# The Side Effects of Safe Asset Creation*

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**Abstract**

We present a model with incomplete markets in order to understand the costs and benefits of increasing government debt in a low interest rate environment. Higher idiosyncratic risk increases the demand for safe assets and can even lower real interest rates below zero. A fiscal authority can issue more debt to meet this increased demand for safe assets and arrest the decline in real interest rates. While such a policy succeeds in keeping real rates above zero, it comes at a cost as higher real interest rates can lead to permanently lower investment. However, in an environment with nominal rigidities and a zero bound on nominal rates, policymakers may not have a choice. On the one hand, without creating additional safe assets, constrained monetary policy is powerless to combat higher unemployment. On the other hand, creating safe assets can make monetary policy potent again, allowing policymakers to lower unemployment, but such a policy only shifts the malaise elsewhere in the economy where it manifests itself as a permanent investment slump.

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# 1 Introduction

The past three decades have been marked by a secular decline in real interest rates in the United States and other advanced economies. A growing literature has argued that this decline can be attributed to an increased demand of safe assets, which pushed the natural rate of interest well below zero (Caballero and Farhi (2016)). This in turn caused the zero lower bound to bind in many advanced economies, rendering conventional monetary policy unable to prevent a deep and lasting recession. At the same time, an empirical literature has documented that safe assets such as U.S. Treasury debt enjoy a safety premium, which is responsive to the aggregate supply of debt (Krishnamurthy and Vissing-Jorgensen (2012)). This suggests that by increasing the supply of safe debt, fiscal authorities may be able to reduce the safety premium, satiate the demand for safe assets, and raise the natural rate of interest, allowing monetary policy to regain its potency. Can governments increase the supply of safe assets in this way? And if they can, should they?

In models where Ricardian Equivalence holds, the supply of government debt does not affect equilibrium prices and allocations unless it is assumed to provide nonpecuniary benefits; while such a modelling strategy is convenient, it does not provide satisfactory answers to the questions we are interested in. Thus in order to answer these questions, we present an analytically tractable incomplete markets model with risky capital, safe government debt, and nominal rigidities. The economy has a simple two period overlapping generations (OLG) structure. Young households supply labor inelastically, pay lump sum taxes (which may be negative) and invest in both risky capital and safe government debt. Each old household operates a firm, hiring labor to produce output with its capital. Old households face uninsurable productivity risk. This creates a risk premium between the marginal product of capital and the return on short term government debt.

In order to understand the forces at play, we first study a model without nominal rigidities. An increase in idiosyncratic risk reduces real interest rates, as households attempt to substitute away from risky capital towards safe debt. A sufficiently large increase in risk can push real interest rates below zero. An increase in government debt can offset the decline in real interest rates by satiating the demand for safe assets. However, this comes at the cost of crowding out investment in physical capital. Absent nominal rigidities, this cost is so strong that it is never optimal to prevent real interest rates from falling below zero.

We then introduce downward nominal wage rigidity and a zero lower bound on nominal interest rates. Under conventional monetary policy rules, a (permanently) negative natural rate of interest generates a (permanent) recession, as nominal rates cannot fall sufficiently to restore full employment. This decline in employment in turn causes a persistent investment slump. An increase in the supply of safe assets can raise the natural rate of interest above zero, allowing conventional monetary policy to preserve full employment. As in the model without nominal rigidities, this comes at the cost of crowding out investment. In this sense, the costs of a risk-induced recession may persist even after the economy has returned to full employment, taking the form of sluggish investment and low labor productivity.

**Related Literature** Our investigation relates to a large recent literature which studies the macroeconomic consequences of the secular decline in safe rates of interest and the supply of safe assets. Most relevant to our work, Caballero and Farhi (2016) study an endowment economy in which safe asset shortages can generate a persistent recession. Relative to their paper, our contribution is to consider the interaction between safe asset shortages and investment in physical capital. While one might have thought that giving households access to a physical storage technology prevents saving gluts from having any adverse effects, this turns out not to be the case when investment in physical capital is risky. Our focus on investment also reveals a new tradeoff. In Caballero and Farhi (2016), increasing government debt can prevent a safety trap without any adverse consequences. In our model, it comes at the cost of crowding out investment in physical capital.

Our approach to modeling safe assets differs from Gorton and Ordonez (2013), for whom a 'safe asset' is an information-insensitive asset. Since individuals are willing to trade information-insensitive assets without fear of adverse selection, such assets circulate widely. The liquidity or moneyness of these assets implies the existence of a convenience yield, so that the pecuniary return on safe assets lies below the yield on comparable securities without these properties. The existence of such a convenience yield is documented empirically by Krishnamurthy and Vissing-Jorgensen (2012, 2015). One important strand of this literature focuses on the financial stability consequences of low real interest rates, and the role of public debt management in regulating these. Greenwood et al. (2016) and Woodford (2016) ask whether a central bank should increase its supply of short term claims in order to promote financial stability, by reducing the private sector's tendency to engage in socially excessive maturity transformation. Like this literature, our model predicts that government debt trades at a spread below other assets, and this spread varies with the supply of Treasury debt. However, our model attributes this spread entirely to the risk properties of debt - we abstract altogether from liquidity.

In this respect, our paper is related to a growing literature discussing the macroeconomic consequences of the liquidity properties of government debt. A large literature, following Woodford (1990), has studied the role of government debt in relaxing private borrowing constraints. In a model without nominal rigidities, Angeletos et al. (2016) study the optimal provision of government debt when debt provides liquidity services and taxes are distortionary. Their Ramsey planner trades off the benefits of increasing debt and relaxing financial frictions against the cost of raising interest rates, tightening the government's borrowing constraint. We focus on a different role for government debt - providing a safe asset, rather than liquidity services - and a diifferent set of tradeoffs. Our government has access to lump sum taxes, so relaxing the government budget constraint is irrelevant. The cost of issuing more government debt is instead that this crowds out investment; a potential benefit is that it avoids liquidity traps. In this regard, our result is reminiscent of Yared (2013) who shows that while increasing government debt can in principle relax private borrowing constraints, it is not always optimal to do so since this distorts investment decisions.

The role of government debt in providing liquidity services becomes even more important in the presence of nominal rigidities. Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) were among the first to present models in which an exogenous shock to borrowing constraints causes a recession due to the zero lower bound; Guerrieri and Lorenzoni (2011) noted that government debt can in principle completely offset a shock to private borrowing constraints, while Bilbiie et al. (2013) demonstrated a similar result in a Eggertsson and Krugman (2012)-type model. Although it is not central to our narrative, our model allows for the possibility of permanently negative real rates; in that regard, it is more closely related to Eggertsson and Mehrotra (2014). In models without capital where government debt is valued for its liquidity rather than its safety, government debt is generally an extremely powerful tool. Again, in our setting government debt has an additional cost in that it crowds out capital investment.

In an economy with capital and nominal rigidities, Boullot (2016) shows that rational bubbles can ameliorate liquidity traps which are driven by a contraction in private borrowing constraints (as in Eggertsson and Krugman (2012) and Korinek and Simsek (2016)). Depending on whether the zero lower bound binds, rational bubbles (and government debt, which is a perfect substitute for bubbles) may either expand or contract output and the stock of capital. Our focus is on the effect of government debt, rather than bubbles, on the natural rate of interest, and we study the safety channel rather than the liquidity channel. Bacchetta et al. (2016) also study the interaction between government debt and capital in a liquidity trap, albeit in a flexible price economy. Like them, we show that safe assets crowd out capital even in a liquidity trap. Unlike them, we focus on goverment debt's safety properties rather than its liquidity properties, and we study nominal rigidities, giving policymakers a motive for increasing the natural rate of interest which is absent in their flexible price economy. Finally, like us, Auclert and Rognlie (2016) study an incomplete markets model, in which they study the consequences of labor income inequality for aggregate demand. Their results are consistent with our findings: when monetary policy is constrained, public debt issuances are expansionary and crowd in investment. Rather than studying inequality, we instead investigate the role of public debt in moderating increases in *risk premia* driven by idiosyncratic capital income risk.

Our stylized model predicts that an increase in the dispersion of firm-specific productivity causes an increase in the risk premium, which pushes real interest rates below zero while the aggregate marginal product of capital remains high. This is broadly consistent with the empirical literature. Decker et al. (2017) document that the volatility of total factor productivity increased since 1980 for U.S. manufacturing firms. Duarte and Rosa (2015) present evidence from a variety of asset pricing models that the equity risk premium increased significantly between 2000 and 2013. Using the methodology of Gomme et al. (2011), Caballero et al. (2017) document that the real return on productive capital remained flat or even increased over the past three decades, while the return on U.S. Treasuries declined dramatically.

## 2 Model

**Households** Time is discrete. At each date $t$, a cohort of ex-ante identical individuals with measure 1 is born and lives for two periods. Each individual $j \in [0,1]$ has identical preferences which can be described as:

$$\mathbb{U}(c_t^Y, c_{t+1}^O) = (1-\beta)\ln c_t^Y + \beta \mathbb{E}_t \ln c_{t+1}^O$$

where $\beta \in (0, 1/2)$. When young, each household is endowed with one unit of labor which it is willing to supply inelastically and earns a nominal wage $W_t$ per unit. The household also receives a transfer $T_t$ (which could be negative) from the government. Households can invest in two assets: *risky* capital and *safe* government debt. The budget constraints of a household can be written as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} = W_t l_t + P_t T_t \tag{1}$$

$$P_{t+1} c_{t+1}^O(z) = P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} \tag{2}$$

where $i_t$ is the nominal interest rate on government debt and $R_{t+1}^k(z)$ is the return on capital earned by household $i$ at date $t+1$ when it is old, which depends on a random variable $z$ described below. When young, the household must decide in how much to invest in capital without knowing the realization of $z$ in the next period. Notice that a household makes all its decisions when young and just consumes its wealth when old. Lemma 1 below summarizes a households decisions.

**Lemma 1** (Saving and Investment Decision)**.** *The household's decisions are described by*

$$c_t^Y = (1-\beta)(\omega_t l_t + T_t) \tag{3}$$

$$k_{t+1} = \beta \eta_t (\omega_t l_t + T_t) \tag{4}$$

$$\frac{1}{R_t} b_{t+1} = \beta(1-\eta_t)(\omega_t l_t + T_t) \tag{5}$$

*where* $b_t = \frac{B_t}{P_t}$ *denotes real debt,* $R_t = \frac{(1+i_t)P_t}{P_{t+1}}$ *is the real return on government debt and* $\eta_t$*, the portfolio share of risky capital, is implicitly defined by*

$$\mathbb{E}_z \left[ \frac{R_{t+1}^k(z) - R_t}{\eta_t R_{t+1}^k(z) + (1-\eta_t) R_t} \right] = 0 \tag{6}$$

*Proof.* See Appendix A. □

Lemma 1 shows that the young household consumes a fraction $1-\beta$ of its labor income net of transfers when young and invests the rest. Out of the $\beta$ fraction it chooses to save, the household invests a fraction $\eta_t$ in risky capital and $1-\eta_t$ in the safe bond. Equation (6) describes the optimal choice of portfolio weights as the solution to a portfolio choice problem seeking to maximize risk-adjusted returns. Using equations (4)-(6), we can express the optimal portfolio share of capital as:[1]

$$\eta_t = \frac{k_{t+1}}{k_{t+1} + b_{t+1}/R_t} = \mathbb{E}_z \left[ \frac{R^k_{t+1}(z) k_{t+1}}{R^k_{t+1}(z) k_{t+1} + b_{t+1}} \right] \tag{7}$$

Another interpretation of $\eta_t$ is that it is the share of capital in the portfolio which maximizes the risk-adjusted return on the portfolio. In order to see this, notice that the denominator of equation (6) is the return on a portfolio comprising of a fraction $\eta$ of capital and $1-\eta$ of bonds. The portfolio choice function can then be described as

$$\max_{\eta_t \in [0,1]} \mathbb{E}_z \ln \left[ \eta_t R^k_{t+1}(z) + (1 - \eta_t) R_t \right]$$

The first-order necessary condition which characterizes the optimal choice of $\eta$ to maximize the risk-adjusted return is given by equation (6). The following properties of the optimal portfolio choice are standard.

**Lemma 2** (Portfolio Choice)**.** *The optimal portfolio choice satisfies the following:*

1. *The optimal choice of $\eta_t$ depends negatively on $R_t$.*

2. *Compare two distributions of the return on capital $R^k(z)$, $F$ and $G$ where $G$ is a mean-preserving spread of $F$. Then $\eta_F < \eta_G$*

*Proof.* See Hadar and Seo (1990). □

The Lemma above shows that agents demand a higher level of safe assets (lower $\eta$) if bonds are relatively cheap (low $1/R_t$) or when risk is high.

**Firms** At each date $t$, each old household operates a firm with a Cobb-Douglas production technology:

$$Y_t(z) = (z_t k_t)^{\alpha} (\ell_t(z))^{1-\alpha}$$

where $k_t$ is the amount of capital that household $i$ invested when young. $z$ is the firm-specific productivity and is i.i.d across all firms with distribution $\ln z \sim N\left(-\sigma_t^2/2, \sigma_t^2\right)$. Given its productivity

[1]Notice that the Euler equation for capital can be written as:

$$\frac{1-\beta}{c^Y_t} = \beta \mathbb{E}_z \frac{R^k_{t+1}(z)}{c^O_{t+1}(z)}$$

Notice that $c^O_{t+1}(z) = R^k_{t+1}(z) k_{t+1} + b_{t+1}$. Using the fact that $c^Y_t = \frac{1-\beta}{\beta}(k_{t+1} + b_{t+1}/R_t)$ and multiplying both sides of the Euler equation by $k_{t+1}$ yields the desired expression for $\eta_t$.

and capital, the firm hires labor in order to maximize profits:

$$R_t^k(z)k_t = \max_{\ell} (zk_t)^{\alpha} \ell_t^{1-\alpha} - \omega_t \ell_t$$

where $\omega_t$ denotes the real wage. Labor demand is given by:

$$\ell_t(z) = \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1}{\alpha}} zk_t \tag{8}$$

and we can write the return to capital as:

$$R_t^k(z) = \alpha \left(\frac{1-\alpha}{\omega_t}\right)^{\frac{1-\alpha}{\alpha}} z \tag{9}$$

**Government** At date $t$, the government issues non-defaultable nominally safe one period debt $B_{t+1}$ at price $1/(1+i_t)$ and uses the proceeds to repay outstanding debt $B_t$ and to disburse transfers $P_t T_t$ to the young:

$$\frac{1}{1+i_t} B_{t+1} = B_t + P_t T_t \tag{10}$$

The monetary authority sets nominal interest rates $i_t$ according to some rule which we specify later.

**Labor Market** We begin by assuming that nominal wages can adjust freely to achieve full-employment.[2] Consequently, labor market clearing determines real wages:

$$l_t = 1 \text{ and } \omega_t = (1-\alpha)k_t^{\alpha} \tag{11}$$

**Return on capital** Given equilibrium wages (11), the return to investing in capital can be written as:

$$R_t^k(z) \;\; = \;\; \alpha z k_t^{\alpha-1} \tag{12}$$

**Goods Market Clearing** The aggregate resource constraint of this economy can be written as:

$$c_t^Y + \int_z c_t^O(z) dF_t(z) + k_{t+1} = \int_z (zk_t)^{\alpha} \ell_t(z)^{1-\alpha} dF_t(z) = k_t^{\alpha} \tag{13}$$

where $F_t(z)$ is the cdf of the log-normal distribution defined above. The LHS of the equation above is the sum of total consumption and investment in capital in period $t$ while the RHS is the GDP.

[2] In section 4, we also consider the case in which nominal wages are sticky downwards but flexible upwards.

## 2.1 Characterizing Equilibrium

**Definition 1** (Equilibrium)**.** *Given a sequence $\{B_{t+1}, i_t, T_t\}_{t=0}^{\infty}$ and initial conditions $\{B_0, k_0\}$, an equilibrium is a sequence $\{c_t^Y, c_t^O(z), k_{t+1}, l_t, \ell_t(z), R_t^k(z), P_t, W_t\}_{t=0}^{\infty}$ such that*

1. *$\{c_t^Y, c_t^O(z), k_{t+1}, B_{t+1}\}$ solves the household's problem for each cohort $t$, given prices $\{i_t, R_t^k(z), P_t, W_t\}$ and transfers $\{T_t\}$*
2. *$\{\ell_t(z), R_t^k(z)\}$ solve the firm's problem at each date $t$*
3. *the government budget constraints (10), labor market clearing (11) and goods market clearing (13) are satisfied.*

There are two key equations that help us describe the dynamics of the economy. The first of these equations, which we refer to as the aggregate supply of savings equation which can be written as:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta\left[(1-\alpha)k_t^{\alpha} + \frac{b_{t+1}}{R_t} - b_t\right] \tag{14}$$

In order to derive the relationship above, we used the equilibrium expression for labor income (11) and government budget constraint (10) to substitute out for transfers from equation (4). The LHS of (14) denotes the total savings in the economy at date $t$.

The other equation of interest concerns the demand for capital. In order to derive an expression for this equation, we start by solving for $\eta_t$ which denotes the portfolio share of capital for young households. Lemma 3 below provides an expression for the optimal portfolio share of capital and describes it's properties.

