Unemployment Traps[*]

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June 20, 2017

Abstract

We present a model in which temporary shocks can permanently scar the economy's productive capacity. Unemployed workers lose skill and are expensive to re-train, generating multiple steady state unemployment rates. Large temporary shocks push the economy into a liquidity trap, generating deflation. With nominal wages unable to adjust freely, real wages rise, reducing hiring and catapulting the economy towards the high-unemployment steady state. Even after a short-lived liquidity trap, the economy recovers slowly at best; at worst, it falls into a permanent unemployment trap. Since monetary policy may be powerless to escape such a trap ex-post, it is especially important to avoid it ex-ante: policy should be preventive rather than curative. The model can quantitatively account for the slow recovery in the U.S. following the Great Recession, and suggests that lack of swift monetary accommodation by the ECB can help explain stagnation in the European periphery.

JEL Classification: E24, E3, E5, J23, J64
Keywords: hysteresis, monetary policy, multiple steady states, skill depreciation

1 Introduction

In the aftermath of the global financial crisis, economic activity has remained subdued, suggesting that the world economy may be on a lower growth trajectory compared to the pre-2007 period. Many researchers attribute this sluggish growth to permanent, exogenous structural changes - either permanently lower productivity growth (Gordon, 2015) or *secular stagnation*.[1] An alternative, less-explored explanation is that large, temporary downturns can themselves damage to an economy's productive capacity. While these two sets of explanations may be observationally similar, they have very different normative implications. If exogenous structural factors drive slow growth, countercyclical policy may be unable to reverse this trend. If instead temporary downturns themselves lead to permanently slower growth, then countercyclical policy, by reducing the severity of the downturn, may also be able to avoid slow growth.[2] We build a stylized model to understand how hysteresis might operate, and how, if at all, countercyclical policy can moderate its impact.

Our model makes three additions to the canonical Diamond-Mortensen-Pissarides (DMP) search and matching framework: human capital depreciates during unemployment spells,[3] nominal wages are slow to adjust downwards,[4] and monetary policy is constrained by a lower bound on nominal interest rates. Together, these render an economy vulnerable to hysteresis.

Hysteresis is possible in our environment because workers lose human capital whilst unemployed and unskilled workers are costly to retrain as in Pissarides (1992). This generates multiple steady states. One, desirable steady state is a high-pressure economy: job finding rates are high, unemployment is low and job-seekers are high-skilled. While wages are high because tight labor markets improve workers' outside options, firms still find job creation attractive, as higher wages are compensated by the low average cost of training highly skilled job seekers. However, the economy can also be trapped in a low-pressure steady state - job finding rates are low, unemployment is high, and many job-seekers are unskilled as longer unemployment spells have eroded their human capital. Slack labor markets lower the outside

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[1]Here by *secular stagnation* we refer to the literature arguing that a chronic excess of global savings relative to investment has depressed equilibrium real interest rates. This imbalance has been variously attributed to permanent changes in either borrowing constraints, supply of safe assets, demographics, inequality or monopoly power. See for example, Eggertsson and Mehrotra (2014), Caballero and Farhi (2016), Jones and Philippon (2016) among many others.

[2] As Chair Yellen has recently suggested *"...hysteresis would seem to make it even more important for policymakers to act quickly and aggressively in response to a recession, because doing so would help to reduce the depth and persistence of the downturn, thereby limiting the supply-side damage that might otherwise ensue."* (Yellen, 2016).

[3]There is a large empirical literature documenting the scarring effects of unemployment; see, for example Song and von Wachter (2014) and Wee (2016).

[4]See for example, (Fallick et al., 2016). See also Schmitt-Grohe and Uribe (2013) for a survey of the empirical literature documenting wage rigidity.

options of workers and drive down wages, but hiring is still limited as firms find it costly to retrain these workers. Crucially, this is an *unemployment trap* - an economy near the low pressure steady state can never escape to a high pressure economy.

The presence of nominal wage rigidities, together with constraints on monetary policy, allows temporary shocks to permanently move the economy from a high pressure steady state into an unemployment trap. Consider the effect of a large but temporary decline in the households' rate of time preference. This increases desired saving, pushing real interest rates below zero. Monetary policy tries to accommodate this by lowering nominal interest rates but is constrained by the ZLB. Thus, households' desire to postpone consumption lowers the demand for current consumption relative to the future, causing a decline in current prices. With nominal wages unable to freely adjust downwards, real wages rise, reducing hiring. This decline in hiring lengthens the average duration of unemployment and increases the incidence of skill loss, leading to a worsening of the skill composition of the unemployed. Even after the shock has abated, it may take the economy to train workers and return to a high pressure steady state. Worse still, the initial decline, if large enough, may leave the economy in an unemployment trap from which it is powerless to escape.

Is this scenario avoidable? Yes. If policy commits to temporarily higher inflation after the liquidity trap has ended, monetary policy can mitigate both the initial rise in unemployment, and its persistent (or permanent) negative consequences. But this commitment must be made early in the crisis, before the recession has left substantial scars. If policy is slow to act, the economy may enter an unemployment trap which monetary policy cannot escape. Similarly, even if the recession leads to slow recovery rather than permanent stagnation, monetary policy cannot hasten the recovery. In such cases, fiscal policy - such as hiring or training subsidies - is necessary to engineer a faster recovery.

Overall, in the presence of hysteresis, a failure to deliver stimulus early on in a recession can have irreversible costs. This contrasts with standard New Keynesian (NK) models, in which accommodative policies are equally effective at any point in a liquidity trap (Eggertsson and Woodford, 2003). In fact, these models predict that while overly tight policy may be costly in the short-run, it has no long run consequences, since temporary shocks have no permanent effects in stationary models.

Our main qualitative results are obtained in a stylized model, allowing us to analytically characterize equilibria in a monetary economy with multiple steady states.[5] This allows monetary policy to affect not just fluctuations of around steady state, but also the level of steady state output.[6] In comparison, standard NK models study stationary fluctuations

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[5]It is important to distinguish our approach from that of Farmer (2012) who considers economies with a continuum of steady states but focuses on how beliefs cause an economy to transition between these.

[6]This does not mean that monetary policy can manipulate a long-run trade off between inflation and

around a unique steady state. This precludes the possibility that short run disturbances can cause permanent damage, preventing monetary policy from influencing long run outcomes.[7] Furthermore, in standard NK models, failure to equate the real interest rate to the *Wicksellian rate* r^* due to the ZLB typically precipitates recessions; equating these rates prevents a recession. We abstract from this problem altogether by studying an environment in which the real rate is always equal to r^*, and show that this is still not enough to avert a recession.

We also ask whether the model can quantitatively explain the slow recovery in the U.S. following the Great Recession. A calibrated version of the model suggests that allowing for a realistic degree of skill depreciation and training costs, in line with the existing literature, is sufficient to generate multiple steady states. Furthermore, this multiplicity is essential in explaining why the unemployment rate in the U.S. took 7 years to return to its pre-crisis level. In contrast, the standard search model without skill depreciation and/or training costs predicts that the U.S. economy should have recovered by 2011. Under our preferred calibration, the model indicates that had monetary policy been less accommodative during the crisis, leading to a higher peak unemployment rate higher than 11 percent, the economy might have been permanently scarred and stuck in an unemployment trap. Our quantitative analysis also suggests that relatively modest hiring or training subsidies can hasten the recovery. In particular, a 4 % reduction in training costs would have sped up the U.S. recovery by 2 years.

The remainder of the paper is structured as follows. Next we discuss related literature. Section 2 presents the model economy . Section 3 characterizes steady states and equilibria in the flexible wage benchmark. Section 4 introduces nominal rigidities, and studies how demand shocks can cause slow recoveries or permanent stagnation. Section 5 analyses whether the mechanism studied here can account for the slow U.S. recovery since the Great Recession. Section 6 concludes.

Related Literature A large theoretical literature has studied hysteresis in the context of economies without nominal rigidities. Drazen (1985) argues that the loss of human capital due to job-loss in recessions can lead to delayed recoveries. Schaal and Taschereau-Dumouchel (2016) show that a labor search model with aggregate demand externalities can generate additional persistence in labor market variables. Similarly, in Taschereau-Dumouchel and Schaal (2015), large recessions frustrate coordination on a high-activity equilibrium, allowing hence temporary shocks to cause quasi-permanent recessions. Our model instead draws on Pissarides (1992), who presents a model in which skill depreciation gives rise to multiple

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unemployment. One the economy has converged to a particular steady state unemployment rate, monetary policy is powerless to reduce unemployment below this rate.

[7] Summers (2015) expounds on this criticism of New Keynesian models at length.

steady states. Sterk (2016) studies a quantitative version of Pissarides (1992) and argues that it can quantitatively account for the behavior of job finding rates in the United States. Relative to our work, all these studies consider purely real models; instead, motivated by the Great Recession, we study how a temporary liquidity trap interacts with skill depreciation in the presence of nominal rigidities. Importantly, in our framework, without nominal rigidities, hysteresis could never take root.

A few papers study hysteresis and monetary policy in the presence of nominal frictions. Recently, Galí (2016) studies optimal policy in a NK model with insider-outside labor markets drawing on the earlier work of Blanchard and Summers (1986). Closest to our work is Laureys (2014), who studies optimal monetary policy when skill depreciates during unemployment spells.[8] These papers find that finds that monetary policy should deviate from strict inflation targeting, putting more emphasis on unemployment stabilization. Relative to our analysis, these papers use first order methods to study economies with a unique steady state, in which hysteresis adds persistence, but does not affect long run outcomes. In contrast, we consider a model with multiple steady states, and study how monetary policy affects long run outcomes.

While we focus on hysteresis operating through the labor market, a recent literature studies the *innovation channel* of hysteresis. Bianchi et al. (2014) find that declines in R&D during recessions can explain persistent effects of cyclical shocks on growth while Garga and Singh (2016) study the conduct of optimal monetary policy in such a model. Benigno and Fornaro (2015) also study an economy in which pessimism can drive the economy to the ZLB and lead to persistent or permanent slowdowns driven by a fall in innovation. A commitment to alternative monetary policy rules or subsidies to innovation can help avoid or exit such *stagnation traps*. While we study a different channel through which hysteresis might operate, our results resonate with Benigno and Fornaro (2015): a commitment to an alternative monetary policy can avoid an unemployment trap as long as it comes swiftly. However, if an economy is already stuck in an unemployment trap, monetary policy may be unable to exit the trap but fiscal policy such as hiring or training subsidies can still do so.

On the empirical side, a large literature finds evidence in support of hysteresis: productive capacity falls after a recession. Dickens (1982) finds that recessions can permanently lower productivity; Haltmaier (2012) finds that trend output falls by 3 percentage points on average in developed economies four years after a pre-recession peak. Using cross-country data, Martin et al. (2014) find that severe recessions have a sustained and sizable negative impact on output. Similarly, Ball (2009) finds that large increases in the natural rate of unemployment are associated with disinflations, and large decreases with inflation.[9] Song and von Wachter

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[8]Kapadia (2005) performs a similar exercise in the 3 equation NK model by incorporating hysteresis in output in a reduced form fashion.

[9]In other work, Ball (2014) finds that countries with a larger fall in output during the Great Recession

([2014](#)) find that the persistent decline in employment following job displacement is larger during recessions, suggesting that a spike in job-destruction rates can persistently affect unemployment.[10]

Aside from the literature on hysteresis, our analysis connects to a few recent developments in monetary economics. Like us, Dupraz et al. (2017) study a *plucking model* in which downward-nominal wage rigidity gives rise to asymmetric effects of monetary policy: while deflation can lead to an increase in real wages and a fall in hiring, inflation has limited ability to reduce unemployment. This asymmetry increases the costs of business cycles; but since their economy features a unique steady state, shocks have at most a temporary effect. In contrast, we show that this asymmetry becomes especially dangerous when combined with hysteresis: temporary deflation can lead to permanently higher unemployment which cannot be reversed by higher inflation at a later date. Thus, in our setting it is especially important for monetary policy to stabilize employment, even at the cost of compromising price stability. This result resonates with Berger et al. (2016), who find that monetary policy should prioritize employment stabilization over price stability when households are imperfectly insured against layoff risk. Our analysis provides another reason why employment fluctuations might have higher costs, and warrant more attention.

Our paper also relates to the secular stagnation literature. Eggertsson and Mehrotra (2014) and Caballero and Farhi (2016) present models in which the market clearing interest rate is persistently or permanently negative, leading to persistently low output, as the ZLB prevents nominal rates from falling to clear markets. In these models, typically a permanent change in fiscal or monetary policy (such as an increase in target inflation) is required to prevent stagnation. We share this literature's concern with long run outcomes, but consider a different mechanism: *temporary* falls in market clearing interest rates have permanent effects, which *temporary* monetary accommodation can prevent.

2   The Model Economy

We use a standard DMP model of labor-market frictions. Time is discrete and there is no uncertainty. The only addition to the standard model is that we assume that workers can lose skill following an unemployment spell.

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experienced a larger decline in *potential output*. See also Blanchard et al. (2015) for a similar account.

[10]These papers also resonate with an older literature asking whether cyclical fluctuations could be studied independently of factors affecting longer-run outcomes. See e.g. Plosser (1989).

Workers   There exists a unit mass of ex ante identical workers, who are risk neutral and discount the future at a rate β. Workers can either be employed or unemployed. We denote the mass of employed workers as n and the mass of unemployed as $u = 1 - n$. All unemployed workers produce $b > 0$ as home-production. Households can borrow and save in a nominal bond which pays a certain nominal return of $1 + i_t$, yielding the Euler equation:

$$\beta(1 + i_t)\frac{P_t}{P_{t+1}} = 1 \tag{1}$$

where P_t is the price of consumption at date t. Employment evolves according to

$$n_t = [1 - \delta(1 - q_t)]\, n_{t-1} + q_t u_{t-1} \tag{2}$$

where δ is the exogenous rate at which workers get separated from their current jobs and q_t is the job-finding rate. Note that equation (2) implies that a worker that gets separated at the beginning of period t can find another job in the same period. Next, let $\mathbb{W}_t$ denote the value of an employed worker and $\mathbb{U}_t$ denote the value of an unemployed worker at time t. These can be expressed as follows:

$$\mathbb{W}_t = \omega_t + \beta\Big\{ [1 - \delta(1 - q_{t+1})]\, \mathbb{W}_{t+1} + \delta(1 - q_{t+1})\mathbb{U}_{t+1} \Big\} \tag{3}$$

$$\mathbb{U}_t = b + \beta\Big\{ q_{t+1}\mathbb{W}_{t+1} + (1 - q_{t+1})\mathbb{U}_{t+1} \Big\} \tag{4}$$

where ω_t denotes the real wage at date t and b is the value of home production. The net value of being employed for a worker can be written as:

$$\mathbb{W}_t - \mathbb{U}_t = \omega_t - b + \beta(1 - \delta)\,(1 - q_{t+1})(\mathbb{W}_{t+1} - \mathbb{U}_{t+1})$$

All else being equal, the worker's gain from matching is smaller (her outside option is higher) if q_{t+1} is higher.

Labor Market   As in Pissarides (1992), a worker who gets separated from her job and is unable to transition back to employment immediately loses skill that she acquired while employed. That is, any worker unemployed for at least 1 period becomes *unskilled*. If a firm hires an unskilled worker, it must pay a training cost $\chi > 0$. Once the firm trains the worker, she remains *skilled* until the next unemployment spell of at least 1 period. μ_t denotes the fraction of unskilled workers in the pool of job-seekers (l_t) at date t and is defined as:

$$\mu_t = \frac{u_{t-1}}{l_t} \equiv \frac{u_{t-1}}{1 - (1 - \delta)(1 - u_{t-1})} \tag{5}$$

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Equation 5 shows that a higher level of unemployment in the past corresponds to a lower fraction of skilled job-seekers. As such, there is a one-to-one mapping between u_{t-1} and μ_t.

