup>28</sup> More generally, our results suggest that the effect of incomplete markets on outcomes in a NK model depends less on whether households expect to be on their Euler equation in the future, and more on the extent to which expectations of higher (or lower) future output cause households to expect higher income risk, and to reduce their consumption.<sup>29</sup>

Our analysis has abstracted from liquidity constraints and MPC heterogeneity. This need not affect our conclusions concerning determinacy and the effects of forward guidance. Suppose only a fraction  $\eta$  of households in our economy were unconstrained, while a fraction  $1 - \eta$  are hand-to-mouth consumers with  $c^i, t = y_t^i$ . In this case, Appendix E shows that the aggregate Euler equation can still be written as:<sup>30</sup>

$$Y_t - \frac{1}{\eta}G_t - \frac{1-\eta}{\eta}T_t = -\frac{1}{\gamma} \ln \beta (1 + r_t) + Y_{t+1} - \frac{1}{\eta}G_{t+1} - \frac{1-\eta}{\eta}T_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \quad (47)$$

which is the same as equation (26) if  $\eta = 1$ . But even if  $\eta < 1$ , the response of output to changes in interest rates is the same as in the model without hand-to-mouth agents. To see this, set  $G_t = G$  and  $T_t = T$ : then (47) is identical to (26). For any  $\eta \in (0, 1)$ , the responsive of aggregate output to changes in current and future interest rates is the same as in the economy with  $\eta = 1$ . Thus, introducing a large number of hand-to-mouth agents who are “off their Euler equation” need not affect the strength of the intertemporal channel, the forward guidance puzzle and so forth.<sup>31</sup> Varying  $\eta$  from 1 to 0 increases the average MPC in

<sup>28</sup>See (Carroll and Kimball, 2001) for a thorough discussion of the relation between precautionary savings and liquidity in an individual decision problem.

<sup>29</sup>In continuous time, Achdou et al. (2017) show that even though all agents are on their Euler equation, the presence of a borrowing constraint generates precautionary savings motive even if the third derivative of period utility functions is not positive.

<sup>30</sup>Here without loss of generality we set  $\tau = 0$ , and so  $S_t = -G_t - T_t$ . Appendix E contains the general specification.

<sup>31</sup>This property holds not just in our model but also in TANK models with a fraction  $\eta$  of unconstrained agents and  $1 - \eta$  hand-to-mouth agents. To see this, suppose the consumption of the unconstrained household satisfies an euler equation:

$$u'(c_t^u) = \beta R_t E_t u'(c_{t+1}^u)$$

the economy from  $\mu_t$  to 1 but this has no effect on the response of aggregate variables to monetary policy.

While this result is by no means new to our setting (Werning, 2015), it might seem counterintuitive. Even if only an arbitrarily small fraction of agents are unconstrained ( $\eta \approx 0$ ) and can adjust borrowing and saving while everyone else is excluded from the bond market, the response of aggregate consumption to a change in interest rates is exactly the same as if everyone was on their Euler equation. One way to understand this, in the spirit of Kaplan et al. (2016), is as follows. Even though the direct effect of changes in interest rates is small since it only affects a small fraction ( $\eta \approx 0$ ) of people, this small increase in spending of the unconstrained increases the income of constrained households who have an MPC of 1. These constrained households spend all of this increase in income further increasing output and so forth. Thus, the small “direct” or “partial equilibrium” effect of monetary policy is exactly counterbalanced by a larger “indirect” or “general equilibrium” effect.<sup>32</sup> In this sense, while  $\eta$  matters greatly for the decomposition of the response of aggregate output to changes in interest rates into direct and indirect effects, this decomposition is completely uninformative about the net aggregate response in this example.

While the introduction of hand-to-mouth agents need not affect the response of output to interest rates, as we have just seen, it may in fact affect this response if fiscal surpluses respond to changes in real interest rates. For example, higher real interest rates would tend to worsen the government’s fiscal position which might be met by a cut in public transfers  $T$  (an increase in fiscal surpluses). If all agents were unconstrained, such a cut would have no effect on aggregate demand, but in the presence of hand-to-mouth agents, the cut in transfers would further reduce spending and output, amplifying the effect of a change in monetary policy. This is the main channel emphasized in Kaplan et al. (2016).

While hand-to-mouth households may or may not affect the response of aggregate variables to interest rates, they certainly affect the average MPC of the economy, and thus the government spending and transfer multipliers. In our baseline CARA-HANK economy with no hand-to-mouth agents, a lumpsum transfer today financed by lumpsum taxes in the future has no effect on output. In the economy with hand-to-mouth agents, (47) shows that this would increase output today since it  $\frac{1-\eta}{\eta} > 1$ . Higher transfers today increase the spending of hand-to-mouth agents one-for-one with the transfer, which in turn leads to second-round effects, and so forth. Equally, it is straightforward to see from (47) that while the balanced budget government spending multiplier<sup>33</sup> is the same as if  $\eta$  was 1, the deficit-financed government spending multiplier is larger than the balanced budget multiplier if  $\eta < 1$ .<sup>34</sup> The point we want to emphasize is that, whether or not a large fraction of agents are hand-to-mouth, the cyclicalty of income risk remains a key force determining how heterogeneity and incomplete markets affect aggregate outcomes.

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while hand-to-mouth households consume  $c_t^c = y_t$  and market clearing imposes that  $\eta c_t^u + (1 - \eta)c_t^c = y_t$ . It is immediate that

$$u'(y_t) = \beta R_t E_t u'(y_{t+1})$$

for any  $\eta \in (0, 1]$ . Assuming that the income of constrained and unconstrained individuals are differentially sensitive to aggregate output, can break this result as Bilbiie (2017b) discusses.

<sup>32</sup>Perhaps a simpler explanation is that, in equilibrium, since hand-to-mouth agents consume their income, unconstrained agents must do the same. One way or another, the magic of general equilibrium must ensure that output moves one-for-one with the consumption of unconstrained households. Thus aggregate output is described by the the Euler equation of the unconstrained households, just as is everyone was unconstrained. See the previous footnote for equations.

