

LIQUIDITY TRAPS, CAPITAL FLOWS*

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Abstract

Motivated by debates surrounding international capital flows during the Great Recession, we conduct a positive and normative analysis of capital flows when a region of the global economy experiences a liquidity trap. Capital flows reduce inefficient output fluctuations in this region by inducing exchange rate movements that reallocate expenditure towards the goods it produces. Restricting capital mobility hampers such an adjustment. From a global perspective, constrained efficiency entails subsidizing capital flows to address an aggregate demand externality associated with exchange rate movements. Absent cooperation, however, dynamic terms-of-trade manipulation motives drive countries to inefficiently restrict capital flows, impeding aggregate demand stabilization.

Keywords: Capital flows, international spillovers, liquidity traps, capital flow management, policy coordination, optimal monetary policy

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1 Introduction

Following the global financial crisis, a large number of advanced economies (including the U.S., the U.K., and the Eurozone) entered a period of anemic economic activity and very low interest rates that resembled a liquidity trap. At the same time, the crisis marked a break in a trend of widening current account imbalances that had been a central feature of the global economy for the previous two decades. After the crisis, on the deficit side, the U.S. experienced an increase its savings rate and a significant reduction in its current account deficit. Meanwhile, on the surplus side, many emerging market economies experienced a surge in capital inflows and a deterioration of their current account position.¹ Some observers and policymakers at the time argued that this incipient unwinding of global imbalances could promote a rebalancing of demand across countries and help facilitate a swifter global recovery (Blanchard, 2009, Blanchard and Milesi-Ferretti, 2009, IMF, 2010). This rebalancing was to crucially rely on the willingness of surplus countries to allow more capital inflows, let their currency appreciate, and thus suffer a loss in external competitiveness (Blanchard and Milesi-Ferretti, 2012). Fearing such prospects, several emerging market countries, likely emboldened by a shift in the stance of multilateral institutions that broke with the Washington consensus view (see Ostry et al., 2010, 2011 and IMF, 2012), adopted forms of capital controls to put the brakes on inflows, with some apparent success (Ahmed and Zlate, 2014).

This narrative raises several questions regarding the foundations and multilateral aspects of capital flow management policies in a liquidity trap. What precise role do capital flows play in global macroeconomic adjustment at the zero lower bound (ZLB)? Do they fulfill this role efficiently, or are capital flow management policies warranted? Are such policies associated with adverse spillover effects? Is coordinating such policies more crucial than in normal times? If so, why? Our goal in this paper is to address these questions.

To this end, we use a general equilibrium two-country model of the world economy in the New Open Economy Macroeconomics tradition, featuring imperfect competition, nominal rigidities, and an explicit zero bound on nominal interest rates. With the global economy’s experience of the Great Recession in mind, we interpret Home as the set of advanced economies and Foreign as the set of emerging economies. In line with the recent literature on policy at the zero lower bound, we consider a large unanticipated negative shock to the home discount rate (a negative “demand” shock) that pushes Home, but not Foreign, into a liquidity trap, defined as a situation where the “natural rate” turns negative.² Assuming that monetary policy is conducted

¹This capital inflows surge followed a brief sudden stop in the last quarter of 2008. See Jeanne et al. (2012) for a detailed description of capital flow patterns during this period.

² The natural rate is defined as the real interest rate prevailing in an equilibrium with flexible prices and exchange rates, under appropriately specified production subsidies that eliminate monopolistic competition distortions.

optimally, we then compare the global macroeconomic adjustment to the shock under a variety of capital flow regimes.

Three results emerge from our analysis. (i) In a liquidity trap, capital flows help reduce inefficient output fluctuations by decoupling output dynamics from consumption dynamics. Capital flows facilitate this decoupling by generating exchange rate movements that promote expenditure switching in favor of the goods whose provision is the most depressed. (ii) At the ZLB, even a regime of free capital mobility is constrained inefficient. Constrained efficiency calls for subsidizing capital flows, so as to encourage even more decoupling and expenditure switching. Thus, in a liquidity trap, managing capital flows has the potential to increase global welfare. (iii) Despite the desirability of capital account interventions in a liquidity trap, uncoordinated capital flow management policies are not generally warranted. The reason is that dynamic terms-of-trade manipulation incentives partly driving these policies work against macroeconomic stabilization.

To build intuition on the role of capital flows in a liquidity trap, consider the case of a closed economy. When a discount rate shock results in a negative natural rate, monetary policy is constrained by the zero bound. This results in an excessively high real interest rate and output must fall below its efficient level on impact in order to eliminate excess supply of savings.³ Optimal monetary policy, by committing to keep interest rates at zero past the liquidity trap episode, can engineer a future boom and thereby dissipate excess demand for current savings without as large a fall in output (Eggertsson and Woodford, 2003, Werning, 2012).

In an open economy context, excess savings can be channeled to other economies, which can further limit the initial output drop. The strength of this equilibrating force, however, crucially depends on the degree of capital mobility. Under free capital mobility, the adjustment features large trade imbalances: a more patient Home initially runs a trade surplus and accumulates claims vis-à-vis Foreign. Meanwhile, a negative interest rate differential between Home, for which the ZLB binds, and Foreign, for which it does not, induces a continuous appreciation of the home currency, following a depreciation on impact. This exchange rate response helps redirect expenditure in favor of the Home good early in the liquidity trap, precisely at the time when its provision is the most depressed. In contrast, under closed capital accounts—much like the closed economy—Home’s excess savings cannot be channeled to Foreign. Furthermore, close capital accounts preclude the stabilizing exchange rate movements that occurred under free capital mobility. Dissipating excess savings in Home requires home output to fall more on impact. Thus, curtailing capital mobility reduces the potency of the equilibrating force associated with openness.

Next, we investigate whether a regime of free capital mobility fulfills the stabilizing role described above efficiently. To this end, we formulate a planning problem in which a global

³This is often referred to as a “demand-driven recession.”

planner chooses a path of taxes or subsidies on capital flows to maximize world welfare. We find that while a regime of free capital mobility is constrained efficient when away from the zero bound, it is constrained *inefficient* when a region of the world economy faces a binding ZLB. In the same way that the real interest rate is excessively high in a closed economy liquidity trap, under free capital mobility the home real exchange rate is also excessively appreciated. The constrained inefficiency of the free capital mobility regime can hence be traced back to an aggregate demand externality resulting from the combination of two factors: output is demand determined and monetary policy is constrained by the zero bound in Home. Atomistic agents do not internalize that their savings decisions lead to adjustments in both inter- and intra-temporal prices. In the presence of nominal rigidities, however, such price adjustments aren't always feasible and quantity adjustments are instead required, resulting in aggregate demand externalities associated with private decisions. Away from the ZLB, optimal monetary policy is able to address this externality. However, at the ZLB it is unable to do so, and capital flow management policy can serve as a useful complement.

We provide a sharp analytical characterization of the constrained efficient capital flow regime, including a closed form expression for the optimal tax wedge on capital flows. During the liquidity trap, this regime entails a subsidy on flows from Home to Foreign and smaller fluctuations in the home output gap.⁴ The managed regime also features a steeper exchange rate path, and a more expansionary foreign monetary policy stance during the liquidity trap. Intuitively, capital flow taxes allow exchange rate dynamics to decouple from interest rate differentials, and thereby relax the ZLB constraint in Home without inflicting much harm on Foreign, for whom monetary policy can adjust.

While our result stands in contrast to the findings of [Devereux and Yetman \(2014\)](#) that capital flow taxes are not desirable in a liquidity trap, the optimal tax formula we derive allows us to reconcile the two views. It shows that a free capital mobility regime is only constrained efficient in knife-edge cases where natural interest rates are equal across countries, the scenario Devereux and Yetman focus on exclusively. Our optimal tax formula also helps distinguish our results from the work of [Farhi and Werning \(2016\)](#), whose general prescription is that optimal financial market taxes should redirect purchasing power toward agents with the highest marginal propensity to consume (MPC) on goods whose provision is relatively more depressed. In fact, our model's prescription entails discouraging spending by home agents at the precise time when the provision of the home good (on which they have a higher MPC) is the most depressed (i.e., early in the liquidity trap). The reason is that such a diversion supports an exchange rate trajectory that induces all agents to redirect expenditure toward the home good at that time. This, in our view, emphasizes the fundamental role of the exchange rate regime in determining

⁴We define the output gap at any date t as the difference between the level of output and its efficient level at the same date. For more details on the path of output under the efficient benchmark, see [Section 2.6](#).

the direction of the inefficiency on capital flows in a liquidity trap.

At first glance, our finding that capital does not flow sufficiently in a liquidity trap may seem difficult to reconcile with a recent literature on capital flow management that argues that free capital flows might instead be excessively volatile (see our literature review below). This literature, however, studies capital flow management from the perspective of individual inflow recipient countries, whereas we take a global efficiency standpoint. To illustrate that this distinction is crucial, we also consider a setting where countries manage capital flows non-cooperatively. In this case, we show that the incentives of individual countries to alter capital flows also respond to a desire to manage their dynamic terms-of-trade (dToT), as in [Costinot et al. \(2014\)](#).⁵ We show that this dToT manipulation motive leads countries to restrict capital flows and thus conflicts with macroeconomic stabilization in a liquidity trap. Furthermore, for commonly used parameterizations of this model, the dToT manipulation motive can easily dominate the macroeconomic stabilization force in a Nash equilibrium where countries manage their capital account non-cooperatively. In such cases, output gap fluctuations are larger, not only than under the efficient regime, but also than under free capital mobility. This result resonates with the argument in [Blanchard and Milesi-Ferretti \(2012\)](#) that adverse spillover effects of capital controls by recipient countries may be particularly severe in a liquidity trap, and provides a theoretical underpinning for efforts to better coordinate capital flow management policies across countries during such episodes (see [IMF, 2011](#), [Ostry et al., 2012](#)).

The rest of the paper is organized as follows. We conclude the introduction with a review of the related literature. We then describe the model in [Section 2](#). [Section 3](#) highlights the role of capital flows at the zero bound, [Section 4](#) analyzes capital flow efficiency, [Section 5](#) studies non-cooperative capital flow management, [Section 6](#) discusses potential extensions, and [Section 7](#) concludes.

Related literature The paper relates to a large body of literature on the conduct of monetary policy in liquidity traps that has developed following the seminal work of [Krugman \(1998\)](#) and [Eggertsson and Woodford \(2003\)](#).⁶ In the open economy context, the literature has mainly emphasized spillovers and interdependence of monetary policy across countries ([Jeanne, 2009](#), [Haberis and Lipinska, 2012](#), [Cook and Devereux, 2013](#), [Fujiwara et al., 2013](#)). By using [Cole and Obstfeld \(1991\)](#) preferences, we intentionally abstract from such monetary policy spillovers, and instead focus on the role played by capital mobility in shaping the dynamics of key macro variables in a liquidity trap. Nevertheless, we provide along the way a first analytical char-

⁵This motive arises in every open economy model where countries have some degree of market power over a good they trade. It applies to capital exporters and importers alike, and prevails independently from zero lower bound considerations.

⁶See, for instance, [Eggertsson and Woodford \(2004b,a\)](#), [Eggertsson \(2006, 2010\)](#), [Christiano et al. \(2011\)](#), [Guerrieri and Lorenzoni \(2011\)](#), [Eggertsson and Krugman \(2012\)](#), [Werning \(2012\)](#), [Correia et al. \(2013\)](#), and [Benigno and Fornaro \(2015\)](#).

acterization of the optimal ZLB exit time in an open economy, extending the closed economy analysis of [Werning \(2012\)](#). Our focus on capital mobility is thus similar to that of [Devereux and Yetman \(2014\)](#), although unlike us they argue that capital controls are not desirable in terms of welfare in a liquidity trap. As mentioned above, we are able to clarify that their result only holds in knife-edge cases where natural interest rates happen to be equal across countries. From an optimal policy perspective, our analysis highlights the role of capital flow taxes/subsidies as an additional tool to overcome the limitations of monetary policy at the ZLB.⁷ By analytically characterizing and comparing cooperative and non-cooperative capital flow management regimes, we further uncover a key source of distortion associated with non-cooperativeness and point to the importance of international cooperation during liquidity trap episodes.

Our paper also connects to a wealth of literature on capital flow regulation in emerging market economies. Several recent papers have developed arguments in favor of capital account interventions based on imperfections in financial markets (e.g., [Caballero and Krishnamurthy \(2001\)](#), [Korinek \(2007, 2010\)](#), [Jeanne and Korinek \(2010\)](#), [Bianchi \(2011\)](#)).⁸ Others have shown that imperfections in goods markets may also provide a rationale for the use of capital controls. [DePaoli and Lipinska \(2012\)](#) and [Costinot et al. \(2014\)](#) emphasize the role of market power and dynamic terms of trade management. [Farhi and Werning \(2012, 2014\)](#) and [Schmitt-Grohe and Uribe \(2016\)](#) stress the role of nominal rigidities. All these papers study optimal capital flow management from the perspective of individual countries. In contrast, we study the desirability of managing capital flows from a global efficiency perspective and highlight how such a regime differs from one where individual countries manage capital flows non-cooperatively.