Matching Technology   Search is random. The number of successful matches m_t between searchers l_t and vacancies v_t is given by a CRS matching technology $m(v_t, l_t)$. We define market tightness θ_t as the ratio of vacancies to searchers. The job-finding rate q_t is then:

$$q(\theta_t) = \frac{m(v_t, l_t)}{l_t} \in [0, 1] \tag{6}$$

In a similar fashion, we can define the job-filling rate as:

$$f(\theta_t) = \frac{m(v_t, l_t)}{v_t} = \frac{q(\theta_t)}{\theta_t} \in [0, 1] \tag{7}$$

Firms   A representative competitive CRS firm uses labor as an input to produce the final good. The production function is $y_t = A n_t$ where $A > b$ is aggregate productivity and n_t is the number of employed workers in period t. A firm must incur a vacancy posting cost of $\kappa > 0$ and an additional training cost of χ for each unskilled worker. A firm with n_{t-1} workers at the beginning of period t chooses vacancies (taking wages as given) to maximize lifetime discounted profit:

$$
\begin{aligned}
\mathbb{J}_t = \quad & \max_{v_t \geq 0} (A - \omega_t) n_t - (\kappa + \chi \mu_t f_t) v_t + \beta \mathbb{J}_{t+1} \\
\text{s.t.} \quad & n_t = (1 - \delta) n_{t-1} + f_t v_t
\end{aligned}
\tag{8}
$$

where ω_t is the wage paid to all workers.[11] This is the standard problem of a firm in search models, with one difference: the total cost of job creation depends on the skill composition of job-seekers. Since the firm pays a cost χ to train each unskilled job-seeker it hires, the effective average cost of creating a job increases in the the fraction of unskilled job-seekers μ_t. Recall from equation (5) that μ_t depends on past unemployment rates, making the cost of job creation *increasing* in the unemployment rate. The job-creation condition is then:

$$\frac{\kappa}{f_t} + \chi \mu_t + \lambda_t = A - \omega_t + \beta(1 - \delta) \left\{ \frac{\kappa}{f_{t+1}} + \chi \mu_{t+1} + \lambda_{t+1} \right\} \tag{9}$$

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[11]Firms pay the same wage to both initially skilled and unskilled hires as well as existing skilled workers. We discuss this in more detail in the section on wage determination.

where $\lambda_t f_t \geq 0$ is the multiplier on the non-negativity constraint on vacancies. Using the Envelope Theorem, the firm's value of a filled vacancy, $J_t = \partial \mathbb{J}_t / \partial n_t$, can be written as:[12]

$$J_t = A - \omega_t + \beta(1-\delta)J_{t+1} \tag{10}$$

Resource Constraint The resource constraint in the real economy can be written as:

$$c_t = An_t + b(1-n_t) - \kappa v_t - \chi \mu_t f_t v_t$$

To close the model, we now need to specify how wages and prices are determined.

Wage and Price Determination Ultimately, we will analyze the conduct of monetary policy in an environment with sticky nominal wages. However, it is useful to define a *flexible wage benchmark economy*, in which wages are determined by Nash bargaining every period. Because bargaining occurs after all hiring and training costs have been paid, all workers are paid the same wage.[13] Formally, the Nash bargaining problem is:

$$\max_{\omega_t} J_t^{1-\eta} \left(\mathbb{W}_t - \mathbb{U}_t\right)^\eta$$

where $\eta \in [0,1)$ denotes the bargaining power of the workers. The Nash-Bargained wage is:

$$\omega_t^* = \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1}J_{t+1} \tag{11}$$

Plugging in the Nash bargained wage into the expression for J_t yields:

$$J_t = a + \beta(1-\delta)(1 - \eta q_{t+1})J_{t+1} \tag{12}$$

where we define $a = (1-\eta)(A-b)$. Again, at every future date, next period's job finding rate puts upward pressure on the Nash wage because it increases the worker's outside option. The result is a smaller profit to the firm or a lower J_t. Iterating this forward and using equation (9), the job creation condition can be rewritten as :

$$J_t = a \sum_{s=0}^{\infty} \beta^s (1-\delta)^s \prod_{\tau=0}^{s} (1 - \eta q_{t+\tau}) \leq \kappa + \chi \mu_t \; , \; \theta_t \geq 0 \; , \; \text{with at least one strict equality} \tag{13}$$

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[12] Using the notation J_t, the job creation condition can also be written as $f_t[J_t - \chi \mu_t] \leq \kappa$, $\theta_t \geq 0$

[13] Given random search and the fact that the training cost is sunk at the time of bargaining, all workers have the same probability of finding a job, and hence share the same outside options.

In this benchmark, the classical dichotomy holds and the price level does not affect real allocations. Thus, it is not necessary to describe the conduct of monetary policy which determines prices.[14] Equilibrium in the benchmark economy is completely characterized by:

$$\mu_{t+1} = \frac{1 - q(\theta_t)}{1 + \gamma[1 - q(\theta_t) - \mu_t]} \tag{14}$$

$$J_t = a + \beta\gamma(1 - \eta q(\theta_{t+1}))J_{t+1} \tag{15}$$

$$J_t \leq \frac{\kappa}{f(\theta_t)} + \chi\mu_t, \theta_t \geq 0, \text{ at least one strict equality} \tag{16}$$

where we define $\gamma = 1 - \delta$ and (14) is derived by combining equations (5) and (8). These equations imply that the value of a filled vacancy to a firm lies in an interval:

Lemma 1. *The value of a filled vacancy to the firm is contained in the closed interval:*

$$J_{min} \leq J_t \leq J_{max} \tag{17}$$

where $J_{min} := \dfrac{a}{1 - \beta(1 - \delta)(1 - \eta)}$ *and* $J_{max} := \dfrac{a}{1 - \beta(1 - \delta)}$. *Moreover, if* $J_t = J_{min}$, *then* $q_{t+1} = 1$, *i.e.* $\theta_{t+1} \geq 1$ *and* $J_{t+1} = J_{min}$

Proof. The first part claim is immediate since $q_t \in [0, 1]$. For the second claim, we have $J_t = a + \beta(1 - \delta)(1 - \eta q_{t+1})J_{t+1}$. The only way to attain $J_t = J_{min}$ is $q_{t+1} = 1$ and $J_{t+1} = J_{min}$, since $q_{t+1} \leq 1$, $J_{t+1} \geq J_{min}$, and the expression is decreasing in q_{t+1} and increasing in J_{t+1}. □

3 Frictionless Wage Benchmark

Our ultimate goal is to describe an economy in which transitory increases in unemployment can permanently scar the economy and to ask whether monetary policy can do anything about it. We choose to interpret these permanent changes through the lens of a model economy in which there are multiple steady state unemployment rates and temporary shocks can move the economy between these steady states. The different steady-state unemployment rates can be interpreted as different possible values of the NAIRU. To this end, we now describe conditions under which multiple steady states can exist in this economy.

In our economy, multiplicity of steady state unemployment rates is possible because workers' skills depreciate during spells of unemployment and firms must pay a cost to train unskilled workers. Consider an economy plagued with high unemployment. Since the average duration of unemployment is high, the average skill composition of the workforce is low.

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[14]In Section 4, we describe the economy with sticky nominal wages and specify how monetary policy is conducted in that economy.

Consequently, firms need to spend more on training workers which makes them less willing to post vacancies. Thus, a high rate of unemployment can be self-sustaining. Conversely, if unemployment is low, average unemployment duration is low and the average skill of the workforce is high. Since the required outlay on training workers is lower, firms are more willing to post vacancies, thus sustaining a low level of unemployment.

In this section and Section 4, we assume a particular form of the matching function, $m_t = \min\{l_t, v_t\}$, which implies $q(\theta_t) = \min\{\theta_t, 1\}$, $f(\theta_t) = \min\{1/\theta_t, 1\}$. This simplifies the analysis without losing any generality. In particular, it implies that the short side of the market matches with probability 1. We refer to the case with $\theta_t < 1$ the *slack labor market regime* and $\theta_t \geq 1$ the *tight labor market regime*.

3.1 Steady States

Whether or not there exist other steady states, a full employment steady state always exists under conditions described next. Under full employment $n = 1$, so from equation (2), we need $q = 1$ (and $f = 1/\theta \leq 1$). Job seekers are on the short side of the market, and always find a job within one period; thus skill depreciation never occurs, and equation (5) implies $\mu = 0$. The complementary slackness condition (13) becomes $\kappa \theta^{FE} = J_{min}$. This has a solution with $\theta^{FE} \geq 1$ provided $J_{min} \geq \kappa$. In what follows, we will assume this is always true, so a employment steady state (FESS) exists.

Assumption 1 (Full Employment Steady State)**.**

$$J_{min} > \kappa.$$

For some parameters, there may also exist a steady state with zero hiring and employment. The following assumption rules out this uninteresting possibility. Qualitatively, none of our results would change if we allowed for a zero employment steady state.

Assumption 2 (No Zero Employment Steady State)**.**

$$J_{max} > \kappa + \chi$$

This assumption broadly requires that training costs are not too large.

Finally, we look for steady states in which firms are on the short side of the labor market, and there is some skill depreciation. In this section and in what follows, it will be convenient to work with the fraction of unskilled workers μ rather than the unemployment rate u as the state variable of interest.[15] Similarly, it is convenient to work with a quasi-value function

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[15]Notice that equation (5) defines a one-to-one map between μ_t and u_{t-1}.

defined in terms of μ as opposed to J_t.

Definition 1 (Quasi-Value Function). *Define the quasi-value function $\mathcal{Q}(\mu)$ as:*

$$\mathcal{Q}(\mu) = \frac{a}{1 - \beta\gamma\left[1 - \eta(1 - \mu)\right]}$$

By construction, $\mathcal{Q}(\mu)$ is the value of the firm as long as the job-finding rate is $1 - \mu$ forever. Note that $\mathcal{Q}'(\mu) > 0$ and $\mathcal{Q}''(\mu) > 0$. Any interior steady state must satisfy $\mathcal{Q}(\mu) = \kappa + \chi\mu$. This describes a quadratic in μ which has at most two solutions. In general, these solutions may not be economically meaningful and may lie outside the closed interval $[0, 1]$; the following assumption guarantees that economically meaningful solutions exist.

Assumption 3 (High η and χ).

$$\eta > \max\left\{\frac{1 - \beta\gamma}{\beta\gamma}, \frac{\gamma}{1 + \gamma}\right\} \quad and \quad \chi \in [\underline{\chi}, J_{\max} - \kappa]$$

where $\underline{\chi} := e[2 - k + 2\sqrt{1 - k}]J_{min}$ and $e = \dfrac{\beta\gamma\eta}{1 - \beta\gamma(1 - \eta)}$, $k = \dfrac{\kappa}{J_{min}}$.

The first part of Assumption 3 states that workers' bargaining power η must be high enough; the second part states that training cost χ must be high enough. High bargaining power increases workers' share of the surplus, and increases the sensitivity of wages and profits to labor market conditions. When unemployment is low, wages are high because workers' outside option is relatively favorable; however, firms happily pay high wages because training costs are low. When unemployment is high, workers are relatively unskilled and expensive to train; firms are willing to pay the high training costs because wages are relatively low, owing to the weak labor market. The following Lemma summarizes.

Lemma 2 (Existence of Multiple Steady States). *Under Assumptions 1, 2, and 3 there exist two interior steady states $0 < \widetilde{\mu} < \overline{\mu} < 1$.*

Proof. See Appendix B. □

Figure 2 graphically depicts the arguments above. The red curve plots the quasi-value function $\mathcal{Q}(\mu)$, the blue line plots $\kappa + \chi\mu$ for different values of χ. When χ is too low, the two curves do not intersect and there are no interior steady states. When χ is too high, the blue line lies above the red curve at $\mu = 1$ and there exists a zero-employment steady state, violating Assumption 2. When χ is in the appropriate range, then there are two steady states, $\widetilde{\mu}$ and $\overline{\mu}$. Finally, recall that there is always a full employment steady state at $\mu = 0$.
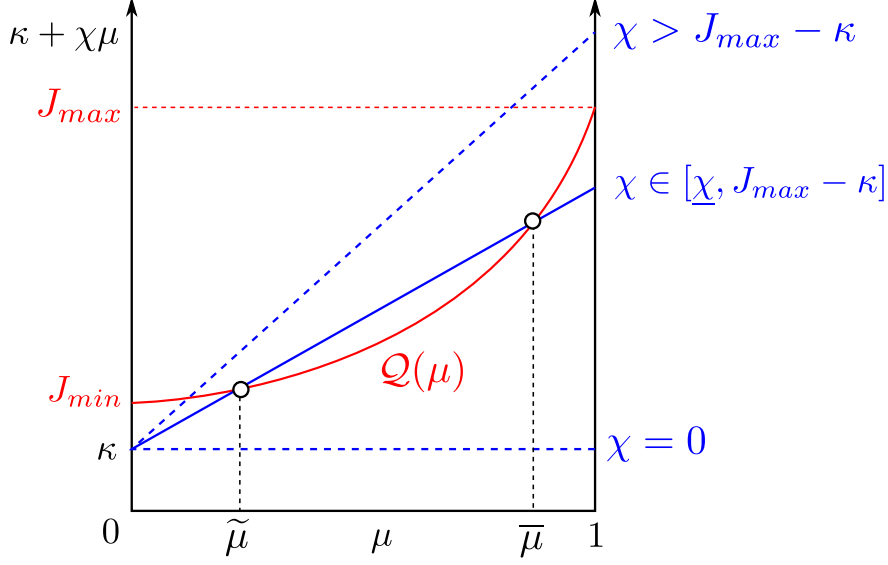
Figure 1. Multiple Steady States

3.2 Dynamics

Having described steady states in the flexible wage economy, we now study its transitional dynamics. We can split initial conditions on μ into 3 regions: (i) healthy (ii) convalescent and (iii) stagnant.

Healthy Region (High Pressure Economy) The healthy region is defined as the set $\mu \in [0, \underline{\mu})$ where $\underline{\mu} := (J_{min} - \kappa)/\chi$ is the highest value of μ for J_{min} is still attainable. For any $\mu < \underline{\mu}$, $q = 1$. The following Lemma says that if the economy starts in the healthy region, then labor markets are tight, $\theta > 1$ and the economy immediately converges to the full employment steady state.

Lemma 3. *Suppose $\mu_0 < \underline{\mu}$. Then the equilibrium is unique, with $n_t = 1$, $\mu_t = 0$ for all $t \geq 1$, and*

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\theta_t = \begin{cases} \dfrac{J_{min} - \chi\mu_0}{\kappa} & \text{for } t = 0 \\[2mm] \dfrac{J_{min}}{\kappa} & \text{for } t \geq 1 \end{cases}
$$

The value of a filled vacancy J_t is constant for all $\mu \in [0, \underline{\mu})$, and equals J_{min}.

Proof. See Appendix C. □

Intuitively, when the skill composition of job-seekers is very high, firms are willing to post a large number of vacancies. This creates a tight labor market, and job-seekers who are on

the short side of the market find jobs very easily. Consequently, unemployment duration is short (and equal to zero after the first period), and the skill quality of the work-force is high. While we have not yet introduced any shocks, one interpretation is that the full-employment steady state is stable with respect to shocks which only cause a small deterioration in the average skill composition of job-seekers μ_0. In particular, if μ_0 rises to a level in the interval $(0, \underline{\mu})$, this shock will be immediately reversed as job-seekers are still largely skilled, and firms are willing to post enough vacancies to hire and retrain them all.

If there is zero hiring for one period, starting from full employment, then the fraction of unskilled job seekers is $\mu_R = 1/(1 + \gamma)$. The following result, which will be useful later, states that $\mu_R > \underline{\mu}$. That is, a hiring freeze takes the economy out of the healthy region.

Lemma 4. *Under Assumption 3, if multiple steady states exist, then* $\mu_R = \dfrac{1}{1 + \gamma} > \underline{\mu}$.

Proof. See Appendix D. □

Convalescent Region   We now describe dynamics in the convalescent region, defined as the set $[\underline{\mu}, \widetilde{\mu})$. In this region, the economy eventually returns to full employment, and thus we can solve the model by backward induction. First we characterize equilibrium at $\underline{\mu}$, the point at which the labor market just becomes tight.