**Lemma 3** (Equilibrium Portfolio Shares)**.** *In equilibrium, the optimal portfolio choices of a young household satisfy:*

1. *The optimal portfolio share of capital can be written as:*

$$\eta_t = \mathbb{E}_z\left[\frac{\alpha z k_{t+1}^{\alpha}}{\alpha z k_{t+1}^{\alpha} + b_{t+1}}\right] \tag{15}$$

2. *The optimal portfolio share of capital is decreasing in $\sigma$, i.e. $\frac{\partial \eta_t}{\partial \sigma} < 0$*

*Proof.* See Appendix B. □

Equation (15) follows immediately from equation (7) with the expression for $R_{t+1}^k(z)$ plugged in. Thus, the equilibrium portfolio share of capital is the same as the expected value of the share of capital income of the old to their total income. Notice that equation (15) shows that the equilibrium portfolio share of capital only depends on capital and bonds only via debt to GDP. That is, we can write $\eta_t = \mathbb{E}_z\left[\alpha z/(\alpha z + \tilde{b}_{t+1})\right]$ where $\tilde{b}_{t+1}$ is defined as $\tilde{b}_{t+1} = b_{t+1}/k_{t+1}^{\alpha}$. In what follows, it will be convenient to work with $\tilde{b}$ instead of $b$ as our measure of fiscal policy.

Throughout our analysis, we will refer to increases in $\sigma$ as increases in risk. It is important to note that given our specification of $\ln z \sim N(-\sigma^2/2, \sigma^2)$, any increase in $\sigma$ corresponds to a mean-preserving spread to the distribution of idiosyncratic productivity, leaving the average return on capital (12) unchanged. Lemma 3 above shows that an increase in risk in this sense reduces the equilibrium portfolio share of capital. Finally, using the expression for $\eta_t$, the demand for capital can be expressed as:[3]

$$\alpha k_{t+1}^{\alpha-1} \quad = \quad g(\tilde{b}_{t+1}, \sigma) R_t \qquad \text{where } g(\tilde{b}, \sigma) = \frac{\mathbb{E}_z\left[(\alpha z + \tilde{b}_{t+1})^{-1}\right]}{\mathbb{E}_z\left[z(\alpha z + \tilde{b}_{t+1})^{-1}\right]} > 1 \tag{16}$$

This equation states that the demand for capital is decreasing in the safe real interest rate, as is standard. However, it also depends on the supply of safe assets and the level of idiosyncratic risk. In order to get some intuition for this relation, notice that the LHS of equation (16) is the expected return on capital $\mathbb{E}_z R^k(z)$. Thus, $g(\tilde{b}, \sigma)$ can be interpreted as the premium earned by capital relative to bonds owing to the inherent risk in holding capital.

**Lemma 4** (Risk premium)**.** *$g(\tilde{b}, \sigma)$ is decreasing in its first argument and increasing in the second argument.*

*Proof.* See Appendix C. □

Unsurprisingly, an increase in the riskiness of capital, $\sigma$, decreases the demand for capital and widens the spread between the expected return on capital and the safe rate. As capital becomes more risky, investors would like to substitute away from capital towards government debt; if no increase in the supply of debt is forthcoming, either the price of debt must rise or investmewnt in capital must fall. Importantly, the Lemma also states that increasing $\tilde{b}_{t+1}$ reduces the safety premium by satiating the demand for safe assets. In this sense, our model provides a micro-founded channel through which the supply of public safe assets affects the risk premium, as found empirically by Krishnamurthy and Vissing-Jorgensen (2012). Krishnamurthy and Vissing-Jorgensen (2012) and many others have used models in which bonds earn a convenience yield arises because (by assumption) they provide direct utility to the holder; as in models with money in the utility function, these utility benefits are a reduced form for the transaction services provided by this asset. In our model, bonds earn a premium relative to capital despite not being in the utility function and this premium responds to the public supply of safe assets. However, this premium is more a *safety* premium than a *liquidity* premium: it arises endogenously due to incompleteness of markets.

The intersection of the aggregate supply of savings (14) and the demand for capital (16) determines the equilibrium level of investment $k_{t+1}$ and real interest rates $R_t$ given today's capital stock

[3] See Appendix C for details.

and government debt policy. Using equations (14) and (16), we can describe the evolution of this economy by the single equation:

$$k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) k_t^\alpha \qquad \text{where } s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) = \frac{\beta(1-\alpha-\tilde{b}_t)}{\beta + (1-\beta)\mathbb{E}_t\left[\dfrac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]^{-1}} \tag{17}$$

Notice that the equation above looks a lot like the Solow model where the aggregate savings rate is given by $s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)$. Note also the aggregate savings rate is different from the private savings rate of the young which is given by $\beta$.

**Lemma 5** (Savings Function)**.** *The savings function is decreasing in all its arguments.*

It is straightforward to show that the savings rate is decreasing in both the current period and next period's debt to GDP ratio. Intuitively, higher government debt requires higher taxes on young saves reducing their disposable income and thus, the amount they save. A high government tomorrow, however crowds out investment in physical capital. In equilibrium, a higher $\tilde{b}_{t+1}$ requires that young households hold more bonds in their portfolio, thus reducing the amount they invest in capital. Finally, an increase in risk reduces the aggregate savings rate. In a riskier environment, young savers shift their portfolio away from riskier capital towards safe government debt reducing aggregate savings and hence investment.

## 2.2 Steady State

In steady state, the aggregate supply of savings (14) can be written as:

$$k^{1-\alpha} = \beta(1-\alpha) - \left[\frac{1-\beta}{R} + \beta\right]\tilde{b} \tag{18}$$

where $\tilde{b} = b/k^\alpha$ is the steady state debt to GDP ratio. We use the convention that quantities and prices without time sub-scripts denote their values in steady state. Equation (18) implies that government debt crowds out capital. Intuitively, for a fixed supply of savings, the higher government debt, the less savings remain to finance physical investment. In addition, if $R > 1$ in steady state, a higher debt increases the tax burden on young households, reducing the total supply of savings further. Also note that holding fixed debt to GDP, this equation describes an increasing relationship between steady state $k$ and $R$. Higher interest rates make the same amount of debt cheaper for young savers, leaving ample funds available to invest in capital. If there is zero government debt outstanding, capital attains its highest attainable steady state level, which is invariant to interest rates. This relationship is captured in the upwards sloping curve in Figure(1).

The upward sloping curves in Figure 2a describes this relationship for a positive level of debt to GDP; the vertical line describes the relationship when debt is zero. Figure 2a describes this relationship for a positive level of debt to GDP; the vertical line describes the relationship when debt is zero.

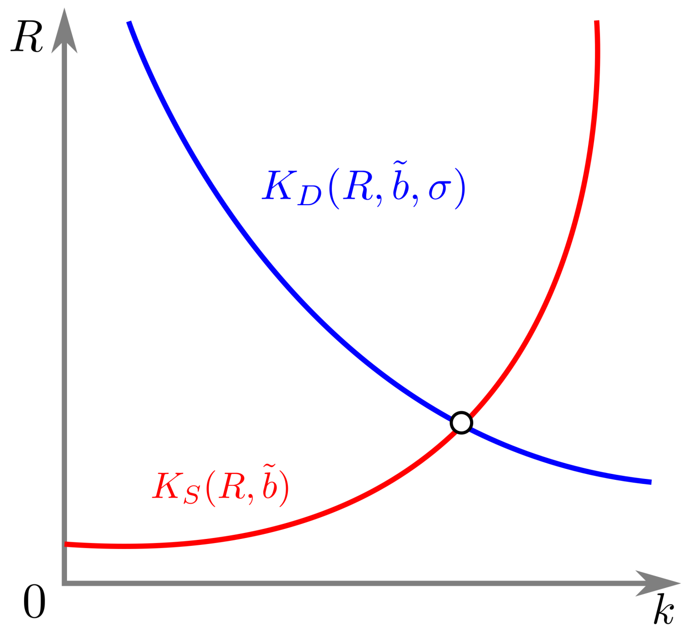



**Figure 1.** Steady state

On the other hand, the downward sloping curves depicts the demand for capital given by (16) evaluated at steady state $\tilde{b}$. A higher debt to GDP ratio shifts the curve up; an increase in risk shifts the curve down. The intersection of the two curves determines steady state capital and interest rates. The following Lemma solves explicitly for the steady state.

**Lemma 6.** *Given a steady state debt-to-GDP ratio, $\tilde{b} \in [0, 1-\alpha)$ and steady state risk $\sigma$, steady state capital and interest rates are given by:*

$$k\left(\tilde{b}, \sigma\right) = \left[\frac{\beta(1-\alpha-\tilde{b})}{\beta + (1-\beta)\mathbb{E}\left[\frac{\alpha z}{\alpha z + \tilde{b}}\right]^{-1}}\right]^{\frac{1}{1-\alpha}} \tag{19}$$

$$R\left(\tilde{b}, \sigma\right) = \frac{1}{1-\alpha-\tilde{b}}\left[\beta^{-1}\mathbb{E}\left[\frac{1}{\alpha z + \tilde{b}}\right]^{-1} - \tilde{b}\right] \tag{20}$$

# 3 Inspecting the Mechanism

Our analysis of the model is going to be centered around changes in idiosyncratic risk $\sigma$ and the supply of safe assets. To this end, we start by describing how a permanent increase in risk affects allocations in both the the short run and the long run. We then investigate how changes in the level of government debt can influence the path the economy takes.

## 3.1 The effects of an increase in risk

**Lemma 7.** *For a given level of $\tilde{b}$, the steady state level of capital $k$ and is weakly decreasing in $\sigma$ while steady state real interest rate $R$ is strictly decreasing in $\sigma$.*

The proof of the Lemma is a straightforward application of Jensen's inequality on equations (19) and (20). An increase in risk reduces the demand for capital, as households seek to hold safe bonds rather than risky capital. Given a fixed supply of safe assets, however, the price $1/R$ of these assets must rise in order to equilibrate demand and supply of safe assets. This is the easiest to see in the case where bonds are in zero net supply. In this case, using equation (20), the relationship between the real interest rate and $\sigma$ can be written as:[4]

$$R = \frac{\alpha}{\beta(1-\alpha)} e^{-\sigma^2}$$

Facing higher prices of safe assets, young savers, who save a fixed fraction $\beta$ of their total income, have less resources left over to invest in physical capital. Consequently, the aggregate saving rate and the capital stock fall. Figure 2a depicts this graphically. An increase in $\sigma$ shifts the capital demand schedule to the left while leaving the aggregate supply of savings unchanged, reducing the steady state levels of capital and real interest rates. Importantly, a high enough $\sigma$ can result in negative real interest rates in steady state, $R < 1$. For example, in the case with $\tilde{b} = 0$, real interest rates will be negative if risk is higher than a threshold $\sigma > \underline{\sigma} := \sqrt{\ln\left[\frac{\alpha}{\beta(1-\alpha)}\right]}$. Note that if $\frac{\alpha}{\beta(1-\alpha)} < 1$, we would have steady state $R < 1$ even in the absence of risk. In this case, the economy would be *dynamically inefficient* in the sense of Diamond (1965). For the rest of the paper, we rule out this case by making the following assumption:

**Assumption 1** (Dynamic efficiency with $\sigma = 0$)**.** *The riskless economy is dynamically efficient.*[5]

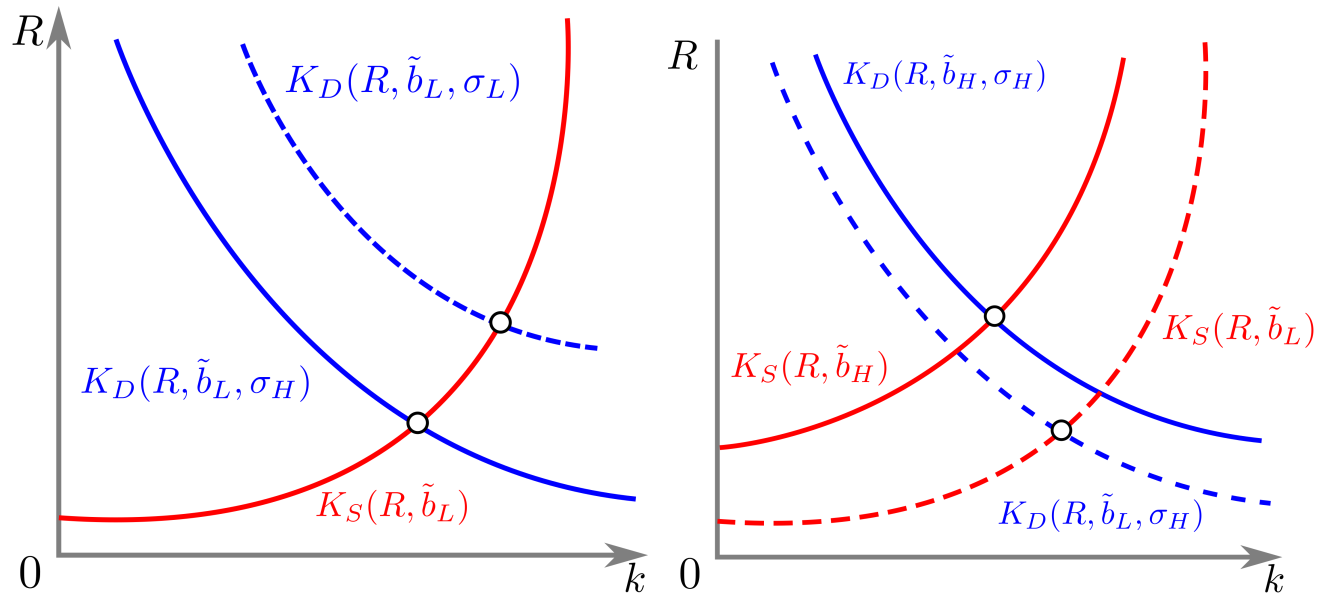
$$\frac{\alpha}{\beta(1-\alpha)} > 1$$

## 3.2 The effects of an increase in safe assets

Next, we ask whether an increase in the supply of safe assets can mitigate the fall in real interest rates induced by an increase in risk. The following Lemma states that an increase in the debt to GDP ratio always increases interest rates. This crowds out investment, reducing the steady state capital stock.

[4] Recall that if a random variable $z$ is distributed log-normally with the underlying normal distribution $N(-\sigma^2/2, \sigma^2)$, then the mean of $z^{-1}$ is given by $e^{\sigma^2}$.

[5] We refer the reader to the text surrounding Definition **??** in Section 3.4 for a definition and discussion of dynamic efficiency.




**(a)** A permanent increase in $\sigma$

**(b)** A permanent increase in $\tilde{b}$

**Figure 2.** Steady States

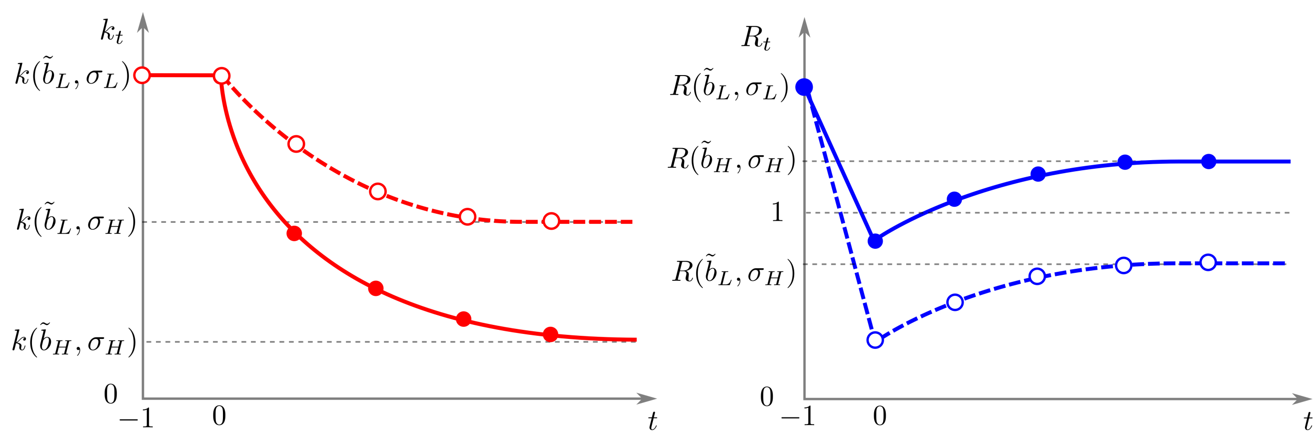
**Lemma 8.** *The steady state levels of capital $k$ is strictly decreasing in $\tilde{b}$ while the steady state real interest rates $R$ is strictly increasing in $\tilde{b}$.*

Figure 2b depicts the claim of Lemma 8 graphically. An increase in the supply of safe assets satiates the demand for safe assets and reduces the safety premium in equation (16). This makes young households willing to hold more capital for a given real interest rate, shifting the capital demand schedule to the right. However, a higher supply of government debt diverts savings away from capital crowding out investment, shifting the aggregate supply of savings to the left. Overall, capital is unambigously lower and real interest rates are higher in the steady state with a higher supply of safe assets. It follows that in response to any increase in risk, a high enough increase in the debt to GDP ratio can always keep real interest rates positive. However, this comes at the cost of crowding out investment in physical capital.