More generally, our work also speaks to a recent literature on optimal policy interventions in economies with aggregate demand externalities (see, for example, [Farhi and Werning, 2012](#), [Farhi and Werning, forthcoming](#) and [Korinek and Simsek, 2016](#)). While our approach shares several features with this work, our findings stand out from its general message that optimal policy should induce agents with higher MPC on goods that are relatively depressed in some states to tilt their wealth toward these states ([Farhi and Werning, 2016](#)).

Finally, the paper also relates to contemporaneous work by [Caballero et al. \(2015\)](#) (CFG), [Eggertson et al. \(2016\)](#) (EMSS) and [Fornaro and Romei \(2016\)](#). Like us, these authors study the interplay between international capital flows and liquidity traps. However, the focus of CFG and EMSS is on the steady state analysis of permanent liquidity traps resulting in secular stagnation, while we emphasize transitional dynamics during temporary liquidity trap episodes. As a result, while interest rate policy is permanently impotent in their frameworks, it remains

⁷[Korinek \(2014\)](#) (section 5.2) also briefly analyzes the use of capital flow taxes at the ZLB but does so only from the point of view of a small open economy.

⁸[Gabaix and Maggiori \(2015\)](#) also show that in the presence of financial frictions, capital controls can increase the potency of currency market interventions as a tool to combat exchange rate movements generated by financial turmoil.

a key determinant of the short-run dynamics in our analysis through forward guidance. With respect to dealing with the multilateral effects of using tools other than monetary policy in a liquidity trap, our papers are complementary: while CFG and EMSS emphasize public debt issuance and fiscal policy, we focus on capital flow management policy and, in particular, the conflict arising between the dictates of global efficiency and the incentives of individual countries in that regard. Our analysis also shares similarities with [Fornaro and Romei \(2016\)](#). Like us, they consider the use of taxes on financial transactions to deal with liquidity traps in an open economy setting, and contrast non-cooperative with cooperative outcomes. However, while we contemplate the ex-post use of these policies for stimulatory purposes, they consider them from an ex-ante precautionary standpoint.

2 Model

The world economy consists of two equally sized countries, labeled “Home” and “Foreign.”⁹ In each country, households consume goods and supply labor, while firms hire labor to produce output. Foreign variables are denoted with asterisks. Following a large body of literature, we adopt the [Cole and Obstfeld \(1991\)](#) parameterization which features unitary inter- and intra-temporal elasticities of substitution. As we shall see, this parameterization eliminates international spillovers from monetary policy (see, e.g., [Corsetti and Pesenti, 2001](#)) and allows us to streamline the role of capital flow regimes. The model is deterministic, and a liquidity trap is generated using a time-varying discount rate for Home.

2.1 Households

Preferences of the representative household in Home are represented by the utility functional

$$\int_0^\infty e^{-\int_0^t (\rho + \zeta_s) ds} \left[\log C_t - \frac{(N_t)^{1+\phi}}{1+\phi} \right] dt,$$

where C_t is consumption, N_t is labor supply, ϕ is the inverse Frisch elasticity of labor supply, ρ is the (steady state) discount rate and ζ_t is a time-varying and country-specific discount rate shifter. Although our model does not feature uncertainty (as of date 0), we will refer to a negative realization of ζ_t as a *negative demand shock*, as such a realization lowers the demand for current consumption relative to future consumption (and hence increases the desire to save).

⁹In the context of the Great Recession, we think of Home as representing the set of *demand deficient* economies and of Foreign as standing for the rest of the world.

C_t is a consumption index defined as

$$C_t \equiv \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} (C_{H,t})^{1-\alpha} (C_{F,t})^\alpha$$

where $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$ denotes an index of domestically produced varieties, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$ is an index of foreign produced varieties, and $\alpha \in (0, 0.5]$ is a home bias parameter representing the degree of openness.

Households have access to markets for bonds issued under home and foreign jurisdiction, but they potentially face taxes for investing abroad. Home bonds are denominated in home currency, and foreign bonds are denominated in foreign currency. Since the model does not feature uncertainty, each of the two bonds trivially spans the space of states of nature. The home household's budget constraint is given by

$$\begin{aligned} \dot{D}_{H,t} + \mathcal{E}_t \dot{D}_{F,t} &= i_t D_{H,t} + (i_t^* + \tau_t - \tau_t^*) \mathcal{E}_t D_{F,t} + W_t N_t + T_t + \Pi_t - \int_0^1 P_{H,t}(l) C_{H,t}(l) dl \\ &\quad - \int_0^1 P_{F,t}(l) C_{F,t}(l) dl \end{aligned} \quad (1)$$

where $D_{H,t}$ denotes home currency bond holdings, $D_{F,t}$ denotes foreign currency bond holdings, \mathcal{E}_t is the nominal exchange rate (the price of the foreign currency in terms of the home currency), W_t is the nominal wage, T_t denotes a lump-sum transfer and Π_t denotes the payout of domestic firms. We explicitly allow for taxes and subsidies on capital flows: τ_t is a tax on capital inflows (or a subsidy on capital outflows) in Home, and similarly τ_t^* is a tax on capital inflows (or a subsidy on capital outflows) in Foreign.¹⁰ The proceeds of these taxes are rebated lump sum to domestic households.

Expenditure minimization leads to a home consumer price index (CPI) defined as $P_t \equiv (P_{H,t})^{1-\alpha} (P_{F,t})^\alpha$, where $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(l)^{1-\epsilon} dl \right]^{\frac{1}{1-\epsilon}}$ is Home's producer price index (PPI) and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(l)^{1-\epsilon} dl \right]^{\frac{1}{1-\epsilon}}$ is Home's price index of imported goods.¹¹ The household's demand for a differentiated good l is given by $C_{j,t}(l) = (P_{j,t}(l)/P_{j,t})^{-\epsilon} C_{j,t}$, for $j = H, F$. The law of one price (LOP) implies $P_{j,t}(l) = \mathcal{E}_t P_{j,t}^*(l)$ for $j = H, F$. At the final good level, it implies

¹⁰A more sophisticated capital flow tax system could feature independent tax rates for inflows and outflows. It would potentially give rise to corner solutions and no-trade equilibria for non-singleton sets of exogenous variables and taxes, thus significantly complicating the analysis. For this reason, we follow the vast majority of the normative literature on capital flow management in assuming that for each country, the tax rate on outflows is constrained to be equal to minus the tax rate on inflows.

¹¹Similarly, $P_t^* \equiv (P_{F,t}^*)^{1-\alpha} (P_{H,t}^*)^\alpha$ is Foreign's CPI, with $P_{F,t}^* \equiv \left[\int_0^1 P_{F,t}^*(l)^{1-\epsilon} dl \right]^{\frac{1}{1-\epsilon}}$ being Foreign's PPI and $P_{H,t}^* \equiv \left[\int_0^1 P_{H,t}^*(l)^{1-\epsilon} dl \right]^{\frac{1}{1-\epsilon}}$ being Foreign's price index of imported goods.

$P_{j,t} = \mathcal{E}_t P_{j,t}^*$ for $j = H, F$. The terms-of-trade between Home and Foreign are defined as the relative price of the foreign index $S_t \equiv P_{F,t}/P_{H,t} = \mathcal{E}_t P_{F,t}^*/P_{H,t}$, while the real exchange rate is defined as the ratio of CPIs: $Q_t \equiv \mathcal{E}_t P_t^*/P_t$.

The home household chooses consumption, labor supply and bond holdings to maximize utility. His optimal labor supply condition is given by $W_t/P_t = N_t^\phi C_t$, and his Euler equation for the home and foreign currency bond holdings are given by

$$\begin{aligned} \frac{\dot{C}_t}{C_t} &= i_t - \pi_t - (\rho + \zeta_t), \\ \frac{\dot{C}_t}{C_t} &= i_t^* + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t - (\rho + \zeta_t). \end{aligned} \tag{2}$$

where $\pi_t \equiv \dot{P}_t/P_t$ is home CPI inflation. The combination of these two Euler equations implies a distorted interest parity condition $i_t = i_t^* + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t}$. Foreign households are symmetric. Their preferences, constraints and optimality conditions are laid out in online Appendix C.2.

2.2 Firms

Technology Firms in Home and Foreign produce differentiated goods $l \in [0, 1]$ with a linear technology: $Y_t(l) = AN_t(l)$, resp. $Y_t^*(l) = A^*N_t(l)$. Without loss of generality and to streamline the notation, we set the level of productivity in both countries to $A = A^* = 1$.

Price setting We assume that the price of each variety is fully rigid, and normalize this price to 1. As a result, the producer price index (PPI) of both countries in their own currencies are fixed at 1. The consumer prices indices (CPI) are thus given by $P_t = S_t^\alpha$ and $P_t^* = S_t^{-\alpha}$. Furthermore, the real exchange rate is related to the terms-of-trade by $Q_t = S_t^{1-2\alpha}$, and the terms-of-trade coincide with the nominal exchange rate: $S_t = \mathcal{E}_t$. The assumption of fully rigid prices can be regarded as an extreme one, but it has the virtue of significantly improving the analytical tractability of the model and making our results transparent.¹²

2.3 Government

Each country's government transfers lump-sum the proceeds from capital flow taxes to the domestic household. The home transfer is thus given by

$$T_t = \tau_t D_{H,t}^* - \tau_t \mathcal{E}_t D_{F,t}.$$

¹²Rigid prices rule out PPI inflation or deflation, but do not eliminate the deflation-recession feedback loop that is a key characteristic of liquidity trap episodes. This is because the relevant measure for that mechanism is CPI inflation rather than PPI inflation, and CPI inflation does respond to fluctuations in the nominal exchange rate. We discuss the consequences of relaxing this rigid price assumption in Section 6.

2.4 Equilibrium

International “risk”-sharing Combining the home and foreign households’ Euler equations for the home bond, it is possible to derive an international consumption smoothing condition relating the ratio of marginal utility in both countries to the real exchange rate¹³

$$C_t = \Theta_t C_t^* Q_t, \quad (3)$$

where $\Theta_t \equiv \Theta_0 \exp \left[\int_0^t (\zeta_s^* - \zeta_s + \tau_s - \tau_s^*) ds \right]$. Θ_0 is a constant related to initial relative wealth positions. Absent preference shocks and capital flow taxes, (3) would indicate a constant ratio of marginal utilities out of nominal income in both countries. Given our logarithmic utility assumption, this translates into the ratio of home expenditure to foreign expenditure being equal to a constant (i.e., $P_t C_t = \mathcal{E}_t P_t^* C_t^*$, as $Q_t = \mathcal{E}_t P_t^* / P_t$). Preference shock and capital flow taxes, however, make this expenditure ratio *time-varying*, following a law of motion given by

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta_t^* - \zeta_t + \tau_t - \tau_t^* \quad (4)$$

Under free capital mobility (i.e., $\tau_t - \tau_t^* = 0$), a scenario where $\zeta_t^* - \zeta_t > 0$ features a relatively more patient home agent who experiences a shrinking trade deficit (or growing trade surplus) and sees its expenditure ratio rise over time: $\dot{\Theta}_t / \Theta_t > 0$. By lending to the foreign agent, the home agent is able to postpone consumption to when it values it relatively more. In this context, the imposition of a (mild) tax on capital inflows by Foreign (i.e., $\tau_t^* > 0$) discourages capital flows from Home into Foreign, leading to smoother trade imbalances and a smoother expenditure ratio (i.e., it makes $\dot{\Theta}_t / \Theta_t$ less positive).

Market clearing In equilibrium, bond markets, goods markets and labor markets all have to clear. Market clearing requires $D_{H,t} + D_{H,t}^* = 0$ for the home currency bond, and $D_{F,t} + D_{F,t}^* = 0$ for the foreign currency bond.¹⁴ Equilibrium in the market for each good l in Home implies that aggregate home output, defined as $Y_t \equiv \left[\int_0^1 Y_t(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$, is given by¹⁵

$$Y_t = (1 - \alpha) S_t^\alpha C_t + \alpha S_t^\alpha Q_t C_t^*. \quad (5)$$

¹³In models featuring uncertainty and complete markets, this condition is often labeled as an international risk-sharing condition. We therefore refer to this condition accordingly, even though risk is absent from our model. (3) is a (potentially distorted) version of what is commonly referred to as the Backus-Smith condition (see Kollmann (1991) and Backus and Smith (1993)) in which Θ_t would represent a Pareto weight in a planning problem. A detailed derivation of (3) is provided in online Appendix C.3.