Lemma 5 (On the verge of a full recovery). *If $\mu_0 = \underline{\mu}$ then $\theta_0 \in [1 - \underline{\mu}, 1]$. Furthermore,* $\mu_1 = \dfrac{1 - \theta_0}{1 + \gamma[1 - \theta_0 - \mu_0]} \leq \mu_0$ *and* $\theta_1 = \dfrac{J_{min} - \chi \mu_1}{\kappa} \geq 1$. *For all $t > 1$, $\theta_t = \theta^{FE}$ and $\mu_t = 0$.*

Proof. See Appendix E. □

The Lemma states that the economy at $\underline{\mu}$ today will reach the healthy region in the next period. This in turn implies that θ today must be greater than $1 - \underline{\mu}$. In fact, any θ between $1 - \underline{\mu}$ and 1 is consistent with equilibrium.

Next, we describe how the economy evolves starting in the interior of the convalescent region. We proceed by backward induction. First, we characterize the set of μ's in the convalescent region from which the economy can reach $\underline{\mu}$ in one period - call this set as I^1. We then proceed to construct the sets $I^2, I^3, \ldots$ from which the economy can reach $\underline{\mu}$ in $2, 3, \cdots$ periods respectively. Crucially, the union of all these sets is the entire convalescent region. That is, starting from any point in the convalescent region, there is an equilibrium in which the economy converges to $\underline{\mu}$ in finite time; conversely, any equilibrium that converges to full employment must start from some point in the convalescent region. Furthermore, starting from any μ_0 in the convalescent region, there is a unique path returning to $\underline{\mu}$ and subsequently to steady state. This is formalized in Proposition 1.

Proposition 1 (Dynamics in the Convalescent Region). *For β sufficiently close to 1, there exists a unique strictly increasing sequence $\{\mu^n\}_{n=-1}^{\infty}$ with $\mu^{-1} = \mu^0 \equiv \underline{\mu}$, $\lim_{n\to\infty} \mu^n = \widetilde{\mu}$, such that if $\mu_0 \in I^n \equiv (\mu^{n-1}, \mu^n]$, the economy escapes the convalescent region in $n+1$ periods and reaches the full-employment steady state in $n+2$ periods, i.e., $\mu_n = \underline{\mu}$, $\mu_{n+1} \in (0, \underline{\mu})$ and $\mu_{n+2} = 0$.*

Proof. Appendix F proves the existence of such a path starting from any μ_0 in the convalescent region, shows that it is unique, and provides an algorithm to construct the path. □

Figure 2 illustrates Proposition 1 by depicting the equilibrium starting from a point μ_0 in the convalescent region which lies between $\underline{\mu}$ and $\widetilde{\mu}$. μ_0 lies in the interval $(\mu^{n-1}, \mu^n]$, so it takes $n + 2$ periods for the economy to reach the full employment steady state. The red circles depict the trajectory at dates $t = 0, 1, \cdots$ while the blue line depicts the nullcline associated with equation (14). When $\theta_t > 1 - \mu_t$, $\mu_{t+1} < \mu_t$, i.e. employment is growing over time and the proportion of unskilled individuals in the pool of job-seekers is shrinking. The light-gray line describe the equilibrium mapping from μ_t to θ_t. Also note that the horizontal dashed line at $\theta = 1$ separates the slack and tight labor market regimes. For $t \leq n$ μ_t is in the convalescent region and $\theta_t < 1$, implying a slack labor market. After n periods, μ_t reaches the healthy region and $\theta_t > 1$, implying a tight labor market.
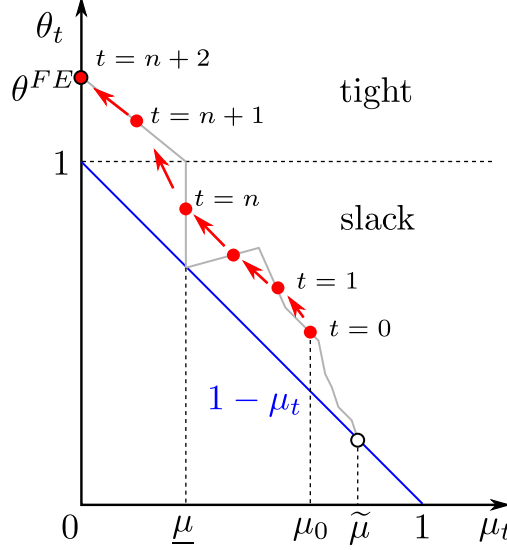
Figure 2. Equilibrium in Convalescent Region

Note that the economy can spend an arbitrarily long time in the the convalescent region before transitioning to the healthy region. In particular, as μ_0 gets closer to $\widetilde{\mu}$, the economy takes longer to recover, particularly in the early stages. Contrast this with Lemma 3: the economy quickly returns to full employment following small shocks, but it takes much longer to recover from a large deterioration in the proportion of skilled job-seekers.

Lemma 6 (Slow Recoveries). *Take any $T \in \mathbb{N}$. Then there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > 0$ for all $t < T$. That is, recoveries can be arbitrarily long. More generally, take any $\delta > 0$, $T \in \mathbb{N}$. Then there exists $\varepsilon > 0$ such that if $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu})$, $\mu_t > \mu_0 - \delta$ for all $t < T$. That is, recoveries can be arbitrarily slow.*

Proof. First we prove the second part. Fix $\delta > 0, T \in \mathbb{N}$ and let n be the smallest integer such that $\mu^n \geq \widetilde{\mu} - \delta$ (this exists, since $\mu^n \to \widetilde{\mu}$ and $\delta > 0$. Set $\varepsilon = \widetilde{\mu} - \mu^{n+T}$. Take any $\mu_0 \in (\widetilde{\mu} - \varepsilon, \widetilde{\mu}) = (\mu^{n+T}, \widetilde{\mu})$. Then $\mu_0 \in (\mu^{m-1}, \mu^m]$ for some $m > n + T + 1$. We know from 1 that $\mu_T \in (\mu^{m-T-1}, \mu^{m-T}]$. In particular,

$$\mu_T > \mu^{m-T-1} > \mu^n \geq \widetilde{\mu} - \delta > \mu_0 - \delta$$

Finally, since $\{\mu_t\}$ is monotonically decreasing, we have $\mu_t > \mu_0 - \delta$ for all $t < T$, as claimed. Next, note that the first part of the lemma is a special case of the second part with $\delta = \widetilde{\mu}$. $\square$

Stagnant Region (Low-pressure Economy)   An important corollary of these results is that any equilibrium converging to full employment must start either in the healthy or the convalescent region. To see this, note that any trajectory which starts to the right of $\underline{\mu}$ and reached full employment at some date T has to be at $\underline{\mu}$ at date $T - 2$. But Proposition 1 showed that *all* trajectories that reach $\underline{\mu}$ lie entirely within the convalescent region. It follows that if the economy starts in the stagnant region, it can never converge to full employment - this region is an *unemployment trap*.

Corollary 1 (Unemployment Traps). *If $\mu_0 \geq \widetilde{\mu}$, the economy never reaches the full employment steady state.*

Intuitively, consider the unstable steady state $\widetilde{\mu}$. Starting from here, can we ever reach full-employment rather than remaining at this steady state? In steady state, firms are just breaking even, and post just enough vacancies to keep skill composition constant at $\tilde{\mu}$. To return to full employment, firms must create more vacancies. But if the economy were to eventually reach full employment, labor markets would be tight, and wages high. Thus, firms would expect lower profits along this trajectory than in the unstable steady state. Since firms just break even in steady state, they would be unwilling to tolerate losses along this alternative trajectory and would post fewer vacancies. Thus, such a recovery is not feasible. Clearly, this holds a fortiori for $\mu > \widetilde{\mu}$.

4 Nominal Rigidities

We now turn to our main objective: analyzing how temporary shocks affecting the economy can have permanent effects, depending on the conduct of monetary policy. We assume that the monetary authority sets nominal interest rates i_t, subject to the zero lower bound $i_t \geq 0$. Because of risk neutrality, the real interest rate is fixed and equal to $r_t = \beta_t^{-1} - 1$, where we now allow β_t to be time varying. Inflation satisfies the Fisher equation:

$$\frac{P_{t+1}}{P_t} = \frac{1+i_t}{1+r_t} = \beta_t(1+i_t) \tag{18}$$

For monetary policy to be relevant, we need to introduce nominal rigidities. We do so by assuming nominal wages are downwardly rigid. At any date t, $W_t \geq \varphi W_{t-1}$ where $\varphi \in (0,1]$ limits how much nominal wages can fall between dates $t-1$ and t. In particular, in the spirit of Eggertsson and Mehrotra (2014) and Schmitt-Grohe and Uribe (2013), nominal wages are set to $W_t = \omega_t^* P_t$ whenever possible, where $\omega^*(\mu)$ is the real wage in the flexible wage benchmark given μ. However, if $W_t = \varphi W_{t-1} > \omega_t^* P_t$, then $W_t = \varphi W_{t-1}$. That is:

$$W_t = \max\left\{\varphi W_{t-1}, P_t \omega_t^*\right\} \tag{19}$$

For $\varphi < 1$ nominal wages can fall but not too fast. With $\varphi = 1$, nominal wages cannot fall, but even so, *real* wages can fall if inflation is high enough:

$$\omega_t = \max\left\{\frac{\varphi P_{t-1}}{P_t}\omega_{t-1}, \omega_t^*\right\} \tag{20}$$

where $\omega_t = \frac{W_t}{P_t}$ is the prevailing real wage and may differ from the Nash-Bargained wage. Real wages can never fall below the Nash-bargained wage, but can exceed the Nash wage.

Shocks We study the response to temporary demand shocks, modeled as a transitory decrease in households' time preference rate: $\beta_0 > 1$, $\beta_t = \beta < 1$ for all $t > 0$. The NK literature uses such a shock to capture an increase in the supply of savings which pushes real interest rates below zero.[16] We consider this shock (rather than, e.g., productivity shocks) to contrast the cases of flexible and inflexible wages. As we will see, these shocks have no real effects when wages are flexible, but can have have adverse effects with nominal rigidities.[17]

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[16] In a richer model such a shock can arise from a tightening of borrowing limits or an increase in precautionary savings (See for example Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2015)).

[17] We consider unanticipated shocks which affect the economy at date 0. We assume that all shocks die out at date 1 and that all agents in the economy know this. This highlights the model's strong propagation mechanism through which even one-period shocks can persistently or permanently affect real outcomes.

4.1 Monetary policy

Because nominal wages are not perfectly flexible downwards, monetary policy can affect real wages by determining the price level. A large literature argues that monetary policy should seek to replicate allocations that would arise in an economy without nominal rigidities (Goodfriend and King (1997), Woodford (2003)).[18] In this spirit, we assume that - wherever possible - monetary policy sets nominal interest rates so that, given the current state of the economy, real wages attain their flexible wage benchmark level, and nominal wages remain stable. From Section 3 we know that in the frictionless wage benchmark economy, real wages are given by some function $\omega^*(\boldsymbol{\mu}_t)$ of the history $\boldsymbol{\mu}_t = \{\mu_0, \mu_1, ..., \mu_t\}$, and μ_t evolves according to some function $\mu_{t+1} = \mathcal{M}^*(\boldsymbol{\mu}_t)$.[19] Given $\boldsymbol{\mu}_t$ and last period's nominal wages W_{t-1}, policy attempts to implement a price level $P_t = \frac{W_{t-1}}{\omega^*(\boldsymbol{\mu}_t)}$ which replicates allocations in the benchmark economy (starting from any initial condition $\boldsymbol{\mu}_t$) while keeping nominal wages constant.[20] However, the ZLB may prevent this, as we now discuss.

Why does the ZLB prevent the monetary authority from implementing the desired price level? Combining (18) with the ZLB reveals that the ZLB imposes a lower bound on inflation, i.e. an upper bound on date t prices, given date $t + 1$ prices:

$$P_t \leq \frac{P_{t+1}}{\beta_t} \tag{21}$$

Households choose between consuming today and saving in the form of nominal government debt. An increase in their desire to save (higher β_t) raises the demand for bonds relative to

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[18]Clearly, this would be Pareto optimal if nominal rigidities were the only distortion rendering equilibrium inefficient. Such a policy may be optimal even in models with an inefficient real equilibrium. For example, the canonical NK model features inefficiencies due to both nominal rigidities and monopolistic competition.

[19]Here there are three technical points worth clarifying. First, the reader might wonder why the whole history $\boldsymbol{\mu}_t$, rather than the current value of μ_t, is a state variable. Recall that when the economy is on the cusp of the healthy region ($\mu_t = \underline{\mu}$) there are multiple equilibria consistent with this value of μ_t. However, given a particular value of μ_{t-1}, the equilibrium is unique. Second, for some initial values of μ_0 (for example, $\mu_0 = \underline{\mu}$), there may be multiple equilibria. In this case, $\omega^*(\boldsymbol{\mu}_t)$ refers to any selection from the set of equilibria. None of our results depend on equilibrium selection. Finally, some histories $\boldsymbol{\mu}_t$ cannot be generated by *any* frictionless wage benchmark equilibrium. However, these histories can arise in the presence of nominal rigidities, so we still need to define the benchmark wage $\omega^*(\boldsymbol{\mu}_t)$ corresponding to these histories. (As an example, take the history $\boldsymbol{\mu}_1 = \{0, \mu_1\}$ where $\mu_1 > \underline{\mu}$. This can never arise in the benchmark economy, because the full employment steady state is stable, but it can happen in the presence of nominal rigidities, as we will see.) In these cases, we define $\omega^*(\boldsymbol{\mu}_t)$ as follows. Delete elements from the history $\boldsymbol{\mu}_t = \{\mu_0, \mu_1, ..., \mu_{t-1}, \mu_t\}$ until the remaining history, say $\{\mu_s, ..., \mu_{t-1}, \mu_t\}$ is consistent with some equilibrium in the frictionless wage benchmark economy. Define $\omega^*(\boldsymbol{\mu}_t)$ as the equilibrium consistent with that truncated history. (Again, if there are multiple equilibria consistent with the truncated history, our results hold for any selection from this set.)

[20]The fact that our characterization features a flat path of nominal wages is not essential. More generally, monetary policy could also implement the same allocations if it wanted a constant rate of nominal wage inflation $\overline{\Pi} > 1$. We discuss the implications of this in Section 4.4.

consumption. Lower demand for consumption tends to reduce the price of consumption today, P_t. Away from the ZLB, monetary policy can prevent deflation by lowering the nominal rate of return on bonds, encouraging households to substitute back towards consumption. This is no longer possible when nominal interest rates are constrained by the ZLB. In this case, since the nominal return on bonds cannot be lowered anymore, the price of consumption today must decline relative to the price tomorrow to make the real return on bonds lower, dissipating the excess demand for bonds. Thus when the ZLB binds, it curtails the ability of the monetary authority to implement its desired price level.
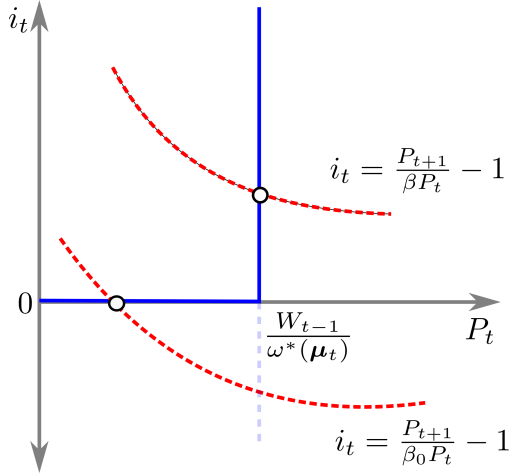
Figure 3. ZLB and the Price Level

Figure 3 describes the determination of nominal interest rates and prices at date t given an anticipated future price level P_{t+1}. The red-dashed downward-sloping curve represents the combinations of (i_t, P_t) consistent with bond market clearing for a given P_{t+1}. When the price level is higher, goods are more expensive today relative to tomorrow and households would rather save; a lower nominal interest rate discourages them from doing so, restoring bond market clearing. The higher curve represents this locus when β is at its steady state level and the desire to save is moderate while the lower one represents the locus for an increased desire to save, $\beta_0 > \beta$. When desired saving is higher, lower nominal interest rates or prices are to clear the bond market. The solid blue curve depicts the combination of (i_t, P_t) which monetary policy can attain given the ZLB. Whenever possible, monetary policy stands ready to adjust nominal rates so as to attain the price level $P_t = W_{t-1}/\omega^*(\boldsymbol{\mu}_t)$ such that the prevailing real wage is the same as in the flexible benchmark given the history $\boldsymbol{\mu}_t$. When desired savings are very high, however, this might require a negative nominal interest rate (intersection of the blue dashed curve and the lower red curve) which is unattainable given the ZLB. Instead, the price level must fall to clear the bond market (the intersection of the solid blue line and the lower red curve).