<sup>33</sup>As before we assume that the increase in spending is financed entirely by a fall in transfers  $T$  within period.

<sup>34</sup>See Mehrotra (2017) for a fuller discussion.

## 6.2 The pervasive importance of fiscal policy in HANK models

In our analysis, fiscal policy emerged as the key force mediating the effect of monetary policy on equilibrium outcomes. Procyclical income risk makes indeterminacy less likely, weakens the forward guidance puzzle and reduces the government spending multiplier in a liquidity trap. Countercyclical risk makes indeterminacy more likely, worsens the forward guidance puzzle and increases the fiscal multiplier in a liquidity trap. Holding fiscal policy fixed, the cyclicalities of income risk depends on structural characteristics of the economy: the cyclicalities of real wages, how unemployment risk behaves in a recession, etc.<sup>35</sup> But holding these structural features, as discussed in section 3.4, fiscal policy can change the cyclicalities of income risk or even wholly flip the sign, drastically changing the transmission mechanism of monetary policy.

In order to focus on this particular interaction between fiscal and monetary policy, we purposely abstracted from two other important interactions. The first concerns whether fiscal policy adjusts surpluses to remain solvent along any price path (Leeper, 1991). Whether fiscal policy is active or passive in this sense crucially determines the effects of monetary policy (Sims, 2011; Cochrane, 2017a). We abstract from this issue by assuming that the government always adjusts lumpsum taxes/transfers to ensure solvency. Even in this case, fiscal policy, by determining  $\Theta$ , can affect everything. More generally these two channels of monetary-fiscal interaction, while logically distinct, may interact. For example, if the government had limited ability to vary lump-sum taxes and instead adjusted labor income taxes  $\tau_t$  to ensure solvency, these movements in  $\tau_t$  could change the cyclicalities of income risk affecting the properties of equilibrium. The other type of monetary-fiscal interaction arises in models where incomplete markets break Ricardian Equivalence, e.g. Kaplan et al. (2016). As discussed in the previous section, even if fiscal policy is passive in the sense of Leeper (1991), changes in interest rates may force changes in lumpsum transfers to constrained agents, affecting demand.<sup>36</sup> In addition to these two types of fiscal-monetary interaction, we identify a third novel channel through which fiscal policy affects monetary policy – by determining  $\Theta$ .

## 7 Conclusion

The fast growing literature on HANK economies suggests that monetary policy works differently in such environments, relative to the standard RANK economy. Much of this literature has relied on computational methods which makes it hard (though not impossible) to understand precisely which features of incomplete markets drive the differences between particular HANK and RANK economies. Our goal has been to shed light on this question. To this end we presented a general HANK economy which can be solved in closed form. We are certainly not the first to do this, but our strategy complements the approaches pursued in the theoretical literature on HANK models in two ways. First, we do not rely on the zero liquidity limit. This allowed us to uncover a new channel through which monetary policy affects aggregate demand: tighter monetary policy increases the sensitivity of consumption to individual income shocks, raising consumption risk for a given level of income risk and reducing aggregate demand via the precautionary savings channel. This channel would be absent in a zero liquidity economy in which consumption risk is trivially the same

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<sup>35</sup>Indeed, as we have shown above, whereas McKay et al. (2015), McKay et al. (2017) present a HANK economy with procyclical risk, Ravn and Sterk (2017b), Challe (2017) present one with countercyclical risk resulting in very different predictions.

<sup>36</sup>Eusepi and Preston (2017) describe how even in an active-money passive-fiscal regime, the scale and composition of government debt can affect determinacy and the behavior of inflation when Ricardian equivalence fails due to imperfect knowledge.

as income risk. Second, our framework nests several different specifications of HANK economies in the theoretical and computational literature so far, and explains how differences in their conclusions arise from the differences in the cyclicalities of income risk in these economies.

[Werning \(2015\)](#) has already shown that the cyclicalities of income risk crucially affects the sensitivity of consumption to interest rates in a zero liquidity HANK economy. We confirm that this holds more generally. However, in addition we also show how cyclicalities of income risk affects a whole host of other issues such as determinacy, fiscal multipliers etc. To our knowledge our closed form characterization of determinacy properties of equilibria in terms of the cyclicalities of income risk is the first of its kind. [Auclert et al. \(2017\)](#) confirm this finding numerically in their CRRA economy with borrowing constraints indicating that our characterization is applicable more generally. Since the properties of equilibrium depend so closely on a low dimensional statistic - the cyclicalities of income risk - measuring this object empirically might help us better understand the monetary transmission mechanism in reality.<sup>37</sup>

## References

- Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll**, “Income and wealth distribution in macroeconomics: A continuous-time approach,” Technical Report, National Bureau of Economic Research 2017.
- Aiyagari, S. Rao**, “Uninsured Idiosyncratic Risk and Aggregate Saving,” *The Quarterly Journal of Economics*, 1994, 109 (3), 659–684.
- Ascari, Guido and Lorenza Rossi**, “Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo\*,” *The Economic Journal*, 2012, 122 (563), 1115–1141.
- Auclert, Adrien, Matt Rognlie, and Ludwig Straub**, “The Intertemporal Keynesian Cross,” Technical Report 2017.
- Basu, Susanto**, “Intermediate Goods and Business Cycles: Implications for Productivity and Welfare,” *American Economic Review*, June 1995, 85 (3), 512–531.
- Bhandari, Anmol, David Evans, and Mikhail Golosov**, “Risk and Monetary Policy in a New Keynesian Model,” Technical Report 2017.
- , —, —, and **Thomas J. Sargent**, “Public debt in economies with heterogeneous agents,” *Journal of Monetary Economics*, 2017, 91 (C), 39–51.
- Bilbiie, Florin**, “Limited asset markets participation, monetary policy and (inverted) aggregate demand logic,” *Journal of Economic Theory*, May 2008, 140 (1), 162–196.
- , “A Catch-22 for HANK Models: No Puzzles, No Amplification,” Technical Report December 2017.
- , “The New Keynesian Cross: Understanding Monetary Policy with Hand-to-Mouth Households,” CEPR Discussion Papers 11989, C.E.P.R. Discussion Papers April 2017.