¹⁴Given the redundancy of one of the two bonds, bond portfolios are indeterminate in equilibrium. However, net foreign asset positions as well as prices and allocations are determinate.

¹⁵For a detailed derivation of the home and foreign aggregate goods market clearing conditions, see online Appendix C.4.

Similarly, market clearing for each good in Foreign requires foreign aggregate output to be given by

$$Y_t^* = (1 - \alpha) S_t^{-\alpha} C_t^* + \alpha S_t^{-\alpha} Q_t^{-1} C_t. \quad (6)$$

Finally, for aggregate employment defined as $N_t \equiv \int_0^1 N_t(l) dl$ and $N_t^* \equiv \int_0^1 N_t^*(l) dl$, equilibrium in the home and foreign labor markets require $N_t = Y_t$ and $N_t^* = Y_t^*$.¹⁶

The above equilibrium conditions can be combined in a way that greatly simplifies the structure of the optimal policy problems we consider in the next sections. Combining the home and foreign aggregate market clearing conditions (5) and (6) with the international “risk”-sharing condition (3) and the equation linking the real exchange rate to the terms-of-trade, $Q_t = S_t^{1-2\alpha}$, yields expressions for home and foreign aggregate consumption:

$$C_t = Y_t^{1-\alpha} (Y_t^*)^\alpha \Theta_t^\alpha (\alpha \Theta_t^{-1} + 1 - \alpha)^{-(1-\alpha)} (\alpha \Theta_t + 1 - \alpha)^{-\alpha}, \quad (7)$$

$$C_t^* = Y_t^\alpha (Y_t^*)^{1-\alpha} \Theta_t^{-\alpha} (\alpha \Theta_t^{-1} + 1 - \alpha)^{-\alpha} (\alpha \Theta_t + 1 - \alpha)^{-(1-\alpha)}, \quad (8)$$

Differentiating these equations with respect to time, and substituting the home Euler equations (2), its foreign analogue, as well as the laws of motion for the consumption ratio (4) yields “Euler equations” in terms of output

$$\frac{\dot{Y}_t}{Y_t} = i_t - (\rho + \zeta_t) - \frac{\alpha}{(1 - \alpha) \Theta_t + \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \quad (9)$$

$$\frac{\dot{Y}_t^*}{Y_t^*} = i_t^* - (\rho + \zeta_t^*) + \frac{\alpha \Theta}{(1 - \alpha) + \alpha \Theta_t} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \quad (10)$$

Lastly, substituting the various equilibrium conditions into the home agent’s budget constraint (1) yields an intertemporal budget constraint written as function of the path of the expenditure ratio

$$b_0 = \alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (\Theta_t - 1) dt, \quad (11)$$

where b_0 is Home’s net foreign assets expressed in terms of the foreign agent’s marginal utility.¹⁷

Equations (7)-(11) summarize the optimal decisions of private agents and can therefore be regarded as implementability conditions. These equations, along with an description of monetary and capital flow management policy, constitute an equilibrium.

Equations (9)-(10) are non-linear dynamic New-Keynesian IS curves that respectively describe the path of output in the home and foreign country. These equations relate output growth to the domestic nominal interest rate, the domestic discount rate and the factors influencing the growth rate of the expenditure ratio. They are among the model’s key equations

¹⁶Price dispersion inefficiencies are absent due to our rigid price assumption.

¹⁷See online Appendix C.5 for a detailed derivation of this constraint and its analogue for Foreign.

and contain important information about the international spillovers at work. Crucially, these equations reveal that home output is independent of foreign monetary policy and vice versa. A foreign monetary expansion stimulates foreign consumption (through a standard inter-temporal substitution channel) and therefore stimulates demand for the home good. At the same time, by generating a home currency appreciation, it switches expenditure (by home and foreign consumers alike) away from the domestic good. As noted by [Corsetti and Pesenti \(2001\)](#) in a related model, under the joint assumption of unitary intra- and inter-temporal elasticity of substitution, these two effects exactly cancel out.¹⁸ However, the equations reveal that demand shocks and capital flow taxes do result in international spillovers. For instance, a negative home demand shock is contractionary for Foreign through its intertemporal substitution effect on home consumption. In addition, a tax on capital inflows by Home induces a current appreciation of the Home currency, and thus is contractionary for Home but expansionary for Foreign through the expenditure switching effect.¹⁹

2.5 Demand shock episode

Our analysis concerns a liquidity trap episode. To this end, we assume that just before date $t = 0$, the world economy is in a symmetric steady state where both countries have zero net foreign asset positions. Next, as is standard practice in the literature, we generate a liquidity trap via a large unanticipated temporary demand shock which we model as a transitory decrease (from date 0 to T) in the rate of time preference of home households. Formally, the rate of time preference at Home is $\rho + \zeta_t$, where ζ_t is given by:

$$\zeta_t = \begin{cases} -\bar{\zeta} & \text{for } t \in [0, T), \\ 0 & \text{for } t \geq T \end{cases} \quad \text{with } \bar{\zeta} > 0$$

The foreign economy is not hit by a demand shock directly and thus, $\zeta_t^* = 0 \forall t \geq 0$. As our analysis of Section 3 will make clear, for large enough $\bar{\zeta}$, replicating the efficient allocation will require negative nominal interest rates in one or both economies up till date T . Therefore, a monetary authority constrained by the ZLB will fail to achieve the efficient allocation, which will result in a situation akin to a liquidity trap.²⁰ In the rest of the paper, we refer to the period between dates 0 and T as the *demand shock episode* in the Home economy.

¹⁸[Benigno and Benigno \(2003\)](#) refer to this case as one where the two economies are “insular.”

¹⁹The mechanism is as follows. From the (distorted) interest parity condition, other things being equal, a positive tax requires an expected appreciation of the home currency vis-à-vis the foreign currency. This expected appreciation of the home currency leads to an expected improvement of the foreign terms-of-trade, and accordingly to an expected deterioration of the home terms-of-trade. This shifts current expenditure toward the home good and away from the foreign good. Note that a positive effect on output growth indicates a contractionary effect, while a negative effect on output growth indicates an expansionary effect.

²⁰We refer the reader to Section 3 for a formal definition of a liquidity trap in our model.

2.6 Socially optimal allocation

A natural way to assess the desirability of decentralized outcomes is to compare these with a socially optimal allocation, which we label *first-best*. The first-best allocation maximizes a symmetrically weighted average of home and foreign agents' utilities subject to worldwide resource constraints and can be described as:²¹

$$N_t^{\text{fb}} = Y_t^{\text{fb}} = [\alpha (\Xi_t)^{-1} + 1 - \alpha]^{\frac{1}{1+\phi}}, \quad \text{and} \quad N_t^{*\text{fb}} = Y_t^{*\text{fb}} = [\alpha \Xi_t + 1 - \alpha]^{\frac{1}{1+\phi}}, \quad (12)$$

and

$$C_t^{\text{fb}} = \Xi_t^\alpha \left[(N_t^{\text{fb}})^{1-\alpha} (N_t^{*\text{fb}})^\alpha \right]^{-\phi}, \quad \text{and} \quad C_t^{*\text{fb}} = \Xi_t^{-\alpha} \left[(N_t^{\text{fb}})^\alpha (N_t^{*\text{fb}})^{1-\alpha} \right]^{-\phi}, \quad (13)$$

where Ξ_t is a time-varying Pareto weight that denotes the relative weight that the social planner assigns to Home at date t . We assume that, at date 0, the planner weighs the discounted lifetime utility in both economies equally, which for the demand shock episode described in Section 2.5, results in a Pareto weight path given by $\Xi_t \equiv \Xi e^{\min\{T, t\}\bar{\zeta}}$ for $\Xi = (\bar{\zeta} - \rho)/(\bar{\zeta}e^{(\bar{\zeta}-\rho)T} - \rho) < 1$.²² In other words, the weight assigned to home agents is initially below one, grows during the demand shock episode (reaching one at $\tilde{T} \equiv -\ln \Xi / \bar{\zeta} < T$) and settles above one from T onwards.

Equations (12)-(13) show that the social planner assigns high employment (i.e. low leisure) and low consumption to Home (resp. low employment and high consumption to Foreign) when Ξ_t is low and, accordingly, low employment (i.e. high leisure) and high consumption to Home (resp. high employment and low consumption to Foreign) when Ξ_t is high. The paths of these variables are depicted graphically in the panels (a) and (b) of Figure 1.

The path of the planner's shadow values also offer a useful benchmark against which to contrast decentralized outcomes. The *shadow terms-of-trade*, defined as the ratio of the planner's shadow values of the foreign good to the home good, is given by

$$\vartheta_t = \frac{Y_t^{\text{fb}}}{Y_t^{*\text{fb}}} \times \frac{\alpha \Xi_t + 1 - \alpha}{\alpha + (1 - \alpha) \Xi_t}. \quad (14)$$

The first term of this expression reflects relative scarcity considerations (i.e., supply factors), while the second term accounts for preference asymmetries (i.e., demand factors). Both elements work in favor of a higher relative valuation of the foreign good initially, and a lower valuation of it later on, as displayed in panel (c) of Figure 1.²³

²¹See Appendix A.1 for a formal description of the planning problem yielding this allocation. The problem is written for an arbitrary Pareto weight but our analysis focuses on a symmetric weight.

²² Ξ is the Pareto weight assigned by the planner to Home at date 0. Due to differences in discounting, the weight giving both countries equal importance, which we refer to as the *symmetric weight*, is given by $\Xi = \int_0^\infty e^{-\int_0^s (\rho + \zeta_s^*) ds} dt / \int_0^\infty e^{-\int_0^s (\rho + \zeta_s) ds} dt$.

²³This is true in the presence of home bias. Without home bias (i.e., when $\alpha = 0.5$), preferences for con-

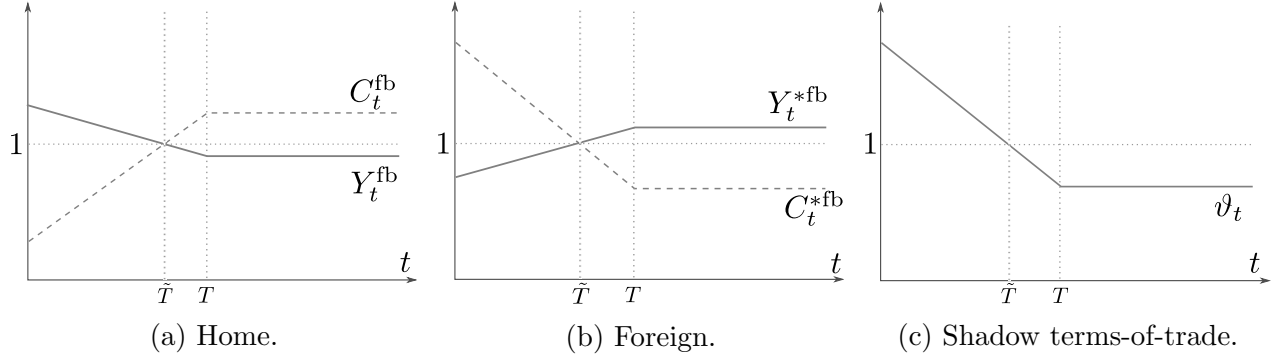


Figure 1: First-best.

In the rest of the paper, we refer to the first-best level of output as *potential output* and to the deviation of actual output from potential output as the *output gap*. Our analysis revolves around the costs imposed by the ZLB under alternative capital flow regimes. These costs can be summarized by three wedges between the decentralized and first-best allocations: the Home labor wedge, $\omega_t \equiv -\ln(MRS_t/MPL_t)$, the Foreign labor wedge, $\omega_t^* \equiv -\ln(MRS_t^*/MPL_t^*)$, and the international “risk-sharing” wedge (which we abbreviate as the international wedge), $\varpi_t \equiv \ln(\Theta_t) - \ln(\Xi_t)$.²⁴ The following lemma relates the labor wedges to home output, foreign output and the expenditure ratio Θ_t .²⁵

Lemma 1 (Labor wedges). *In equilibrium, the labor wedges are given by*

$$\omega_t = -\ln\left(\frac{(Y_t)^{1+\phi}}{\alpha\Theta_t^{-1} + 1 - \alpha}\right), \quad \omega_t^* = -\ln\left(\frac{(Y_t^*)^{1+\phi}}{\alpha\Theta_t + 1 - \alpha}\right). \quad (15)$$

Proof. See Appendix B.1. □

3 Positive analysis

In order to characterize decentralized outcomes, we need to specify how policy is conducted. Since our interest lies in assessing the performance of alternative capital flow management regimes, we find it convenient to assume that monetary policy is always set optimally by a global monetary authority.²⁶ Under this assumption, our goal in this section is to shed light on

sumption goods are symmetric and the second term is always equal to 1. In this case, shadow terms-of-trade movements only reflects the relative scarcity of the two goods.