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Having described monetary policy, we are ready to formally define equilibrium in the presence of nominal rigidities.

Definition 2 (Monetary Equilibrium given nominal rigidities). *A monetary equilibrium is a collection of functions* $\mathcal{P}(\boldsymbol{\mu}_t, W_{t-1})$, $\mathcal{J}(\boldsymbol{\mu}_t, W_{t-1})$, $\mathcal{M}(\boldsymbol{\mu}_t, W_{t-1})$, $\mathcal{I}(\boldsymbol{\mu}_t, W_{t-1})$ *and* $\mathcal{W}(\boldsymbol{\mu}_t, W_{t-1})$ *defining the equilibrium price level, value of a filled vacancy, next period's fraction of unskilled workers, the nominal interest rate between* t *and* $t+1$, *and today's nominal wage, given a history* $\boldsymbol{\mu}_t$ *and last period's nominal wages* W_{t-1}. *These are defined recursively by:*

$$\mathcal{P}(\boldsymbol{\mu}_t, W_{t-1}) = \min\left\{\frac{W_{t-1}}{\omega^*(\boldsymbol{\mu}_t)}, \frac{\mathcal{P}\Big(\boldsymbol{\mu}_{t+1}, \mathcal{W}(\boldsymbol{\mu}_t, W_{t-1})\Big)}{\beta_t}\right\}, \tag{22}$$

$$\boldsymbol{\mu}_{t+1} = \{\boldsymbol{\mu}_t, \mathcal{M}(\boldsymbol{\mu}_t, W_{t-1})\}$$

$$\mathcal{J}(\boldsymbol{\mu}_t, W_{t-1}) = A - \frac{\mathcal{W}(\boldsymbol{\mu}_t, W_{t-1})}{\mathcal{P}(\boldsymbol{\mu}_t, W_{t-1})} + \beta_t \gamma \mathcal{J}\Big(\boldsymbol{\mu}_{t+1}, \mathcal{W}(\boldsymbol{\mu}_t, W_{t-1})\Big) \tag{23}$$

$$\mathcal{W}(\boldsymbol{\mu}_t, W_{t-1}) = \max\left\{\varphi W_{t-1}, \mathcal{P}(\boldsymbol{\mu}_t, W_{t-1})\,\omega^*(\boldsymbol{\mu}_t)\right\} \tag{24}$$

$$\mathcal{I}(\boldsymbol{\mu}_t, W_{t-1}) = \frac{\mathcal{P}\Big(\boldsymbol{\mu}_{t+1}, \mathcal{W}(\boldsymbol{\mu}_t, W_{t-1})\Big)}{\beta_t \mathcal{P}(\boldsymbol{\mu}_t, W_{t-1})} - 1 \geq 0 \tag{25}$$

Both the conditions: $\mathcal{J}(\boldsymbol{\mu}_t; W_{t-1}) \leq \kappa + \chi\mu_t$ *and* $\mathcal{M}(\boldsymbol{\mu}_t; W_{t-1}) \leq \frac{1}{1+\gamma(1-\mu_t)}$ *must hold; at least one must hold with a strict equality. If* $\mathcal{P}(\boldsymbol{\mu}_t, W_{t-1}) = \frac{W_{t-1}}{\omega^*(\boldsymbol{\mu}_t)}$, *then* $\mathcal{M}(\boldsymbol{\mu}_t; W_{t-1}) = \mathcal{M}^*(\boldsymbol{\mu}_t)$.[21]

Intuitively, given yesterday's nominal wage there is a particular price level today, $W_{t-1}/\omega^*(\boldsymbol{\mu}_t)$, which replicates the Nash-bargained flexible real wage while keeping nominal wages (and, in steady state, prices) stable. Monetary policy seeks to ensure that this price level is attained in the economy. When the ZLB does not bind, the desired real wage can be attained and the economy perfectly mimics the flexible wage outcomes. In particular, the fraction of unskilled job seekers, μ_{t+1}, evolves exactly as in Section 3.2: i.e. $\mu_{t+1} = \mathcal{M}^*(\boldsymbol{\mu}_t)$. However, when the ZLB binds, prices must be lower relative to the target to dissipate the demand for saving and restore equilibrium in the goods market. If the fall in prices required to clear bond markets is sufficiently severe, nominal wages hit the constraint $W_t = \varphi W_{t-1}$, resulting in higher real wages. This lowers firm profits and the value of a filled vacancy, discouraging hiring relative to the flexible wage benchmark. This in turn increases unemployment and lowers job-finding rates. The fall in job-finding rates raises the average duration of unemployment, increasing the fraction of unskilled workers relative to the flexible benchmark. Indeed, real wages might rise so much that the net value of hiring an additional worker for a firm becomes negative, i.e. $\mathcal{J}(\boldsymbol{\mu}_t; W_{t-1}) < \kappa + \chi\mu_t$. In this case, there is no vacancy posting, the job-finding rate is zero

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[21]Recall that $\mathcal{M}^*(\boldsymbol{\mu}_t)$ describes transition in the economy without nominal rigidities.

and following from equation (5) the fraction of unskilled workers next period is given by:

$$\mu_{t+1} = \mathcal{M}(\boldsymbol{\mu}_t; W) = \frac{1}{1 + \gamma(1 - \mu_t)} > \mathcal{M}^*(\boldsymbol{\mu}_t)$$

This is the fastest rate of aggregate skill depreciation given a history $\boldsymbol{\mu}_t$ since all workers who lose their job are unable to find jobs. Even when there is not a complete hiring freeze, firms still post fewer vacancies relative to the natural level resulting in a lower job-finding rate. In this case $\mathcal{J}(\boldsymbol{\mu}_t, W_{t-1}) = \kappa + \chi\mu_t$ and thus, firms remain willing to hire. Consequently, the fraction of unskilled workers next period is lower than with a complete hiring freeze (but still higher than the natural level):

$$\mu_{t+1} = \mathcal{M}(\boldsymbol{\mu}_t; W_{t-1}) < \frac{1}{1 + \gamma(1 - \mu_t)}$$

Lemma 7 (Nominal Wages in the Monetary Equilibrium). *Given the monetary policy described above, in equilibrium nominal wage growth always lies between $\varphi \leq 1$ and 1, i.e., nominal wages do not grow. If the ZLB does not bind in period t, then the nominal wage inflation between period $t-1$ and t is 1.*

$$\varphi W_{t-1} \leq W_t \leq W_{t-1} \tag{26}$$

Proof. $\varphi W_{t-1} \leq W_t$ holds by construction. Given monetary policy, we have $P_t\omega^*(\boldsymbol{\mu}_t) \leq W_{t-1}$. We also have $W_t = \max\{\varphi W_{t-1}, P_t\omega^*(\boldsymbol{\mu}_t)\}$. Combining the two yields $W_t \leq W_{t-1}$. $\square$

Intuitively, whenever the ZLB does not bind, monetary policy implements a price level consistent with nominal wages remaining constant. When the ZLB does bind, deflationary pressures may fall owing to the deflationary pressure but cannot decline at a rate greater than φ. It is also straightforward to see that if $\varphi = 1$, then nominal wages are constant across time, i.e. $W_t = W_{-1}$ for all $t \geq 0$.

4.2 Temporary Shocks and Permanent Effects

In the presence of nominal rigidities, temporary shocks can have permanent real effects. We now show that, starting from full employment, a one period demand shock can lead to persistently or even permanently higher unemployment.

First we explain how a demand shock can cause an initial spike in unemployment. Even in the presence of the ZLB, without nominal rigidities, this would never happen. Since a filled vacancy is a long-lived asset yielding dividends in the future, while the cost of posting a vacancy is paid today, an increase in β increases the net present value of vacancy posting,

encouraging vacancy creation. When wages are not perfectly flexible, this is no longer the case. The deflation engendered by the ZLB causes real wages to rise (since nominal wages can't fall) discouraging hiring. Lemma 8 formalizes this notion.

Lemma 8. *Suppose Assumption 3 holds and the economy is at the FESS at date 0, i.e., $\boldsymbol{\mu}_0 = \{0\}$. Then if $\beta_0 > 1$ and $\beta_t = \beta < 1$ for all $t > 0$, and if the economy returns to FESS after date 0, the zero lower bound binds at date 0 only, and J_0 is a decreasing function of β_0:*

$$\mathcal{J}(\boldsymbol{\mu}_0, W_{-1}) = A - \beta_0 \omega_{fe}^* + \beta_0 \gamma J_{min}$$
$$\frac{\partial \mathcal{J}(\boldsymbol{\mu}_0, W_{-1})}{\partial \beta_0} = -\omega_{fe}^* + \gamma J_{min} < 0$$

$\underline{\beta} \equiv \dfrac{A - \kappa}{\omega_{fe}^* - \gamma J_{min}}$ *is the largest possible demand shock such that a firm would be willing to post vacancies at date* 0 *if the economy were expected to remain at full employment at date* 1.

Proof. See Appendix G. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

The easiest way to understand Lemma 8 is by considering the case where $\varphi = 1$. Recall that Lemma 7 showed that nominal wages are constant over time in this case. Since by assumption, the economy remains at full employment at date 1, monetary policy is able to attain its desired price level given by $P_{-1} = W_{-1}/\omega_{fe}^*$. At date 0 however, the demand shock causes price level to fall to equilibrate the bond market:

$$P_0 = \frac{P_1}{\beta_0} < P_1 = P_{-1}$$

The resulting rise in real wages at date 0 reduces the value of the firm:

$$\mathcal{J}(\boldsymbol{\mu}_0, W_{-1}) = A - \frac{W_0}{P_0} + \beta_0 \gamma J_{min} = A - \beta_0 \omega_{fe}^* + \beta_0 \gamma J_{min}$$

Here β_0 affects the value of a filled vacancy in two ways. First, a higher discount factor makes firms more willing to invest in vacancy creation. Second, a higher β_0 - a larger decline in real interest rates - requires higher inflation going forward, when nominal interest rates are pinned down by the ZLB. The resulting decline in prices (increase in real wages) at date 0 lowers the value of a filled vacancy, discouraging hiring. Under Assumption 3, the second effect always outweighs the first, and J_0 is decreasing in β_0 for $\beta_0 > 1$. As β increases towards 1 from below, the value of a filled vacancy increases. If nominal wages were perfectly flexible, this positive relationship between J_0 and β_0 would hold for all β_0 as shown by the black dashed line in Figure 4. This would be true even with downwardly rigid nominal wages,

absent the ZLB. However, in the presence of downward rigid nominal wages and the ZLB, the relationship between J_0 and β_0 is non-monotonic, as shown by the solid blue curve in Figure 4. Once β is higher than 1, the ZLB binds and the resulting deflation raises real wages and lowers the value of a filled vacancy even though firms value dividends in the future more.
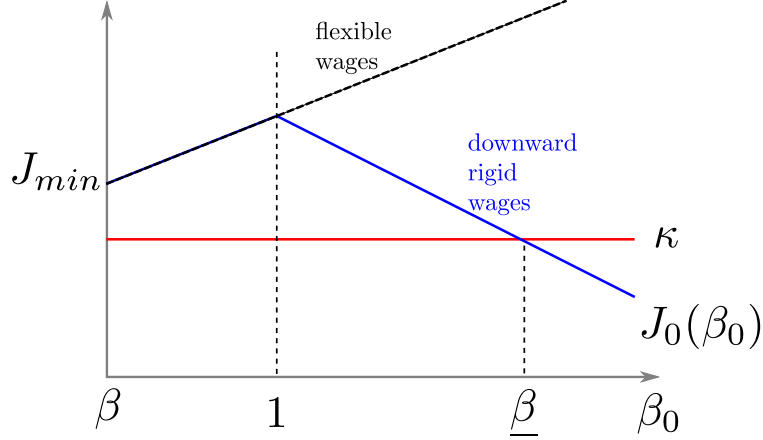
Figure 4. J_0 as a function of β_0

Interestingly, the result is the same for the case in which $\varphi < 1$ and nominal wages can decline a bit. In this case, deflationary forces drive nominal wages down to the lowest allowable level at date 0, i.e., $W_0 = \varphi W_{-1}$. Since monetary policy is unconstrained at date 1, nominal wages remain constant between date 0 and 1, i.e. $W_1 = W_0 < W_{-1}$. As a result, to implement the full employment steady state natural real wage ω_{fe}^* at date 1, the monetary authority must ensure a lower price level:

$$P_1 = \frac{W_0}{\omega_{fe}^*} < \frac{W_{-1}}{\omega_{fe}^*} = P_{-1}$$

Anticipating lower prices at date 1, households are even more unwilling to consume today, further depressing the demand for current production. Thus, even lower prices must prevail at date 0 to clear goods markets. Lower prices imply higher real wages despite they fall in nominal wages, lowering firm profitability. This is a version of the *paradox of flexibility* (Eggertsson and Krugman, 2012) - a lower φ, i.e. more flexibility, does not help avoid the increase in real wages following a shock, even though nominal wages fall.

Overall, an increase in $\beta_0 > 1$ reduces the value of a filled vacancy for a firm. For a moderate demand shock $\beta_0 \leq \underline{\beta} > 1$, this reduction in value is not sufficiently severe to altogether discourage firms from maintaining a level of hiring consistent with full employment. However, if β_0 is greater than $\underline{\beta}$, the value of a filled vacancy would be negative, contradicting the conjecture that the economy remains at full employment. Proposition 2 below characterizes

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equilibrium when the demand shock is large enough to create unemployment, i.e. $\beta_0 > \underline{\beta}$. It is convenient to make the following assumption:

Assumption 4. $\eta A + (1-\eta)b > \gamma(\kappa + \chi)$

The LHS of this expression is the lower bound on wages in the flexible benchmark while the RHS is proportional to the upper bound on the value of a filled vacancy at date 1. This assumption ensures that wages in the flexible benchmark are never too small, even at zero employment.[22] Thus, this assumption ensures that $\omega^*(\mu^1) > \gamma J_1$ which in turn ensures that the value of a filled vacancy is decreasing in β_0 at the ZLB.[23]

Proposition 2 (Monetary Equilibrium with Demand Shocks)**.** *Given Assumption 4, there exists $\overline{\beta} > \underline{\beta} > 1$ such that:*

1. *If $\beta_0 \in (1, \underline{\beta}]$, there exists an equilibrium with $\theta^{FE} > \theta_0 \geq 1$ and $\mu_t = 0$ for all t.*

2. *If $\beta_0 \in (\underline{\beta}, \overline{\beta})$, there exists an equilibrium with $\theta_0 \in (0,1)$ and $\mu_1 \in (\underline{\mu}, \mu_R)$*

3. *If $\beta_0 \geq \overline{\beta}$, there exists an equilibrium with $\theta_0 = 0$ and $\mu_1 = \mu_R$*

In addition, in cases 2 and 3 above, if $\mu_1 < \widetilde{\mu}$, then $\lim_{T \to \infty} \mu_T = 0$ and the economy returns to full employment. However, if $\mu_1 \geq \widetilde{\mu}$, then the economy never returns to full employment.

Proof. See Appendix H. $\square$

As we have already seen, small increases in β_0 above 1 raise real wages, but not enough to generate unemployment. For large enough shocks, however, higher real wages cause firms to reduce hiring. This decreases the job finding rate, increasing the fraction of unskilled job-seekers at date 1. This deterioration in human capital would lower real wages in the flexible wage benchmark economy. Recognizing this, the monetary authority targets a higher price level at date 1 to reduce real wages. The higher price level at date 1 tends to raise the

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[22]Using the definition $a = (1-\eta)(A - b)$, this assumption can be re-written as:

$$\eta > \frac{\gamma(\kappa + \chi) - b}{\gamma(\kappa + \chi) + a - b}$$

which is decreasing in b. In conjunction with Assumption 3, this requires that

$$\eta > \max\left\{ \frac{1 - \beta\gamma}{\beta\gamma}, \frac{\gamma}{1+\gamma}, \frac{\gamma(\kappa + \chi) - b}{\gamma(\kappa + \chi) + a - b} \right\}$$

Note that there is a non-empty set of parameters for which this requirement is satisfied. Given an η and a, γ, β, κ and χ satisfying Assumptions 1-3, we can always choose b (increasing A at the same time to keep a fixed) such that Assumption 4 is true.