### 3.3 Transitions

Having characterized how steady state interest rates and capital stocks depend on the level of idiosyncratic capital income risk and the quantity of outstanding safe assets, we now describe how permanent, unanticipated shocks to risk affect the transition of interest rates and capital stocks towards their new steady state levels. We also discuss how the level of government debt affects this transition path. We assume that the economy is initially in steady state at date 0 when it is hit by an unanticipated and permanent increase in $\sigma$ from $\sigma_L$ to $\sigma_H$.

It is convenient to first consider the case in which the government keeps the debt to GDP ratio $\tilde{b}_t = \tilde{b}_L$ constant and positive, adjusting the level of government debt as necessary in response to changes in the capital stock and the level of output. The solid lines in figure 3 illustrate the dynamics of capital and real interest rates under such a policy. Prior to date 0, capital and gross real interest rates are at their steady state levels, $k(\tilde{b}_L, \sigma_L)$, $R(\tilde{b}_L, \sigma_L)$. Following from Lemma 2, the permanent increase in $\sigma$ decreases the portfolio share of risky capital $\eta$, and thus reduces the economy's aggregate savings rate $s(\tilde{b}_L, \tilde{b}_L, \sigma_H)$. With a lower aggregate savings rate, the economy gradually transitions towards a lower steady state capital stock, $k(\tilde{b}_L, \sigma_H)$. Real interest rates fall on impact, undershooting their new steady state level $R(\tilde{b}_L, \sigma_H)$, as the economy initially has a high level of capital (and thus a low marginal product of capital) but also has a high risk premium. As the capital stock gradually declines, both the expected return on risky capital and the safe real interest rate gradually transition to a somewhat higher level, although the real interest rate is still lower than in the low-risk steady state. If the increase in $\sigma$ is sufficiently large, the long run real interest rate may be negative. Interestingly, even if this is not the case, interest rates may be negative during the transition to the new steady state.




**(a)** Capital **(b)** Gross real interest rate

**Figure 3.** Dashed lines denote equilibrium without an increase in safe assets, $\tilde{b}_t = \tilde{b}_L \forall t$. Bold lines denote equilibrium with an increase in safe assets, $\tilde{b}_t = \tilde{b}_H$ for $t \geq 1$.

An increase in the supply of safe assets can mitigate the decline in real interest rates. The dashed lines in figure 3 show dynamics of capital and interest rates under a policy which sets $\tilde{b}_t = \tilde{b}_H > \tilde{b}_L$ for $t \geq 1$. The right panel shows that under this policy, real interest rates fall by a smaller amount on impact, and converge to a higher steady state level, $R_{ss}(\tilde{b}_H, \sigma_H)$, than they would in the absence of a change in the debt to GDP ratio. However, this comes at the cost of a sharper decline in the capital stock, as shown in the left panel. In equilibrium, investors must be induced to accommodate an increased supply of safe assets by allocating a larger share of their portfolio to these assets, and a smaller share to risky capital. This immediately reduces the aggregate savings rate. In addition, starting at date 1, young savers must pay higher taxes to finance the increase in debt, further reducing aggregate savings.

### 3.4 When does safe asset creation increase welfare?

Our analysis above showed that an increase in risk can reduce equilibrium real interest rates, even pushing them below zero. We also showed that a sufficiently large increase in the supply of safe assets can mitigate this decline, keeping real interest rates above zero in the new steady state with lower capital. Just because policy can do this, however, does not mean that it should. As we show below, a planner who wishes to maximize steady state welfare would choose not to create additional safe assets in order to counter the negative real interest rates.

We consider a constrained social planner who seeks to maximize steady state welfare, subject to the implementability constraint (19):

$$\max_{k,\tilde{b}} \quad (1-\beta)\ln\left[(1-\alpha-\tilde{b})k^{\alpha}-k\right]+\beta\mathbb{E}_z\ln\left[(\alpha z+\tilde{b})k^{\alpha}\right] \text{ s.t. } k=s(\tilde{b},\tilde{b},\sigma)k^{\alpha} \tag{21}$$

In order to understand why the planner chooses not to increase the supply of safe assets in response to a increase in risk, it is important to understand the trade-off he faces. In equilibrium, an increase in government debt is essentially a forced transfer from the young to the old.[6] Since the planner cannot directly insure old people against low realizations of $z$, the only way to raise their consumption in low $z$ states is to give them a unconditional transfer. We label this the *insurance motive* for creating safe assets. However, if the old accumulate financial claims to consume a greater share of GDP, the young must consume a smaller share. Whether it is desirable to redistribute from the young to the old depends on the planner's preferences for redistribution, and also the amount of risk faced by the old. The higher the level of risk, the higher the expected marginal utility of the average old individual, and thus the stronger the insurance motive. However, safe asset production also crowds out physical capital investment. This harms both young households, who earn lower wages, and old households, who earn less capital income. In the absence of crowding out, the planner would like to create just enough safe assets that the real interest rate is zero, as the following Lemma states.

**Lemma 9.** *Consider the unconstrained planner's problem in which the planner maximizes (21), ignoring constraint. The solution to this unconstrained problem is unique, with either $R \geq 1$ and $\tilde{b}=0$, or $R=1$ and $\tilde{b}>0$.*

*Proof.* See Appendix D. □

If there was no crowding out, it would be optimal to produce enough safe assets to prevent real interest rates from becoming negative. Intuitively, the real interest rate $R$ measures how much an individual values a unit of consumption when young relative to when old. If risk is ralatively low

[6] Recall that we have assumed that debt is financed by lump sum taxes on the old. If in addition there were lump sum taxes on the old, our results would be unchanged, provided that $b_t$ is redefined to be government debt net of taxes on the old. Allowing the government to levy taxes conditional on the realization of an old household's idiosyncratic productivity would (trivially) change our results.

and $R > 1$, a unit of consumption is worth more when young than when old, because impatiance outweighs the private desire to insure againsts consumption risk when old. Conversely, when risk is high and $R < 1$, a yound individual would strictly prefer to give up one unit of consumption when young to recceive one unit when old. While the social planner shares the individuals preferences, unlike the individual, she has the technology to transfer one unit of consumption from young to old, namely issuance of government bonds - safe assets. Consequently, the planner would never permit $R < 1$; because that would signal a unmet desire for transfers from young to old which the planner could easily satiate by creating more safe assets.[7]

However, safe asset creation does crowd out investment. Thus, it is in fact *not constrained optimal* to produce enough safe assets to keep real interest rates positive, as we will now see. In this sense, negative real interest rates are not a signal of a *shortage of safe assets* per se. Of course, for a sufficiently high level of risk, increasing government debt does increase social welfare. But even in this case, it is never desirable to produce enough safe assets that the real interest rate becomes positive. The proposition below summarizes the optimal response of the planner depending on the level of riskiness in the economy.

**Proposition 1** (Constrained Optimal Level of Safe Assets). *If the riskless steady state is dynamically efficient,*[8] *there exist $\underline{\sigma}$, $\overline{\sigma}$ such that the solution to the constrained problem has the following properties:*

*i. If risk is low enough, i.e. $\sigma \leq \underline{\sigma}$, then the optimal choices of the planner satisfy $\widetilde{b} = 0$, $R \geq 1$.*

*ii. If risk is in the intermediate range $\sigma \in (\underline{\sigma}, \overline{\sigma}]$, the planner still does not choose to create safe assets and his optimal choices satisfy $\widetilde{b} = 0$, $R < 1$.*

*iii. If risk is very high, $\sigma > \overline{\sigma}$, then the planner would optimally choose to create some safe assets but still not enough such that real interest rates are positive. In this case, the optimal choices satisfy $\widetilde{b} > 0$, $R < 1$.*

*Proof.* See Appendix E. □

It is also important to realize that the result is not driven by our assumption that the planner maximizes steady state welfare. It is possible to show that irrespective of the Pareto weights the planner puts on the welfare of young and old households respectively, it is never optimal to prevent real interest rates from becoming negative. In one sense, these results are not surprising. In the absence of nominal rigidities, negative real interest rates are of no particular significance. However, what these results do highlight is that the potential for safe assets to crowd out investment in physical capital is particularly strong in this world.

[7] In contrast when $R > 1$, housholds would like to transfer resources from tomorrow to today but the planner has no technology to facilitate such a transfer.

[8] This is ensured by Assumption 1.

The result above is not driven by the particular social welfare function we considered above (steady state welfare of a representative cohort). Instead of maximizing welfare, we also consider the Ramsey problem of a planner who puts arbitrary Pareto weights on the welfare of different cohorts. This allows us to define constrained (Pareto) efficient allocations:

**Definition 2** (Constrained Efficiency). *The ex-ante welfare of cohort $t$, given an allocation $\{k_t, b_t\}_{t=0}^{\infty}$, is*

$$U_t = (1-\beta)\ln((1-\alpha)k_t^{\alpha} - k_{t+1} - b_t) + \beta \mathbb{E}_z \ln(\alpha z k_{t+1}^{\alpha} + b_{t+1}).$$

*In the spirit of Negishi (1960), an allocation $\{k_t, b_t\}_{t=0}^{\infty}$ is constrained efficient if it solves:*

$$\max_{\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \phi_t U_t + \phi_{-1}\mathbb{E}_z \ln c_0^O(z)$$

*subject to $k_{t+1} = s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma)k_t^{\alpha}, \tilde{b}_t = \frac{b_t}{k_t^{\alpha}}$ and given $(k_0, b_0)$ for some sequence of Pareto weights $\{\phi_t\}$ with $\sum_{t=0}^{\infty} \phi_t < \infty$ with each $\phi_t \geq 0$ and at least one $\phi_t > 0$.*

Starting from the steady state associated with zero safe assets, it remains constrained efficient to refrain from producing safe assets as long as risk remains below a certain level $\sigma^{\diamond} > \underline{\sigma}$, as the following Lemma states.

**Lemma 10.** *There exists $\sigma^{\diamond} > \underline{\sigma}$ such that, if $\sigma < \sigma^{\diamond}$, it is constrained efficient to choose $\tilde{b}_t = 0$ for all $t$. Importantly, $\sigma^{\diamond} > \underline{\sigma}$.*

*Proof.* See Appendix F. □

Lemma 10 establishes that for $\sigma \in (\underline{\sigma}, \sigma^{\diamond})$, the allocation $(k_t, b_t) = \left(s(0,0,\sigma)^{\frac{1}{1-\alpha}}, 0\right)$ is not Pareto dominated by any other allocation. Thus, despite the prevailing negative real interest rates, not creating safe assets is constrained Pareto optimal. The Lemma, however does not state that creating bonds is unambiguously bad. Rather, it highlights that there are distributional consequences associated with creating more safe assets and that doing so will not increase the welfare of all future generations because of crowding out of capital despite providing insurance. However, if risk is large enough, $\sigma > \sigma^{\diamond}$, then the welfare gains from increased insurance outweigh the crowding out of capital and not creating any safe assets is Pareto dominated.

**Dynamic Efficiency** It is important to note that our findings in Proposition 1 and Lemma 10 are not driven by dynamic inefficiency: our economy is always dynamically efficient. In economies without risk such as Diamond (1965), dynamic inefficiency takes the form of over accumulation of capital. Indeed, in our economy with $\sigma = 0$, over accumulation of capital could only arise if Assumption 1 was violated. In this case, real interest rates and the return to capital would be negative in the absence of safe assets, and producing safe assets could increase the welfare of all subsequent cohorts precisely by crowding out capital. Since we impose Assumption 1, we rule out this possibility. In fact, once we introduce risk, our economy features under-accumulation of capital.[9] In the presence of risk, negative real interest rates do not necessarily indicate the presence of dynamic inefficiency (Abel et al., 1989). Abel et al. (1989) consider a general neoclassical economy with overlapping generations in which there is aggregate risk but no idiosyncratic risk. In their setting, an allocation is identified as dynamically efficient if capital income is larger than investment. Our economy satisfies this criterion: even when the safe real interest is negative, the expected return on capital is positive because capital earns a risk premium.[10] Thus, whether or not it is desirable to produce safe assets is not driven by an over-accumulation of capital.

# 4 Nominal rigidities

We have seen that in an economy without nominal distortions, even if increases in risk cause interest rates to become negative, it is not necessarily desirable for fiscal policy to reverse this. However, one important motivation for raising the equilibrium real interest rate which we have not discussed so far is that the zero lower bound on nominal rates may prevent the monetary authority from lowering real rates sufficiently to preserve full employment. We now introduce nominal rigidities, by assuming that nominal wages are sticky downwards but flexible upwards. In the spirit of Eggertsson and Mehrotra (2014) and Schmitt-Grohé and Uribe (2016), we assume that workers are unwilling to work for wages below a wage norm given by:

$$\ln \widetilde{W}_t = (1-\gamma)\ln W_{t-1} + \gamma \ln P_t \omega_t^* \text{ with } \gamma \in [0,1) \tag{22}$$

where $\omega_t^* := (1-\alpha)k_t^\alpha$ is the real wage that delivers full employment at date $t$. With $\gamma = 0$, nominal wages are rigid downwards; with $\gamma = 1$, wages are fully flexible. Formally, we assume that the actual nominal wage is given by:

$$W_t = \max\left\{\widetilde{W}_t, P_t \omega_t^*\right\} \tag{23}$$

[9]More precisely, it can be shown that starting from a steady state with zero safe assets (which features the highest level of capital attainable in equilibrium), there is a Pareto improving deviation which involves higher capital in every period. Importantly, though this deviation cannot be supported as an equilibrium in our setting. This result is somewhat similar to Davila et al. (2012) who find that an appropriately calibrated Aiyagari (1994) economy actually features under-accumulation of capital from the perspective of a utilitarian planner: higher capital would raise wages and depress returns on capital, benefiting poor individuals who hold less capital.

[10] Abel et al. (1989) call an allocation dynamically efficient if it is not possible to increase the ex-ante welfare of any generation without reducing the ex-ante welfare of another. Again, in their setting this is ensured whenever capital income is greater than investment. While this definition of dynamic efficiency may be appropriate in their setting with only aggregate risk, it is not suitable to evaluate outcomes in our model economy: Trivially, starting from any feasible allocation in equilibrium in our economy, an unconstrained planner could equalize consumption across all old agents in a given cohort, increasing welfare of each cohort. Thus, this criterion is not very useful in evaluating economies with idiosyncratic risk rather than aggregate risk.

Using equations (22) and (23), the evolution of real wages can be described by the following equation:

$$\frac{W_t}{P_t} = \max\left\{ \left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\gamma} (\Pi_t)^{\gamma-1} (\omega_t^*)^{\gamma}, \omega_t^* \right\} \tag{24}$$

where $\Pi_t = P_t/P_{t-1}$ denotes the gross rate of inflation. When real wages $W_t/P_t$ are higher than $\omega_t^*$, aggregate labor demand is less than the endowment of labor resulting in unemployment: $\int_0^1 \ell_{i,t} di < 1$. Whenever there is unemployment, we assume that households are proportionally rationed, so that each young household supplies the same amount of labor implying that labor supply by each young household is given by $l_t = \int_z \ell_t(z) dF_t(z)$ .

Regardless of whether there is unemployment or not, since firms are always on their labor demand curve in equilibrium, the actual real wage satisfies $W_t/P_t = (1-\alpha)k_t^{\alpha} l_t^{-\alpha}$. Then using equations (24) and the definition of the market clearing real wage ($\omega_t^* = (1-\alpha)k_t^{\alpha}$), we can derive a relation between changes in employment and the rate of inflation:

$$l_t = \min\left\{ \left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma} l_{t-1}^{1-\gamma} \Pi_t^{\frac{1-\gamma}{\alpha}}, 1 \right\} \tag{25}$$

Equation (25) can be interpreted as a *wage Phillips curve* or the aggregate supply relationship and reveals that the labor market can be in one of two regimes. When last period's nominal wage lies below the wage that would clear markets today, and full employment requires nominal wages today to rise, wages jump to their market clearing level and and there is full employment, $l_t = 1$. However, when last period's wage lies above today's market clearing wage, and full employment requires wages to fall, the wage norm binds not allowing nominal wages to adjust fully. Wages only partially fall towards their market clearing level, resulting in unemployment. In this unemployment regime, employment will be higher, all else equal, if employment was higher in the previous period (as this signals that wages were not too high and do not have too far to fall); if capital is higher today than last period (as this means that the market clearing wage is higher today than last period); or, most importantly, if inflation is higher in the current period.

Finally, in order to close the model with nominal rigidities, we need to specify a monetary policy rule. We assume that monetary policy sets nominal interest rates according to the following flexible inflation targeting rule subject to the zero lower bound (ZLB):

$$\begin{aligned} \Pi_t \left(\frac{Y_t}{Y_t^*}\right)^{\psi} &\leq 1 \\ i_t &\geq 0 \\ \left[\Pi_t \left(\frac{Y_t}{Y_t^*}\right)^{\psi} - 1\right] i_t &= 0 \end{aligned} \tag{26}$$

where $Y_t^* = k_t^{\alpha}$ is the level of output consistent with full-employment, given the level of capital.