²⁴By definitions, these three wedges are zero in the first-best.

²⁵We will occasionally refer to a period with a negative labor wedge as a recession and to a period with a positive labor wedge as a boom.

²⁶This amounts to assuming that it is set cooperatively, and allows us to abstract from any inefficiencies arising from non-cooperative or other suboptimal monetary policy setting. Note that cooperative and non-cooperative monetary policy outcomes differ despite our adopted Cole-Obstfeld parametrization due to level (i.e., steady

the role played by capital flows in a liquidity trap, by comparing the world economy's adjustment to the demand shock episode described in Section 2.5 under two stylized capital flow regimes: free capital mobility and closed capital accounts.

3.1 Optimal monetary policy with free capital mobility

We assume that a benevolent global monetary authority operates under commitment and specifies the path of nominal interest rates i_t for Home and i_t^* for Foreign in order to maximize a symmetrically weighted sum of welfare in both countries.²⁷ Importantly, the monetary authority is constrained to set non-negative nominal interest rates in each country at all times.²⁸ While Appendix A.2 describes and characterizes the optimal monetary policy problem for any capital flow regime, in what follows, we describe the optimal policy problem under a regime of free capital mobility. The problem is given by

$$\max_{i_t \geq 0, i_t^* \geq 0, C_t, C_t^*, Y_t, Y_t^*} \int_0^\infty e^{-\int_0^t (\rho + \zeta_h^*) dh} \left\{ \Xi_t \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] \right\} dt$$

subject to the implementability constraints (7), (8), (9) and (10) with $\tau_t - \tau_t^* = 0$ and thus, $\Theta_t = \Xi_t$ for all t . The following lemma sets the stage by characterizing the optimal policy away from (or absent) the ZLB.

Lemma 2 (Unconstrained optimal monetary policy). *Absent the ZLB, optimal monetary policy implements the first-best allocation by choosing an initial exchange rate of $\mathcal{E}_0 = \vartheta_0$ and an interest rate path given by:*

$$\mathcal{I}_t = \rho + \frac{(1-\alpha)\Xi_t + \frac{\alpha}{1+\phi}}{(1-\alpha)\Xi_t + \alpha} \zeta_t \quad \text{and} \quad \mathcal{I}_t^* = \rho + \frac{\alpha\phi}{1+\phi} \frac{\Xi_t}{\alpha\Xi_t + (1-\alpha)} \zeta_t, \quad (16)$$

implying $\text{sign}(\mathcal{I}_t^* - \mathcal{I}_t) = -\text{sign}(\zeta_t)$ and an exchange rate path of $\mathcal{E}_t = \vartheta_t$.

Proof. See Appendix B.2. □

Intuitively, optimal policy responds to the demand shock episode described in Section 2.5 by lowering the nominal interest rates in both countries, but more so in Home. Owing to the interest parity condition, the resulting interest rate differential is accompanied by a continuous appreciation of the home currency during the episode ($\dot{\mathcal{E}}_t/\mathcal{E}_t < 0$ for $0 \leq t < T$), following depreciation on impact (i.e., at $t = 0$, \mathcal{E}_0 jumps up from 1). The resulting terms-of-trade path

state) effects related to countries' market power that cannot be undone via labor subsidies since firms do not set prices.

²⁷The symmetric Pareto weight is given explicitly in Section 2.6 (see in particular footnote 22).

²⁸We are agnostic about whether such ZLB constraints exist because the monetary authority is truly unable to set negative rates or because it has imposed such a constraint on itself voluntarily.

coincides with that of the shadow terms-of-trade in the first-best: home goods are relatively cheaper initially (from 0 to \tilde{T}) and then more expensive (from \tilde{T} to T). Since output is demand determined, this relative price path naturally allows the monetary authority to achieve the first-best allocation.

Home's trade balance is given in terms of the home good by

$$TB_t = \alpha(1 - \Theta_t)C_t^* \mathcal{E}_t^{1-\alpha}. \quad (17)$$

For $\Theta_t = \Xi_t$, it is positive initially (from 0 to \tilde{T}), and then negative (from \tilde{T} onwards). After the shock has dissipated (i.e., after T), Home runs a permanent trade deficit financed by the foreign assets accumulated during the episode. Hence, trade imbalances and capital flows are a key part of the adjustment allowing the first-best to be achieved. They allow temporarily more patient home agents to reduce both consumption and leisure simultaneously, and catch up later with accordingly higher consumption and leisure.

However, the unconstrained policy described in Lemma 2 is not always implementable. That is the case when one of the interest rate expressions in (16) results in a negative nominal rate. We refer to such a situation as a *liquidity trap* in the country in question. For the rest of the paper, we focus on a situation where the home demand shock is large enough to make Home experience a liquidity trap, yet small enough not to make Foreign experience one. We thus make the following assumption:

Assumption 1 (Liquidity trap in Home only). *The demand shock size $\bar{\zeta}$ satisfies:²⁹*

$$\rho + \frac{\alpha\rho}{(1-\alpha)(\bar{\zeta}-\rho)}e^{\bar{\zeta}T} \left[\bar{\zeta}e^{(\bar{\zeta}-\rho)T} - \rho \right] < \bar{\zeta} < \rho + \frac{(1-\alpha)\rho}{\alpha(\bar{\zeta}-\rho)}e^{-\bar{\zeta}T} \left[\bar{\zeta}e^{(\bar{\zeta}-\rho)T} - \rho \right]. \quad (18)$$

Under these circumstances, the optimal policy is described by the following lemma.

Lemma 3 (Optimal monetary policy at the ZLB). *Consider a demand shock scenario for which Assumption 1 holds. Then the optimal policy is characterized as follows:*

1. *In Home, the ZLB binds, the interest rate path is described by $i_t = 0$ for $t \in [0, \hat{T})$ and $i_t = \mathcal{I}_t = \rho$ for $t \geq \hat{T}$, while the output path and optimal ZLB exit time $\hat{T} > T$ are jointly determined by:*

$$0 = \int_0^{\hat{T}} e^{-\int_0^t (\rho + \zeta_s) ds} \left[(Y_t^{fb})^{1+\phi} - (Y_t)^{1+\phi} \right] dt \quad (19)$$

²⁹Notice that in the limiting case of extreme home bias ($\alpha \rightarrow 0$), this condition trivially reduces to $\rho < \bar{\zeta}$ -the closed economy condition under which the natural rate becomes negative. Thus, a small α is enough to ensure that the condition (18) is satisfied if $\rho < \bar{\zeta}$. A necessary condition for the parameter set satisfying condition (18) to be non-empty is $\bar{\zeta}T < \ln\left(\frac{1-\alpha}{\alpha}\right)$ (remember that home bias requires $\alpha < 1/2$). Loosely speaking, for a given duration of the liquidity trap \hat{T} the shock $\bar{\zeta}$ cannot be too large, or equivalently, for a given shock $\bar{\zeta}$, the duration of the trap T cannot be too long.

and the differential equation (9) with terminal condition $Y_{\hat{T}} = Y_T^{fb}$. Furthermore, the output gap is negative on impact: $Y_0 < Y_0^{fb}$.

2. In Foreign, the ZLB does not bind, the interest rate path is described by $i_t^* = \mathcal{I}_t^*$, and output is at its first best level $Y_t^* = Y_t^{*fb}$ at all time.

3. The initial exchange rate is given by $\mathcal{E}_0 = \frac{Y_0}{Y_0^{*fb}} \times \frac{\alpha\Xi_0 + 1 - \alpha}{(1 - \alpha)\Xi_0 + \alpha} < \vartheta_0$.

Proof. See Appendix B.3. □

Some aspects of the optimal policy outcome common to closed economy frameworks are worth mentioning. First, it is optimal for the monetary authority to make a commitment to keep the home interest rate at zero even after the demand shock scenario has ended at date T . This commitment, known to be a feature of optimal monetary policy at the ZLB and often referred to as “forward guidance,” generates a boom in demand after the end of the liquidity trap. This future boom in turn dampens the initial decline of output via the intertemporal channel. Second, the ZLB exit time \hat{T} is precisely chosen so as to minimize (weighted) average deviations from the first-best output path. We will refer to the period from 0 to T as phase I of the liquidity trap, and to the period from T to \hat{T} as phase II. The ZLB therefore implies that home output falls short of its first-best level on impact, grows continuously during phase I, overshooting its first-best level late during that phase, before reverting back to it by the end of phase II, as shown in the left panel of Figure 2 (dark solid line). This characterization of optimal policy is reminiscent of earlier results in the closed economy ZLB literature (e.g., Eggertsson and Woodford, 2003 and Werning, 2012).

Owing to our unitary elasticities assumptions, each country’s interest rate is set only with regard to its own output path. Our model therefore abstracts from the monetary policy interdependence and spillovers that have been the focus of most of the open economy literature on the ZLB (e.g., Haberis and Lipinska, 2012 and Fujiwara et al., 2013). This feature delivers a sharp characterization of exchange rate dynamics at the ZLB. Under a regime of free capital mobility, the exchange rate path is tightly linked interest rate differentials through the interest parity condition. Relative to what would prevail absent the ZLB, the differential is smaller during phase I and larger during phase II. As a result, the home currency appreciates too slowly during phase I and too fast during phase II. Lemma 3 indicates that as a result, the home exchange rate does not depreciate sufficiently on impact. This is shown in the right panel of Figure 2 (dark solid line).

The above characterization indicates that early in phase I, in addition to being too expensive relative to the future home good (a notion familiar from the closed economy analysis), the current home good is also too expensive relative to the current foreign good. Thus, in analogy with the mechanism by which output has to drop on impact to make consumers content with their savings

choice when facing an excessively high interest rate in a liquidity trap, here home output has to drop relative to foreign output to make consumers content with their intra-temporal expenditure allocation decision when facing an excessively appreciated home currency.

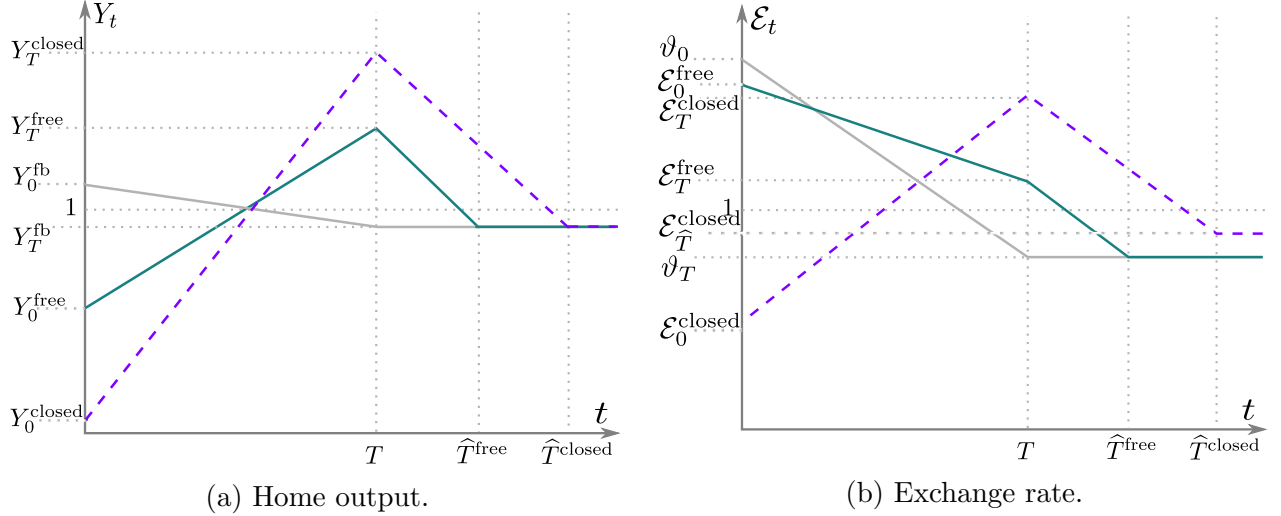


Figure 2: Home output and exchange rate paths under ZLB with free capital mobility (solid dark), ZLB with closed capital accounts (dashed dark) and unconstrained policy with free capital mobility (solid light).

Regarding consumption dynamics, it is easy to establish that home consumption falls on impact, tilts up during phase I and tilts down during phase II. Foreign consumption, on the other hand, jumps up on impact, before tilting down during phase I and II. The paths of the main model variables are shown for illustration purposes in Figure 3 (dark solid line).³⁰

The distortions caused by the ZLB constraint can be summarized by the three wedges defined in Section 2.6. Under a regime of free capital mobility, a binding ZLB constraint in Home translates into an opening of the home labor wedge, but not of the foreign labor wedge and the international wedges. More precisely, the home labor wedge path mirrors the output gap path. It jumps up on impact, decreases during phase I, and increases during phase II. This corroborates the finding that the ZLB causes a recession-boom cycle in Home, and suggests that this cycle is the key source of efficiency losses relative to the unconstrained policy outcome when capital flows freely across countries.