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<sup>37</sup>A caveat is that, as mentioned above, the relevant measure of cyclicalities is not the *correlation* between aggregate output and idiosyncratic risk - as measured by [Storesletten et al. \(2004\)](#), [Guvenen et al. \(2014\)](#), among others - but the *effect* of an increase in aggregate output on idiosyncratic risk, holding other variables fixed.

- Blundell, Richard, Luigi Pistaferri, and Ian Preston**, “Consumption Inequality and Partial Insurance,” *American Economic Review*, December 2008, 98 (5), 1887–1921.
- Broer, Tobias, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg**, “The New Keynesian Transmission Mechanism: A Heterogenous-Agent Perspective,” CEPR Discussion Papers 11382, C.E.P.R. Discussion Papers July 2016.
- Bullard, James and Kaushik Mitra**, “Learning about monetary policy rules,” *Journal of Monetary Economics*, September 2002, 49 (6), 1105–1129.
- Caballero, Ricardo J.**, “Consumption puzzles and precautionary savings,” *Journal of Monetary Economics*, 1990, 25 (1), 113 – 136.
- Carroll, Christopher D. and Miles S. Kimball**, “Liquidity Constraints and Precautionary Saving,” Working Paper 8496, National Bureau of Economic Research October 2001.
- Challe, Edouard**, “Uninsured unemployment risk and optimal monetary policy,” Technical Report 2017.
- **and Xavier Ragot**, “Fiscal Policy in a Tractable Liquidity-Constrained Economy\*,” *The Economic Journal*, 2011, 121 (551), 273–317.
- **and –**, “Precautionary Saving Over the Business Cycle,” *Economic Journal*, February 2016, 126 (590), 135–164.
- **, Julien Matheron, Xavier Ragot, and Juan F. Rubio-Ramirez**, “Precautionary saving and aggregate demand,” *Quantitative Economics*, 2017, 8 (2), 435–478.
- Cochrane, John**, “Michelson-Morley, Fisher, and Occam: The Radical Implications of Stable Quiet Inflation at the Zero Bound,” in “NBER Macroeconomics Annual 2017, volume 32,” National Bureau of Economic Research, Inc, 2017.
- **, “The new-Keynesian liquidity trap,”** *Journal of Monetary Economics*, 2017, 92, 47 – 63.
- Debortoli, Davide and Jordi Galí**, “Monetary Policy with Heterogeneous Agents: Insights from TANK models,” Technical Report 2017.
- Del Negro, Marco, Marc Giannoni, and Christina Patterson**, “The forward guidance puzzle,” Staff Reports 574, Federal Reserve Bank of New York 2015.
- Eggertsson, Gauti B.**, “What Fiscal Policy is Effective at Zero Interest Rates?,” in “NBER Macroeconomics Annual 2010, Volume 25” NBER Chapters, National Bureau of Economic Research, Inc, July 2011, pp. 59–112.
- Eusepi, Stefano and Bruce Preston**, “Fiscal foundations of inflation: Imperfect knowledge,” CAMA Working Papers 2017-34, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University May 2017.
- Galí, J.**, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications, Second Edition*, Princeton University Press, 2015.



- Gornemann, Nils, Keith Kuester, and Makoto Nakajima**, “Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy,” International Finance Discussion Papers 1167, Board of Governors of the Federal Reserve System (U.S.) May 2016.
- Guvenen, Fatih, Serdar Ozkan, and Jae Song**, “The Nature of Countercyclical Income Risk,” *Journal of Political Economy*, 2014, 122 (3), 621–660.
- Hagedorn, Marcus, Iourii Manovskii, and Kurt Mitman**, “The Fiscal Multiplier,” 2017.
- Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante**, “Monetary Policy According to HANK,” NBER Working Papers 21897, National Bureau of Economic Research, Inc January 2016.
- Leeper, Eric M.**, “Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies,” *Journal of Monetary Economics*, February 1991, 27 (1), 129–147.
- Leland, Hayne E**, “Saving and Uncertainty: The Precautionary Demand for Saving,” *The Quarterly Journal of Economics*, 1968, 82 (3), 465–473.
- McKay, Alisdair and Ricardo Reis**, “Optimal Automatic Stabilizers,” NBER Working Papers 22359, National Bureau of Economic Research, Inc June 2016.
- and —, “The Role of Automatic Stabilizers in the U.S. Business Cycle,” *Econometrica*, January 2016, 84, 141–194.
- , **Emi Nakamura, and Jon Steinsson**, “The Power of Forward Guidance Revisited,” NBER Working Papers 20882, National Bureau of Economic Research, Inc January 2015.
- , —, and —, “The Discounted Euler Equation: A Note,” *Economica*, 2017, 84 (336), 820–831.
- Mehrotra, Neil**, “Fiscal Policy Stabilization: Purchases or Transfers?,” Technical Report 2017.
- Nakamura, Emi and Jon Steinsson**, “Monetary Non-neutrality in a Multisector Menu Cost Model,” *The Quarterly Journal of Economics*, 2010, 125 (3), 961–1013.
- Oh, Hyunseung and Ricardo Reis**, “Targeted transfers and the fiscal response to the great recession,” *Journal of Monetary Economics*, 2012, 59 (S), 50–64.
- Ravn, Morten O. and Vincent Sterk**, “Job uncertainty and deep recessions,” *Journal of Monetary Economics*, 2017, 90 (C), 125–141.
- and —, “Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach,” Discussion Papers 1633, Centre for Macroeconomics (CFM) 2017.
- Rotemberg, Julio J.**, “Monopolistic Price Adjustment and Aggregate Output,” *Review of Economic Studies*, 1982, 49 (4), 517–531.
- Sandmo, Agnar**, “The effect of uncertainty on saving decisions,” *The Review of Economic Studies*, 1970, 37 (3), 353–360.