Intuitively, the monetary authority aims to implement zero inflation and full employment whenever the ZLB does not prevent this. The monetary authority is willing to tolerate positive inflation if employment is below target; the relative weight that the monetary authority places on output stabilization, relative to price stability, is denoted by $\psi$.[11] For example, $\psi = 0$ implies a strict inflation targeting regime. However, the ZLB may constrain monetary policy in which both inflation and output may be below target.

The remaining equations governing the dynamics of the economy with nominal rigidities are similar to those in flexible wage economy with the exception that the economy might not always be at full-employment. Thus, the aggregate supply of saving is given by:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta \left[ (1-\alpha) k_t^{\alpha} l_t^{1-\alpha} + \frac{b_{t+1}}{R_t} - b_t \right] \tag{27}$$

Notice that the aggregate supply of savings relation (14), described in the flexible wage economy, is the same as (27) evaluated at full employment $l_t = 1$. Equation (27) shows that unemployment today ($l_t < 1$) reduces the labor income of the young, reducing their savings and therefore investment in capital and demand for bonds. In a similar fashion, the optimal portfolio decisions of savers can be written as:

$$\eta_t = \mathbb{E}_z \left[ \frac{\alpha z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha}}{\alpha z k_{t+1}^{\alpha} l_{t+1}^{1-\alpha} + b_{t+1}} \right] \tag{28}$$

As before the equilibrium portfolio share of capital depends on the expected ratio of capital income to total income of the old. Unemployment reduces the marginal product of capital and thus increases the equilibrium portfolio share of safe assets (reduces $\eta_t$) for a given $k_{t+1}$ and $b_{t+1}$.

As before, we can use the expression (28) to derive the demand for capital:

$$\alpha \left( \frac{k_{t+1}}{l_{t+1}} \right)^{\alpha-1} = g(\tilde{b}_{t+1} l_{t+1}^{\alpha-1}, \sigma) R_t \tag{29}$$

where the average marginal product of capital is now $\mathbb{E}_z R_{t+1}^k(z) = \alpha k_t^{\alpha-1} l_t^{1-\alpha}$. Notice that the demand for capital in the flexible wage economy (16) is simply equation (29) evaluated at $l_{t+1} = 1$. The possibility of unemployment at date $t+1$ affects the demand for capital in two ways. First, a value of $l_{t+1}$ below full employment lowers the average marginal product of capital, reducing the demand for capital for a given $R_t$. However, a lower $l_{t+1}$ also increases the portfolio share of safe assets, narrowing the spread between the safe rate on bonds and the risky return on physical capital. Intuitively, the consumption of household contains a risky component (capital income) and a safe component (bonds). Thus, the risk premium that young households demand depends on the share

[11] This specification of monetary policy can be thought of as a limit of a interest rate rule of the form: $1 + i_t = \max \left\{ 1, R^{\star} \Pi_t^{\phi_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\phi_y} \right\}$ as $\phi_\pi \to \infty$ and $\phi_y / \phi_\pi \to \psi$.

of risky to total income when old; when risky income is a large share of total income, the covariance of the return on capital and consumption (when old) will be large. A higher unemployment in the future lowers the risky share of income, leaving old households less exposed to risk, causing them to demand a lower risk premium. Overall, the dynamics of the economy with nominal rigidities are described by equations (25)-(29).

## 4.1 The possibility of risk-induced stagnation

The presence of nominal rigidities allows for unemployment to persist in equilibrium, perhaps even permanently. In what follows, first we show that risk can induce stagnant steady states which feature permanently lower employment. Then we describe how an increase in risk can transition an economy in a full employment steady state to such a stagnant steady state.

### 4.1.1 Stagnant steady states

In Section 3 we established that even if an increse in risk pushes real interest rates below 0 and lowers steady state investment, thre is always full employment. The presence of nominal rigidities opens the door to the possibiility of steady states featuring permanently high unemployment. To understand how such dire conditions can linger permanently, it is useful to revisit our analysis of the steady states in Section 3. We start by analyzing the differences in the supply side of the two economies. Unlike the flexible wage economy, which was always at full employment, in the presence of nominal wage rigidities, the labor market can be in one of two regimes. Notice that in steady state, equation (25) can be written as:

$$l = \min\left\{\Pi^{\frac{1-\gamma}{\alpha\gamma}}, 1\right\} \tag{30}$$

When inflation is nonnegative ($\Pi \geq 1$), wages are effectively flexible, and we have full employment, $l = 1$. When there is deflation ($\Pi < 1$), in order for real wages to be constant in steady state, nominal wages must be falling. Wages cannot be at their market clearing level, since the nominal wage that would have cleared markets in the last period no longer clears markets today, now that prices have fallen. In fact, equation (30) defines an increasing relationship between inflation and employment in this regime, which can be thought of as a *long-run Phillips curve*. The degree of wage flexibility $\gamma$ determines the slope of the Phillips curve in the unemployment regime. When $\gamma = 1$ (perfect flexibility), the Phillips curve is vertical at full employment. At the other extreme, when $\gamma = 0$ (perfect downward nominal wage rigidity), the Phillips curve is inverse-L shaped and is horizontal at zero inflation ($\Pi = 1$). Thus, with $\gamma < 1$, in the deflation regime, the inflation rate affects real allocations in the long run.

Like the long run Phillips curve, our monetary policy rule (26) implies 2 steady state regimes: either $i = 0$ and $\Pi < l^{-(1-\alpha)\psi}$ or $i \geq 0$ and $\Pi = l^{-(1-\alpha)\psi}$ . In fact, the two regimes of the Phillips curve and the monetary policy rule necessarily coincide in steady state: we either have $l = 1$ or $i = 0$ and $R = \Pi^{-1}$. Figure 4 illustrates. The red upward sloping line describes the long run Phillips curve. The blue solid line depicts the stance of monetary policy whenever the ZLB does not bind. A higher $\psi$, i.e., a higher weight on output stabilization relative to inflation makes the blue curves steeper. For example, in the case of a strict inflation target, $\psi = 0$, the blue curve is horizontal. As can be seen, when the ZLB does not bind, the blue and red curves intersect at $(1, 1)$ implying a steady state with full employment and zero inflation. However, when the ZLB binds, the monetary authority may be unable to attain zero inflation and full employment (depicted by the intersection of the dashed blue curve and the red curve).

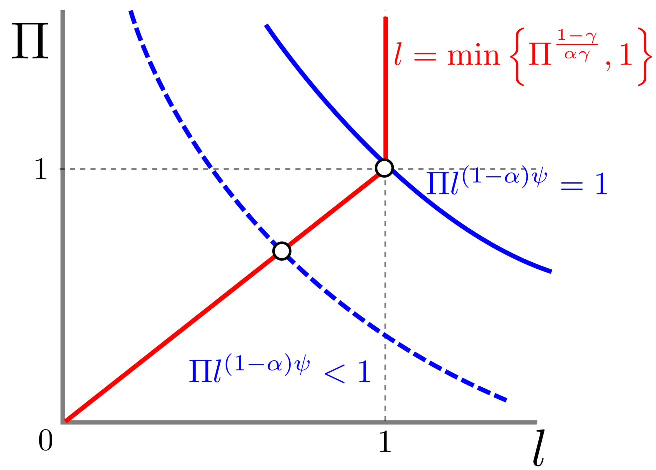



**Figure 4.** Determination of steady state inflation and employment

At full employment, inflation must be nonnegative, given the Phillips curve; thus, given our monetary policy rule, it must be zero. Zero inflation is possible provided that real interest rates are positive, so that the nominal interest rate does not violate the zero lower bound. When there is less than full employment, the Phillips curve requires that steady state unemployment must be accompanied by deflation. The monetary policy rule only tolerates both deflation and unemployment when it is forced to do so by the ZLB. The combination of deflation with the ZLB generates a positive real interest rate $\frac{1}{\Pi}$. In fact, a higher level of steady state unemployment generates more deflation and a higher steady state real interest rate. Mathematically, combining the monetary policy rule and the Phillips curve yields the following set-valued map:

$$R = \begin{cases} l^{-\frac{\alpha\gamma}{1-\gamma}} & \text{if } l < 1 \\ r & \text{for any } r \geq 1 \text{ if } l = 1 \end{cases} \tag{31}$$

We refer to this relationship as the *'LM'* curve (Labor markets and Monetary policy); it is depicted by the red curve in the left panel of Figure 5.

The remaining ingredient to complete the characterization of steady state is the young house-

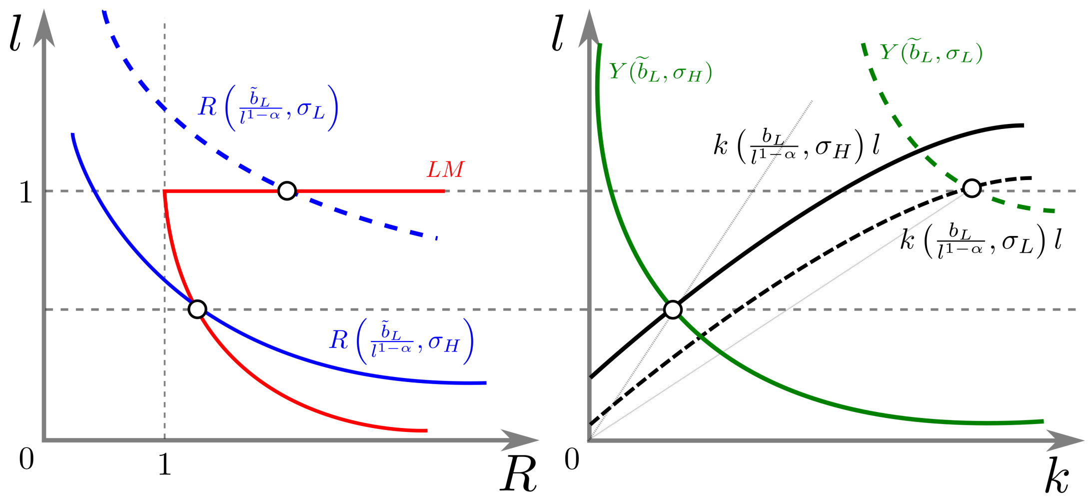



**Figure 5.** A permanent increase in $\sigma$ keeping $\tilde{b}$ fixed

holds' investment and savings decisions. Evaluating (27) and (29) at steady state, we have

$$k^{1-\alpha} = \beta(1-\alpha)l^{1-\alpha} - \left[\beta + \frac{1-\beta}{R}\right]\tilde{b} \tag{32}$$

$$\alpha\left(\frac{k}{l}\right)^{\alpha-1} = g(\tilde{b}l^{\alpha-1}, \sigma)R \tag{33}$$

where $g(\cdot,\cdot)$ is defined in (16). We can solve explicitly to get $k$ and $R$ as a function of steady state employment $l$. In fact, these are the same functions defined in Lemma 4:

$$k = k\left(\tilde{b}l^{\alpha-1}, \sigma\right) l \tag{34}$$

$$R = R\left(\tilde{b}l^{\alpha-1}, \sigma\right) \tag{35}$$

Recall from Lemma 6 that $k(\cdot,\sigma)$ is decreasing in its first argument, and $R(\cdot,\sigma)$ is increasing in its first argument. Consequently, (34) defines an increasing relationship between the capital stock and employment, depicted in the dashed black line on the right panel of Figure 5. A higher rate of employment raises labor income, increasing savings and the steady state capital stock. Similarly, (35) defines a decreasing relationship between the interest rate and employment, depicted in the dashed blue curve on the left panel of Figure 5. A higher rate of employment increases the steady state capital stock and investment. In order to induce households to save more in the form of risky capital, rather than in safe government debt, the rate of return on safe debt (that is, the real interest rate) must fall. In this sense, the two blue curves show the relation between employment and the rate of interest which must be maintained to make saving equal to investment in steady state. It is therefore convenient to refer to them as IS curves, following Hicks (1937). The dashed-blue curve denotes the IS curve when risk is relatively low while the solid-blue curve denotes the same relationship with a high $\sigma$.

The intersection of the IS and LM curves determines the steady state real interest rate and level of employment. The dashed line in Figure 5 (with $\sigma = \sigma_L$) intersects the full employment portion of the LM curve at a positive real interest rate. The right panel shows that this high rate of employment generates a high steady state capital stock. The green curves on the right panel indicate isoquants of the aggregate production function, $Y = k^{\alpha}l^{1-\alpha}$. With a high rate of employment and a high capital stock, the level of output is relatively high in this low risk steady state, shown by the dashed-green higher isoquant.

The solid lines in Figure 5 shows the case in which risk has risen to a higher level $\sigma_H$. This shifts the IS curve to the left, as savers shift their portfolios away from increasingly risky capital towards safe debt, so that it would take a lower return on bonds to push them back towards capital. Indeed, the IS curve intersects the dashed horizontal full employment line at a negative real interest rate, indicating that this economy requires negative rates to sustain full employment. Given the constraints on monetary policy, however, the LM curve does not permit negative real rates. Instead, the zero lower bound binds, and the IS and LM intersect at a level of employment below 1. This unemployment in turn generates persistent deflation, raising real interest rates with the nominal rate stuck at zero. The economy enters a stagnant steady state. The stagnant level of employment has consequences for the steady state capital stock and rate of investment. Lower employment implies lower income for young savers, and less investment, implying a smaller steady state capital stock, as the solid black line in the right panel in Figure 5 indicates. With a decline in both capital and employment, the economy falls to a much lower level of output, indicated by a lower isoquant in the right panel. In particular, the increase in risk is associated with a lower capital labor ratio as indicated by the diagonal gray lines passing through the the origin in the right panel.

At this point, it is important to note a difference from standard New Keynesian models of liquidity traps. In standard three equation New Keynesian models with no physical storage technology such as capital, a large enough increased supply of savings pushes the economy to the ZLB. With real interest rates constrained by the ZLB to be *too high* output is the variable which must adjust to equilibrate the supply and demand of savings. However, if a physical storage technology such as capital was available, the higher desire of households to save can be accommodated by an increase in investment - leading to a boom instead of a recession. These scenarios are different from our liquidity trap because the shock we consider - an increase in risk - does not lead to an overall increase in the the level of desired savings relative to consumption. Instead, it shifts the desired composition of savings by increasing the demand for safe assets relative to capital. Thus, even though physical storage technology is available capital is available, an increase in the demand for *safe* assets can cause a recession, and in fact a permanent investment slump.

It is also worth noting that in our model, stagnation is accompanied by a *higher* expected marginal product of capital, since the capital-labor ratio falls. Gomme et al. (2015) present evidence that while the return on government debt has remained at a low level following the financial crisis, the real return on productive capital has rebounded, with the after-tax return on business capital at its highest level over the past three decades. They interpret this as evidence against the variants of the secular stagnation hypothesis which emphasize a scarcity of investment opportunities. It is, however, entirely consistent with our risk-based view of stagnation. Higher risk may deter investment even though the *average* return on capital remains high. Furthermore, through the lens of our model, an increase in $\sigma$ would be consistent with a decline in the safe rate and a larger risk premium, as documented empirically by Duarte and Rosa (2015) and more recently Caballero et al. (2017).

#### 4.1.2 The transition to a stagnant steady state

Having seen that a steady state featuring unemployment is possible, we now describe the transition to such a steady state. In Section 3, we have already seen that a permanent increase in risk can lower the equilibrium stock of capital permanently, causing a modest decline in output. This increase in risk also pushes down the safe real interest rate, potentially below zero. In the absence of nominal rigidities, negative real interest rates are not a cause for concern; while an increase in the supply of safe assets can return the real rate to positive territory, there is no reason to do so. However, in the presence of nominal rigidities and a lower bound on nominal rates, permanently negative real interest rates may be incompatible with full employment as we now describe. In such a situation, with monetary policy constrained, an increase in safe assets may be unavoidable if full employment is to be restored.

Our experiment is the same as in Section 3. At date 0, there is a permanent unanticipated increase in $\sigma$ from $\sigma_L$ to $\sigma_H > \sigma_L$. As before, we assume that $\sigma_L, \sigma_H$ are such that the associated real interest rates in the flexible-wage steady states are, respectively, positive and negative, given the supply of safe assets $\tilde{b}_L$. For now, we assume that fiscal policy keeps $\tilde{b}$ constant. Recall that $\tilde{b}_t = b_t/k_t^\alpha$ is the ratio of debt to the the level of GDP which would prevail under full-employment.[12]

**Proposition 2** (Stagnation)**.** *Suppose that $\tilde{b}_t = \tilde{b}_L$ for all $t \geq 0$ and prior to date 0 the economy is in steady state with $\sigma = \sigma_L$ such that steady state frictionless real interest rate $R(\tilde{b}_L, \sigma_L) > 1$. At date 0, $\sigma_t$ unexpectedly and permanently jumps to a higher level $\sigma_H$ with $R(\tilde{b}_L, \sigma_H) < 1$. Then:*

1. *There is no bounded equilibrium in which the economy returns to a steady state with full-employment.*

2. *For $\psi$ sufficiently high and $\gamma$ sufficiently low, there exists a unique equilibrium in which $i_t = 0$ for all $t \geq 0$ and the economy converges to a steady state with unemployment.*

*Proof.* See Appendix G. □

[12] Notice that the definition of $\tilde{b}$ in the model with nominal rigidities is slightly different than in Section 3 because of the possibility that the economy might not be at full employment. Thus, in the case with nominal rigidities, $\tilde{b}$ might be smaller than the ratio of debt to actual GDP.