[23]Recall that μ^1 denotes the sequence $\{\mu_1, \mu_0\}$ where $\mu_0 = 0$ and $\mu_1 = \mathcal{M}(\mu_0, W_0)$.

price level at date zero, mitigating the deflationary impact of the original increase in β_0. As a result of this counteracting force, an increase in β_0 above $\underline{\beta}$ may not result in a *complete* hiring freeze. If the increase in β_0 is sufficiently large, however, there is a complete hiring freeze at date 0, and the fraction of unskilled job-seekers at date 1 rises to μ_R.[24]

Slow Recoveries At best, the reduction in hiring drives the economy into the convalescent region, i.e. $\mu_1 \in (\underline{\mu}, \tilde{\mu})$. In this case, a temporary fall in demand results in a persistently slow recovery. As we have already seen in the flexible wage benchmark in Section 3.2, when μ_1 lies in the convalescent region, there is a moderate level of unskilled job seekers. Firms internalize the fact that when they meet an applicant they will likely have to pay a training cost to make them suitable for production. Facing this higher incidence of training costs, firms will only post vacancies if they are compensated by lower wages, at least in the near term. However, the only way low wages can be an equilibrium outcome is if the job-finding rates are depressed for a period of time, keeping the workers outside option low and forcing them to settle for a lower wage during the wage bargaining process. With low job-finding rates, the unemployment rate, and thus the fraction of unskilled job seekers only declines gradually. Since the flexible benchmark itself features an arbitrarily slow recovery, monetary policy - which tries to replicate outcomes in this benchmark - can do no better.

Permanent Stagnation Worse still, if the economy enters into the stagnant region ($\mu_1 > \tilde{\mu}$), it never returns to full employment. The fraction of job-seekers in this case is so high that real wages must be very low for firms to post *any* vacancies. Such low real wages only obtain if slack labor markets are expected to persist forever. In this scenario the economy converges to the low-pressure steady state with a high fraction of unskilled job-seekers $\overline{\mu}$ and high unemployment. In this steady state, even though high unemployment depresses wages, firms are reluctant to post vacancies because the average job-seeker is likely to be unskilled and costly to retrain. These low vacancy posting rates support high unemployment. A monetary policy that tries to replicate these allocations can not help but get stuck in the stagnant region. Thus, a transitory demand shock which only lasts one period can permanently depress employment and output, even once the ZLB no longer binds.

What determines whether the economy ends up in the stagnant or the convalescent region? It depends on two factors. The first is the size of the demand shock - the larger the shock, the larger the decline in hiring and skill deterioration, pushing the economy towards the stagnant region. The second factor is the strength of the forces generating

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[24]Again, these results hold even if nominal wages can adjust downwards to some extent.

multiplicity. If χ - the cost of training unskilled workers - equalled zero, there would be a unique steady state, and no adverse shock could ever prevent a return to full employment after the shock abates. For sufficiently high χ, there are multiple steady states as firms' job-creation decisions become sensitive to the cost of re-training unskilled workers. The higher is χ, the lower $\widetilde{\mu}$, and the more likely it is that the economy enters the stagnant region. Figure 5 illustrates all these possibilities.[25]
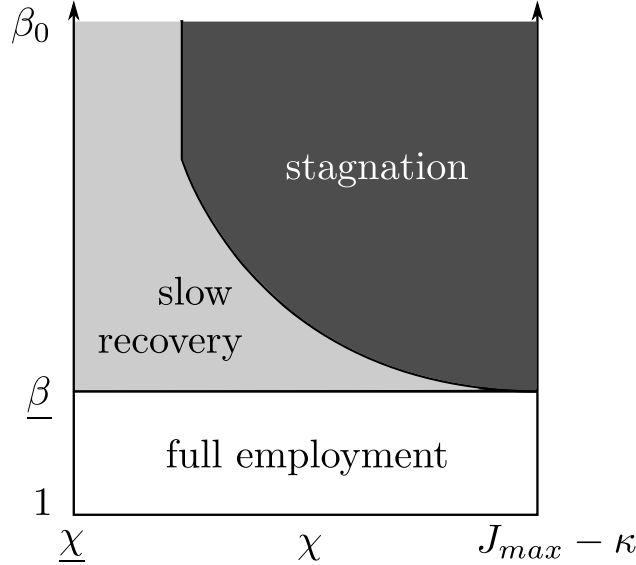
Figure 5. Parameters for which demand shocks lead to slow recoveries or stagnation.

Note that the ZLB works through a different channel here than in the standard NK model. In the standard model, the lower bound on nominal rates, together with an upper bound on expected inflation, make real interest rates too high, and consumption growth too large. If long run consumption is pinned down (by the assumption that the economy returns to the unique steady state), high consumption *growth* requires consumption to fall today. In our model with linear utility, real rates are fixed. The ZLB requires positive inflation to generate a lower real rate in response to demand shocks. If long run prices are pinned down (by monetary policy), high inflation requires prices to fall today. With sticky nominal wages, this fall in prices raises real wages, and potentially reduces hiring.

In NK models of the ZLB, the goal of policy is to set the real interest rate equal to the Wicksellian rate $r^\star$ - through fiscal policy, commitment to higher inflation and so on. We abstract from this problem altogether by studying an environment in which this is *always* accomplished, through commitments to higher inflation. Our point is that even if policy does

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[25]Interestingly, the degree of nominal wage (downward) flexibility φ has no bearing on whether the economy ends up in the slow recovery or stagnation scenario. The discussion following Lemma 8 makes clear why this is the case.

track the Wicksellian rate, this may not prevent a persistent - or even permanent - slump.

4.3 What is the role of multiplicity and the ZLB?

The take-away from the analysis above is that large transitory shocks can permanently damage the economy. This is in strong contrast to standard models with nominal rigidities, in which transitory shocks do not affect long run outcomes. What explains this discrepancy? The following Lemma shows that multiplicity is key.

Proposition 3 (No Multiplicity of Steady States). *If $\chi < J_{min} - \kappa$, FESS is the only steady state. Furthermore, after a temporary demand shock, the economy returns to full employment by date 2 at the latest. Thus, temporary shocks only have short lived effects on the economy.*

Proof. With $\chi < J_{min} - \kappa$, the economy is always in the healthy region, i.e. $\underline{\mu} > 1$. Thus the natural real wage always equals ω_{fe}^*, and $P_t = W_0/\omega_{fe}^*$ from date 1 onwards. Thus $J_1 = J_{\min}$, whatever the value of μ_1. This produces the same dynamics as in Lemma 3 albeit moved one period forward: $\theta_1 = \frac{J_1 - \chi\mu_1}{\kappa} > 1$ and hence $\mu_t = 0$ for all $t \geq 2$. $\qquad\square$

Why does the outlook become less sunny once multiplicity is introduced? For larger χ, the firm's job-creation decision becomes sensitive to the extent of skill depreciation in the economy as the effective cost of hiring rises with the level of unemployment. For low χ, while demand shocks still cause a spike in unemployment, the economy can quickly retrain unskilled job-seekers once the shock abates, since training is cheap. Thus, transitory shocks only have transitory effects. In contrast, when χ is high, the economy is less able to absorb these displaced workers back into employment once the shock has abated.

Absent the ZLB, monetary policy could perfectly insulate the economy from demand shocks, stabilizing it at full employment. A temporary increase in β_0 above 1 drives real interest rates below zero. Without the ZLB, negative real rates can be accommodated with negative nominal rates without a fall in prices. Thus demand shocks lose their bite, and the full employment steady state is stable in response to these shocks, as in the flexible wage benchmark. The ZLB changes the stability properties of the model, and in particular of the full employment steady state. Negative real interest rates can no longer be accommodated by negative nominal rates, and instead require positive inflation, a fall in current prices, a rise in real wages and a contraction in hiring. This in turn leads to a slow recovery or stagnation. Even absent the ZLB and nominal rigidities, if the economy started out near the low-pressure steady state, a higher rate of unemployment may persist forever. But it is only because of the ZLB that demand shocks can create unemployment, bringing multiplicity into play.[26]

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[26]More generally, such a situation could arise even if the ZLB does not bind, provided that monetary policy

4.4 Can unconventional policies prevent hysteresis?

We have seen that in the presence of hysteresis, even temporary demand shocks can lead to permanent stagnation. Can monetary policy avoid such an outcome?

In the scenarios we have described, monetary policy is constrained at date 0, allowing prices to fall and real wages to rise. Recall that our baseline policy tries to implement natural allocations while ensuring zero nominal wage inflation.[27] Thus, one might think that a monetary policy which instead tried to implement natural allocations while allowing nominal wages (and prices in steady state) to grow at a constant rate, say $\overline{\Pi} > 1$, might be able to avoid slow recoveries or stagnation. Indeed, a higher inflation target $\overline{\Pi}$ would decrease the probability of a ZLB episode - the ZLB only binds if $\beta_0 > \overline{\Pi}$. Higher trend inflation target allows negative real rates despite the ZLB. However, such a policy would not altogether insulate the economy from the persistent effects of demand shocks. Even with $\overline{\Pi} > 1$, a sufficiently large demand shock $\beta_0 > \overline{\Pi}$ would still cause the ZLB to bind, and potentially generate a persistent or permanent downturn as described above. Thus, even with trend inflation, our results would be qualitatively the same - it would take larger demand shocks to push the economy to the ZLB, but once at the ZLB, the dynamics would be identical.

Instead, as emphasized by a large literature (Eggertsson and Woodford, 2003; Werning, 2011), avoiding adverse outcomes in liquidity trap episodes requires commitments to temporarily more accommodative policy even when the liquidity trap is over. In our environment, this takes the form of temporarily higher wage inflation at date 1 - or, equivalently, a permanently higher nominal wage level from date 1 onwards. Proposition 4 below describes how such commitments can keep the economy at full employment following an adverse shock. Note that such a policy does not require a commitment to higher trend inflation.

Proposition 4 (Hysteresis-proof policy). *Suppose $\beta_0 > \underline{\beta}$ and the economy starts at full employment, i.e. $\mu_0 = 0$ and $W_{-1} = \omega_{fe}^* P_{-1}$. Then, the hysteresis proof monetary policy implements a price sequence given by $P_t = P_{-1}$ if $t = 0$ and $P_t = \beta_0 P_{-1} > P_{-1}$ if $t > 0$. The unique equilibrium consistent with this price path features full employment for all t and a nominal wage path $W_t = \omega_{fe}^* P_t$.*

Proof. Date 1 inflation equals $P_1/P_0 = \beta_0 > 1$ which clearly satisfies the ZLB constraint (21). Real wages are given by equation (20) which implies that $W_t/P_t = \omega_{fe}^*$ along the price path defined above, since $\varphi P_{t-1}/P_t \leq 1$ for all $t \geq 0$. $\qquad \square$

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does not cut nominal interest rates enough to prevent deflation - for example, because policy is insufficiently responsive to deflationary forces, or faces other tradeoffs (such as cost push shocks, in the standard NK model) which are not modeled here. In our stylized model, this is not possible: policy always cuts nominal rates to maintain price stability, unless prevented from doing so by the ZLB.

[27]Such a preference for nominal stability defines the mandates of many central banks globally.

Commitment to higher date 1 prices - which involves deviating from nominal wage stability, even though the shock has passed - prevents prices from falling at date 0. Even with nominal rates stuck at zero, higher future prices discourage households from saving in nominal bonds, propping up the goods demand at date 0 and preventing deflation. Without a fall in prices, real wages do not rise, filled vacancies remain profitable, and ment remains low. Thus the economy never enters the convalescent or stagnant regions and hysteresis is averted.

Forward guidance and the strength of the intertemporal substitution channel
The policy just described involves commitment to expansionary policy once the liquidity trap has abated. In this regard, it resembled *forward guidance* policies discussed in the NK literature.[28] That literature argues that committing to keep interest rates low in the future, once shocks have abated, can alleviate demand driven recessions. In NK models, this policy works through an intertemporal consumption smoothing channel. To see this, consider a workhorse NK model which features households with CRRA utility with intertemporal elasticity of substitution σ^{-1}. Date 0 consumption is given by:[29]

$$c_0^{-\sigma} = \left[\prod_{s=1}^{T} \beta_{s-1} \frac{P_{s-1}(1 + i_{s-1})}{P_s} \right] c_T^{-\sigma} \tag{27}$$

Forward guidance in the NK model operates through two channels: (i) holding consumption at some future date T fixed, lower nominal interest rates at any date $s < T$ raise consumption at date 0 and (ii) if interest rates are constrained at the ZLB for all dates $s < T$, commitments to keep interest rates low after date T can raise c_T and further stimulate demand at date 0. Because of intertemporal substitution, both these forces encourage households to consume more during the recession at date 0, strengthening demand and reducing the severity of the contraction in output (Eggertsson and Woodford, 2003; Werning, 2011).

There is an important debate about whether the strength of forward guidance in NK models is realistic[30] and about the strength of the intertemporal substitution channel.[31] Importantly, the power of commitment to higher future prices in our framework does not depend on the intertemporal substitution channel, which is entirely absent. To see this, recall that the Euler equation in our model is:

$$1 = \beta_0 \frac{P_0(1 + i_0)}{P_1}$$

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[28] To be clear though, the policy described here is not a commitment to keep nominal rates at zero for an extended period of time.

[29] This expression is obtained by iterating forward the standard Euler equation $c_0^{-\sigma} = \beta_0 c_1^{-\sigma} \frac{P_0(1+i_0)}{P_1}$.

[30] See for example Del Negro et al. (2015) and McKay et al. (2015).

[31] see for example Kaplan et al. (2016)

As this equation makes clear, real interest rates are pinned down solely by β_0; the intertemporal channel of monetary policy has no bite. With real rates fixed, at the ZLB, the rate of inflation is also pinned down. However, with inflation fixed, a higher price level at any date in the future entails a higher price today, lowering real wages and encouraging hiring. Since our hysteresis-proof policy operates through the effect of the price level on firms' hiring decisions, rather than the time path of real interest rates, it does not depend on the strength of the intertemporal substitution channel.

4.5 Escaping Unemployment Traps?

The alternative monetary policy just described prevents adverse shocks from causing a recession, and the associated decline in skill composition of the workforce. If the monetary authority instead maintains the benchmark policy, this leads to unemployment and deterioration of human capital. Suppose this deterioration is severe enough that the economy enters the stagnant region, scarred by high unemployment forever. Are these scars permanent or can monetary policy heal them?

Our model delivers the stark conclusion that monetary policy cannot escape an unemployment trap. Since nominal wages are fully flexible upwards, real wages can never fall below their natural level, given the current state of the economy $\boldsymbol{\mu}_t$. Once the economy has entered the stagnant region, employers are unwilling to create more vacancies even given the low real wages in the natural allocation. A temporary real wage cut would accelerate hiring, but monetary policy *cannot* reduce real wages below their natural level. Any rise in prices is met by a proportional increase in nominal wages.[32] Thus if the economy enters the stagnant region at date 1, monetary policy is powerless to escape. Similarly, even if the recession generates a slow recovery rather than permanent stagnation, monetary policy cannot speed up the recovery; temporarily lower real wages would in principle stimulate hiring, but monetary policy cannot reduce real wages.

In the presence of hysteresis, insufficient accommodation early on in the crisis may have persistent or permanent costs which cannot be reversed by more accommodative policy later on, once the rot has set in. In our model, this dichotomy is particularly extreme: monetary policy can prevent *any* increase in unemployment if it acts at date 0, but is completely powerless to improve outcomes if it waits until date 1. In reality, both predictions are likely too strong. Monetary policy can avert the recession altogether at date 0 only because we assume full commitment, and that there are no costs associated with permanently higher nominal prices and wages. If higher prices were costly, and the monetary authority was

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[32]If anything, real wages may even be higher than their natural level owing to the ZLB.

unable to commit, such a policy would not be implementable. Within our model, if nominal wages were inflexible in both directions, a higher future price level may involve a deviation from natural allocation and welfare losses.[33] These tradeoffs would make a commitment to higher prices, once the shock has passed, time-inconsistent.