- Sargent, Thomas J and Neil Wallace**, “Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule,” *Journal of Political Economy*, April 1975, 83 (2), 241–254.
- Sims, Christopher A.**, “Stepping on a rake: The role of fiscal policy in the inflation of the 1970s,” *European Economic Review*, January 2011, 55 (1), 48–56.
- Storesletten, Kjetil, ChrisI. Telmer, and Amir Yaron**, “Cyclical Dynamics in Idiosyncratic Labor Market Risk,” *Journal of Political Economy*, 2004, 112 (3), 695–717.
- Toda, Alexis Akira**, “Huggett economies with multiple stationary equilibria,” *Journal of Economic Dynamics and Control*, 2017, 84 (C), 77–90.
- Weil, Philippe**, “Precautionary Savings and the Permanent Income Hypothesis,” *The Review of Economic Studies*, 1993, 60 (2), 367–383.
- Werning, Ivan**, “Managing a Liquidity Trap: Monetary and Fiscal Policy,” NBER Working Papers 17344, National Bureau of Economic Research, Inc August 2011.
- , “Incomplete Markets and Aggregate Demand,” Working Paper 21448, National Bureau of Economic Research August 2015.
- Woodford, M.**, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.
- Woodford, Michael**, “Control of the Public Debt: A Requirement for Price Stability?,” Working Paper 5684, National Bureau of Economic Research July 1996.
- , “Simple Analytics of the Government Expenditure Multiplier,” *American Economic Journal: Macroeconomics*, January 2011, 3 (1), 1–35.

## Appendix

### A Deriving the consumption decision rule

Given a path of real interest rates  $\{r_t\}$  and aggregate output  $\{y_t\}$ , household  $i$ ’s problem is:

$$\begin{aligned} \max_{\{c_t^i\}} \quad & -\gamma^{-1} E_0 \sum_{t=0}^{\infty} \beta^t e^{-\gamma c_t^i} \\ \text{s.t.} \quad & c_t^i + \frac{1}{1+r_t} a_{t+1}^i = a_t^i + y_{i,t} \end{aligned} \tag{48}$$

$$y_t^i \sim N\left(y_t, \sigma_t^2(y_t)\right) \tag{49}$$

where  $a_t^i = A_t^i/P_t$  denotes the real value of household  $i$ ’s wealth at the beginning of date  $t$ . The optimal choices of the household can be summarized by the standard euler equation:

$$e^{-\gamma c_t^i} = \beta (1+r_t) E_t e^{-\gamma c_{t+1}^i} \tag{50}$$

Taking logs on both sides, the equation above can be written as:

$$-\gamma c_t^i = \ln \beta (1 + r_t) + \ln \mathbb{E}_t e^{-\gamma c_{t+1}^i} \quad (51)$$

Next, we guess that the consumption decision rule of household  $i$  takes the form:

$$c_t^i = \chi_t + \mu_t (a_t^i + y_t^i) \quad (52)$$

where  $\chi_t$  and  $\mu_t$  are deterministic processes that are common across all households. Given this guess, we can use the budget constraint (48) to write:

$$a_{t+1}^i = (1 + r_t) (1 - \mu_t) (a_t^i + y_t^i) - (1 + r_t) \chi_t \quad (53)$$

Using equation (53), one can express consumption at date  $t + 1$  as:

$$\begin{aligned} c_{t+1}^i &= \chi_{t+1} + \mu_{t+1} (a_{t+1}^i + y_{t+1}^i) \\ &= \chi_{t+1} + \mu_{t+1} [(1 + r_t) (1 - \mu_t) (a_t^i + y_t^i) - (1 + r_t) \chi_t + y_{t+1}^i] \end{aligned} \quad (54)$$

Then it is straightforward to see that:

$$E_t [-\gamma c_{t+1}^i] = -\gamma \chi_{t+1} - \gamma \mu_{t+1} [(1 + r_t) (1 - \mu_t) (a_{i,t} + y_{i,t}) - (1 + r_t) \chi_t + y_{t+1}] \quad (55)$$

$$E_t (-\gamma c_{t+1}^i - E_t [-\gamma c_{t+1}^i])^2 = \gamma^2 \mu_{t+1}^2 \sigma_{y,t+1}^2 \quad (56)$$

and using the property of log-normals:

$$\ln \mathbb{E}_t e^{-\gamma c_{t+1}^i} = -\gamma \chi_{t+1} - \gamma \mu_{t+1} [(1 + r_t) (1 - \mu_t) (a_{i,t} + y_{i,t}) - (1 + r_t) \chi_t + y_{t+1}] + \frac{\gamma^2 \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \quad (57)$$

Using this in the Euler equation (51) and matching coefficients:

$$\mu_t = \frac{\mu_{t+1} (1 + r_t)}{1 + \mu_{t+1} (1 + r_t)} \quad (58)$$

$$\chi_t [1 + \mu_{t+1} (1 + r_t)] = -\frac{1}{\gamma} \ln \beta (1 + r_t) + \chi_{t+1} + \mu_{t+1} y_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \quad (59)$$

which verifies the guess. Next, solving (59) forwards and using (58) yields equation (24) in the main text:

$$\chi_t = \sum_{s=1}^{\infty} Q_{t+s|t} \frac{\mu_t}{\gamma \mu_{t+s}} \ln \left[ \frac{1}{\beta (1 + r_{t+s-1})} \right] + \mu_t \sum_{s=1}^{\infty} Q_{t+s|t} y_{t+s} - \frac{\gamma \mu_t}{2} \sum_{s=1}^{\infty} Q_{t+s|t} \mu_{t+s} \sigma_{y,t+s}^2 \quad (60)$$

where  $Q_{t+s|t} = \prod_{k=0}^{s-1} \left( \frac{1}{1 + r_{t+k}} \right)$ . If real interest rates are constant at a level  $r > 0$ , then (58) implies that

$$\mu_t = \mu = \frac{r}{1 + r} > 0, \forall t \quad (61)$$

which confirms the claim in Corollary 1.