At date 0, following Lemma 2 the increase in risk makes young savers want to reallocate their portfolios away from increasingly risky capital, towards safe government debt. Given that supply of safe assets, $\tilde{b}$ is fixed, this excess demand for bonds necessitates a fall in the real return on bonds in order to equilibrate the bond market. In the absence of inflation, this requires reduction in the nominal interest rate. However, the zero lower bound (ZLB) on nominal interest rates prevents such an adjustment. Consequently, the real return on holding safe assets is *too high* thus, lowering the demand for investment in capital and thus, the price of the final output (which is the price of both consumption and capital). With sticky nominal wages, the fall in price is only partially met resulting in higher real wages and consequently lower labor demand by firms. Lower labor demand by firms results in unemployment of labor services in the economy. Overall, total output and labor income decline making both young and old households poorer. The young households, now poorer, have a lower desire to save and hence reduce their demand for bonds, thus equilibrating the bond market but also reduces investment in capital. This is only the beginning of a *risk-induced recession*.

It gets worse. In the subsequent period, the aggregate capital stock in the economy is lower, thus reducing the marginal product of labor and hence demand for labor. In addition, nominal wages are not free to adjust downwards and continue to stay above their market clearing level. This further reduces the demand for labor and results in persistent unemployment. Since nominal wages are slow to adjust, young households also anticipate that real wages will remain above the level consistent with full employment in the future. Thus, anticipating a lower marginal product of capital and hence a lower average return on capital, young households have even less incentive to invest in capital rather than the attractive alternative of safe government debt. Furthermore, with risk permanently high, young households continue to demand more safe assets. Given that fiscal policy holds $\tilde{b}$ fixed, this creates an excess demand for bonds which requires negative real rates for the bond market to clear. Again the ZLB binding prevents this adjustment and causes a further decline in investment. Unemployment remains high in subsequent periods; the excess demand for safe assets is permanent, and so the economy needs permanently lower income in order to equate the demand and supply of safe assets.

Figure 6 shows a numerical example. We set $\alpha = 1/3$, $\beta = 0.495$, $\sigma_L = 0.49$, $\sigma_H = 0.55$, $\gamma = 0.22$, $\tilde{b}_L = 0.065$.[13] In the figure, $k_{-1}, y_{-1}, w_{-1}, \tilde{b}_{-1}$ and $P_{-1}$ are each normalized to unity. The dashed grey lines in figure 6 illustrate dynamics in the flexible wage benchmark economy. An increase in risk, absent any increase in safe assets, pushes real rates into permanently negative territory (shown in the middle left panel). While this causes a very slight decline in capital, output, and real wages (shown in the top left, top middle, and center panels respectively), the economy naturally remains at full employment throughout (as shown in the top right panel).

[13]Since this is not intended as a quantitative exercise, we choose these particular values of $\sigma_L, \sigma_H, \gamma, \tilde{b}_L$ purely to make the qualititative features of equilibrium described in Proposition 2 and the foregoing description of steady state - negatitve real rates in the flexible wage benchmark economy, long run deflation in the economy with nominal rigidities, and so forth - easy to see. These properties of equilibrium do not depend qualitatively on the choice of parameters.

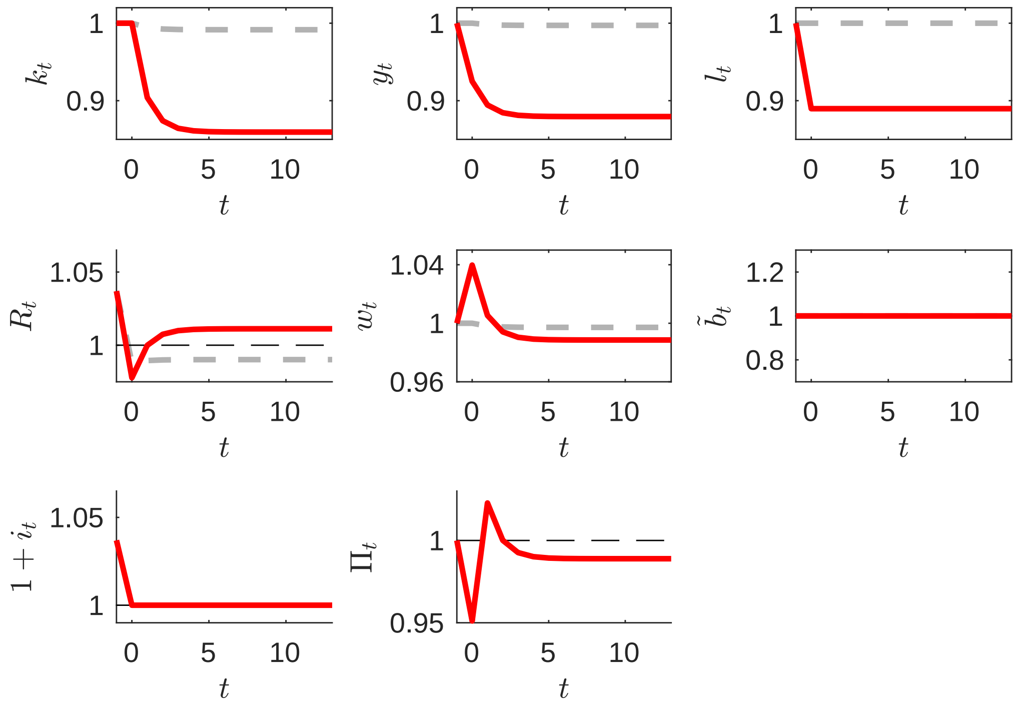



**Figure 6.** Dashed lines denote equilibrium in the absence of nominal rigidities. Solid lines denote the equilibrium trajectory of the economy with nominal rigidities following the permanent increase in $\sigma$.

In contrast, the solid red lines in figure 6 illustrate dynamics in the absence of an increase in safe assets. The top left panel depicts the evolution of the capital stock. The increase in risk, and the associated fall in employment, cause a permanent decline in the aggregate saving rate. This causes capital to gradually decline to a new, lower steady state level. The middle left panel shows the trajectory of the real interest rate. Real rates fall on impact, as the spread between the safe rate and the expected marginal product of capital increases. As the capital stock gradually declines, the expected marginal product of capital rises while the spread remains wider, leading the real rate to increase to its new steady state level. The top right panel displays the behavior of employment, which falls immediately to its steady state level following the increase in risk. The fall in capital and employment combine to create a sustained decline in output, shown by the top middle panel. Finally, the bottom middle pane shows the dynamics of inflation. The collapse in demand at date zero causes a large fall in prices, pushing up real wages (shown by the middle panel) and creating unemployment. Inflation then recovers somewhat before gradually declining to its new steady state level. Intuitively, this economy requires relatively lower interest rates early on in the transition to a new steady state, as the capital stock remains high and the marginal product of capital remains low. With the nominal rate of interest stuck at zero (as shown in the bottom left panel), the only way to create a temporarily low real rate is to have a relatively high rate of inflation. As the capital stock gradually declines, the real interest rate rises somewhat, and the rate of inflation falls further.

In the case illustrated in Figure 6, inflation remains negative and the real return on the safe asset remains positive throughout the transition. This need not always be the case. If the economy is hit with a large enough shock, the real return on bonds may actually be negative at date zero, as the economy's capital stock is far above its new steady state level. This in turn requires positive inflation in the short run, even though the economy will eventually arrive at a deflationary steady state. Even though the monetary policy rule we have specified rules out positive inflation in the long run, it may be consistent with keeping rates at zero even in the presence of short run inflation. Recall that the monetary authority's target depends on both inflation and the shortfall of output relative to its flexible price level, which will be negative throughout the transition. Whether the monetary authority is willing to keep rates at zero will depend on the weight it attaches to output stabilization (represented by $\psi$). If the weight on output stabilization is too small, the monetary authority might be unwilling to keep rates at zero early on in the transition if the economy experiences positive inflation. In this case, no equilibrium may exist, given the configuration of fiscal and monetary policy that we have specified. The economy desperately requires at least a few periods of negative real rates to smooth the transition to the new steady state, since capital is high in the short run, depressing interest rates even beyond the effect of the increase in risk. A monetary rule which is unable to accommodate temporarily negative real interest rates cannot even engineer a transition to a steady state with inflation and unemployment. Instead, employment spirals towards zero eventually leaving the government unable to meet its fiscal obligations: Either fiscal or monetary policy must adjust. Even if the monetary rule does allow for temporarily positive inflation, however, all is not well. Instead of imploding, it slumps into a steady state characterized by permanently higher unemployment, low investment and deflation.

## 4.2 How safe assets can preserve full employment

The previous section showed that in an economy with nominal rigidities, an increase in risk can lead to undesirable outcomes such as permanently high unemployment and low investment. The essential problem is that when the return on capital is very risky, the only way to sustain investment in capital is to have the real return on safe assets fall below zero. The ZLB prevents this adjustment leading to lower permanently lower economic activity.

This raises the question as to whether there is a way out of this conundrum? We have already seen that in an economy without nominal wage rigidities, an increase in government debt can offset an increase in idiosyncratic risk and keep real interest rates positive. In the absence of nominal rigidities, there is no reason to be concerned about real interest rates falling below a particular threshold. However, we have just seen that a negative natural rate $R^*$ causes problem in the presence of nominal rigidities. It is therefore natural to ask whether issuing government debt can redress this situation by raising $R^*$.

The answer is a qualified yes. Suppose that in response to the increase in idiosyncratic income risk, the fiscal authority raises $\widetilde{b}$ from $\widetilde{b}_L$ to a higher level $\widetilde{b}_H$. The blue and black solid lines in Figure 7 depict the equilibrium in this case; we set $\widetilde{b}_H = 0.077$. In order to show how the stance of monetary policy affects transitions, the blue line shows equilibrium when the monetary authority puts a relatively low weight on output gap stabilization ($\psi = 0.1$), while the black line depicts equilibrium when the monetary authority puts an equal weight on inflation gap and output stabilization.

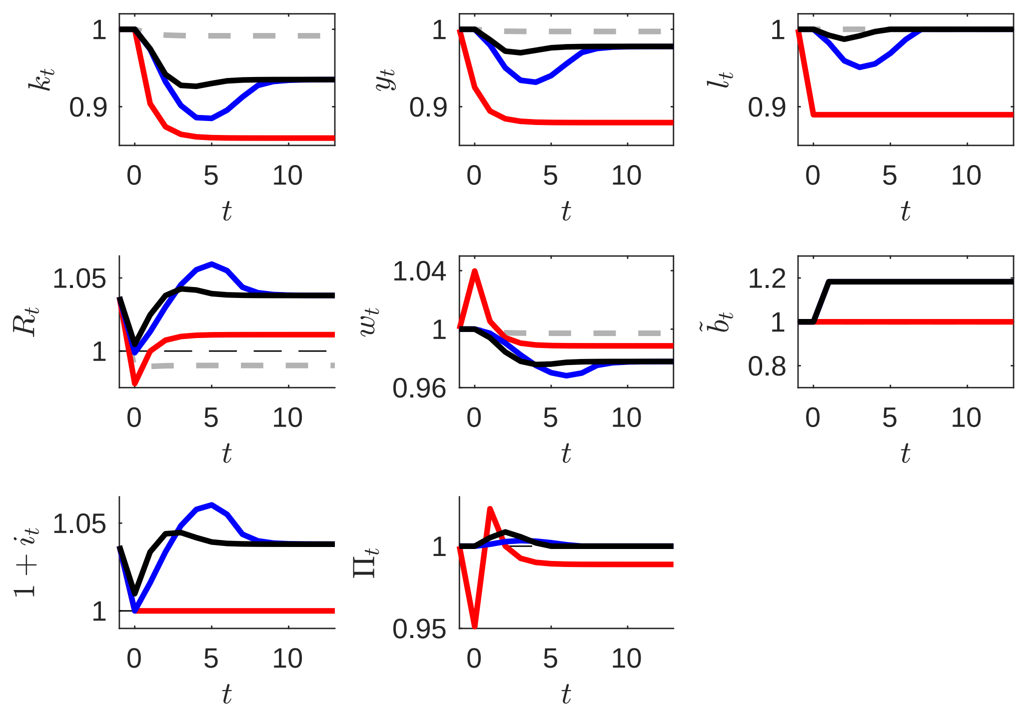



**Figure 7.** Dashed lines denote equilibrium in the absence of nominal rigidities. Solid red lines denote equilibrium with nominal rigidities and no increase in safe assets. Solid blue and black lines denote equilibrium with an increase in safe assets under hawkish ($\psi = 0.1$) and dovish ($\psi = 1$) monetary policy rules, respectively

As the middle left panel of Figure 7 shows, this increase in the supply of safe assets accommodates increased demand, equilibrating asset markets without the need for a fall in incomes. Real rates fall somewhat on impact, as the economy suddenly finds itself with a high level of capital relative to the new steady state. However, as capital gradually falls towards the new steady state (as shown in the top left panel), real interest rates rise again to their new steady state level. Thus while nominal rates fall close to zero on impact (as shown in the bottom left panel), the economy quickly escapes from the zero bound.

As in the flexible wage economy, an increase in safe assets still crowds out investment in physical capital - in the precise sense that investment is lower than it would be if there were no increase in safe assets *and if wages were fully flexible.* Both the blue and black lines in the top left panel show that the trajectory of capital lies below its trajectory in a flexible wage economy without any increase in safe assets. This causes a decline in real wages, as shown in the center panel of Figure 7. But in the presence of nominal rigidities, increasing safe assets 'crowds in' investment, in that investment is higher than it would be if there were no increase in safe assets, given that nominal wages are not fully flexible. Thus while the blue and black lines lie below the dashed line describing the flexible wage benchmark, they lie above the red line describing outcomes in the economy with nominal rigidities and no increase in safe assets. As a result, while producing safe assets reduces the capital-labor ratio, thus reducing real wages and labor productivity, this is more than compensated by an increase in employment, and so producing safe assets increases long-run output (shown in the top middle panel).

The decline in investment due to the increase in safe assets reduces the demand for labor. Since wages are sticky downwards, this tends to cause unemployment in the short run. Given that the monetary authority puts some weight on output gap stabilization, and is not contrained by the zero lower bound, it responds by loosening monetary policy somewhat, resulting in moderate inflation shortly after the initial shock (shown in the bottom middle panel). This helps facilitate the decline in real wages towards their new steady state level (shown in the center panel). Indeed, a more hawkish policy which puts a smaller weight on output gap stabilization relative to inflation ($\psi = 0.1$, shown by the blue lines) features a larger temporary decline in employment and output, relative to a dovish policy which puts equal weight on both objectives ($\psi = 1$, shown by the black lines). In either case, however, in the long run an increase in safe assets results in full employment and zero inflation, although at a lower level of capital and output than prevailed before the initial shock.

As Figure 7 shows, a permanently higher level of safe assets $\widetilde{b}_H$ does more than just smooth transitions; it also affects long run outcomes inducing a steady state characterized by full employment. The dashed lines in Figure 8 depict the steady state without the increase in debt; the solid lines depict steady state with higher debt. Higher debt satiates investors' demand for safe assets, reducing the risk premium and raising the real interest rate. The IS curve shifts up and to the right. A sufficiently large increase in $\widetilde{b}$ restores full employment and zero inflation.

However, as we have already seen, this is not costless. As the lower panel of Figure 8 shows, the increase in safe assets shifts down the steady state capital stock for any level of employment. While an increase in safe assets raises real interest rates and restores full employment, it crowds out investment in physical capital, in the precise sense that investment is lower than it would be if there were no increase in safe assets *and if wages were fully flexible*. Thus, the creation of additional safe assets ultimately leaves the capital stock and investment lower before the increase in risk. With a lower level of capital, output remains lower than its pre-crisis level even though full employment has been restored, as the isoquants show. Indeed the new full employment equilibrium has a lower capital-labor ratio, and thus lower real wages and labor productivity, not just relative to the low risk steady state but also the low debt, high risk, high unemployment steady state (see the grey lines in the lower panel which pass through the origin). In this sense, our model suggests that a risk-induced recession can continue to depress output, wages and productivity even when fiscal

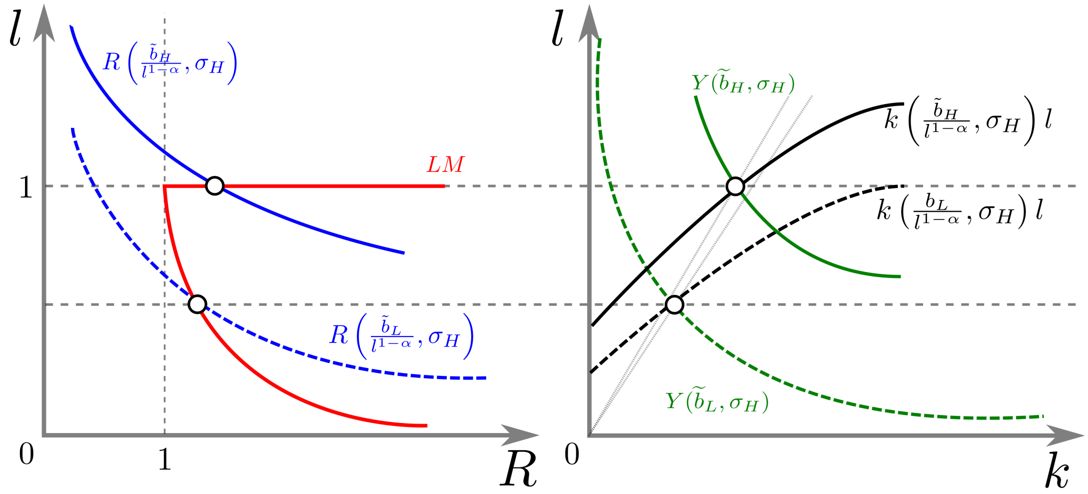



**Figure 8.** A permanent increase in $\tilde{b}$ in a high $\sigma$ environment

policy has restored full employment.