³⁰ The parametrization used to generate the figure relies on standard values from the literature. We set the discount rate to $\rho = 0.04$, the openness parameter to $\alpha = 0.2$, and the inverse Frisch elasticity of labor supply to $\phi = 3$. For parameters pertaining to our demand shock trap scenario, we follow Werning (2012). The duration of the shock is set to $T = 2$ years, and the size of the demand shock is set to $\bar{\zeta} = 2\rho$. In a closed economy benchmark, such a shock size would result in a natural real interest rate of -4% for the duration of the liquidity trap. These parameter values satisfy Assumption 1. Unless noted otherwise, they are used for all our subsequent figures.

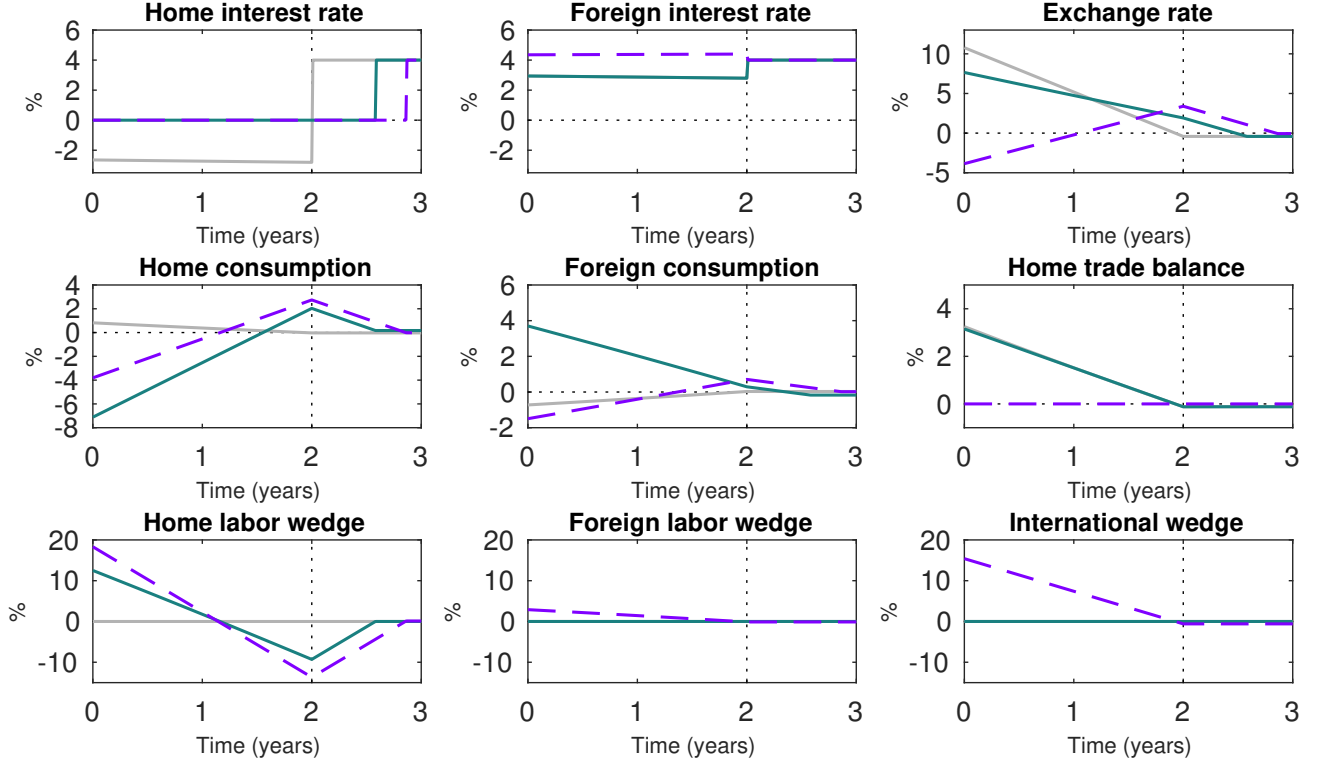


Figure 3: Variable paths under ZLB with free capital mobility (solid dark), ZLB with closed capital accounts (dashed dark) and unconstrained policy with free capital mobility (solid light).

3.2 Capital flows at the ZLB

To shed light on the role played by international capital flows in a liquidity trap, we conduct the experiment of shutting down capital accounts and contrast the resulting allocations and prices to those obtained under a regime of free capital mobility. In doing so, we allow for intra-temporal trade but require it to be balanced period by period. The price implementation of shutting down international capital flows entails setting a tax wedge of $\tau_t - \tau_t^* = \zeta_t$.³¹ This implies that the expenditure ratio is fixed at 1. The optimal monetary policy problem under closed capital accounts is thus isomorphic to the case of free capital mobility, but with $\tau_t - \tau_t^* = \zeta_t$ and $\Theta_t = 1 \forall t \geq 0$. In these circumstances, the IS equations (9) and (10) are given by their closed economy counterparts and the unconstrained interest rate expressions in (16) become³²

$$\mathcal{I}_t^{\text{closed}} = \rho + \left(1 + \frac{\alpha}{1 + \phi} \frac{\Xi_t^{-1}}{1 - \alpha + \alpha \Xi_t^{-1}}\right) \zeta_t \quad \text{and} \quad \mathcal{I}_t^{*\text{closed}} = \rho - \frac{\alpha}{1 + \phi} \frac{\Xi_t}{\alpha \Xi_t + 1 - \alpha} \zeta_t. \quad (20)$$

The following lemma adapts Lemma 3's description of optimal monetary policy at the ZLB to the case of closed capital accounts.

³¹More generally, it entails setting $\tau_t - \tau_t^* = \zeta_t - \zeta_t^*$ in order to achieve $\dot{\Theta}_t/\Theta_t = 0$ according to (4).

³²See Appendix A.2 for an analysis of optimal monetary policy for an arbitrary capital flow regime.

Lemma 4 (Optimal monetary policy at the ZLB with closed capital accounts). *Consider a liquidity trap scenario for which Assumption 1 holds. Then the optimal policy outcome is isomorphic to that of Lemma 3, with the following modifications: (i) the foreign interest rate is given in (20), and (ii) the initial exchange rate is given by $\mathcal{E}_0^{\text{closed}} = Y_0^{\text{closed}}/Y_0^{*fb}$.*

Proof. The proof of Lemma 3 applies. \square

The unconstrained interest rate expressions in (20) indicate that in our demand shock scenario, the following inequalities hold for $t \in [0, T)$: $\mathcal{I}_t^{\text{closed}} < \mathcal{I}_t^{\text{free}} < 0$ and $\mathcal{I}_t^{*\text{closed}} > \mathcal{I}_t^{*\text{free}} > 0$. In other words, closing capital accounts makes Home experience a deeper liquidity trap, while pushing Foreign further away from experiencing one. We will now argue that closing capital accounts hampers the adjustment process in a number of additional dimensions.

Proposition 1 (Capital flows in a liquidity trap). *Relative to the free capital mobility regime, a regime of closed capital accounts results in*

1. *a further delay of the optimal ZLB exit time ($\hat{T}^{\text{closed}} > \hat{T}^{\text{free}}$),*
2. *a more variable path of home output and output gap.*

Proof. See Appendix B.5. \square

This comparison is illustrated in Figure 2, where the free capital mobility and closed capital accounts regime are respectively represented by a dark solid line and a dark dashed line. The proposition indicates that the adjustment process happening in Home takes longer *and* is features larger inefficient output fluctuations when capital flows are constrained. We interpret this as evidence that capital flows play a fundamentally smoothing role in a liquidity trap. In what follows, we describe the paths of other key macro variables under closed capital accounts to shed light on the mechanisms behind this result.

Our first observation is that under a closed capital account, exchange rate movements lose their stabilizing role during the liquidity trap; rather than facilitating expenditure switching in favor of the home good early on, they hamper it. Despite a positive interest rate differential between Foreign and Home – in fact, a larger one than under free capital mobility – distorted interest parity does not require the home currency to continuously appreciate during phase I. Instead, it continuously depreciates during phase I ($\dot{\mathcal{E}}_t/\mathcal{E}_t = -\mathcal{I}_t^{*\text{closed}} + \tau_t^* - \tau_t > 0$ for $t \in [0, T)$), and may even appreciate (rather than depreciate) on impact if the drop in Home output is severe enough.³³ Hence, the exchange rate “moves the wrong way” from the perspective of stabilizing expenditure reallocation during the liquidity trap. Under free capital mobility, a continuous appreciation of the home currency during phase I contributed to encourage expenditure switching

³³To establish the depreciation during phase I, note that for $t \in [0, T)$, $\zeta_t = -\bar{\zeta}$ so that $-\mathcal{I}_t^{*\text{closed}} + \tau_t^* - \tau_t = \frac{\frac{\phi}{1+\phi}\alpha\Xi_t + (1-\alpha)}{\alpha\Xi_t + (1-\alpha)}\bar{\zeta} - \rho > \frac{(1-\alpha)}{\alpha\Xi_t + (1-\alpha)}\bar{\zeta} - \rho > 0$, where the last inequality follows from Assumption 1.

in favor of the home good in the early stage of the trap, precisely at the time when the home good was the most under-provided. Without capital flows, in contrast, a continuous depreciation of the home currency during phase I diverts expenditure away from the home good when its provision is the most depressed.

Our second observation is that the absence of capital flows prevents the decoupling of consumption from output which is characteristic of the first-best allocation. Under free capital mobility, trade surpluses in the early stage of phase I allowed Home to satisfy its desire to shift consumption forward, while letting current demand for its good be supported by foreign consumers. This channel is unavailable when capital is not able to flow. Instead of experiencing an initial consumption boom, Foreign experiences a consumption bust on impact.³⁴ The contrast in the exchange rate paths between both regimes can be further interpreted from the perspective of the trade balance. Under free capital mobility, Foreign was initially running a trade deficit. Thus, for trade to be balanced under closed capital accounts, the relative price of home goods must initially increase to reduce Foreign's incentives to import them.

The paths of the main model variables are contrasted with their free capital mobility counterparts in Figure 3 (dark dashed line). The figure shows an opening of the foreign labor wedge and of the international wedge, along with a more variable home labor wedge than under the free capital mobility regime.

4 Efficient capital flows

The positive analysis of the preceding section emphasized the stabilizing role played by capital mobility in a scenario where a region of the world economy experiences a liquidity trap. Its concluding paragraph hinted at the additional distortions caused by impediments to cross-border capital flows in such an episode. In this section, we adopt a normative perspective and analyze the constrained efficiency properties of the free capital mobility regime.

4.1 Constrained planning problem

We frame this constrained efficiency question by formulating a Ramsey planning problem. We endow the global planner with the ability to tax or subsidize international financial transactions, in addition to its ability to set monetary policy under the zero bound constraint. The planning

³⁴The behavior of home consumption on impact relative to the free capital flow regime depends on two counteracting forces. On the one hand, the lack of savings opportunity pushes consumption up on impact. On the other hand, the fact that Home output is more depressed pushes consumption down.

problem is given by

$$\max_{\{i_t \geq 0, i_t^* \geq 0, \tau_t, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_h^*) dh} \left\{ \Xi_t \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] \right\} dt$$

subject to (4), (7), (8), (9) and (10), with Θ_t only allowed to jump at $t = 0$. τ_t is the tax on capital inflows into (or subsidy on outflows out of) Home. Without loss of generality, we can assume that the planner sets the foreign capital flow tax τ_t^* to zero.³⁵ (4) and (7)-(10) are implementability conditions, while $i_t \geq 0$ and $i_t^* \geq 0$ are zero bound constraints.

4.2 Characterization of efficient regime

Framing the efficiency question via the above Ramsey problem has several advantages. First, we can evaluate the constrained efficiency of the free capital mobility regime by asking a very simple question, namely: *Is the planner's optimal choice characterized by $\tau_t = 0$, $\forall t$?* Second, anticipating a negative answer to our first question, we can learn about the direction of the inefficiency by analyzing the sign of the optimal capital flow tax. The following lemma provides a characterization of the efficient capital flow regime.