## B Deriving the Aggregate Euler Equation

In order to derive the aggregate Euler equation, we start with the individual consumption decision rules. Since  $\mu_t$  and  $\chi_t$  do not have  $i$  superscripts, i.e. they are the same across all households, independent of wealth of income. Thus, we can linearly aggregate this economy to get an *aggregate consumption function*:

$$\begin{aligned} c_t &= \int c_t^i di = \chi_t + \mu_t \int (a_t^i + y_t^i) di \\ &= \chi_t + \mu_t \left( \frac{B_t}{P_t} + y_t \right) \end{aligned} \quad (62)$$

where we have used asset market clearing  $\left( \frac{B_t}{P_t} = \int a_t^i di \right)$  and the fact that  $y_t = \int y_t^i di$  in the second line. Then, using (62) in (59):

$$\begin{aligned} \left[ c_t - \mu_t \left( \frac{B_t}{P_t} + y_t \right) \right] [1 + \mu_{t+1} (1 + r_t)] &= -\frac{1}{\gamma} \ln \beta (1 + r_t) + c_{t+1} - \mu_{t+1} \left( \frac{B_{t+1}}{P_{t+1}} + y_{t+1} \right) + \mu_{t+1} y_{t+1} \\ &\quad - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \end{aligned}$$

Next, using (18) and (58), we can rewrite the equation above as:

$$\begin{aligned} \left[ c_t - \mu_t \left( \mathcal{S}_t + \frac{B_{t+1}}{P_{t+1}} \frac{1}{1 + r_t} + y_t \right) \right] \left[ \frac{1}{1 - \mu_t} \right] &= -\frac{1}{\gamma} \ln \beta (1 + r_t) + c_{t+1} - \mu_{t+1} \left( \frac{B_{t+1}}{P_{t+1}} + y_{t+1} \right) \\ &\quad + \mu_{t+1} y_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} \end{aligned}$$

Recall that  $Y_t = \mathcal{S}_t + G_t + y_t$  and in general equilibrium,  $c_t + G_t = Y_t$ . Combining this information with the information above:

$$Y_t = -\frac{1}{\gamma} \ln \beta (1 + r_t) + Y_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2} + G_t - G_{t+1} \quad (63)$$

This is the same as equation (26) in the main text and (45) is the linearized version of this equation.

## C The 4 Equation HANK Model

In this section we present the linearized model. Recall that the aggregate dynamics of our HANK economy can be fully described by:

$$Y_t = -\frac{1}{\gamma} \ln \beta (1 + r_t) + Y_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma^2(Y_t)}{2} + G_t - G_{t+1} \quad (64)$$

$$\mu_t = \frac{\mu_{t+1} (1 + r_t)}{1 + \mu_{t+1} (1 + r_t)} \quad (65)$$

$$\Xi \Pi_t (\Pi_t - 1) = 1 - \theta \left( 1 - x_t^{\frac{1-\alpha}{\alpha}} \right) + \Xi (\Pi_{t+1} - 1) \Pi_{t+1} \left[ \frac{1}{1 + r_t} \frac{x_{t+1}}{x_t} \right] \quad (66)$$

$$1 + i_t = (1 + r) \Pi_t^{\Phi_\pi} \quad (67)$$

where  $1 + r_t = \frac{1+i_t}{\Pi_{t+1}}$  and  $Y_t = x_t - x_t^{\frac{1}{\alpha}}$ . Next, we linearize the model. In what follows,  $i_t, \pi_t$  and  $\hat{\mu}_t$  denote the log-deviations of  $1 + i_t, \Pi_t$  and  $\mu_t$  from their steady state values  $1 + i = 1 + r, \Pi = 1$  and  $\mu = \frac{r}{1+r}$  respectively.  $\hat{Y}_t$  and  $\hat{G}_t$  denote the deviations in levels of  $Y_t$  and  $G_t$  from the steady state levels  $Y = \left( \frac{1}{\theta-1} \right) \left( 1 - \frac{1}{\theta} \right)^{\frac{1}{1-\alpha}}$  and  $G$ . The linearized model is given by:

$$\hat{Y}_t = \Theta \hat{Y}_{t+1} - \gamma^{-1} (i_t - \pi_{t+1} - \rho_t) - \Lambda \hat{\mu}_{t+1} + \hat{G}_t - \hat{G}_{t+1} \quad (68)$$

$$\hat{\mu}_t = \tilde{\beta} (\hat{\mu}_{t+1} + i_t - \pi_{t+1}) \quad (69)$$

$$\pi_t = \tilde{\beta} \pi_{t+1} + \kappa \hat{Y}_t \quad (70)$$

$$i_t = \Phi_\pi \pi_t \quad (71)$$

where  $-\rho_t$  is the log-deviation of  $\beta$  from steady state,  $\Lambda = \gamma \mu^2 \sigma^2(Y^*)$ ,  $\Theta = 1 - \frac{\gamma \mu^2}{2} \frac{d\sigma^2(Y^*)}{dY_t}$ ,  $\tilde{\beta} = \frac{1}{1+r}$  is the inverse of the steady state real interest rate, and  $\kappa = \frac{\theta-1}{\Xi} \left[ \frac{\theta(1-\alpha)}{1-\theta(1-\alpha)} \right] \left( \frac{\theta}{\theta-1} \right)^{\frac{\alpha}{1-\alpha}}$ . Notice that as  $\Xi \rightarrow \infty$  (prices become perfectly rigid)  $\kappa \rightarrow 0$ .