Similarly, monetary policy is unable to escape unemployment traps or speed-up slow recoveries only because it cannot raise employment above its natural level given a history $\boldsymbol{\mu}_t$. If instead monetary policy has some ability to create temporary hiring booms, then it might be possible to reverse some of the damage done to the economy. Again, even within our model, if nominal wages were not fully flexible upwards, targeting higher prices once the shock has passed would stimulate hiring by temporarily lowering real wages - potentially speeding up a recovery or escaping the stagnant region.[34]

Relative to the existing literature our model is designed to highlight the notion, not that monetary policy is extremely powerful or weak, but that - in the presence of hysteresis - delaying policy action might be very costly. By contrast, in standard models such as Eggertsson and Woodford (2003), such policies are equally effective at any point in a crisis - and in any case, even if no such policy is forthcoming, the economy always returns to steady state in the long run. Another contrast is that in the presence of hysteresis, failing to stabilize employment can be extremely costly, while the existing literature generally finds that policy should put little weight on stabilizing output or employment.

Finally, it is worth noting that even within our model, fiscal policy, such as hiring or training subsidies, could potentially ameliorate the effects of hysteresis when monetary policy cannot. Committing to compensate firms for each worker they train would be equivalent to lowering χ, speeding up recovery or even removing the possibility of permanent stagnation. Subsidizing job-creation could have similar effects.

5 Hysteresis since the Great Recession

We now ask whether the mechanisms described in the paper can help quantitatively explain the sluggish economic recovery following the Great Recession.

While the particular form of the matching function assumed above facilitates analytical results, it has the counterfactual prediction that the "high-pressure" steady state has 0

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[33] At the FESS, lower real wages than the natural level lead to welfare losses. Lower wages encourage firms to create more vacancies. With the economy already at full-employment, higher market tightness only leads to higher vacancy posting costs, and thus lower consumption, without creating more jobs. This is the manifestation of the Hosios (1990) externality in our setting.

[34] In a similar vein, Ball (2015) has argued that policy should be able to reverse the effects of hysteresis if monetary policy is able to recreate a high-pressure economy.

percent unemployment. In what follows, we use a more standard matching function $m(v,l) = vl/(v^\iota + l^\iota)^{\frac{1}{\iota}}$.[35] This allows us to consider a high pressure economy with an empirically plausible unemployment rate. We calibrate the model to the U.S. economy. In our model unemployed workers lose skill after one period. We calibrate one period to six months, so "unskilled" workers correspond to those unemployed for 27 weeks or more. Using resume audit studies, Kroft et al. (2013) find that the likelihood of receiving a call-back declines significantly after an unemployment spell, with most of the decline occurring within the first 8 months. Similarly, Ghayad (2014) finds a sharp decline in the call-back rate after 6 months. It therefore seems reasonable to posit that most of the skill loss upon losing a job occurs within the first 6 months.[36] We set $\beta = 0.98$, implying a 4 % annualized steady state real interest rate. A is normalized to 1. We set $\iota = 0.5$ following Menzio and Shi (2011) and $\eta = 0.7$ (Shimer, 2005). b is chosen to imply a steady state replacement ratio of 70 percent (Hall, 2009). We set $\delta = 0.2105$ so that 5 percent state unemployment steady is consistent with 20 percent of job seekers being long term unemployed in steady state as observed in the U.S. before 2008. This leaves us with two parameters, κ and χ, to target one remaining moment. We consider a range of values for these parameters.

After the Great Recession, U.S. unemployment peaked at close to 10 percent in the second half of 2009 before beginning a slow decline, not returning to 5 percent for another 6 years, as shown by the black line in Figure 6a. We have already seen analytically (Lemma 6)that our model can in principle generate an arbitrarily slow recovery. To see whether a slow recovery is plausible quantitatively, we solve our model for a range of (κ, χ) combinations consistent with multiple interior steady states, starting from the unemployment rate of 9.8% observed in the second half of 2009. These trajectories are shown by the gray lines in Figure 6a. Moving from left to right, as we increase χ and decrease κ, the forces generating multiplicity become stronger and the recovery becomes slower. Quantitatively, the model can match the sluggish recovery observed in the data. The red line in Figure 6a indicates our preferred calibration, $\chi = 0.52$, which fits the data most closely. While direct evidence on training costs is hard to come by, this lies well within the empirical estimates found in the literature.[37]

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[35]Notice that as $\iota \to \infty$, this matching function converges to the $\min\{\cdot, \cdot\}$ matching function employed in the previous sections.

[36]Strictly speaking, in our simple model, skilled and unskilled job-seekers have the same job-finding rates and are paid the same wages conditional on employment: skill depreciation only shows up in the training costs faced by employers. For our purposes, all that matters is that the findings of these studies is consistent with a bulk of the skill depreciation occurring within 6 months of job-loss. Our rate of skill depreciation is also consistent with Ljungqvist and Sargent (1998); their calibration implies a 95% probability of losing skill after a 6 month unemployment spell.

[37]Barron et al. (1989) find that, on average, a new hire spends 151 hours on training in the first 3 months of the job. If only unskilled workers require training, as assumed in our model, this implies an upper bound of $\chi = 151/(0.2 * 1043.5) = 0.72($ since the average fraction of unskilled workers in the US prior to 2008 was

The blue line in Figure 6a shows the trajectory of unemployment when $\chi = 0$, i.e. a model without training costs - essentially the standard DMP model, which has a unique steady state. Absent any further shocks, this model predicts a rapid recovery with unemployment returning to 5 percent by the end of 2010. This suggests that in the absence of persistent shocks, our mechanism is *necessary* to match the sluggish recovery observed in the data.

(a) Model vs data

(b) Different starting points
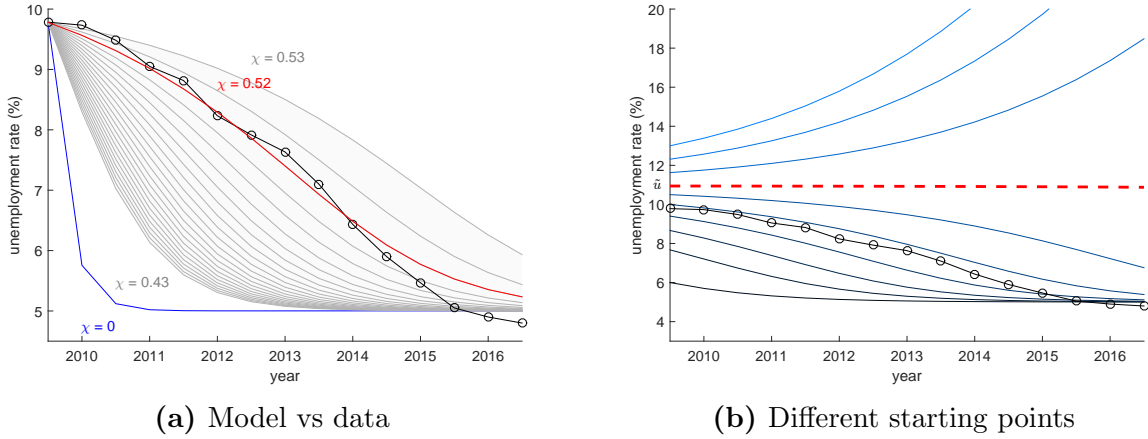
Figure 6. Trajectories of unemployment

Figure 6a suggests that, given the magnitude of shocks that hit the U.S. during the Great Recession, monetary policy was accommodative enough to avert permanent stagnation, but not enough to prevent a slow recovery. The model allows us to evaluate how the economy would have responded had shocks been larger or policy less accommodative. Figure 6b shows the trajectory of unemployment under our preferred calibration given different initial unemployment rates (in the second half of 2009).[38] Again, the black line plots data. The dark blue lines show trajectories starting from lower initial unemployment; light blue lines indicate trajectories starting from higher unemployment. The red-dashed line shows the trajectory starting from $\tilde{u} = 10.9\%$ (the unstable steady state) which divides the regions of slow recovery and permanent stagnation. The figure shows that, had monetary policy been more accommodative after the initial shocks and kept the initial rise in unemployment below 8 percent, unemployment would have returned to 6% two years earlier, in 2012. More strikingly, had policy been less accommodative allowing unemployment to rise to 12%, the

20 percent, and assuming 2087 hour work-year, as is standard. Barron et al. (1989) also find that the median worker spends 81 hours in training. If we instead calibrate χ to match the difference between training costs between unskilled and skilled (median) worker, we get $\chi = (151 - 81)/(0.2 * 1043.5) = 0.34$.) The American Society for Training and Development (Paradise, 2009) estimated the average annual learning expenditure to be 2.24% of total annual payroll in 2008. In our model total training expenditures equal $\chi\mu\delta(1 - u)$ in steady state while payroll equals $w(1 - u)$ implying $\chi = 0.48$. Our preferred value of $\chi = 0.52$ lies comfortably within this range.

[38] These initial combinations refer to different combinations of shocks and policy.

economy would have been unable to return to full-employment absent fiscal policy.

Figure 6b suggests that had monetary policy not been so quick to respond following the shocks in 2007, the U.S. economy could have fallen into permanent stagnation. In this regard, Europe presents a cautionary tale. Figure 7 shows the fraction of long-term unemployed in Ireland, Greece, Spain, the Euro area and the U.S. from 2008 to 2016. While the fraction of long-term unemployed increased in the U.S. following 2007, timely monetary policy accommodation ensured that this increase was temporary. In contrast, the fraction of long-term unemployed increased following the European recessions of 2008 and 2011 and has remained elevated since.[39] Many commentators[40] have argued that the European Central Bank's response was insufficient from the point of view of these economies or came too late. The model suggests that this delayed or insufficient monetary policy response could explain why long-term unemployment has remained persistently high in these economies. From this perspective, ECB President Draghi's "whatever it takes" speech in July 2012 - which can be seen as a commitment to very accommodative policy - may have come too late to reverse the effects of hysteresis. This underlines that to prevent hysteresis, monetary stimulus must be not just large, but also timely.

We already saw from Figure 6a how training costs drive the slow recovery; when training costs are zero, and the only costs of job creation are those associated with vacancy posting, the model predicts a fast recovery. As highlighted in Pissarides (2009), when firms post fewer vacancies due to poor aggregate conditions, competition for workers amongst recruiters declines, shortening the average duration of a vacancy. Absent any other hiring costs, this lowers the average cost of job creation, mitigating the recession's adverse impact on hiring. Training costs, χ, undo this phenomenon, creating a protracted recovery. Unlike the average cost of vacancy creation, κ/f, which is pro-cyclical and rises when there is more competition amongst recruiters, training costs are counter-cyclical and rise when the composition of job-seekers tilts towards the unskilled. Figure 8 highlights the inverse relationship between the average expected vacancy posting cost κ/f (red line) and the expected training costs $\chi\mu$ (black line) in the years following the Great Recession. Notably, the increase in job creation costs (blue line) solely stemmed from the large spike in training costs. The sharp rise in training costs more than counteracted the benefit of lower average expected vacancy posting costs following the Great Recession, depressing job growth and stalling the recovery.

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[39]Of course, experiences differed widely across European countries; for example, the fraction of long -term unemployed actually declined in Germany over this period. Here we focus on those countries most severely affected during the European crisis, from whose perspective the ECB's monetary policy was arguably insufficiently accommodative. Having said that, despite the heterogeneous experiences of different countries, on average, the Euro area did experience an increase in the fraction of long-term unemployed.

[40]See for example Kang et al. (2015).
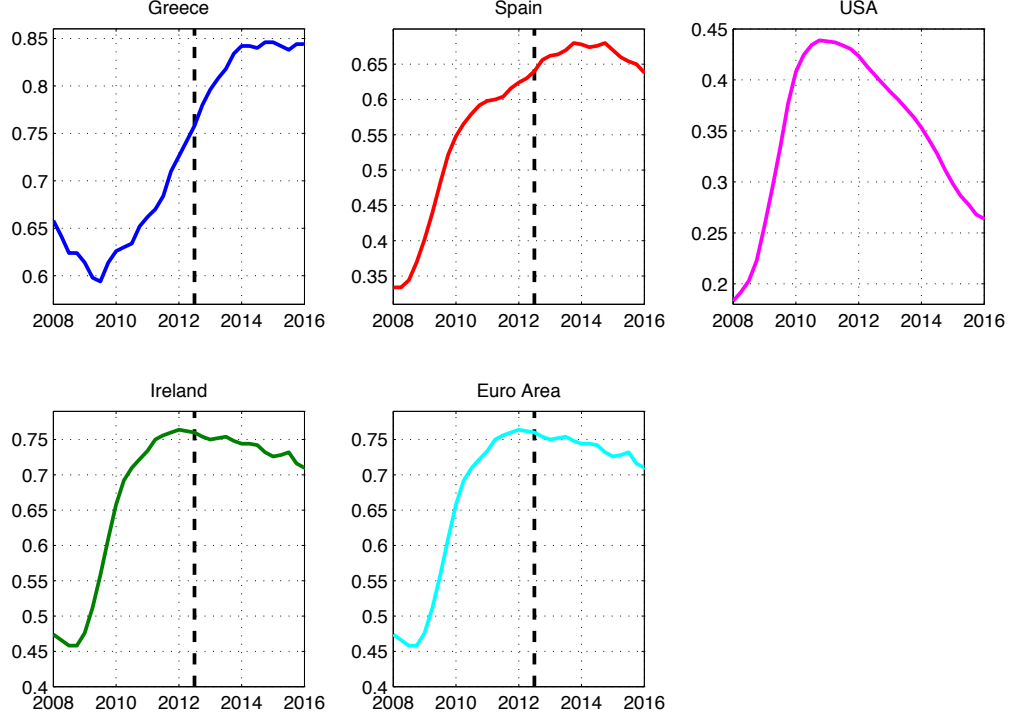
Figure 7. Fraction of Long-term unemployed (>27 weeks) in select countries. The figure plots five quarter moving averages of quarterly data. The dashed-line indicates the timing of Draghi's "whatever it takes" speech. Source: Eurostat and FRED.
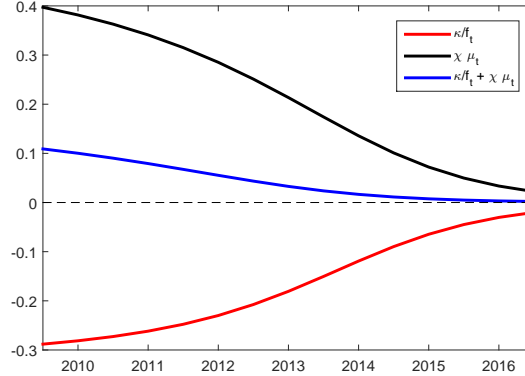
Figure 8. Components of job-creation costs

Even when monetary policy cannot ameliorate the scarring effects of recessions, fiscal policy - hiring or training subsidies - can help. Figure 9 shows the duration of the recovery (starting from 9.8% unemployment) as a function of χ with the cross denoting our preferred value of χ. The effect of χ on duration is highly non-linear. A modest subsidy which lowers the per worker training cost χ by 4% from 0.52 to 0.5 could have hastened the recovery by over 2 years.
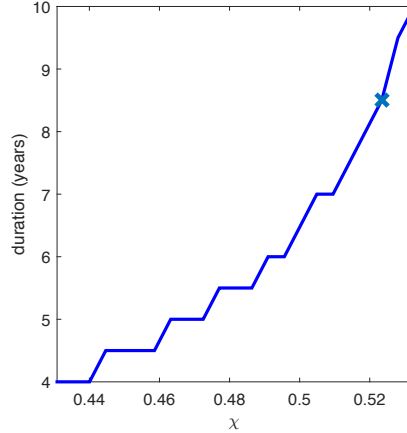
Figure 9. Duration as function of χ

6   Conclusion

We presented a model to understand the positive and normative implications of hysteresis. Skill depreciation, nominal rigidities and constraints on monetary policy together allow temporary shocks to create slow recoveries or even permanent stagnation. Aggresive countercyclical policy may be able to avoid these outcomes, but only if it comes early enough; once the rot has set in, monetary policy cannot rescue the economy from an unemployment trap.