Lemma 5 (Targeting rule in efficient capital flow regime). *The constrained efficient capital flow regime is characterized by the targeting rule*

$$1 - e^{-\omega_t - \varpi_t} = \Xi_t (1 - e^{-\omega_t^* + \varpi_t}). \quad (21)$$

Proof. See part of Appendix A.3 leading up to equation (A.19). \square

As is standard with targeting rules in New Keynesian models (see, e.g., [Woodford, 2003](#), [Gali, 2015](#)), this rule does not directly describe what optimal policy should be, but rather what it should target. It indicates that the planner aims for a balance between the distortions experienced by Home (left-hand-side of (21)) and the ones experienced by Foreign (right-hand-side of (21)), with a weight reflecting the time-varying Pareto weight Ξ_t . Absent (or away from) the ZLB, all three wedges, i.e., the home labor wedge ω_t , the foreign labor wedge ω_t^* and the international wedge ϖ_t , are zero. As a result, intervening in international financial markets is undesirable in that case as $\tau_t = \varpi_t = 0$ and the free capital mobility regime is constrained efficient.³⁶ This is no longer true when the ZLB binds for at least one country. When the ZLB

³⁵We follow the literature in normative open-economy macroeconomics in assuming that the planner has access to a date 0 transfer across the two countries. This assumption allows us to drop the country resource constraint (11) from the planning problem and makes the tax differential $\tau_t - \tau_t^*$ (rather the individual taxes τ_t, τ_t^*) the only relevant instrument. Normalizing $\tau_t^* = 0$ is thus without loss of generality.

³⁶It actually turns out to be simply efficient (i.e., without the “constrained” qualification), since the first-best allocation is achieved in this case.

binds in Home but not in Foreign, free capital mobility is constrained inefficient, for it would imply an opening of the home labor wedge only, a contradiction with the targeting rule (21).³⁷ This result, together with a characterization of the constrained efficient regime, constitute our main normative contribution.

To gain insights into the logic of the optimal policy intervention, it is useful to resort a first-order Taylor approximation of some key equations around the symmetric steady state.³⁸ First, linearizing the targeting rule (21) yields

$$\omega_t + \varpi_t = \omega_t^* - \varpi_t. \quad (22)$$

Hence, faced with asymmetric labor wedges across countries, the planner wants to distort consumption allocations in favor of the country with the smallest labor wedge, i.e., experiencing the less severe recession (or the larger boom). Second, linearizing the labor wedge expressions in (15) and writing them in gaps relative to the efficient (or flexible price) allocation delivers:

$$\omega_t = -\alpha\varpi_t - (1 + \phi)\tilde{y}_t, \quad \text{and} \quad \omega_t^* = \alpha\varpi_t - (1 + \phi)\tilde{y}_t^* \quad (23)$$

for the output gaps $\tilde{y}_t \equiv \hat{Y}_t - \hat{Y}_t^{fb}$, $\tilde{y}_t^* \equiv \hat{Y}_t^* - \hat{Y}_t^{*fb}$, where $\hat{\Xi}_t \equiv \ln \Xi_t$, $\hat{Y}_t \equiv \ln(Y_t/Y)$, $\hat{Y}_t^* \equiv \ln(Y_t^*/Y^*)$ and $\hat{Y}_t^{fb} = -\alpha\hat{\Xi}_t/(1 + \phi)$, $\hat{Y}_t^{*fb} = \alpha\hat{\Xi}_t/(1 + \phi)$.³⁹ In a scenario of interest where the ZLB does not bind in Foreign, the foreign output gap is zero at all times, $\tilde{y}_t^* = 0$, and the foreign labor wedge is therefore proportional to the international wedge: $\omega_t^* = \alpha\varpi_t$. Taking a first-order Taylor approximation of the IS curves (9)-(10) and expressing these in gaps relative to the efficient allocation yields

$$\dot{\tilde{y}}_t = i_t - r_t^n - \alpha(\tau_t - \tau_t^*), \quad \text{and} \quad \dot{\tilde{y}}_t^* = i_t^* - r_t^{*n} + \alpha(\tau_t - \tau_t^*) \quad (24)$$

where $r_t^n \equiv \rho + \left(1 - \alpha + \frac{\alpha}{1+\phi}\right)\zeta_t$ and $r_t^{*n} \equiv \rho + \left(\alpha - \frac{\alpha}{1+\phi}\right)\zeta_t$ are the home and foreign natural real interest rate, respectively. Differentiating the two equations in (23) and the targeting rule (22) with respect to time, and combining them with the IS curves in (24) when Home is at the

³⁷This logic generically holds in the case where the ZLB binds in both countries. We briefly discuss this case at the end of this section.

³⁸This symmetric steady state is described in Appendix A.3.1.

³⁹Thus, absent the international wedge, the labor wedges are simply proportional to the negative output gaps, as in the standard closed economy model (coinciding with the limit where $\alpha \rightarrow 0$). However, for given output gaps, a positive international wedge is associated with a smaller (i.e., more negative) home labor wedge and a larger (i.e., more positive) foreign labor wedge. The intuition is that for given output levels, a larger international wedge translates into higher home consumption, and therefore lower home marginal utility and a higher home marginal rate of substitution (MRS) of consumption for leisure, thus reducing the labor wedge. Likewise, in Foreign, a larger international wedge translates into lower consumption, higher marginal utility, a lower MRS, and thus a higher labor wedge.

ZLB but Foreign is not, yields:

$$\tau_t - \tau_t^* = -\Psi r_t^n, \quad i_t^* = r_t^{n*} + \alpha \Psi r_t^n, \quad \dot{e}_t = -r_t^{n*} + (1 - \alpha) \Psi r_t^n, \quad \text{and} \quad \dot{y}_t = -(1 - \alpha \Psi) r_t^n \quad (25)$$

where $e_t \equiv \ln(\mathcal{E}_t)$, for $\Psi \equiv (1 + \phi)/[2 + \alpha(\phi - 1)] > 0$.⁴⁰ This compares with $\tau_t - \tau_t^* = 0$, $i_t^* = r_t^{n*}$, $\dot{e}_t = -r_t^{n*}$ and $\dot{y}_t = -r_t^n$ in the free capital mobility regime. Meanwhile, in either regime, home monetary policy is set such that the home output gap averages out to zero over time: (19) (which applies irrespective of the capital flow regime) is given in linearized form by $\int_0^{\hat{T}} e^{-\rho t} \tilde{y}_t dt = 0$. (22) and (23) then show that in the constrained efficient regime, the home labor wedge, foreign labor wedge and international wedge must also average out to zero over time. Given these properties, one can think about the degree of distortions associated with a regime as being related to the slope of these variables: smoother/less variable gaps or wedges (as represented by smaller growth rates in absolute values) indicate smaller distortions.⁴¹ With this in mind, we can summarize our main normative results in the following proposition.

Proposition 2 (Constrained efficient capital flow regime). *The free capital mobility regime is constrained efficient if and only if the ZLB constraints never bind. Furthermore, when the ZLB binds in Home but not in Foreign from 0 to T, up to a first-order, the constrained efficient regime compares with the free capital mobility regime as follows:*

1. *Capital flows out of Home are subsidized in phase I and taxed in phase II.*
2. *The home output gap is smoother, while the foreign output gap is still zero.*
3. *The home labor wedge is smoothed out at the expense of the foreign labor wedge and international wedge, which both open.*
4. *The home exchange rate appreciates at a faster rate in phase I and at a slower rate in phase II.*
5. *Monetary policy in Foreign is more expansionary in phase I and less expansionary in phase II.*

Proof. See argument in text. □

Point 1. reflects the expression for τ_t in (25) according to which the optimal tax wedge on flows from Home to Foreign is proportional to the negative of the home natural rate during the time spent by Home at the ZLB. Since the home interest rate is zero at the ZLB, its negative represents the gap between the home interest rate and its ideal level. This suggests that the

⁴⁰Note that $\Psi \leq 1$ and $\partial \Psi / \partial \alpha \geq 0$ for $\phi \leq 1$. Furthermore, $0 < \alpha \Psi < 1$.

⁴¹This representation admittedly abstracts from the fact that the ZLB exit time differs across capital flow regimes.

capital flow tax is used as a substitute for deficient home monetary policy, but only to the extent that the policy deviates from its frictionless target.⁴² Since $r_t^n < 0$ during phase I and $r_t^n = \rho > 0$ during phase II, the optimal policy consists in subsidizing flows out of Home during phase I, and taxing such flows during phase II.⁴³ The intervention yields a smoother real adjustment in Home, as evidenced by a reduction in the growth rates of the home output gap and home labor wedges (points 2. and 3.).⁴⁴ This improvement is achieved at the expense of a mild destabilization of the foreign labor wedge and international wedge. Thus, from an optimal taxation perspective, the planner's intervention in the efficient capital flow regime can be seen as reflecting wedge management: it is desirable to strike a balance between fluctuations in the model's three wedges so as to satisfy (21). Notice that under free capital mobility, (21) is violated since only the home labor wedge was open, while the foreign labor wedge and the international wedge were zero at all times.

How does the described path of capital flow tax result in a smoother adjustment in Home? The intuition has to do with the tilting of the exchange rate path induced by the optimal tax (points 4.). Through the distorted interest parity condition, a positive τ_t increases the required rate of appreciation of the home currency in phase I, while a negative τ_t decreases it in phase II. This tilting of the exchange rate path stabilizes demand for the home good, by switching expenditure in its favor in the early stage of phase I, and at its expense in the late stage of phase I and in phase II. Accordingly, and since expenditure switching occurs vis-à-vis the foreign good, the capital flow tax is contractionary in Foreign when it is expansionary in Home and vice-versa (see IS curves in (24)). And as the ZLB on the foreign nominal rate is not binding, foreign monetary policy is optimally adjusted to a more expansionary (contractionary) stance in phase I (II) so as to align foreign output with its natural level. Thus, in analogy to the manner in which delaying exit from the ZLB in a closed economy allows borrowing monetary policy room from the future, constrained efficient capital flow management can be interpreted as enabling a transfer of monetary policy room across regions.

This stabilizing effect of efficient capital flow management is depicted in Figure 4, which contrasts the paths of key variables in the efficient regime with their counterparts in the free capital mobility regime and unconstrained benchmark.⁴⁵ It is evident that the capital account intervention results in a steeper exchange rate (and hence terms-of-trade) path, a smoother home output gap, more pronounced consumption fluctuations and larger current account imbalances. Accordingly, the home labor wedge is stabilized at the expense of the opening of the foreign

⁴² "Frictionless" here refers to the absence of a ZLB constraint.

⁴³ For $t \geq \hat{T}$, expression (25) does not hold any more, since the home output gap is back to zero. As a result, and consistently with the targeting rule (21), all wedges are zero and so is the optimal capital flow tax.

⁴⁴ The home labor wedge growth rate is given by $\dot{\omega}_t = \frac{2(1-\alpha)+\alpha}{2(1-\alpha)+\alpha(1+\phi)} (1+\phi) r_t^n$ in the efficient regime, while it was given by $\dot{\omega}_t = (1+\phi) r_t^n$ in the free capital mobility regime.

⁴⁵ The picture is produced using an exact non-linear solution to the planning problem, using the parametrization described in Footnote 30.