### C.1 The 3 equation RANK model

The standard 3 equation RANK model is a special case of our 4 equation HANK model. In the case of a representative agent model,  $\sigma^2(Y) = \frac{d\sigma^2(Y)}{dY} = 0$  and  $\tilde{\beta} = \beta$ . Thus, in the RANK model,  $\Lambda = 0$  and  $\Theta = 1$ . Thus, we can write the system as:

$$\hat{Y}_t = \hat{Y}_{t+1} - \gamma^{-1} (i_t - \pi_{t+1} - \rho_t) + \hat{G}_t - \hat{G}_{t+1} \quad (72)$$

$$\pi_t = \tilde{\beta} \pi_{t+1} + \kappa \hat{Y}_t \quad (73)$$

$$i_t = \Phi_\pi \pi_t \quad (74)$$

Notice that the dynamics of  $\hat{\mu}_t$  given by  $\hat{\mu}_t = \tilde{\beta} (\hat{\mu}_{t+1} + i_t - \pi_{t+1})$  no longer affect the dynamics of  $\hat{Y}_t$  and  $\pi_t$ . Thus, we can ignore that equation in the RANK model.

## C.2 Determinacy properties of the RANK model under a peg

It is commonly known that if the monetary authority follows a nominal interest rate peg,  $\Phi_\pi = 0$  then the standard RANK model features local indeterminacy. In other words, with  $\Phi_\pi = 0$  there are multiple bounded path of  $\hat{Y}_t$  and  $\pi_t$  which satisfy equations (72)-(74). More generally, as long as  $|\Phi_\pi| < 1$ , the standard RANK model features local indeterminacy. See [Sargent and Wallace \(1975\)](#), [Bullard and Mitra \(2002\)](#) and [Galí \(2015\)](#) for a detailed exposition. This indeterminacy is generally associated with unanchored inflation. If prices are sticky  $\kappa > 0$ , the indeterminacy in prices also manifests itself in output. However, if prices are perfectly rigid, indeterminacy under a nominal peg manifests only in output since prices cannot move. To see this, notice that with perfectly rigid prices, the RANK model can be written as (with  $\hat{G}_t = 0$  wlog):

$$\hat{Y}_t = \hat{Y}_{t+1} \quad (75)$$

$$\pi_t = 0 \quad (76)$$

As can be clearly seen, any constant level of output is consistent with a bounded equilibrium in this case. With fixed prices, a fixed nominal interest rate translates into a fixed real interest rate. In this extreme case, output is demand determined expectations of higher income in the future are self-fulfilling and raise the level of income today by the same amount.

## D Determinacy in HANK

Setting  $\hat{G}_t = \rho_t = 0$ , equations (68)-(71) can be written in matrix form as

$$\underbrace{\begin{bmatrix} \Theta & \frac{1}{\gamma} & -\Lambda \\ 0 & \tilde{\beta} & 0 \\ 0 & -\tilde{\beta} & \tilde{\beta} \end{bmatrix}}_A \underbrace{\begin{bmatrix} \hat{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{\mu}_{t+1} \end{bmatrix}}_B = \underbrace{\begin{bmatrix} 1 & \frac{\Phi_\pi}{\gamma} & 0 \\ -\kappa & 1 & 0 \\ 0 & -\tilde{\beta}\Phi_\pi & 1 \end{bmatrix}}_B \underbrace{\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \\ \hat{\mu}_t \end{bmatrix}}_B$$

and so

$$A^{-1}B = \begin{bmatrix} \frac{1}{\theta} + \frac{\kappa(\gamma^{-1}-\Lambda)}{\beta\theta} & \frac{(\gamma^{-1}-\Lambda)(\Phi_\pi-\beta^{-1})}{\theta} & -\frac{\Lambda}{\beta\theta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} - \Phi_\pi & \frac{1}{\beta} \end{bmatrix}$$

The characteristic polynomial of this system is:

$$\mathcal{P}(z) = \frac{1 + \gamma^{-1}\kappa\Phi_\pi}{\tilde{\beta}^2\Theta} - z^3 + \frac{\tilde{\beta}^2 + 2\tilde{\beta}\Theta + \tilde{\beta}\kappa(\gamma^{-1} - \Lambda)}{\tilde{\beta}^2\Theta} z^2 - \frac{2\tilde{\beta} + \tilde{\beta}\kappa\Phi_\pi(\gamma^{-1} - \Lambda) + \Theta + \kappa\gamma^{-1}}{\tilde{\beta}^2\Theta} z \quad (77)$$

Determinacy requires that all the roots of  $\mathcal{P}$  lie within the unit circle. Clearly we have  $\lim_{z \rightarrow -\infty} \mathcal{P}(z) = +\infty$ ,  $\lim_{z \rightarrow +\infty} \mathcal{P}(z) = -\infty$ . Under the assumption that  $\gamma^{-1} > \Lambda$ ,  $\mathcal{P}(z) > 0 \forall z \leq 0$ . Thus, determinacy requires that  $\mathcal{P}(1) > 0$ . Otherwise, the polynomial would have at least one real root within the unit circle. Thus

we need

$$\mathcal{P}(1) = \frac{(1-\Theta)(1-\tilde{\beta})^2}{\tilde{\beta}^2\Theta} + \frac{\kappa\gamma^{-1}(1-\tilde{\beta}) + \tilde{\beta}\kappa\Lambda}{\tilde{\beta}^2\Theta}(\Phi_\pi - 1) > 0$$

which implies

$$\Phi_\pi > 1 + \frac{1}{\kappa} \left[ \frac{\gamma(1-\tilde{\beta})^2}{(1-\tilde{\beta}) + \gamma\tilde{\beta}\Lambda} \right] (\Theta - 1) \quad (78)$$

Under our maintained assumptions, this condition is always necessary for determinacy. Next, we show that it is sufficient for determinacy under the following assumptions.