As we saw in the model without nominal rigidities, an economy with a high level of nominal risk *needs* negative real interest rates in order to sustain a high level of investment. In the presence of nominal rigidities and a monetary policy rule that targets zero inflation whenever unconstrained by the zero lower bound, negative rates are simply not a possibility. Instead, an economy with a negative natural rate experiences a recession, as monetary policy loses its potency at the zero lower bound. Issuing public debt satiates the demand for safe assets and raises the natural rate of interest, relaxing the ZLB constraint and rendering monetary policy potent again. But this does not change the fact that a risky economy **requires negative real rates** in order to sustain high *investment*. The same increase in debt which restores full employment crowds out investment in physical capital.

From the perspective of our model, rather than increasing government debt, a better policy would be to engineer negative real interest rates, either through negative nominal rates or a permanently higher inflation target. Negative real rates would permit both full employment and high investment. Our model is not the place for a full cost-benefit analysis of such alternative policies, since it abstracts both from the costs associated with a higher steady state rate of inflation (Coibion et al. (2016)), and the potential consequences of persistently low rates for financial stability (Greenwood et al. (2016)). If these costs are large, risk-induced secular stagnation may present problems which cannot be solved by either monetary or fiscal policy.[14] Nonetheless, such policies are worth considering, because safe asset creation is no panacea.

This provides a new perspective on an argument made by economists such as Paul Krugman and Brad DeLong during the Great Recession. Their argument was that when governments can borrow at negative real interest rates while unemployment is high, they should obviously do so.

[14] Another possibility, within the lens of our model, would be to produce safe assets but keep investment at its efficient level by other means, for example with an appropriately designed subsidy.

Higher deficits reduce unemployment in the short run; moreover, negative real rates make it an exceptionally good time for the government to borrow (since the private sector is effectively paying them to take its money). In our model, this argument is correct as far as it goes: increasing deficits prevents unemployment, and actually results in higher investment. This is preferable to the alternative of tight fiscal policy and maintaining the monetary policy rule. However, a better alternative to both these options would be to keep a low level of public debt, but allow for a higher inflation target. Such a policy would engineer negative real interest rates, permitting a high level of investment without the need for safe assets.

# 5 Rational bubbles

Our analysis above showed how an increase in risk can lead to the equilibrium real interest rate becoming negative. As is commonly known, negative real interest rates permit the existence of *rational bubbles* (Tirole, 1985). A rational bubble is defined as as asset with no intrinsic value, pays no dividend but is in finite supply. In general, there exist equilibria in which such assets trade at a positive price when real interest rates are negative. This is also true in our setting. In this section we explore how the existence of rational bubbles interacts with and is affected by the supply of safe assets.

Suppose there exists a stock of intrinsicly useless assets in measure 1. At date 0, these are all owned by the date 0 old generation. Let $Q_t$ denote the price of this asset and let $x_{t+1}$ denote the quantity of this asset purchased by a young household at date $t$. Then the budget constraints of cohort $t$ household can be written as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} + Q_t x_{t+1} = W_t l_t + P_t T_t \tag{36}$$

$$P_{t+1} c_{t+1}^O(z) = P_{t+1} R_{t+1}^k(z) k_{t+1} + B_{t+1} + Q_{t+1} x_{t+1} \tag{37}$$

where $Q_t x_{t+1}$ in (36) denotes the expenditure on bubbles by the young household and $Q_{t+1} x_{t+1}$ in (37) denotes the payoff of holding $x_{t+1}$ bubbles when old. The market clearing conditions in the market for bubbles is $x_t = 1$. All of our analysis considered equilibria in which $Q_t = 0$ for all $t$.

**Definition 3** (Equilibrium with bubbles)**.** *Given a sequence $\{B_{t+1}, i_t, T_t\}_{t=0}^{\infty}$ and initial conditions $\{B_0, k_0\}$, an equilibrium is a sequence $\{c_t^Y, c_t^O(z), k_{t+1}, x_{t+1}, l_t, \ell_t(z), R_t^k(z), P_t, W_t, Q_t\}_{t=0}^{\infty}$ such that*

1. *$\{c_t^Y, c_t^O(z), k_{t+1}, B_{t+1}, x_{t+1}\}$ solves the household's problem for each cohort $t$, given prices $\{i_t, R_t^k(z), P_t, W_t, Q_t\}$ and transfers $\{T_t\}$*
2. *$\{\ell_t(z), R_t^k(z)\}$ solve the firm's problem at each date $t$*
3. *the government budget constraints (10), labor market clearing (11), goods market clearing (13) and bubble market clearing $x_{t+1} = 1$ are satisfied*

**Safe bubbles** Following Weil (1987), we consider equilibria in which the bubble bursts with a constant probability $1-\rho$ for $\rho \in (0,1]$ in each period. The simplest case to consider is one in which bubbles never burst, $\rho = 1$. We refer to this case as a *"safe" bubble.* In such a setting, safe bubbles and government debt are perfect substitutes from the point of view of a young household (assuming that bubbles have positive value). Thus, these two assets must offer the same return:

$$R_t = \frac{q_{t+1}}{q_t}$$

where $q_t = Q_t/P_t$ is the real price of a bubble at date $t$. In particular, if such a bubble is traded at a positive price in steady state, it must be the case that $R = 1 = q_{t+1}/q_t$. It follows then that if the level of risk is such that there are positive real interest rates in the absence of bubbles, $R^s s(\tilde{b}, \sigma) \geq 1$ in (20), then no equilibrium exists in which safe bubbles trade at a positive price in steady state. If safe bubbles traded at a positive price in steady state and $R > 1$, then this would require $q_{t+1}/q_t = R > 1$ the price of bubbles to grow. As a result, the value of traded bubbles would continue to grow in such a steady state and eventually outgrow the size of the economy exceeding the income of young households who are the purchasers of these assets. However, if $R < 1$ in the absence of safe bubbles, then there exists a steady state in which safe bubbles are traded at a constant positive price $q$ and $R = 1$ where $q$ is the solution to:

$$R^{ss}\left(\tilde{b} + \frac{q}{k^\alpha}, \sigma\right) = 1 \tag{38}$$

where $R^{ss}()$ is defined in (20). Compare two steady states with the same $\sigma$ and $\tilde{b}$, one with bubbles and one without: the bubbly steady state features a higher real interest rate and lower capital stock. Safe bubbles provide insurance and crowd out investment in much the same way as government produced safe assets.

**Lemma 11** (Welfare and Safe Bubbles)**.** *Consider the problem of the planner who maximizes steady state welfare in* (21)*. For any $\sigma$, the solution to this planner's problem strictly dominates any steady state featuring safe bubbles (if such steady states exist) .*

*Proof.* Clearly, (38) implies that any steady state with bubbles and $q > 0$ has the same level of capital and consumption as one with $q = 0$ and $\tilde{b}$ such that $R = 1$. From Proposition 1, recall that the solution to the steady state planner's problem features either $\tilde{b} = 0$ or $R < 1$ and welfare dominates any steady state allocation with $\tilde{b} > 0$ and $R = 1$. □

Notice that this result stands in contrast to the literature that has studied the properties of bubbles in overlapping gernations models. Generally, rational bubbles can only exist when the bubble-less equilibrium would feature overaccumulation of capital. The emergence of bubbles improves welfare in such an economy facilitating a transfer of wealth from the young to the old without neccesitating the inefficiently high levels of capital. In other words, bubbles increase welfare by crowding out capital in such settings. In our environment, bubbles can arise even when the bubble-less equilibrium is dynamically efficient: bubbles decrease welfare by crowding out capital and emerge even when they are not desirable. Thus, our paper can be thought of as providing a counterexample to a conjecture of Abel et al. (1989) that rational speculative bubbles can only exist in economies with over accumulation of capital, even in the presence of risk.[15]

**Leaning against the wind** Optimal policy in our economy often involves creating an environment in which negative real interest rates exist. As we have just shown, such an environment is a breeding ground for bubbles, which reduce welfare. One might worry that even if a planner would like to refrain from producing safe assets in order to prevent crowding out, this will not be possible, because the resulting low interest rate environment may permit bubbles to arise which crowd out productive investment in any case. In fact, this need not be the case if the fiscal authority can make credible commitments regarding what it would do if bubbles were to arise, as we now show. Suppose that instead of commiting to a fixed path of $\tilde{b}_t^*$, the fiscal authority commits to implement this path as an equilibrium outcome using the following policy rule: For any date $t$, if $q_t = 0$, choose $\tilde{b}_{t+1} = \tilde{b}_{t+1}^*$. If instead, $q_t > 0$, revert to a policy which sets $\tilde{b}_s = \tilde{b}_{ss}$ for all $s > t$, where $R(\tilde{b}_{ss}, \sigma) > 1$. The off-equilibrium threat to crowd out rational bubbles with government debt prevents such bubbles from ever emerging.

A large literature has attempted to formalize the notion that monetary policy should lean against the wind to prevent asset price bubbles; however, it has proven challenging to construct models in which bubbles can exist, reduce welfare, and can be prevented with contractionary monetary policy (Gali (2014), Allen et al. (2017)). Our model provides an environment in which welfare-reducing bubbles can exist, but suggests that it may be more appropriate for fiscal policy, rather than monetary policy, to lean against the wind. One interpretation of our results in Sections 2-4 is that such off-equilibrium commitments prevented bubbles from arising.

**Risky bubbles** Consider instead risky bubbles with $\rho < 1$: bubbles may exist, but have constant positive probability $1 - \rho$ of bursting in each period. For simplicity, we assume that once a bubble bursts, a new bubble never appears to replace it. In this case, risky bubbles are no longer a perfect substitute for safe government debt, so we must have $R_t < \frac{q_{t+1}}{q_t}$ (assuming that the bubble does not burst at date $t + 1$). In this case, bubbles still crowd out capital, but now introduce an additional risk: bubbles can burst, leading to consumption losses for the old households whose wealth vanishes. In principle the fiscal authority can prevent such a possibility by committing to a fiscal rule as described above. However, if such commitments are not credible, a government wishing to eliminate bubbles may be forced to increase the supply of publicly produced safe assets on-equilibrium. While our stylized model in no way addresses matters such as financial stability, this result resonates with the argument of Greenwood et al. (2016) that the government should supply short-term safe assets and crowd out socially excessive private safe asset creation which may

[15]See paragraph 3 on page 15 of Abel et al. (1989).

arise to compensate for a shortage of government-provided safe assets. Again, while our model abstracts from the externalities associated with excessive private maturity transformation which are the focus of Greenwood et al. (2016), risky bubbles can be broadly thought of as an example of the excessive private creation of safe assets - which fiscal policy can potentially prevent.

# 6 Conclusion

We presented a model in which the natural rate of interest is affected both by idiosyncratic risk and by fiscal policy. By increasing the supply of safe assets, the government can prevent an increase in risk from driving real interest rates below zero, allowing monetary policy to operate effectively rather than being constrained by the zero lower bound on nominal rates. However, our analysis uncovers that negative real interest rates do not necessarily indicate a *shortage* of safe assets. While it is possible for fiscal policy to keep interest rates above zero, this is generally not desirable in the absence of nominal rigidities, because increasing debt crowds out investment in physical capital. In the presence of nominal rigidities and a lower bound on nominal interest rates, a negative natural rate of interest (along with a low inflation target) can cause persistent investment and employment slumps. An increase in government debt can prevent this, relaxing the zero bound constraint and allowing monetary policy to restore full employment. However, the reprieve comes at the cost of a decline in investment and the steady state capital stock. The return to full employment merely disguises the deeper problem - that the economy requires negative interest rates in order to operate at potential - which manifests itself in the form of sluggish investment and productivity growth. In this sense, the cost of a risk-induced recession may linger even once the economy has returned to full employment. Rather than increasing government debt, it may be preferable to engineer negative real interest rates, either through negative nominal rates or a higher inflation target; such policies sustain a high level of investment while preventing unemployment.

A full empirical evaluation of this theory lies beyond the scope of this paper. Nevertheless, the scenario we have described is in some respects disturbingly similar to the experience of the U.S. and other advanced economies during the recovery from the Great Recession.

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# Appendix

## A Household's Optimal Choices

Using equations (1)-(2), the objective function of the households can be written as:

$$\max_{k_{t+1},b_{t+1}} (1-\beta)\ln\left[\omega_t l_t + T_t - \frac{1}{R_t}b_{t+1} - k_{t+1}\right] + \beta\mathbb{E}_z \ln\left[R^k_{t+1}(z)k_{t+1} + b_{t+1}\right]$$

where $\omega_t = \frac{W_t}{P_t}$ and $b_{t+1} = \frac{B_{t+1}}{P_{t+1}}$ and $R_t = \frac{\Pi_t}{1+i_t}$. The first order conditions w.r.t $k_{t+1}$ and $b_{t+1}$ can be written as:

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R^k_{t+1}}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{39}$$

$$\frac{1-\beta}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \beta\mathbb{E}_z\left[\frac{R_t}{R^k_{t+1}(z)k_{t+1} + b_{t+1}}\right] \tag{40}$$

Next multiply equation (39) by $k_{t+1}$, (40) by $\frac{b_{t+1}}{R_t}$ and add them up:

$$\frac{k_{t+1} + \frac{b_{t+1}}{R_t}}{\omega_t l_t + T_t - \frac{b_{t+1}}{R_t} - k_{t+1}} = \frac{\beta}{1-\beta} \tag{41}$$

which can be rearranged to yield:

$$k_{t+1} + \frac{b_{t+1}}{R_t} = \beta \left[\omega_t l_t + T_t\right] \tag{42}$$

i.e. the young household save a fraction $\beta$ of its labor income net of transfers. Using the budget constraint, it is straightforward to see that

$$c^Y_{t+1} = (1-\beta)\left[\omega_t l_t + T_t\right]$$

Using these equations, we can re-write the objective as:

$$\max_{\eta_t}(1-\beta)\ln\Big[(1-\beta)(\omega_t l_t + T_t)\Big] + \beta\ln\Big[(\omega_t l_t + T_t)\Big] + \beta\mathbb{E}_z\ln\Big[\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t\Big]$$

where we define the portfolio share of capital as $\eta_t$ as $\frac{k_{t+1}}{k_{t+1}+\frac{b_{t+1}}{R_t}}$. The optimal choice of $\eta_t$ can then be written as:

$$\mathbb{E}_z \ln\left[\frac{R^k_{t+1}(z) - R_t}{\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t}\right] = 0$$

# B Proof of Lemma 3

Multiplying (6) by $(\eta_t - 1)$ and rearranging, we have

$$\mathbb{E}_z\left[\frac{R^k_{t+1}}{\eta_t R^k_{t+1}(z) + (1-\eta_t)R_t}\right] = 1$$

Using the expression for $R^k(z)$ in equation (12) and multiplying both sides by $\eta_t$ we can rewrite the equation above as:

$$\eta_t = \mathbb{E}_z\left[\frac{\alpha z k^\alpha_{t+1}}{\alpha z k^\alpha_{t+1} + \frac{1-\eta_t}{\eta_t}R_t}\right] \tag{43}$$

Next, using equations (4) and (5) we know that:

$$\frac{1-\eta_t}{\eta_t} = \frac{b_{t+1}}{R_t k_{t+1}} = \frac{\tilde{b}_{t+1}}{R_t}k^{\alpha-1}_{t+1} \tag{44}$$

where we used the definition of $\tilde{b}$ to go from the first to the second equality. Plugging in this expression into (43) and simplifying, we get:

$$\eta_t = \mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]$$

In order to see that $\eta_t$ is decreasing in $\sigma$, consider instead, the expression for $1-\eta_t$:

$$1-\eta_t \quad = \quad \mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z+\tilde{b}_{t+1}}\right]$$

which is clearly increasing in $\sigma$ because of Jensen's inequality.

# C Deriving an Expression for the Real Interest Rate

In order to derive the expression for the real interest rate, substitute the expression (44) for $\eta_t$ into equation (44) and rearrange:

$$R_t \quad = \quad \tilde{b}_{t+1}\frac{\eta_t}{1-\eta_t}k_{t+1}^{\alpha-1} = \frac{\mathbb{E}_z\left[\dfrac{\alpha z}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\dfrac{1}{\alpha z+\tilde{b}_{t+1}}\right]}k_{t+1}^{\alpha-1}$$

How does $R_t$ change with $\sigma$? Notice that we can also write the expression for $R_t$ as:

$$R_t = \left(\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right)k_t^{\alpha-1}$$

Then since $\mathbb{E}_t\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1}$ is increasing in $\sigma$ (from Jensen's inequality), the whole expression is decreasing and thus, $\frac{\partial R_t}{\partial \sigma} < 0$.