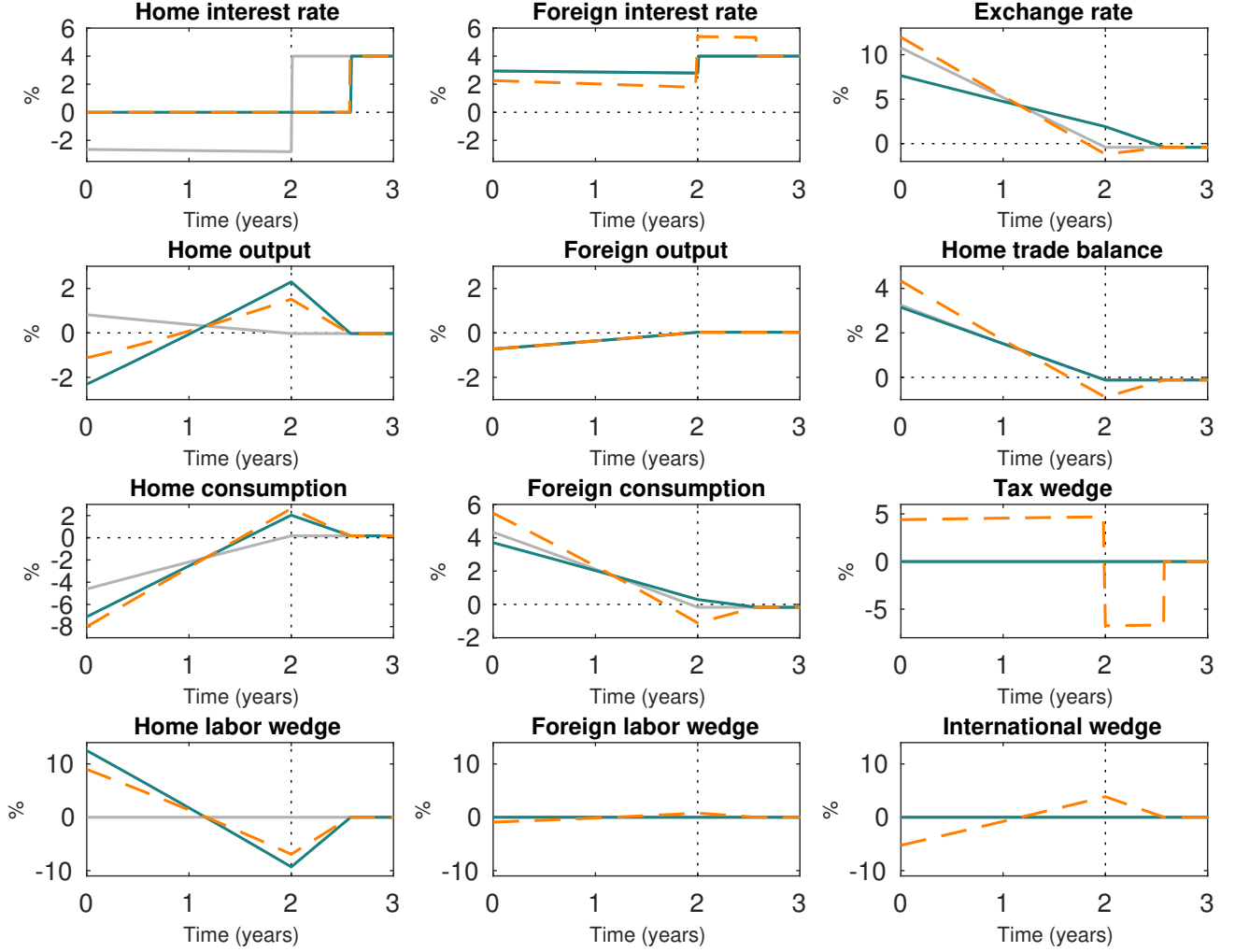


Figure 4: Variable paths under ZLB with free capital mobility (solid dark), ZLB with efficient regime (dashed) and unconstrained policy (solid light).

labor wedge and the international wedge.

The underlying rationale for the constrained inefficiency of the free capital mobility regime is an aggregate demand externality generically present in economies with nominal rigidities where constraints on monetary policy make the socially optimal allocation unattainable (see [Blanchard and Kiyotaki \(1987\)](#) for an early discussion focusing on pricing decisions, and more recently, [Farhi and Werning \(2016\)](#) for a general treatment regarding financial choices). With the nominal rigidities present in our model, prices fail to fulfill their allocative role, and the fall in demand associated with agents' increased desire to save pushes the economy into a recession. The monetary authority internalizes these effects, attempts to nullify them by affecting intertemporal prices, and is successful at correcting the externality with monetary policy alone absent (or away from) the ZLB. But at the ZLB, it cannot lower the nominal rate sufficiently, and distorting international savings decision leads to exchange rate movements that help curtail the severity

of the bust-booms cycle in Home.

While our constrained inefficiency result might appear to fall under the umbrella of the general theory put forward by [Farhi and Werning \(2016\)](#), the multiple-currency model structure called for by our particular application makes the mechanics of the intervention, and thus our contribution, distinct. Rather than aiming to simply direct purchasing power toward agents with the highest marginal propensity to consume (MPC) on relatively more depressed goods, the optimal intervention in our model is guided by a desire to switch expenditure toward more depressed goods by manipulating the only flexible component of relative prices, namely the exchange rate. In fact, the resulting policy prescription is at odds with Farhi and Werning’s general principle: while home agents have a higher MPC on the home good, our model’s prescription entails discouraging spending by these agents at the precise time when this good is relatively more depressed (early in phase I). The reason is that such a diversion supports an exchange rate trajectory that induces all agents to redirect expenditure toward the home good at that time. This underlines the relevance of the exchange rate regime for the direction of the desirable intervention. Were the constraint on monetary policy arising from a peg rather than a ZLB, this expenditure switching channel would be absent, and Farhi and Werning’s general principle would apply (see [Farhi and Werning, 2012](#)).

A further benefit of our two-country model structure is that it naturally lends itself to an investigation of the coordination problem inherent to capital flow policies in a liquidity trap, an issue to which we turn in [Section 5](#).

Global liquidity trap While our focus is on a scenario where only a region of the world economy experiences a liquidity trap, it is worth noting that our normative results carry over to an alternative global liquidity trap scenario (i.e., where the ZLB binds in both countries). Indeed, combining the labor wedge expressions in [\(23\)](#) with the IS curves in [\(24\)](#), under the assumption that $i_t = i_t^* = 0$, yields⁴⁶

$$\tau_t - \tau_t^* = -\dot{e}_t = -\Psi^g (r_t^n - r_t^{n*}), \quad \dot{y}_t = -(1 - \alpha\Psi^g) r_t^n - \alpha\Psi^g r_t^{n*}, \quad \dot{y}_t^* = -\alpha\Psi^g r_t^n - (1 - \alpha\Psi^g) r_t^{n*}. \quad (26)$$

where the g superscript stands for “global” liquidity trap, and $\Psi^g \equiv (1 + \phi)/[2(1 + \alpha\phi)]$. This contrasts with $\dot{e}_t = 0$, $\dot{y}_t = -r_t^n$ and $\dot{y}_t^* = -r_t^{n*}$ under free capital mobility. Thus, when the ZLB binds everywhere, our result translates into one indicating that it is optimal to subsidize outflows out of the country with the lowest natural interest rate. [Devereux and Yetman \(2014\)](#) argue that imposing capital controls necessarily reduce welfare during a liquidity trap, using an environment where due to an absence of home-bias and preference shocks that do not affect the

⁴⁶These expressions hold at any instant where the ZLB binds in both countries. However, as the optimal ZLB exit times does generally not coincide for Home and Foreign, after the first country has exited the ZLB but before the second one has, there is a time interval where expressions analogous to those in [\(25\)](#) will hold.

dis-utility from labor supply, natural interest rates are by construction equal across countries at all times. Our optimal tax expression in (26) reveals the knife-edge property of their results by showing that the free capital mobility regime is only constrained efficient in non-generic cases where natural rates happen to be equal in Home and Foreign.

5 Capital flow management and currency wars

Given the constrained inefficiency of a free capital mobility regime established in Section 4, it is natural to ask whether the constrained efficient outcome can also be achieved in a decentralized (i.e., non-cooperative) setting where each country sets its own capital flow taxes independently. The objective of this section is to tackle this question.

5.1 Game between planners

In order to focus on the potential coordination problem pertaining to capital flow management, we still delegate monetary policy decisions to a global monetary authority but let national planners in Home and Foreign set capital flow taxes optimally.⁴⁷ The global planner sets monetary policy optimally for all future dates and chooses a date 0 transfer b_0 from Foreign to Home to maximize global welfare. The home planner chooses a path for home capital flow taxes to maximize home welfare, and the foreign planner chooses a path for foreign capital flow taxes to maximize foreign welfare. The three planners choose their actions simultaneously at date 0.

The problem of the global planner is given by

$$\max_{\{i_t \geq 0, i_t^* \geq 0, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}, b_0} \int_0^\infty e^{-\int_0^t (\rho + \zeta_h^*) dh} \left\{ \Xi_t \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] + \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] \right\} dt$$

subject to (4), (7), (8), (9), (10) and (11); the problem of the home planner is given by

$$\max_{\{\tau_t, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_h) dh} \left[\ln C_t - \frac{(Y_t)^{1+\phi}}{1+\phi} \right] dt$$

subject to (4), (7), (8), (9), (10) and (11); and the problem of the foreign planner is given by

$$\max_{\{\tau_t^*, C_t, C_t^*, Y_t, Y_t^*, \Theta_t\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_h^*) dh} \left[\ln C_t^* - \frac{(Y_t^*)^{1+\phi}}{1+\phi} \right] dt$$

⁴⁷We retain the assumption of cooperative monetary policy so as to study the implications of non-cooperativeness of capital flow management policies in a transparent fashion. In an earlier version (Acharya and Bengui, 2015), we obtained qualitatively similar results and predictions under the assumption of non-cooperative monetary policy.

subject to (4), (7), (8), (9), (10) and (11).

The three planning problems are analyzed formally in Appendix A.4. In the next two sections, we discuss the motivations faced by national planners and explain how these shape the macroeconomic adjustment in a liquidity trap.

5.2 Best responses

Monetary policy is set by the global planner following the same principles as in Sections 3 and 4. The global planner aims to replicate the first-best output paths in both countries, and whenever the interest rates necessary to achieve that goal violates the ZLB, uses *forward guidance* to minimize average output gaps over time. The new element of the policy setting analyzed in this section is that the path of the expenditure ratio Θ_t , whose growth rate is taken as given by the global monetary authority, is determined by the interaction of the capital flow taxes set by the home and foreign planners. We thus turn to the forces driving optimal capital flow management. The following lemma characterizes the national planners' choices.

Lemma 6 (Targeting rules in non-cooperative capital flow regime). *When capital flow taxes are set non-cooperatively, the home and foreign planners' choices are characterized by the targeting rules*

$$\Gamma_H \Theta_0 e^{\int_0^t \tau_s ds} = 1 + \Xi_t^{-1} e^{-\omega_t - \varpi_t}, \quad (27)$$

$$\Gamma_F \Theta_0^{-1} e^{\int_0^t \tau_s^* ds} = 1 + \Xi_t e^{-\omega_t^* + \varpi_t}, \quad (28)$$

where Γ_H and Γ_F denote the planners' multipliers on their own country's lifetime budget constraints.

Proof. See Appendix B.4. □

In stark contrast with their efficient regime counterpart (21), these targeting rules indicate that non-zero taxes are desirable for the national planners, even when wedges are zero. In order to gain further insights into the logic of the optimal policy by national planners, it is again useful to resort a first-order approximation of some key equations around the symmetric steady state.⁴⁸ Linearizing the home targeting rule (27) and then taking the time derivative yields:

$$\tau_t = -\frac{1}{2}\dot{\omega}_t - \frac{1}{2}\frac{\dot{\Theta}_t}{\Theta_t}. \quad (29)$$

This relation embeds the two types of incentives faced by the home planner when setting the home capital inflow tax. The first term on the right-hand side of (29) represents a macroeconomic stabilization motive: when the home labor wedge is contracting over time ($\dot{\omega}_t < 0$), such

⁴⁸This symmetric steady state is described in Appendix A.3.1.

as in phase I when the output gap is growing ($\dot{y}_t > 0$), smoothing it requires engineering a more depreciated home currency, which is achieved by encouraging outflows ($\tau_t > 0$).⁴⁹ The second term represents a dynamic terms-of-trade (henceforth, dToT) manipulation motive entailing that it is optimal for the home planner to smooth out the expenditure ratio Θ_t . This second motive is related to Costinot et al. (2014)'s result that when a country's trade balance grows or shrinks, managing the capital account provides a subtle way of extracting rents from foreigners by exerting market power differentially across time periods. In the context of our model and scenario, from the trade balance expression (17), $\dot{\Theta}_t > 0$ during phase I indicates that Home's trade surplus (measured in marginal utility terms) is shrinking over time. When the home trade surplus is positive but shrinking, taxing capital outflows induces the home consumer to front load consumption and thereby contributes to smooth surpluses. This allows Home to reap benefits from higher prices early in phase I, at times when the net sales to foreign buyers are the greatest, through a dynamic manipulation of the terms-of-trade. On the margin, curtailing capital outflows at this time makes home output relatively scarcer, allowing the Home planner to earn higher monopoly rents. Similarly, when the home trade surplus turns negative late in phase I, the mechanism is analogous: with growing deficits, subsidizing capital inflows induces the home consumer to front load consumption, this time to smooth deficits. This allows Home to reap benefits (monopsony rents) from lower prices late in phase I when the quantities purchased by Home from Foreign are the greatest. Equation (29) makes clear that the two motives just described conflict during a liquidity trap.

Using the linearized equilibrium labor wedge expression in (23), the linearized home IS curves in (24), and the law of motion for the expenditure ratio Θ_t (4); relation (29) and its foreign counterpart lead to linearized best responses describing a (home or foreign) planner's optimal choice as a function of the other two planners' choices and exogenous variables

$$\tau_t = \frac{1 + \phi}{3 + \phi\alpha} (i_t - r_t^n) + \frac{\zeta_t}{3 + \phi\alpha} + \frac{1 + \phi\alpha}{3 + \phi\alpha} \tau_t^* \quad (30)$$

$$\tau_t^* = \frac{1 + \phi}{3 + \phi\alpha} (i_t^* - r_t^{n*}) - \frac{\zeta_t}{3 + \phi\alpha} + \frac{1 + \phi\alpha}{3 + \phi\alpha} \tau_t \quad (31)$$

In both best response functions, the first term reflects macroeconomic stabilization motives, the second one reflects dToT manipulation motives, and the third one reflects a combination of both.⁵⁰

Next, we look at the interplay between the macroeconomic stabilization and dToT manipu-

⁴⁹The home labor wedge is contracting when the home output gap is growing. Recall that this is the case during phase I, when the home output gap is growing after an initial fall to a negative value.