**Assumption 2.**  $2\tilde{\beta} - 1 - \tilde{\beta}^3 > 0$ ,  $\Lambda < \gamma^{-1}$ , and income risk is not too countercyclical, i.e.

$$\Theta - 1 < \min \left\{ \frac{(1-\tilde{\beta})(1-\tilde{\beta}^2 + \gamma^{-1}\kappa) + \tilde{\beta}(\gamma^{-1}\kappa - \tilde{\beta}^2)\gamma\Lambda}{\tilde{\beta} - \frac{(1-\tilde{\beta})^2}{(1-\tilde{\beta}) + \tilde{\beta}\gamma\Lambda} - \tilde{\beta}^3(1-\gamma\Lambda)}, \frac{1}{\tilde{\beta}\Lambda} - 1 + \frac{\gamma^{-1}\kappa}{\tilde{\beta}^2}, \frac{1-\tilde{\beta} + \gamma^{-1}\kappa}{\tilde{\beta} - \frac{(1-\tilde{\beta})^2}{1-\tilde{\beta} + \tilde{\beta}\gamma\Lambda}} \right\}$$

Note that when  $\Lambda = 0$ , this last assumption reduces to

$$\Theta - 1 < \min \left\{ \frac{(1-\tilde{\beta})(1-\tilde{\beta}^2 + \gamma^{-1}\kappa)}{2\tilde{\beta} - 1 - \tilde{\beta}^3}, \frac{1-\tilde{\beta} + \gamma^{-1}\kappa}{2\tilde{\beta} - 1} \right\}$$

which is strictly positive, under our other assumptions. Thus for  $\Lambda$  sufficiently close to zero, our risk-adjusted Taylor principle is sufficient for determinacy even under moderately countercyclical income risk. The other assumptions in 2 are satisfied for reasonable parameter values of the discount factor  $\beta$  as we now show.

**Lemma 1.** *If  $\beta > \frac{\sqrt{5}}{2} - \frac{1}{2}$ , then  $2\tilde{\beta} - 1 - \tilde{\beta}^3 > 0$  and  $\Lambda < \gamma^{-1}$ .*

*Proof.* Recall that we define  $\tilde{\beta} = \frac{1}{1+r}$ ,  $\Lambda = \gamma \left( \frac{r}{1+r} \right)^2 \sigma_y^2$ , and  $r$  solves

$$\frac{(1+r)^2 \ln \left[ \frac{1}{\tilde{\beta}(1+r)} \right]}{\gamma^2 r^2} = \frac{1}{2} \sigma_y^2$$

It is immediate that  $\tilde{\beta} \in (\beta, 1)$ , so  $\tilde{\beta} > \beta > \frac{\sqrt{5}}{2} - \frac{1}{2}$  and  $2\tilde{\beta} - 1 - \tilde{\beta}^3 > 0$ . Using the definition of  $\Lambda$  and the fact that  $r > 0$ , and rearranging,

$$\gamma\Lambda < 2 \ln \left( \frac{1}{\tilde{\beta}} \right) < 2 \ln \left( \frac{2}{\sqrt{5}-1} \right) < 1$$

So we are done. □

With these assumptions in hand, we can provide sufficient conditions for determinacy. Lemma 2 provides sufficient conditions for a general cubic polynomial to have all its roots outside the unit circle, as required for determinacy. Lemma 3 then concludes by showing that these conditions are satisfied if Assumption 2 holds and the risk-adjusted Taylor principle (78) is satisfied.

**Lemma 2.** *Consider the characteristic polynomial*

$$\mathcal{P}(z) = -z^3 + A_2 z^2 + A_1 z + A_0 = (z_1 - z)(z_2 - z)(z_3 - z)$$

Suppose  $A_2 > 0, A_1 < 0, A_0 > 0$ . Then the following two conditions are sufficient<sup>38</sup> for  $\mathcal{P}$  to have three roots outside the unit circle:

$$A_0^2 - A_0 A_2 - A_1 - 1 > 0 \quad (79)$$

$$-1 + A_2 + A_1 + A_0 > 0 \quad (80)$$

$$A_1 > 1 \quad (81)$$

*Proof.* Assume the conditions hold. Since  $\mathcal{P}(1) = -1 + A_2 + A_1 + A_0 > 0$  and  $\lim_{z \rightarrow +\infty} \mathcal{P}(z) = -\infty$ , there is at least one real root above 1; let this be  $z_3$ . Either  $z_1, z_2$  are complex conjugates, or they are both real. Suppose they are complex conjugates. Note that

$$(z_1 z_2 - 1)(z_2 z_3 - 1)(z_3 z_1 - 1) = A_0^2 - A_0 A_2 - A_1 - 1 > 0$$

$z_2 z_3 - 1$  and  $z_3 z_1 - 1$  are complex conjugates, so their product is a positive real number. So we must have  $z_1 z_2 = |z_1| = |z_2| > 1$ , i.e. all eigenvalues lie outside the unit circle in this case. Suppose then that  $z_1, z_2$  are both real.  $\mathcal{P}(0) = A_0 > 0$ , so  $\mathcal{P}$  has either two real roots in  $(0, 1)$  or none (in which case we are done, since it has no negative real roots). Suppose  $z_1, z_2 \in (0, 1)$ . By (80), we have

$$z_3^2(z_1 z_2 - 1)(z_1 - z_3^{-1})(z_2 - z_3^{-1}) > 0$$

$z_3^2(z_1 z_2 - 1) < 0$  by assumption, so we must have (letting  $z_1 < z_2$  without loss of generality)

$$0 < z_1 < z_3^{-1} < z_2 < 1$$

So  $z_1 z_2 z_3 < z_3^{-1} z_2 z_3 = z_2 < 1$ . Since we have assumed  $A_0 = z_1 z_2 z_3 < 1$ , this case is ruled out. Then it must be that all eigenvalues lie outside the unit circle.  $\square$

Next, we show that under assumption 2, the risk-adjusted Taylor principle (78) is sufficient to ensure that the conditions in Lemma 2 obtain.