Notice that since the spread can be written as:

$$\frac{R_t}{\mathbb{E}_z R_{t+1}^k(z)} = \frac{1}{\alpha}\frac{1-\mathbb{E}_z\left[\frac{\tilde{b}_{t+1}}{\alpha z+\tilde{b}_{t+1}}\right]}{\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]} = \frac{1}{\alpha}\left[\mathbb{E}_z\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]^{-1} - \tilde{b}_{t+1}\right]$$

Next, from Jensen's inequality, we know that:

$$\frac{\partial\left(\frac{R_t}{\mathbb{R}_{t+1}^k(z)}\right)}{\partial \tilde{b}_{t+1}} = \frac{\mathbb{E}\left[\left(\frac{1}{\alpha z+\tilde{b}_{t+1}}\right)^2\right] - \left(\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\right)^2}{\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]\mathbb{E}\left[\frac{1}{\alpha z+\tilde{b}_{t+1}}\right]} > 0$$

## D Proof of Lemma 9

The FOC for the choice of $\tilde{b}$ yields:

$$\begin{aligned} -(1-\beta)\frac{1}{(1-\alpha-\tilde{b})k^{\alpha}-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^{\alpha}}\right] &\leq 0 && (45)\\ \tilde{b} &\geq 0 && (46)\\ \left\{[-(1-\beta)\frac{1}{(1-\alpha-\tilde{b})k^{\alpha}-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^{\alpha}}\right]\right\}\tilde{b} &= 0 && (47)\end{aligned}$$

Also, note that the young household's bond Euler equation in equilibrium can be written as:

$$-(1-\beta)\frac{1}{(1-\alpha-\tilde{b})k^{\alpha}-k}+\beta R\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^{\alpha}}\right]=0$$

Combining this equation with (45), we can write optimality as: $R\geq 1$, $\tilde{b}\geq 0$ and $(R-1)\tilde{b}=0$.

## E Proof of Proposition 1

The problem of the steady state planner can be written as:

$$\mathcal{L}=\max_{k,\tilde{b}\geq 0}(1-\beta)\ln\left[(1-\alpha-\tilde{b})k^{\alpha}-k\right]+\beta\mathbb{E}_z\ln\left[(\alpha z+\tilde{b})k^{\alpha}\right]-\lambda^{ss}\left(k-s(\tilde{b},\tilde{b},\sigma)^{\frac{1}{1-\alpha}}\right)$$

The FOC for $k$ can be written as:

$$\frac{\alpha}{k}-\frac{(1-\beta)(1-\alpha)}{(1-\alpha-\tilde{b})k^{\alpha}-k}-\lambda^{ss}=0 \tag{48}$$

The FOC for $\tilde{b}$ can be written as:

$$\begin{aligned}\frac{-(1-\beta)}{(1-\alpha-\tilde{b})k^{\alpha}-k}+\beta\mathbb{E}_z\left[\frac{1}{\left(\alpha z+\tilde{b}\right)k^{\alpha}}\right]+\frac{\lambda^{ss}s(\tilde{b},\tilde{b},\sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\left[s_1(\tilde{b},\tilde{b},\sigma)+s_2(\tilde{b},\tilde{b},\sigma)\right] &\leq 0 && (49)\\ \tilde{b} &\geq 0 && (50)\end{aligned}$$

To show that $\lambda^{ss}>0$

**Proof of (i)** The objective function can also be written in terms of $(k, b)$:

$$\mathbb{W} = \max_{\{k,b\}} (1-\beta)\ln\left[(1-\alpha)k^{\alpha} - b - k\right] + \beta\mathbb{E}\ln\left[\alpha z k^{\alpha} + b\right]$$

It is straightforward to see that $\mathbb{W}$ is concave in $(k, b)$. Suppose $\sigma < \underline{\sigma}$ and evaluate the derivative of $\mathbb{W}$ at $(k_{\max}, 0)$ where $k_{\max} = s(0,0,\sigma)^{\frac{1}{1-\alpha}}$:

$$\begin{aligned} \frac{\partial \mathbb{W}(k_{\max},0)}{\partial k} &= (1-\beta)\frac{\alpha(1-\alpha)k_{\max}^{\alpha-1} - 1}{(1-\alpha)k_{\max}^{\alpha} - k_{\max}} + \beta\frac{\alpha}{k_{\max}} \\ &= \frac{\alpha}{k_{\max}}\left[1 - \frac{\beta(1-\alpha)}{\alpha}\right] > 0 \end{aligned} \tag{51}$$

where the last inequality stems from Assumption 1. Similarly,

$$\begin{aligned} \frac{\partial \mathbb{W}(k_{\max},0)}{\partial b} &= -(1-\beta)\frac{1}{(1-\alpha)(1-\beta)k_{\max}^{\alpha}} + \beta\mathbb{E}_z\left[\frac{1}{\alpha z k_{\max}^{\alpha}}\right] \\ &= -\frac{1}{(1-\alpha)k_{\max}^{\alpha}}\left[1 - \frac{\beta(1-\alpha)}{\alpha}e^{\sigma^2}\right] < 0 \end{aligned} \tag{52}$$

where the last inequality holds since $\sigma < \underline{\sigma}$.

Next, take any feasible allocation where $b > 0$: it must feature $k < k_{\max}$. Since $\mathbb{W}(k, b)$ is concave,we have:

$$\begin{aligned} \mathbb{W}(k,b) &\leq \mathbb{W}(k_{\max},0) + \frac{\partial \mathbb{W}(k_{\max},0)}{\partial k}(k - k_{\max}) + \frac{\partial \mathbb{W}(k_{\max},0)}{\partial b}b \\ &< \mathbb{W}(k_{\max},0) \end{aligned}$$

Thus, $(k_{\max}, 0)$ must be optimal. Since $\sigma < \underline{\sigma}$ and $b = 0$, we know that $R(0, \sigma) > 1$.

**Proof of (ii)** Substituting the implementability constraint into $\mathbb{W}(k, b)$, we have

$$\mathbf{W}(\tilde{b},\varepsilon) := \mathbb{W}\left(s(\tilde{b},\tilde{b},\underline{\sigma}+\varepsilon)^{\frac{1}{1-\alpha}}, \tilde{b}s(\tilde{b},\tilde{b},\underline{\sigma}+\varepsilon)^{\frac{\alpha}{1-\alpha}}\right)$$

for $\varepsilon > 0$. In order for $\tilde{b} > 0$ to be optimal given $\varepsilon$, we need $\mathbf{W}_b(\tilde{b}, \varepsilon) = 0$ and $\mathbf{W}(\tilde{b}, \varepsilon) \geq \mathbf{W}(0, \varepsilon)$. It is straightforward to show that there exists a function $\tilde{\mathbf{b}}(\varepsilon)$ such that for $\tilde{b} > \tilde{\mathbf{b}}(\varepsilon)$, $\mathbf{W}(\tilde{b}, \varepsilon) < \mathbf{W}(0, \varepsilon)$. Further, $\tilde{\mathbf{b}}(\varepsilon) \to 0$ as $\varepsilon \to 0$. Next, note that $\mathbf{W}_b(\tilde{b}, \varepsilon)$ is a continuous function and is strictly negative at $(0, 0)$. Thus, there exists $(\gamma, \delta)$ such that $\tilde{b} \in (0, \gamma)$, $\varepsilon \in (0, \delta)$ implies $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0.5\mathbf{W}_b(0, 0) < 0$. Choose $\varepsilon_1 < \delta$ such that $\tilde{\mathbf{b}}(\varepsilon_1) < \gamma$. For all $\varepsilon \in (0, \varepsilon_1)$, we have $\mathbf{W}_b(\tilde{b}, \varepsilon) < 0$ for all $\tilde{b} \in (0, \tilde{\mathbf{b}}(\varepsilon))$. Thus, there are no interior optimum and $\tilde{b} = 0$ must be optimal in an open interval around $\underline{\sigma}$.

**Proof of (iii)** First, we show that for $\sigma$ sufficiently large, the following expression is positive

$$
\begin{aligned}
d\mathbb{W}(k_{\max},0) &= \frac{\partial \mathbb{W}(k_{\max},0)}{\partial k}\frac{s(0,0,\sigma)^{\frac{\alpha}{1-\alpha}}}{1-\alpha}\Big[s_1(0,0,\sigma)+s_2(0,0,\sigma)\Big]+\frac{\partial \mathbb{W}(k_{\max},0)}{\partial \tilde{b}} \\
&= -\frac{1-\beta\,(1-\alpha)}{(1-\alpha)^2}+\left[\frac{\beta-\alpha}{\beta(1-\alpha)}+1-\beta\right]\frac{\beta}{\alpha}e^{\sigma^2}
\end{aligned}
$$

For large enough $\sigma$ the second term overwhelms the first term making $d\mathbb{W}(k_{\max},0)>0$ if $\alpha<\beta$. In this case, there exists a finite $\bar{\sigma}$ such that as long as $\sigma>\bar{\sigma}$, it is optimal to create safe assets. If however $\alpha$ is large relative to $\beta$, and $\frac{\beta-\alpha}{\beta(1-\alpha)}+1-\beta<0$, then it may never be optimal to create safe assets for any level of $\sigma$ since crowding out always dominates the benefits from insurance.

It remains to show that at the optimum whenever $\tilde{b}>0$, $R<1$. First, we show that we can never have an interior optimum with $\mathbb{W}_k\le 0$ and $\mathbb{W}_b<0$. Consider any point $(k_0,\tilde{b}_0)$ with $b_0>0$ s.t. $\mathbb{W}_k(k_0,\tilde{b}_0)\le 0$ and $\mathbb{W}_b(k_0,\tilde{b}_0)<0$. For any $\varepsilon>0$, define $k_\varepsilon=s(\tilde{b}_0-\varepsilon,\tilde{b}_0-\varepsilon,\sigma)<k_0$ as the steady state level of capital for $\tilde{b}_0-\varepsilon$. The gain in welfare from decreasing $\tilde{b}$ by $\varepsilon$ is approximately $\mathbb{W}_k(k_0,\tilde{b}_0)(k_\varepsilon-k_0)+\mathbb{W}_b(k_0,\tilde{b}_0)\varepsilon$. For small $\varepsilon$, this gain is positive since $\mathbb{W}_k\le 0$, $k_\varepsilon<k_0$ and $\mathbb{W}_b>0$. So the initial point cannot be optimal. By a similar argument, we cannot have both $\mathbb{W}_b\le 0$ and $\mathbb{W}_k<0$ at an optimum. Finally, since $\mathbb{W}$ is concave and attains its maximum at $\tilde{b}=0$, $k>k_{max}$, we cannot have $\mathbb{W}_k=\mathbb{W}_b=0$ at any feasible point.

Take any interior optimal point. The first order necessary condition for optimality is

$$
\mathbb{W}_b+\mathbb{W}_k\frac{\partial k}{\partial b}=0
$$

If $\mathbb{W}_b\le 0$ at an optimum, then by the above arguments we must have $\mathbb{W}_k>0$, which contradicts the optimality condition. So at any interior optimum, we must have $\mathbb{W}_b>0$, which, again using the household's Euler equation for bonds, implies that $R<1$.

# F Proof of Lemma 10

Define $\sigma^\diamond=\sqrt{\ln\left[\frac{\alpha}{(\beta-\alpha)(1-\alpha)}\right]}$ and $k_{\max}^{1-\alpha}=s(0,0,\sigma)$ We begin by showing that for all $\sigma\in[0,\sigma^\diamond]$, there exists at least one sequence of non-negative Pareto weights $\{\phi_i\}_{i=0}^{\infty}$ which satisfies absolute summability for which $k_t=k_{\max}$ and $\tilde{b}_t=0$ for all $t\ge 0$ solve the following problem:

$$
\mathbb{U}(\phi)=\max_{\{k_{t+1},\tilde{b}_{t+1}\}_{t=0}^{\infty}}\sum_{t=0}^{\infty}\phi_t U_t+\phi_{-1}\mathbb{E}_z\ln c_0^O(z) \tag{53}
$$

subject to

$$
\begin{aligned}
U_t &= (1-\beta)\ln c_t^Y + \beta \mathbb{E}_z \ln c_{t+1}^O(z) && (54)\\
k_{t+1} &= s(\tilde{b}_t, \tilde{b}_{t+1}, \sigma) k_t^\alpha && (55)\\
k_0 &= k_{\max} && (56)
\end{aligned}
$$

For $\sigma > \sigma^\diamond$, there is no sequence which of Pareto weights for which $(k_{\max}, 0)$ solves the problem above.

Plugging the constraints (54)-(56) into the objective function (53) and rearranging yields:

$$
\mathbb{U} = \max_{\{\tilde{b}_{t+1}\}_{t=0}^\infty} \sum_{t=0}^\infty \phi_t \left\{ (1-\beta) \ln \left[ \left(1-\alpha-\tilde{b}_t\right) - s\left(\tilde{b}_t, \tilde{b}_{t+1}\right) \right] + \beta \mathbb{E}_z \ln \left[ \alpha z + \tilde{b}_{t+1} \right] \right\}
$$
$$
+\phi_{-1}\beta \mathbb{E}_z \ln \left[\alpha z + \tilde{b}_0\right] + \sum_{t=0}^\infty \ln s\left(\tilde{b}_t, \tilde{b}_{t+1}\right) \left( \phi_t \beta + \sum_{j=t+1}^\infty \phi_j \alpha^{j-t} \right) + \text{constants independent of } \tilde{b}
$$

The FOC is given by:

$$
\phi_{t-1} \left\{ (1-\beta) \frac{-s_2\left(\tilde{b}_{t-1}, \tilde{b}_t\right)}{\left(1-\alpha-\tilde{b}_{t-1}\right) - s\left(\tilde{b}_{t-1}, \tilde{b}_t\right)} + \beta \mathbb{E}_z \left[ \frac{1}{\alpha z + \tilde{b}_t} \right] \right\} + \frac{s_2\left(\tilde{b}_{t-1}, \tilde{b}_t\right)}{s\left(\tilde{b}_{t-1}, \tilde{b}_t\right)} \left( \phi_{t-1}\beta + \sum_{j=t}^\infty \phi_j \alpha^{j-t+1} \right)
$$
$$
+\phi_t \left\{ (1-\beta) \frac{-1 - s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{\left(1-\alpha-\tilde{b}_t\right) - s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)} \right\} + \frac{s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)} \left( \phi_t \beta + \sum_{j=t+1}^\infty \phi_j \alpha^{j-t} \right) \leq 0 \quad (57)
$$

where

$$
s_1\left(\tilde{b}_t, \tilde{b}_{t+1}\right) = \frac{\partial s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{\partial \tilde{b}_t} = \frac{-\beta}{\beta + (1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]\right)^{-1}}
$$

and

$$
s_2\left(\tilde{b}_t, \tilde{b}_{t+1}\right) = \frac{\partial s\left(\tilde{b}_t, \tilde{b}_{t+1}\right)}{\partial \tilde{b}_{t+1}} = -\frac{\beta\left(1-\alpha-\tilde{b}_t\right)}{\beta + (1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]\right)^{-1}} \frac{(1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]\right)^{-2} \mathbb{E}_z\left[\frac{\alpha z}{\left(\alpha z + \tilde{b}_{t+1}\right)^2}\right]}{\beta + (1-\beta)\left(\mathbb{E}_z\left[\frac{\alpha z}{\alpha z + \tilde{b}_{t+1}}\right]\right)^{-1}}
$$

Evaluating (57) at $\tilde{b}_t = 0$ for all $t \geq 0$ and rearranging yields:

$$
\phi_{t-1} \frac{\beta(1-\alpha) e^{\sigma^2}}{1 + (1-\alpha)(1-\beta) e^{\sigma^2}} \leq \alpha \sum_{s=0}^\infty \alpha^s \phi_{t+s} \quad (58)
$$

Define $y_t = \sum_{s=0}^\infty \alpha^s \phi_{t+s} \in [0, \infty)$. So, $\phi_{t-1} = \alpha\left(\frac{1}{\alpha} y_{t-1} - y_t\right)$. Using these definitions, (58) can be written as:

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)\,e^{\sigma^2}}{1+(1-\alpha)\,e^{\sigma^2}}\right]y_{t-1}\leq y_t$$

Since $y_t<\infty$ for any $\{\phi_s\}$ which satisfies absolute-summability[16], such a sequence $\{y_t\}$ exists iff

$$\frac{\beta}{\alpha}\left[\frac{(1-\alpha)\,e^{\sigma^2}}{1+(1-\alpha)\,e^{\sigma^2}}\right]<1$$

which holds as long as $\sigma<\sigma^{\diamond}$. Conversely, if $\sigma>\sigma^{\diamond}$, the above expression is strictly greater than one and no absolutely-summable positive sequence $\{\phi_t\}$ exists which satisfies (58).

Finally, as is standard following Negishi (1960), an allocation is constrained efficient iff there exists Pareto weights $\{\phi_t\}$ such that the allocation solves the problem in (53) . So we are done.