While we have focused on skill deprecation, more generally recessions may damage productive capacity through multiple channels - reducing capital accumulation, reducing labor force participation, slowing productivity growth, and so on. Many of these effects may also be hard or even impossible to reverse. For example, Wee (2016) shows that recessions can permanently change young workers' search behavior, causing them to stay in careers in which they have a comparative disadvantage but have accumulated sufficient human capital, instead of searching for a career in which they have a comparative advantage; even after the recession, the cost of discarding accumulated human capital to pursue their comparative advantage dissuades workers from searching further. Whenever such mechanisms are operative, it is all the more important for countercyclical policy to nip recessions in the bud; the damage from failing to do so may be irreversible. In a world vulnerable to hysteresis, prevention is better than cure.

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Appendix

A Wages and Nash Bargaining

Recall that the value of an employed worker and of an unemployed worker, respectively, are defined by the recursions (3) and (4). Also, the value of a filled vacancy to a firm is given by equation (10). We can then define the surplus of a match between a worker and a firm as:

$$\mathcal{S}_t = J_t + \mathbb{W}_t - \mathbb{U}_t$$

Wages are determined by Nash bargaining. Denoting workers' bargaining power by η, wages solve

$$\max_{w_t} J_t^{1-\eta}(W_t - U_t)^\eta$$

implying

$$\eta J_t = (1 - \eta)(W_t - U_t)$$

Notice that the match surplus can be rewritten as:

$$
\begin{aligned}
\mathcal{S}_t &= J_t + \mathbb{W}_t - \mathbb{U}_t \\
&= A - b + \beta(1-\delta)J_{t+1} + \beta(1-\delta)(1-q_{t+1})(\mathbb{W}_{t+1} - \mathbb{U}_{t+1}) \\
&= A - b + \beta(1-\delta)(1-q_{t+1})S_{t+1} + \beta(1-\delta)q_{t+1}J_{t+1}
\end{aligned}
$$

Using the fact that $W_t - U_t = \eta S_t$ in the equation above, we have:

$$\omega_t = \eta A + (1-\eta)b + \beta(1-\delta)\eta q_{t+1}J_{t+1}$$

B Proof of Lemma 2

Steady states solve

$$\frac{a}{1 - \beta\gamma(1 - \eta(1-\mu))} = \kappa + \chi\mu$$

Define $x = \frac{\chi}{J_{min}}$, dividing through by J_{min}, this becomes

$$\frac{1}{1 - e\mu} = k + x\mu$$

Assumptions 1 and 2 impose that $k < 1$ and $\dfrac{1}{1-e} > k + x$. Since $e \in (0, 1)$, $\dfrac{1}{1-e\mu}$ is an increasing, strictly convex function. Starting from $x = 0$, as we increase x, either the intersection of these two functions first occurs at $\mu \in (0, 1)$, in which case a slightly higher x would give us multiplicity, or the first intersection has $\mu \geq 1$. Consider the knife edge case in which the first intersection of these two curves is at $\mu = 1$. Then the curves must be tangent and equal to each other at $\mu = 1$, i.e.

$$\frac{e}{(1-e)^2} = k, \frac{1}{1-e} = k + x$$

which implies $k = \dfrac{1-2e}{(1-e)^2}$. In order to have multiple intersections in $(0, 1)$, then, it is clear graphically that we need $k > \dfrac{1-2e}{(1-e)^2}$. Assumption 3 is sufficient (but not necessary) to ensure this, since it implies that $e > 0.5$. If this is true, and if x is just large enough that there is a single slack steady state, then the polynomial

$$1 - (k + x\mu)(1 - e\mu) = 0$$

has a unique solution, i.e. its discriminant equals zero:

$$(x - ek)^2 - 4xe(1 - k) = 0$$
$$x^2 - 2e(2 - k)x + e^2k^2 = 0$$

Graphically, it is clear that this will have two solutions, the larger of which corresponds to $\mu \in (0, 1)$; choosing this one we have

$$x^* = e(2 - k) + \sqrt{e^2(2 - k)^2 - e^2k^2} = e[2 - k + 2\sqrt{1 - k}]$$

Thus there will be multiple steady states if $x > x^*$.

C   Proof of Lemma 3

Suppose $\mu_0 = 0$. Then, note that $\mu_t = 0$ (which implies $n_t = 1$) is consistent with equation 14, since in the tight labor market regime $q_t = 1$, and $n_t = 1, \mu_{t+1} = 0$. Next we show that we cannot have $\theta_0 < \theta^{FE}$ given $\mu_0 = 0$. (Since $\theta^{FE} \geq 1$ by Assumption 1, this implies in particular that we cannot have $\theta_0 < 1$.) In any equilibrium, equation 13 must be satisfied,

which means that

$$J_{min} \leq a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1-\eta\min\{\theta_\tau, 1\}) \leq \kappa\max\{\theta_t, 1\}$$

where the first inequality comes because the left hand side is decreasing in θ_τ. Since we know that $J_{min} > \kappa$ from Assumption 1, it is immediate that this inequality can only be satisfied if $\theta \geq \theta^f \geq 1$. Finally, we show that we cannot have $\theta_0 > \theta^f$. We have shown that

$$a \sum_{t=0}^{\infty} \prod_{\tau=0}^{t} \beta(1-\delta)(1-\eta\min\{\theta_\tau, 1\}) = \kappa\theta_0$$

in any equilibrium, and that this expression is satisfied by $\theta_t = \theta^{FE}$, $\forall t \geq 0$. If $\theta_0 > \theta^{FE}$, it follows that $\theta_t < \theta^{FE}$ for some $t > 0$. Let T be the first date at which this is true. Then up to that date, since the labor market has been tight, $\mu_T = 0$. This is a contradiction, since we have already shown that if $\mu_T = 0$, $\theta_T \geq \theta^{FE}$. It follows that the unique equilibrium has $\theta_t = \theta^{FE}$ for all $t \geq 0$. The proof for any $\mu_0 \in (\mu_0, \underline{\mu})$ is similar and follows from the fact that $q_0 = 1$ which implies that all workers are employed in period 0.

D   Proof of Lemma 4

We prove the Lemma by proving the contrapositive. The first thing to note is that $\mu_R := \frac{1}{1+\gamma} > 0.5$ since $0 < \gamma < 1$. Recall that $\underline{\mu} = \frac{J_{min}-\kappa}{\chi}$. Suppose $\underline{\mu} \geq \mu_R$. This implies that $\underline{\mu}$ must also be greater than 0.5. In this case, no interior steady state can exist. Recall that any interior steady state solves:

$$
\begin{aligned}
\kappa + \chi\mu &= \mathcal{Q}(\mu) \\
&= \frac{a}{1 - \beta\gamma[1 - \eta(1-\mu)]} \\
&= \frac{a}{1 - \beta\gamma(1-\eta)} \frac{1 - \beta\gamma(1-\eta)}{1 - \beta\gamma[1 - \eta(1-\mu)]} \\
&= J_{min} \frac{1}{1 - e\mu}
\end{aligned}
$$

where, as before $e = \frac{\beta\gamma\eta}{1-\beta\gamma(1-\eta)}$.

Thus interior steady states solve:

$$\Omega(\mu) := \frac{J_{min}}{1 - e\mu} - \kappa - \chi\mu = 0$$

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We show that this is not possible if $\underline{\mu} > \mu_R$. In particular, we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$. First, we show that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. Notice that e can also be rewritten as:

$$e = \frac{1}{1 + \frac{1 - \beta\gamma}{\beta\gamma\eta}} > \frac{1}{1 + \frac{\eta}{\eta}} = \frac{1}{2}$$

where the inequality follows since $\eta > \frac{1-\beta\gamma}{\beta\gamma}$ by Assumption 3. Thus, $e > \frac{1}{2}$. To see that $\chi < 2(J_{min} - \kappa)$, note that from the definition of $\underline{\mu}$:

$$\chi = \frac{J_{min} - \kappa}{\underline{\mu}} < 2(J_{min} - \kappa)$$

since $\underline{\mu} > 0.5$ by assumption.

Fix $\kappa \in [0, J_{min})$, $\mu \in [0,1]$. Even though we have shown above that $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$, for a moment, set $e = 1/2$, $\chi = 2(J_{min} - \kappa)$. We claim that

$$\mathcal{Q}(\mu) = \frac{J_{min}}{1 - e\mu} \geq \kappa + \chi\mu = \kappa + 2(J_{min} - \kappa)\mu$$

with strict inequality unless $\kappa = 0$ and $\mu = 1$, in which case the expression holds with equality. When $\kappa = 0$, the RHS becomes $2J_{min}\mu$, and the LHS and RHS are only equal for $\mu = 1$. For any $\mu < 1$, the LHS is larger. When $\kappa > 0$, the RHS is strictly lower for any $\mu > 1/2$. Thus for any $\kappa \in [0, J_{min}]$, the inequality holds for all $\mu \in [0,1)$. Finally, for any $\mu \leq 1/2$, the inequality clearly holds since the LHS is greater than J_{min}, and the RHS smaller than J_{min}.

Next, suppose $e > 1/2$ and $\chi < 2(J_{min} - \kappa)$. If $\mu = 0$, this does not change the inequality, which still holds strictly (since $\mu \neq 1$). If $\mu > 0$, this strictly increases the LHS and strictly decreases the RHS. Thus the expression is still satisfied with strict inequality. Thus we have $\Omega(\mu) > 0$ for all $\mu \in [0,1]$, and there is no interior steady state. Since we have shown that $\underline{\mu} \geq \mu_R$ implies there exists no interior steady state, it follows that if there exist multiple interior steady states, we must have $\underline{\mu} < \mu_R$. $\qquad\square$

E   Proof of Lemma 5

For $\mu_0 = \underline{\mu}$, the value of an employed worker for a firm is given by J_{min}. To see this, notice that $J_0 \leq \kappa + \chi\underline{\mu}$ as long as labor markets are slack, $\theta_0 \leq 1$. In this case, by definition, $J_0 \leq J_{min}$ and by definition this relationship has to hold with equality. If labor markets are tight, $\theta_0 > 1$, then $J_0 = \kappa\theta_0 + \chi\underline{\mu} > J_{min}$ since $\theta_0 > 1$. This is a contradiction since if $\theta_0 > 1$, $\mu_1 = 0$ from Lemma 3 and $J_0 = J_{min}$ from Lemma 3. Furthermore, from Lemma 1, it follows that $J_1 = J_{min}$ and $\theta_1 \geq 1$.

The contradiction above shows that $\theta_0 \leq 1$. We now need to show that $\theta_0 > 1 - \underline{\mu}$. Suppose that $\theta_0 < 1 - \underline{\mu}$. Then μ_1 is given by:

$$\mu_1 = \frac{1 - \theta_0}{1 + \gamma[1 - \theta_0 - \underline{\mu}]} > \underline{\mu}$$

This is a contradiction since

$$J_1 = J_{min} = \kappa + \chi\underline{\mu} < \kappa + \chi\mu_1$$

which requires that $\theta_1 = 0$. Thus, we have shown that $\theta_0 \in [1 - \bar{\mu}, 1]$. The rest follows from equation (??) and the previous Lemmas.

F   Proof of Proposition 1

Definition 3. *Define the functions* $\Theta^1 : I^1 \to [0,1]$, $F^1 : I^1 \to \mathbb{R}_+$, $M^1 : I^1 \to \{\underline{\mu}\}$ *as:*

$$\Theta^1(\mu_{T-1}) \; := \; 1 - \frac{\underline{\mu}}{1 - \gamma\underline{\mu}}(1 - \gamma\mu_{T-1})$$

$$F^1(\mu_{T-1}) \; := \; \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1 - \eta\Theta^1(\mu_{T-1}))(\kappa + \chi\mu_{T-1})\right]$$

$$M^1(\mu_{T-1}) \; := \; \underline{\mu}$$

where $I^1 = [\underline{\mu}, \mu^1]$ *and* $\mu^1 := F^1(\underline{\mu})$.

Intuitively, at any date t, for any $\mu_t \in I^1$, $\Theta^1(\mu_t)$ describes the job-finding rate that ensures that the economy reaches $\underline{\mu}$ at date $t + 1$. $F^1(\mu_t)$ describes the unique value that μ_{t-1} can have in period $t - 1$ such that $\mu_t \in I^1$ and also $\mu_{t+1} = \underline{\mu}$. In other words, given market tightness at date t, $\Theta^1(\mu_t)$, one can compute the value of a filled vacancy at date $t - 1$ and zero and by no-arbitrage, this pins down the value of μ_{t-1} for which firms would have been willing to post the requisite number of vacancies. $M^1(\mu)$ is just a constant function which by definition describes where any $\mu \in I^1$ ends up.

Corollary 2. *It must be true that* $\mu^1 < \widetilde{\mu}$.

By the definition of μ^1, it must be true that

$$\begin{aligned}
\mu^1 &= \frac{1}{\chi}\left[a - \kappa + \beta\gamma\left[1 - \eta(1 - \underline{\mu})\right](\kappa + \chi\underline{\mu})\right] \\
&< \frac{1}{\chi}\left[a - \kappa + \beta\gamma\left[1 - \eta(1 - \widetilde{\mu})\right](\kappa + \chi\widetilde{\mu})\right] \\
&= \widetilde{\mu}
\end{aligned}$$

Lemma 9. *For β sufficiently close to 1, F^1 is increasing in μ for $\mu \in [\underline{\mu}, \widetilde{\mu})$*

Proof. Since $F^1(\mu)$ is composed of constants and a concave part, it suffices to consider the concave polynomial $\xi(\mu) = [1 - \eta\Theta^1(\mu)](\kappa + \chi\mu)$. This function is increasing in μ for

$$\mu < \frac{1}{2}\left[\frac{(1 - \gamma\underline{\mu})(1 - \eta)}{\eta\gamma\underline{\mu}} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right] \tag{28}$$

It is thus sufficient to show that $\widetilde{\mu}$ satisfies this inequality. Recall that $\widetilde{\mu}$ satisfies

$$\mathcal{Q}(\widetilde{\mu}) = \frac{a}{1 - \beta\gamma(1 - \eta + \eta\widetilde{\mu})} = \kappa + \chi\widetilde{\mu}$$

Since the left hand side is convex and the right hand side linear, since $\widetilde{\mu}$ is the smaller of two solutions to this equation, then

$$\mathcal{Q}'(\widetilde{\mu}) = \frac{a\beta\gamma\eta}{[1 - \beta\gamma(1 - \eta + \eta\widetilde{\mu})]^2} < \chi$$

In other words, the LHS cuts the RHS from above. Next, dividing the first equality by the second inequality, we have

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1 - \beta\gamma}{\beta\gamma\eta} + 1 - \frac{\kappa}{\chi}\right] \tag{29}$$

Define:

$$\Xi = \frac{1}{2}\left\{\frac{(1 - \gamma\underline{\mu})(1 - \eta)}{\eta\gamma\underline{\mu}} - \frac{1 - \beta\gamma}{\eta\beta\gamma} + \frac{1}{\gamma} - 1\right\}$$

Assuming that $\beta > \frac{\widetilde{\mu}}{\eta\widetilde{\mu}+1-\eta}$, it can be shown that $\Xi > 0$.[41] Thus, as required:

$$\widetilde{\mu} < \frac{1}{2}\left[\frac{1 - \beta\gamma}{\beta\gamma\eta} + 1 - \frac{\kappa}{\chi}\right] + \Xi = \frac{1}{2}\left[\frac{(1 - \gamma\underline{\mu})(1 - \eta)}{\eta\gamma\underline{\mu}} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right]$$

$\square$

It was already clear that given a $\mu_{t+1} \in I^1$, there exists a unique μ_t which could have led there. In addition, this Lemma shows that given any μ_t, there exists at most one $\mu_{t+1} \in I^1$ is consistent with equilibrium.

---

[41]Note that this assumption is a condition on an endogenous variable, $\widetilde{\mu}$ and can be rewritten as $\widetilde{\mu} < \frac{1 - \eta}{\beta^{-1} - \eta}$. Nonetheless, it is a weak condition: for any $\widetilde{\mu} < 1$, it is satisfied for β sufficiently close to 1.