**Lemma 3.** *Suppose Assumption 2 holds and  $\Phi_\pi$  satisfies (78). Then  $A^{-1}B$  has three eigenvalues outside the unit circle.*

*Proof.* Suppose (78) holds: then (80) holds. Given our assumptions, we have  $A_0, -A_1, A_2 > 0$ . It only remains to show (79) and (81). Using the definition of the characteristic polynomial in (77), some algebra

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<sup>38</sup>The first two conditions are also necessary.

yields

$$\begin{aligned}
A_0^2 - A_0 A_2 - A_1 - 1 &= \underbrace{\beta^{-1} \left( \frac{1 + \gamma^{-1} \kappa \Phi_\pi}{\tilde{\beta} \Theta} - 1 \right)}_{B_1} \underbrace{\left( \frac{1 + \gamma^{-1} \kappa \Phi_\pi}{\tilde{\beta}^2 \Theta} - \frac{\tilde{\beta} + \Theta + \kappa(\gamma^{-1} - \Lambda)}{\tilde{\beta} \Theta} + \beta \gamma (\gamma^{-1} - \Lambda) \right)}_{B_2} \\
&\quad + \underbrace{\frac{\gamma}{\beta \Theta} + \gamma \left( \frac{\gamma^{-1} \kappa}{\tilde{\beta}^2 \Theta} - 1 \right) \Lambda}_{B_3} > 0
\end{aligned}$$

We will show  $B_1, B_2, B_3 > 0$ . First take  $B_2$ . Multiplying through by the positive number  $\tilde{\beta}^2 \Theta$  and using the lower bound on  $\Phi_\pi$  given by (78), we have

$$\tilde{\beta}^2 \Theta B_2 = (1 - \tilde{\beta})[\kappa \gamma^{-1} + 1 - \tilde{\beta}^2] + \tilde{\beta}(\gamma^{-1} \kappa - \tilde{\beta}^2) \gamma \Lambda - (\Theta - 1) \left[ \tilde{\beta} - \frac{(1 - \tilde{\beta})^2}{(1 - \tilde{\beta}) + \tilde{\beta} \gamma \Lambda} - \tilde{\beta}^3 (1 - \gamma \Lambda) \right]$$

The term in square brackets is minimized when  $\Lambda = 0$ , in which case it equals  $2\tilde{\beta} - 1 - \tilde{\beta}^3 > 0$ . So it is positive, and we will have  $B_2 > 0$  provided that

$$\Theta - 1 < \frac{(1 - \tilde{\beta})[\kappa \gamma^{-1} + 1 - \tilde{\beta}^2] + \tilde{\beta}(\gamma^{-1} \kappa - \tilde{\beta}^2) \gamma \Lambda}{\tilde{\beta} - \frac{(1 - \tilde{\beta})^2}{(1 - \tilde{\beta}) + \tilde{\beta} \gamma \Lambda} - \tilde{\beta}^3 (1 - \gamma \Lambda)}$$

which is guaranteed by Assumption 2. Next we show  $B_3 > 0$ . We have

$$B_3 = \frac{\gamma}{\beta} + \frac{\kappa \Lambda}{\beta^2} - \gamma \Lambda \Theta$$

which will be positive provided that

$$\Theta < \frac{1}{\tilde{\beta} \Lambda} + \frac{\gamma^{-1} \kappa}{\tilde{\beta}^2}$$

as ensured by Assumption 2. Next we show  $B_1 > 0$ . Given (78),

$$B_1 = \frac{1 + \gamma^{-1} \kappa \Phi_\pi}{\tilde{\beta} \Theta} - 1 > \frac{1}{\tilde{\beta} \Theta} \left[ 1 + \gamma^{-1} \kappa + \left( \frac{(1 - \tilde{\beta})^2}{(1 - \tilde{\beta}) + \gamma \tilde{\beta} \Lambda} \right) (\Theta - 1) - \beta \Theta \right]$$

which is positive provided that

$$\Theta - 1 < \frac{1 - \tilde{\beta} + \gamma^{-1} \kappa}{\tilde{\beta} - \frac{(1 - \tilde{\beta})^2}{1 - \tilde{\beta} + \gamma \tilde{\beta} \Lambda}}$$

as ensured by Assumption 2. This establishes that (79) is satisfied. To apply Lemma 2 we only need to check condition (81), i.e.

$$A_0 = \frac{1 + \gamma^{-1} \kappa \Phi_\pi}{\tilde{\beta}^2 \Theta} > 1$$

Since  $B_1 = \frac{1 + \gamma^{-1} \kappa \Phi_\pi}{\tilde{\beta} \Theta} - 1 > 0$  and  $\tilde{\beta} \in (0, 1)$ , this is immediate. So we are done.  $\square$



## E A Model with some hand-to-mouth agents

In this section, we augment our standard model to accommodate a mass of agents who are hand-to-mouth, i.e. for these agents  $c_t^i = y_t^i$  for all  $t$ . Relative to our baseline model, we now have a fixed fraction  $1 - \eta \leq 1$  of agents who are hand-to-mouth while the rest are not (who we call unconstrained agents). Proposition 1 is still valid for these unconstrained agents and hence their consumption decision rules can be described by (21). Then, the average consumption of unconstrained agents  $c_t^u$  at date  $t$  can be written as:

$$c_t^u = \chi_t + \mu_t (a_t + y_t)$$

Asset market clearing is now given by  $\eta a_t = \frac{B_t}{P_t}$  since only the unconstrained agents hold assets. Imposing asset market clearing, we can write the above as:

$$c_t^u = \chi_t + \mu_t \left( \frac{1}{\eta} \frac{B_t}{P_t} + y_t \right)$$

Using this expression in (59) and using (18) and (58):

$$\left[ c_t^u - \mu_t \left( \frac{\mathcal{S}_t}{\eta} + y_t \right) \right] \left( \frac{1}{1 - \mu_t} \right) = -\frac{1}{\gamma} \ln \beta (1 + r_t) + c_{t+1}^u - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2}$$

In GE, we have  $Y_t = \mathcal{S}_t + G_t + y_t$  and  $\eta c_t^u + (1 - \eta) c_t^c = Y_t - G_t$  where  $c_t^c = y_t$  is the average consumption of constrained households. Substituting these into the equation above yields equation (47) in the main text:

$$Y_t - G_t + \frac{1 - \eta}{\eta} \mathcal{S}_t = -\frac{1}{\gamma} \ln \beta (1 + r_t) + Y_{t+1} - G_{t+1} + \frac{1 - \eta}{\eta} \mathcal{S}_{t+1} - \frac{\gamma \mu_{t+1}^2 \sigma_{y,t+1}^2}{2}$$