# G Proof of Proposition 2

The first claim follows from our analysis in section 4.1.1 which shows that when $R(\tilde{b}_L,\sigma_H)<1$, then no full employment steady state can exist. To see why the second claim is true, first, we establish that the unemployment steady state is unique for $\gamma$ sufficiently small. After that we construct an equilibrium in which $i_t=0$ for all $t\geq 0$ and show that it is unique.

Steady states are characterized by:

$$R\left(\tilde{b}_L l^{\alpha-1},\sigma_H\right)=l^{-\frac{\alpha\gamma}{1-\gamma}}$$

When $\gamma=0$, this equation has a unique solution. It follows immediately that for $\gamma$ sufficiently close to zero, the steady state remains unique.

Equilibrium with $i_t=0$ for all $t\geq 0$ must satisfy the following conditions:

$$\begin{aligned}
k_{t+1}+(1-\beta)\frac{\tilde{b}_L}{R_t}k_{t+1}^{\alpha} &= \beta\left[(1-\alpha)k_t^{\alpha}l_t^{1-\alpha}-\tilde{b}_L k_t^{\alpha}\right] && (59)\\
\Pi_t^{-1}=R_t &= \frac{1}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)}\alpha\left(\frac{k_{t+1}}{l_{t+1}}\right)^{\alpha-1} && (60)\\
l_t &= \min\left\{\left(\frac{k_t}{k_{t-1}}\right)^{1-\gamma}l_{t-1}^{1-\gamma}\Pi_t^{\frac{1-\gamma}{\alpha}},1\right\} && (61)\\
\Pi_t l_t^{(1-\alpha)\psi} &\leq 1 && (62)
\end{aligned}$$

We proceed that (62) is satisfied with a strict inequality and that (61) holds with $l_t<1$ for all $t$.

[16] $y_t$ is the discounted sum of a absolutely-summable sequence and hence must be finite.

Plug in (60) into (59):

$$\left[1+\left(\frac{1-\beta}{\alpha}\right)\tilde{b}_L l_{t+1}^{1-\alpha} g(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H)\right]\frac{k_{t+1}}{k_t^{\alpha}}=\beta\left[(1-\alpha)l_t^{1-\alpha}-\tilde{b}_L\right]$$

Similarly, using (61) and (60):

$$\frac{k_{t+1}}{k_t^{\alpha}}=\alpha\frac{l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}}}{l_t^{\alpha}}\frac{1}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)} \tag{63}$$

Substitute the second equation into the first to get:

$$\begin{aligned}\left[\alpha\frac{1}{g\left(\tilde{b}_L l_{t+1}^{\alpha-1},\sigma_H\right)}+(1-\beta)\,\tilde{b}_L l_{t+1}^{1-\alpha}\right]l_{t+1}^{\frac{1-\gamma(1-\alpha)}{1-\gamma}} &= \beta\left[(1-\alpha)l_t-\tilde{b}_L l_t^{\alpha}\right] &(64)\\ LHS(l_{t+1}) &= RHS(l_t) &(65)\end{aligned}$$

It is easy to see that $LHS(l)$ is increasing and nonnegative, while $RHS(l)$ is negative for $l_t < l_{min} = \left(\frac{1-\alpha}{\tilde{b}_L}\right)^{\frac{1}{1-\alpha}}$, and positive and increasing after that. Furthermore, for $\gamma$ sufficiently close to 0, the two curves have a unique intersection in $(0,1)$, as we now show.

First let $\gamma=0$. Intersections of the two curves satisfy

$$\begin{aligned}\left[\alpha\frac{1}{g\left(\tilde{b}_L l^{\alpha-1},\sigma_H\right)}+(1-\beta)\,\tilde{b}_L l^{1-\alpha}\right]l &= \beta\left[(1-\alpha)l-\tilde{b}_L l^{\alpha}\right]\\ \alpha\frac{1}{g\left(\tilde{b}_L l^{\alpha-1},\sigma_H\right)}+(1-\beta)\,\tilde{b}_L l^{1-\alpha} &= \beta\left[(1-\alpha-\tilde{b}_L l^{\alpha-1}\right]\\ \frac{\mathbb{E}\left[\alpha z(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right]}{\mathbb{E}\left[(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right]} &= \beta(1-\alpha)-\tilde{b}_L l^{\alpha-1}\\ \mathbb{E}\left[\alpha z(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right]+\mathbb{E}\left[\tilde{b}_L l^{\alpha-1}(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right] &= \beta(1-\alpha)\mathbb{E}\left[(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right]\\ 1 &= \beta(1-\alpha)\mathbb{E}\left[(\alpha z+\tilde{b}_L l^{\alpha-1})^{-1}\right]\end{aligned}$$

which has a unique solution for $l$. Again, by continuity it follows that the solution is also unique for $\gamma$ sufficiently close to 0.

It follows that at the unique intersection $l^*$, $RHS$ cuts $LHS$ from above, i.e. $RHS'(l^*) < LHS'(l^*)$. Thus if $l_0 < l^*$, $LHS(l_1) = RHS(l_0)$ implies $l_1 < l_0$, and so forth: $\{l_t\}$ is monotonically decreasing. The sequence cannot converge to any positive number: if it did converge, that limit would be another steady state, a contradiction. So eventually we must have $l_t < l_{min}$, which cannot be an equilibrium. By a similar argument, if $l_1 > l_0$, we must eventually have $l_t > 1$, which contradicts our assumption that the ZLB binds in every period. Thus the unique equilibrium with $i_t = 0$ features $l_t = l^*$ in every period. It is straightforward to construct the rest of the equilibrium setting $l = l^*$. Iterating forwards on equation (63) delivers the dynamics of capital. Imposing $l_t = l^*$ in (63) for all $t \geq 0$ reveals that the path for capital is monotonically declining towards the new steady state. Plugging these into equation (60) we obtain a sequence of inflation rates. Finally since $l_t = l^* < 1$ for all $t \geq 0$, (62) is always satisfied for high enough $\psi$.

# H Foundations for incomplete markets

In the paper we assumed that capital income risk faced by each household is non-diversifiable. Here, we show that unobservable capital quality can micro-found the incompleteness of markets assumed in the main text.

Suppose a household $i$ invests $k_{t+1}$ when young in physical capital and draws productivity shock $z_i$ when old. This productivity is embodied in the units of capital that household $i$ possesses when old. That is, even if another household $j$ with a different productivity $z_j$ were to operate the capital produced by $i$, that capital would continue to have productivity $z_i$even in the hands of households $j$. We assume that household $i$ cannot directly observe the realization of $z_j$ for $j \neq i$.

**No trade in spot markets for capital** Suppose old household $i$ perceives that it can buy or sell capital after the realization of $z_i$ at a price $q^k$. Then the problem of the firm operated by old household $i$'s can be written as:

$$R^k(z)\,k = \max\left[z\left(k - k_s(z)\right) + \tilde{z}k_b(z)\right]^{\alpha} \ell(z)^{1-\alpha} - \omega\ell(z) - q^k\left(k_b - k_s\right)$$

subject to:

$$\begin{aligned} k_s(z) &\in [0, k] \\ k_b(z) &\geq 0 \end{aligned}$$

where $k$ denotes the amount of capital household $i$ had invested in when young, $k_s(z) \in [0, k]$ is the amount of capital that household $i$ chooses to sell and $k_b(z)$ is the amount of capital chooses to buy. $\widetilde{z}$ denotes the average quality of capital being sold in equilibrium and is given by:

$$\widetilde{z} = \frac{\int_0^\infty z k_s(z) dF(z)}{\int_0^\infty k_s(z) dF(z)}$$

as long as the denominator is positive (i.e. there is some capital being sold) and zero otherwise (if no capital is sold). It is straightforward to see that given the a firm's optimal labor demand, this problem can be re-written as:[17]

$$R^k(z)\,k = \max_{k_s(z)\le k, k_b(z)\ge 0} \alpha\left(\frac{1-\alpha}{\omega}\right)^{\frac{1-\alpha}{\alpha}}\left[z\,(k-k_s)+\tilde{z}k_b\right]-q^k\,(k_b-k_s)$$

which is linear in $k_s$ and $k_b$ implying that all firms who sell their capital are those with productivity:

$$z \le \left(\frac{\omega}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}\frac{q^k}{\alpha} \tag{66}$$

In addition, firms are willing to buy any amount of capital as long as

$$\alpha\left(\frac{1-\alpha}{\omega}\right)^{\frac{1-\alpha}{\alpha}}\tilde{z} \ge q^k \tag{67}$$

and demand an infinite amount of capital if the inequality is strict. Thus, if there is any trade in this spot market, equation (67) must hold with a strict equality. Plugging in the expression for $q^k$ into (66) yields $z_i \le \tilde{z}$ for all sellers. However, this cannot be the case given the definition of $\widetilde{z}$. Thus, there is no trade is such a spot market and hence old households cannot insure themselves against low realizations of $z$ through such a spot market.

**No trade in Arrow securities contingent on productivity realizations** In the main text, we had also made the assumption that households could not insure themselves against low realizations of $z$ by buying Arrow securities (when young) which pay off after such realizations. Since the actual realization of $z_i$ is not publicly observable, the Arrow securities must payoff based on the profile of reports which we denote by $\hat{\mathbf{z}} := (\hat{z}_i, \hat{\mathbf{z}}_{-i})$ where $\hat{z}_i$ denotes the report by household $i$ and $\hat{\mathbf{z}}_{-i}$ denotes the profile of all other household's reports.

Given each household's purchases of Arrow securities when young, and given everyones realization of productivity, all old households of a given generation play the following message game: each household announces $\hat{z}_i$ in order to maximize:

$$c^O(z_i) = R^k(z_i)k_i + b_i + a_i(\hat{z}_i, \hat{\mathbf{z}}_{-i})$$

Observe that household $i$'s best response correspondence does not depend on the actual realization of $z_i$: $i$ always reports whichever state maximizes the net transfers from the rest of the households to her. So do all other households. Thus, the Nash equilibrium of the message game does not depend on the true state of the world, and each household merely receives a constant transfer. Finally,

[17]The labor demand conditional on $k_s$ and $k_b$ can be derived as:

$$\ell(z) = \left(\frac{1-\alpha}{\omega}\right)^{\frac{1}{\alpha}}\left\{z\,(k-k_s(z))+\tilde{z}k_b(z)\right\}$$

it is easy to see that these transfers must be zero. Since the transfers must sum to zero, positive transfers for some households must be balanced by negative transfers from others. The households receiving negative transfers would prefer to deviate by not participating in these markets at all. Thus, Arrow securities cannot provide households insurance against low realizations of $z$.

# I Cashless limit and implementation of monetary policy

In the main text, we assumed that monetary policy followed a Taylor-rule subject to a ZLB on nominal interest rates. We now show that such a rule can be implemented by a monetary authority by controlling money supply in the spirit of Woodford's cashless limit (Woodford, 1998). Appropriately specifying the cashless limit is a delicate matter, given our focus on equilibria in which nominal interest rates are pinned at zero and money offers the same return as government debt. Our exposition closely follows Asriyan et al. (2016).

Consider a slight departure from the model in the main text under which households can not also hold money. In this case, the budget constraint of young households can be written as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} B_{t+1} + M_{t+1} = W_t l_t + P_t T_t$$

where $M_{t+1}$ denotes money holdings and $i_t$. Defining $\hat{B}_{t+1} = B_{t+1} + M_{t+1}$ as the total safe asset holdings of the household, we can rewrite this as:

$$P_t c_t^Y + P_t k_{t+1} + \frac{1}{1+i_t} \hat{B}_{t+1} + \left(\frac{i_t}{1+i_t}\right) M_{t+1} = W_t l_t + P_t T_t \tag{68}$$

and the consumption of old can be written as:

$$P_{t+1} c_t^O = P_{t+1} R_{t+1}^k(z) k_{t+1} + \hat{B}_{t+1} \tag{69}$$

In equilibrium, agents may choose not to old any money if other assets dominate it. If no agents hold money, the monetary authority loses control of the price level. To avoid this technical problem, we impose a cash-holding requirement - each young household must hold at least $\nu$ amount of real balances, i.e.

$$\frac{M_{t+1}}{P_t} \geq \nu \tag{70}$$

Next, we assume that the monetary authority remits all seigniorage revenue to the fiscal authority. Thus, the government budget constraint can be written as:

$$M_{t+1} - M_t + \frac{1}{1+i_t} B_{t+1} = B_t + P_t T_t \tag{71}$$

## I.1 Implementing any desired level of $i_t \geq 0$

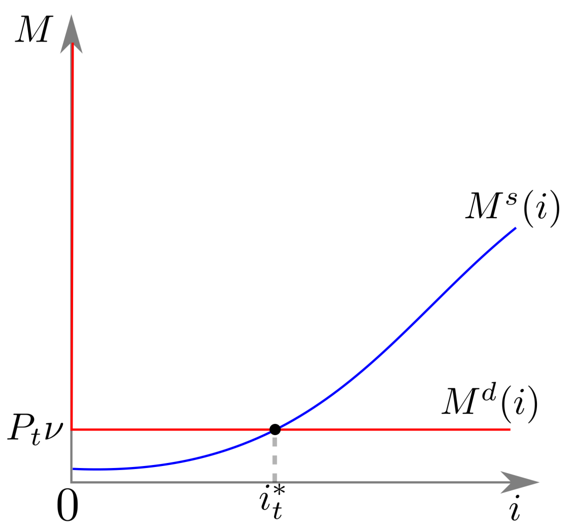
First, it is straightforward to show that the existence of money, which pays a zero nominal return, imposes a ZLB on nominal interest rates.[18] To see this, compare the households euler equations for bonds and money:

$$\frac{1-\beta}{c_t^Y} = \beta\frac{(1+i_t)P_t}{P_{t+1}}\mathbb{E}_t\left[\frac{1}{c_{t+1}^O}\right] \tag{72}$$

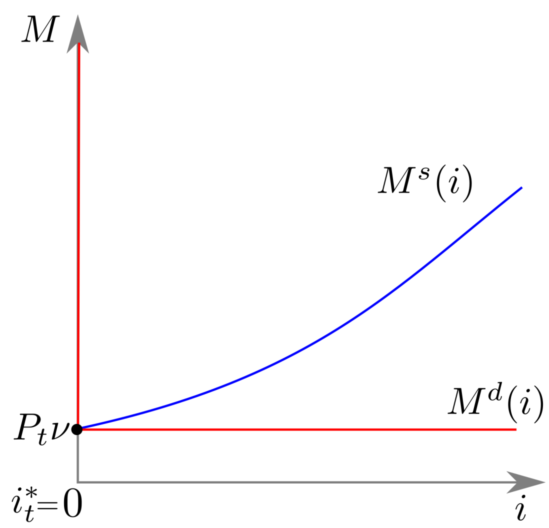
$$\frac{1-\beta}{c_t^Y} \geq \beta\frac{P_t}{P_{t+1}}\mathbb{E}_t\left[\frac{1}{c_{t+1}^O}\right] \tag{73}$$

which together require that $i_t \geq 0$ (ZLB); otherwise households would infinitely short bonds and hold money. Thus, in equilibrium, using equation (68) along with 70 we can express the demand for money and bonds as:[19]

$$\left(\frac{M_{t+1}}{P_t}, \frac{B_{t+1}}{P_t}\right) = \begin{cases} \left(\nu, \frac{\hat{B}_{t+1}}{P_t} - \nu\right) & \text{if } i_t > 0 \\ \left(x, \frac{\hat{B}_{t+1}}{P_t} - x\right) \text{ for } x \in \left[\nu, \frac{\hat{B}_{t+1}}{P_t}\right] & \text{if } i_t = 0 \end{cases} \tag{74}$$




**(a)** $i_t^* > 0$




**(b)** $i_t^* = 0$

**Figure 9.** Implementing the desired level of nominal interest rate $i_t^*$

Next, suppose the monetary authority wants to implement a nominal interest rate $i_t^*$. It does

[18] While this argument assumes that each young household perceives that she can take a negative position in bonds, nothing substantial would change if households were unable to short bonds; it would only be necessary to reinterpret $i_t$ as the shadow cost of funds.

[19] given $\hat{B}_{t+1} \geq \nu P_t$)

this by committing to a money supply schedule $M^s(*i)$ which is strictly increasing in $i$ and satisfies $M^s(i^*) = P_t\nu$. This ensures that in any equilibrium $i_t = i_t^*$. If by contradiction, interest rates were higher than $i_t^*$, the monetary authority would create a quantity of real balances in excess of household's desired holdings of money balances, distributing the excess money to households via transfers $T_t$. Not wishing to hold this excess money in their portfolio, households would seek to use this to buy government debt instead, pushing down nominal interest rates. Similarly, if the nominal rate was less than $i_t^*$, the monetary authority would contract the supply of real balances below the minimum required holdings $\nu$. This leads households to reduce their holdings of bonds in an attempt to accumulate the necessary amount of money, thus raising nominal interest rates. Figure 9 shows equilibrium in the money market with the desired level of interest rate $i_t^*$. Note in particular that nominal rates are uniquely determined even when $i_t^* = 0$. Finally, in the spirit of Woodford (1998) taking limits as $\nu \to 0$, allocations in this economy converge to those described in the main text.