Corollary 3. *Let $I^2 = F^1(I^1)$ and let $M^2(\mu)$ be the inverse of this function. Then $M^2(\mu^1) = M^1(\mu^1) = \underline{\mu}$.*

Since F^1 is increasing and continuous, its inverse M^2 exists and is increasing and continuous. Consequently, $F^1(I^1)$ maps into an interval $(\mu^1, \mu^2]$. Further since $\mu^1 = F^1(\underline{\mu})$, then $M^2(\mu^1) = \underline{\mu}$.

Lemma 10. $\mu^2 = F^1(\mu^1) < \widetilde{\mu}$

Proof. Since $\Theta^1(\underline{\mu}) = 1 - \underline{\mu}$ and Θ^1 is increasing, we have $\Theta^1(\mu^1) > 1 - \underline{\mu} > 1 - \widetilde{\mu}$. It follows that:

$$\frac{1}{\chi}\left[a - \kappa + \beta\gamma(1 - \eta\Theta^1(\mu^1))(\kappa + \chi\widetilde{\mu})\right] \quad < \quad \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1 - \eta(1 - \widetilde{\mu}))(\kappa + \chi\widetilde{\mu})\right]$$

Then, from Corollary 2, since $\mu^1 < \widetilde{\mu}$:

$$
\begin{aligned}
F^1(\mu^1) \;&=\; \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1 - \eta\Theta^1(\mu^1))(\kappa + \chi\mu^1)\right]\\
&<\; \frac{1}{\chi}\left[a - \kappa + \beta\gamma(1 - \eta(1 - \widetilde{\mu}))(\kappa + \chi\widetilde{\mu})\right]\\
&=\; \widetilde{\mu}
\end{aligned}
$$

$\square$

Lemma 11. *Define $\Theta^2(\mu) : I^2 \to [0,1]$ as:*

$$\Theta^2(\mu) := 1 - M^2(\mu)\frac{1 - \gamma\mu}{1 - \gamma M^2(\mu)}$$

Then,

$$\frac{\partial\Theta^2(\mu)}{\partial\mu} \leq \frac{\gamma M^2(\mu)}{1 - \gamma M^2(\mu)}$$

Proof.

$$
\begin{aligned}
\frac{\partial\Theta^2(\mu)}{\partial\mu} \;&=\; M^2(\mu)\frac{\gamma}{1 - \gamma M^2(\mu)} - \frac{\partial M^2(\mu)}{\partial\mu}\left[1 + \frac{\gamma\,(1 - \gamma\mu)\,M^2(\mu)}{[1 - \gamma M^2(\mu)]^2}\right]\\
&\leq\; M^2(\mu)\frac{\gamma}{1 - \gamma M^2(\mu)}
\end{aligned}
$$

where the inequality comes because $M^2(\mu)$ is increasing and the expression in square brackets is positive. $\square$

We are now ready to characterize equilibrium in the entire convalescent region.

Lemma 12 (Induction Step). *Suppose the functions $\Theta^n(\mu)$, $M^n(\mu)$ are defined on some interval $I^n = [\mu^{n-1}, \mu^n]$ and $M^{n-1}(\mu_{T-n+1})$ is defined on an interval $I^{n-1} = [\mu^{n-2}, \mu^{n-1}]$, with $\underline{\mu} < \mu^{n-2} < \mu^n < \widetilde{\mu}$, and that these functions satisfy*

$$\Theta^n(\mu) = 1 - M^n(\mu)\frac{1 - \gamma\mu}{1 - \gamma M(\mu)}$$

$$\frac{\partial \Theta^n(\mu)}{\partial \mu} < \frac{\gamma M^n(\mu)}{1 - \gamma M^n(\mu)}$$

$$M^n(I^n) = I^{n-1}$$

$$M^n(\mu^{n-1}) = M^{n-1}(\mu^{n-1}) = \mu^{n-2}$$

Then, for β sufficiently close to 1, we have the following results:

1. *The function*

$$F^n(\mu) := \frac{1}{\chi}[a - \kappa + \beta\gamma(1 - \eta\Theta^n(\mu))(\kappa + \chi\mu)]$$

   is monotonically increasing in μ for $\mu \leq \widetilde{\mu}$.

2. *Let $I^{n+1} = F^n(I^n)$ and let $M^{n+1}(\mu)$ be the inverse of this function. Then $M^{n+1}(\mu^n) = M^n(\mu^n) = \mu^{n-1}$.*

3. *$I^{n+1} = [\mu^n, \mu^{n+1}]$ with $\mu^{n+1} < \widetilde{\mu}$.*

4. *Define $\Theta^{n+1}(\mu)$ on I^{n+1} by*

$$\Theta^{n+1}(\mu) = 1 - M^{n+1}(\mu)\frac{1 - \gamma\mu}{1 - \gamma M^{n+1}(\mu)}$$

   The derivative of this function satisfies

$$\frac{\partial \Theta^{n+1}(\mu)}{\partial \mu} < \frac{\gamma M^{n+1}(\mu)}{1 - \gamma M^{n+1}(\mu)}$$

Proof. (1.) The derivative of $F^n(\mu)$ is

$$\frac{\partial F^n(\mu)}{\partial \mu} = \frac{\beta\gamma}{\chi}\left[-\eta\frac{\partial \Theta^n(\mu)}{\partial \mu}(\kappa + \chi\mu) + \chi(1 - \eta\Theta^n(\mu))\right]$$

$$> \frac{\beta\gamma}{\chi}\left[-\eta\frac{\gamma M^n(\mu)}{1 - \gamma M^n(\mu)}(\kappa + \chi\mu) + \chi(1 - \eta\Theta^n(\mu))\right]$$

Substituting in the definition of Θ^n and rearranging, we see that this expression will be

positive provided that

$$\mu < \frac{1}{2}\left[\frac{1 - \eta\left(1 - \gamma M^n(\mu)\right)}{\eta}\frac{1}{\gamma M^n(\mu)} + \frac{1}{\gamma} - \frac{\kappa}{\chi}\right]$$

By the same logic as in Lemma 9, for β sufficiently close to 1, this is satisfied for any $\mu \leq \widetilde{\mu}$, since we have $M^n(\mu) \leq \widetilde{\mu}$. So $F^n(\mu)$ is increasing, and hence invertible, for $\mu < \widetilde{\mu}$. Let $M^{n+1}(\mu)$ be the inverse of this function.

(2.) We have

$$
\begin{aligned}
M^n(\mu^{n-1}) &= M^{n-1}(\mu^{n-1}) \\
\Theta^n(\mu^{n-1}) &= \Theta^{n-1}(\mu^{n-1}) \\
F^n(\mu^{n-1}) &= F^{n-1}(\mu^{n-1}) = \mu^n \text{ by definition of } \mu^n \\
M^{n+1}(\mu^n) &= M^n(\mu^n)
\end{aligned}
$$

(3.) Since F^n is a continuous, increasing function, the image of the interval $[\mu^{n-1}, \mu^n]$ under F^n must be an interval $[\mu^n, \mu^{n+1}]$. (We have already shown that $F^n(\mu^{n-1}) = \mu^n$.) We need to show that $\mu^{n+1} = F^n(\mu^n) < \widetilde{\mu}$. We know that $\widetilde{\mu} \geq M^n(\mu^n)$. Then, it must be true that

$$
\begin{aligned}
1 - \widetilde{\mu} &< 1 - M^n(\mu^n) \\
&= 1 - M^n(\mu^n)\frac{1 - \gamma\mu^n}{1 - \gamma\mu^n} \\
&< 1 - M^n(\mu^n)\frac{1 - \gamma\mu^n}{1 - \gamma M^n(\mu^n)} \\
&= \Theta^n(\mu^n)
\end{aligned}
$$

Then, by the same logic as in Lemma 10 we have $F^n(\mu^n) < \widetilde{\mu}$. So we have shown that $I^{n+1} \subset [\underline{\mu}, \widetilde{\mu}]$.

(4.) The bound on the derivative is established in the same way as Lemma 11.

$\square$

Lemma 13. $\lim_{n\to\infty} \mu^n \to \widetilde{\mu}$.

Proof. We have shown that $\{\mu^n\}$ is an increasing sequence bounded above by $\widetilde{\mu}$; thus by the Monotone Convergence Theorem, its limit μ^∞ exists, and $\mu^\infty \leq \widetilde{\mu}$. Suppose by contradiction that $\mu^\infty < \widetilde{\mu}$. Then μ^∞ must be a steady state. But by definition, $\widetilde{\mu}$ is the smallest slack steady state. So we must have $\mu^\infty = \widetilde{\mu}$. $\square$

G Proof of Lemma 8

Suppose that the ZLB does not bind at date 0. Then monetary policy is unconstrained in all periods, and nominal wages and prices remain constant. From the household Euler equation, we have $1 + i_t = \frac{P_1}{P_0 \beta_0} = \frac{1}{\beta_0}$. When $\beta_0 > 1$, this would imply a negative nominal interest rate, violating the ZLB. In this case, monetary policy is constrained at date 0 and we have $P_0 = \frac{P_1}{\beta_0}$. If the economy returns to FESS after date 0 (that is, $\mu_t = 0$ for all t), then real wages will equal ω^*_{fe} at all dates $t \geq 1$. This implies that the ZLB will not bind at date 1: there is an equilibrium in which prices and nominal are constant forever starting at date 1. Consequently, monetary policy is unconstrained at date 1, and $P_1 = \frac{W_0}{\omega^*_{fe}}$, which in turn implies $W_1 = W_0$. This implies that real wages at date 0 are given by:

$$\omega_0 = \frac{W_1}{W_0} \frac{P_0}{P_1} \omega_1 = \frac{\omega^*_{fe}}{\beta_0}$$

and we indeed have

$$J_0 = A - \beta_0 \omega^*_{fe} + \beta_0 \gamma J_{min}$$

as claimed in the Lemma. The full employment steady state Nash wage equals

$$\omega^*_{fe} = \frac{\eta}{1 - \beta\gamma(1 - \eta)} A + \frac{(1 - \beta\gamma)(1 - \eta)}{1 - \beta\gamma(1 - \eta)} b$$

So $\frac{\partial J}{\partial \beta_0} = -\omega^*_{fe} + \gamma J_{min}$ will be negative provided that

$$-\frac{\eta}{1 - \beta\gamma(1 - \eta)} A - \frac{(1 - \beta\gamma)(1 - \eta)}{1 - \beta\gamma(1 - \eta)} b + \gamma \frac{(1 - \eta)(A - b)}{1 - \beta\gamma(1 - \eta)} < 0$$

$$A \left[\gamma - \frac{\eta}{1 - \eta} \right] - (\gamma + 1 - \beta\gamma) b < 0$$

By assumption 3, both terms are negative, so this condition is satisfied.

H Proof of Proposition 2

First we need to prove two lemmas. The first states that wages are lower in the convalescent region than at full employment. We need this result to show that prices will be higher in the convalescent region.

Lemma 14. $\omega(\boldsymbol{\mu}_t) < \omega^*_{fe}$ if $\mu_t \in (\underline{\mu}, \tilde{\mu})$.

Proof. We know that $M(\boldsymbol{\mu}_t) < \mu_t$ if $\mu_t \in (\underline{\mu}, \tilde{\mu})$.

$$
\begin{aligned}
\omega(\boldsymbol{\mu}_t) &= A - (\kappa + \chi\mu_t) + \beta\gamma[\kappa + \chi M(\boldsymbol{\mu}_t)] \\
&= A - \beta\gamma\chi(\mu_t - M(\boldsymbol{\mu}_t)) - (1 - \beta\gamma)(\kappa + \chi\mu_t) \\
&< A - (1 - \beta\gamma)(\kappa + \chi\mu_t) \\
&< A - (1 - \beta\gamma)(\kappa + \chi\underline{\mu}) = \omega_{fe}^*
\end{aligned}
$$

$\square$

Lemma 15. *Under Assumption 4,* $\dfrac{\mathcal{W}(\boldsymbol{\mu}_t; W_{t-1})}{\mathcal{P}(\boldsymbol{\mu}_t; W_{t-1})} > \gamma[\kappa + \chi\mu_t]$ *for all* $\boldsymbol{\mu}_t, W$.

Proof. We know that $\dfrac{\mathcal{W}(\boldsymbol{\mu}_t; W_{t-1})}{\mathcal{P}(\boldsymbol{\mu}_t; W_{t-1})} \geq \omega^*(\boldsymbol{\mu}_t)$ by definition, so it suffices to show that $\omega^*(\boldsymbol{\mu}_t) > \gamma[\kappa + \chi\mu_t]$. In the flexible wage benchmark we have

$$
\omega_t = \eta A + (1 - \eta)b + \beta\gamma q_{t+1}J_{t+1} \geq \eta A + (1 - \eta)b > \gamma(\kappa + \chi\mu_t)
$$

for any $\mu_t \in [0, 1]$, given assumption 4. $\square$

Finally, we need to characterize dynamics of the economy starting at date 1, once the shock has abated. Under neutral monetary policy, if the ZLB never binds, allocations are (by definition) equal to those in the flexible wage benchmark. The following is immediate.

Lemma 16. *If* $\mu_1 \geq \tilde{\mu}$, *the economy never reaches the full employment steady state.*

Proof. If the ZLB never binds, allocations are equivalent to those in the flexible wage benchmark, and we know that the economy never returns to steady state. It only remains to show that the ZLB can never help the economy converge to the FESS. Suppose by contradiction that the economy converges to the FESS. Let μ_t^R, μ_t^N denote allocations in the flexible wage benchmark and in the nominal economy, respectively, given the initial condition $\mu_1 \geq \tilde{\mu}$. Let $T \geq 1$ be the first date at which $\mu_t^N < \mu_t^R$ (there must be some such date, since in the long run $\mu_t^N = 0, \mu_t^R > 0$, by assumption). Then we have

$$
\begin{aligned}
J_{T-1}^N &= \kappa + \chi\mu_{T-1}^N = \kappa + \chi\mu_{T-1}^R = J_{T-1}^R \\
J_T^N &= \kappa + \chi\mu_T^N < \kappa + \chi\mu_T^R = J_T^R
\end{aligned}
$$

This implies that real wages are higher at date $T - 1$ in the flexible wage benchmark than in

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the nominal economy:

$$J_{T-1}^N = J_{T-1}^R$$
$$A - \omega_T^N + \beta\gamma J_T^N = A - \omega_T^R + \beta\gamma J_T^R$$
$$\omega_t^R - \omega_t^N = \beta\gamma(J_T^R - J_T^N) > 0$$

This is a contradiction - given the downward nominal wage rigidities, wages are always weakly higher than in the flexible wage benchmark. Thus the economy cannot converge to the FESS. $\qquad\square$

We are now ready to prove Proposition 2. Part 1. follows for the same reasons as in the previous lemmas.

Define the function

$$B(\mu) = \frac{A - \kappa}{\Omega(\mu) - \gamma(\kappa + \chi\mu)}$$

on $(\underline{\mu}, \mu_R]$, where we define

$$\Omega(\mu_1) = \frac{\mathcal{W}\Big(\{0, \mu_1\}, \varphi W_{-1}\Big)}{\mathcal{P}\Big(\{0, \mu_1\}\}, \varphi W_{-1}\Big)}$$

It is straightforward to show that Ω is continuous, and thus B is continuous. We have $B(\underline{\mu}) = \underline{\beta}$.

Define $\overline{\beta} := B(\mu_R)$. If $\beta_0 > \overline{\beta}$, then if $\mu_1 = \mu_R$, we have

$$J_0 = A - \beta_0\Omega(\mu_R) + \beta_0\gamma(\kappa + \chi\mu_R) < \kappa$$

thus $\theta_0 = 0$, which is consistent with $\mu_1 = \mu_R$. If instead $\beta_0 \in (\underline{\beta}, \overline{\beta})$, then there exists $\mu \in (\underline{\mu}, \mu_R)$ such that $B(\mu) = \beta_0$, and a corresponding $\theta_0 = 1 - \frac{\mu_1}{1 - \gamma\mu_1}$. Then we have

$$J_0 = \kappa = A - \beta_0 w^B(\mu_1) + \beta_0\gamma(\kappa + \chi\mu_1)$$

and firms are indifferent between posting any number of vacancies; thus $\theta_0 \in [0, 1]$ can indeed be an equilibrium. The last part of the Proposition is immediate given the analysis in Section 3.2.