⁵⁰Notice that the third term implies that the capital flow taxes are strategic complements. A larger tax on inflows by Foreign pushes capital to flow into Home, leading to a more appreciated home currency. As a result of both the macroeconomic stabilization motive and the dToT manipulation motive, the home planner responds by adjusting its capital flow tax upwards. The same logic applies to the foreign planner.

lation motives in the Nash equilibrium of the game played by the three planners.

5.3 Nash equilibrium

In the absence of the zero bound, it is straightforward to establish that from 0 to T the global planner would set interest rates so as to perfectly stabilize the output gap in both countries, the home planner would set a tax on outflows and the foreign planner would set a tax on inflows.⁵¹ Without the ZLB, the equilibrium interest rate are given by $i_t = r_t^n + \alpha\zeta_t/(2 - \alpha)$ and $i_t^* = r_t^{*n} - \alpha\zeta_t/(2 - \alpha)$, and the equilibrium capital flow taxes are $\tau_t = \zeta_t/[2(2 - \alpha)]$ and $\tau_t^* = -\zeta_t/[2(2 - \alpha)]$ implying an equilibrium tax wedge $\tau_t - \tau_t^* = \zeta_t/(2 - \alpha)$.⁵² In our demand shock scenario where $\zeta_t < 0$ from 0 to T , the home interest rate is lower than the home natural rate, the foreign interest rate is higher than the foreign natural rate, the home planner sets a tax on outflows, and the foreign planner sets a tax on inflows. Consistent with our discussion of relation (29) in the preceding section, this suggests that independently from ZLB considerations, uncoordinated capital flow management tends to hinder intertemporal trade and capital flows across countries.

The intuition is that demand shocks lead to trade imbalances, but both countries face incentives to reduce these: Initially, Home wants to reduce its trade surpluses to exert monopoly power, while Foreign wants to reduce its trade deficit to exert monopsony power. Current account positions flip later on, but regardless of the sign of imbalances, it is attractive for both countries to choose taxes which restrict capital flows, implying a lower growth rate of Θ_t . In other words, the path of Θ_t is “smoother” than that of Ξ_t , opening up a international wedge.⁵³ Hence, in the absence of the ZLB, despite monetary policy being able to successfully stabilize aggregate demand so as to implement first-best output everywhere, the first-best allocation is not achieved. Relative to the first-best, the path of the terms-of-trade $S_t = (Y_t^{\text{fb}}/Y_t^{*\text{fb}})(1 - \alpha + \alpha\Theta_t)/[(1 - \alpha)\Theta_t + \alpha]$ is smoother than that of the shadow terms-of-trade (14) and as a result, consumption decouples insufficiently from output relative to the first-best allocation.

When monetary policy is constrained by the ZLB, it is unable to stabilize aggregate demand. In this case, the insufficient adjustment in the terms-of-trade brought about by non-cooperative capital flow management distorts not just relative consumption but also output, resulting in even more pervasive efficiency losses. In our case of interest where the ZLB binds in Home but not in Foreign, interest rates are given by $i_t = 0$ and $i_t^* = r_t^{*n} - \alpha(\tau_t - \tau_t^*)$ and the global

⁵¹See equations (A.26) and (A.27) for our statement regarding output levels.

⁵²From the IS equations in (24), the interest rates would be given by $i_t = r_t^n + \alpha(\tau_t - \tau_t^*)$ and $i_t^* = r_t^{*n} - \alpha(\tau_t - \tau_t^*)$. Substituting these expressions into the best responses (30) and (31) leads to the equilibrium tax wedge expression.

⁵³ Using the fact that the $0 < \dot{\Theta}_t/\Theta_t < \dot{\Xi}_t/\Xi_t$ with the lifetime budget constraint (11) implies that $\Xi_0 < \Theta_0 < 1$. This is what we mean when we say that the path of Θ_t is smoother than Ξ_t .

monetary authority is unable to implement the first-best output path in Home.⁵⁴ Substituting the interest rate expressions into the best responses (30) and (31) yields

$$\begin{aligned}\tau_t - \tau_t^* &= -(1 - \Phi) \Psi r_t^n + \Phi \zeta_t, & i_t^* &= r_t^{n*} + \alpha [(1 - \Phi) \Psi r_t^n - \alpha \Phi \zeta_t], \\ \dot{e}_t &= -r_t^{n*} + (1 - \alpha) [(1 - \Phi) \Psi r_t^n - \Phi \zeta_t], & \dot{y}_t &= -[1 - (1 - \Phi) \alpha \Psi] r_t^n - \Phi \zeta_t.\end{aligned}\quad (32)$$

for $\Phi \equiv 2/[4 + \alpha(\phi - 1)]$.⁵⁵ The Nash equilibrium tax wedge expression in (32) is a weighted average of an aggregate demand stabilization term, already present in the efficient regime expression in (25), and a new, conflicting, dToT manipulation term. Home monetary policy is again set such that the home output gap averages out to zero over time (i.e. $\int_0^{\hat{T}} e^{-\rho t} \tilde{y}_t dt = 0$).⁵⁶ The main properties of the Nash regime is summarized in the following proposition.

Proposition 3 (Non-cooperative capital flow regime). *When the ZLB binds in Home but not in Foreign from 0 to T, up to a first-order, the non-cooperative regime compares with the constrained efficient regime as follows:*

1. *Capital flows out of Home are less subsidized in phase I and less taxed in phase II.*
2. *The home output gap is less smooth.*
3. *The home exchange rate appreciates at a slower rate in phase I and at a faster rate in phase II.*

Moreover, in phase I, relative to the free capital mobility regime, capital flows out of Home may even be taxed (rather than subsidized), the home output gap may be less smooth, and the home exchange rate may appreciate at a slower rate.

Proof. See argument in the text above. □

These results show that the idea put forward in the context of the best responses that dToT management motives conflict with macroeconomic stabilization finds its way to the Nash equilibrium. Notably, they indicate that the tax wedge in the non-cooperative regime falls short of its efficient value and may even take the “wrong” sign during phase I of the liquidity trap.

The determination of the Nash equilibrium is illustrated for the special case of a unit Frisch elasticity ($\phi = 1$, implying $\Psi = 1$ and $\Phi = 1/2$) in the (τ_t, τ_t^*) space in Figure 5. The free capital mobility regime, corresponding to a zero tax wedge, is represented by the straight line $\tau_t^* = \tau_t$. Points to the South-East of this line represent regimes associated with net subsidies

⁵⁴Our analysis of non-cooperative capital flow management can easily be extended to accommodate situations where the ZLB binds in both countries.

⁵⁵Note that $0 < \Phi < 1$, $\partial\Phi/\partial\phi < 0$ and $\partial\Phi/\partial\alpha \geq 0$ for $\phi \leq 1$. Moreover, $\lim_{\phi \rightarrow \infty} \Phi = 0$.

⁵⁶Note however that unlike in the constrained efficient and free capital mobility regime, the home labor wedge, foreign labor wedge, and international wedge do not necessarily also average out to zero over time

on flows from Home to Foreign, while points to the North-West of this line represent regimes associated with net taxes on such flows. The closed capital account regime, corresponding to a tax wedge of ζ_t , is represented by the straight line $\tau_t^* = -\zeta_t + \tau_t$. The constrained efficient regime, corresponding to a tax wedge of $-r_t^n$, is represented by the straight line $\tau_t^* = r_t^n + \tau_t$. Finally, the home and foreign planners' best responses (30) and (31), drawn at the Nash equilibrium interest rates, are represented by the straight lines $\tau_t^* = [(3 + \alpha) \tau_t + 2r_t^n - \zeta_t] / (1 + \alpha)$ and $\tau_t^* = [(1 + \alpha) \tau_t + \alpha r_t^n - (1 + \alpha) \zeta_t] / (3 + \alpha)$. The figure shows that the Nash outcome features too small a subsidy to flows from Home to Foreign in phase I, and too small a subsidy to flows from Foreign to Home in phase II. In phase I, it even illustrates a case where flows from Home to Foreign end up being taxed, while efficiency consideration would require them to be subsidized. With unit Frisch elasticity, the sign of the tax wedge is unambiguously negative in phase I of the liquidity trap (unlike in the efficient regime where it was positive). More generally, it is unambiguously negative in phase I when $\phi < (\rho + \bar{\zeta}) / [(1 - \alpha) \bar{\zeta} - \rho]$, i.e., labor supply is sufficiently elastic. The intuition is that with sufficiently elastic labor supply, labor wedge or output gap fluctuations are not too costly for the home planner, and the dToT manipulation force more easily dominates the macroeconomic stabilization force in equilibrium. For the liquidity trap scenario parametrization of Werning (2012), for which $\bar{\zeta} = 2\rho$, the condition becomes $\phi < 3 / (1 - 2\alpha)$. This condition is satisfied for most values of the Frisch elasticity and trade openness used in the open economy literature.

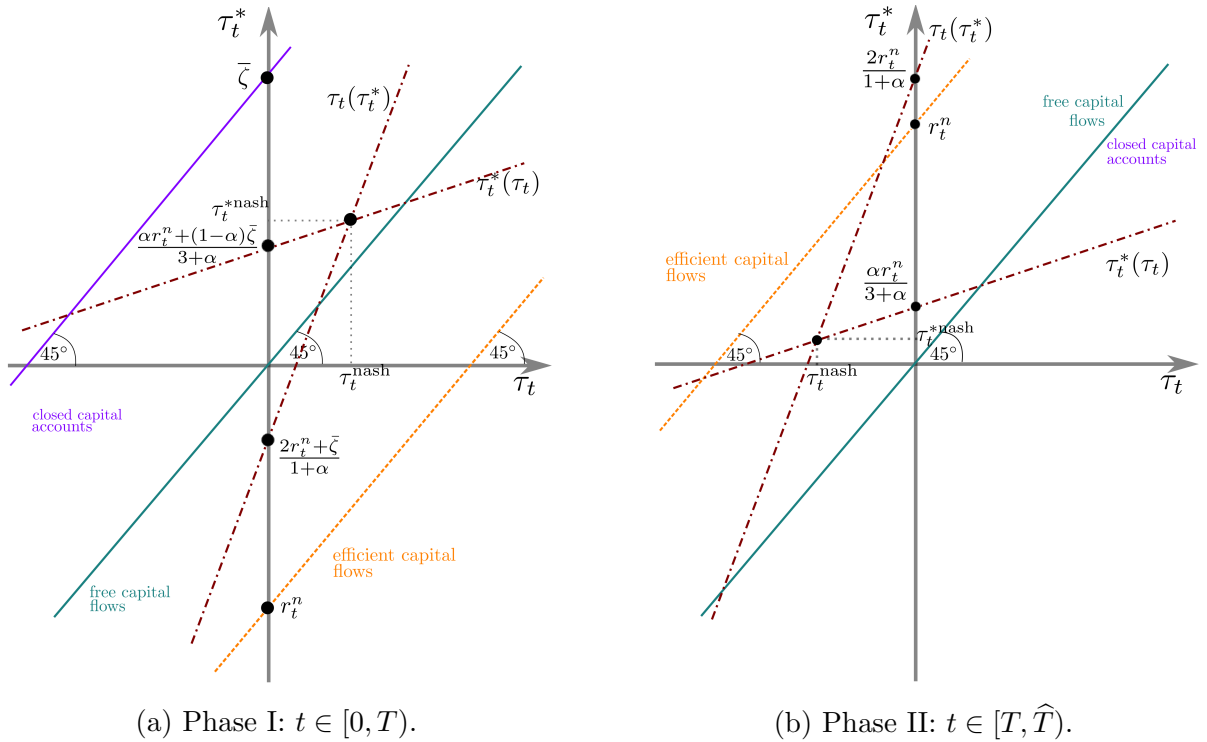


Figure 5: Linearized best-responses and Nash equilibrium when $\phi = 1$.

Figure 6 illustrates the effects of the distortions induced by the non-cooperative regime by plotting the paths of the model's main variables.⁵⁷ It is again apparent that relative to the efficient regime, the tax wedge has the wrong sign during phase I, and is too small during phase II. As a consequence, during the liquidity trap, the exchange rate path is smoother during phase I than under both the efficient regime *and* the free capital mobility regime. The uncoordinated capital flow management regime thus hampers smooth adjustment, as suggested by the more variable home output gap, home labor wedge, foreign labor wedge, and international wedge paths than under the free capital mobility regime.

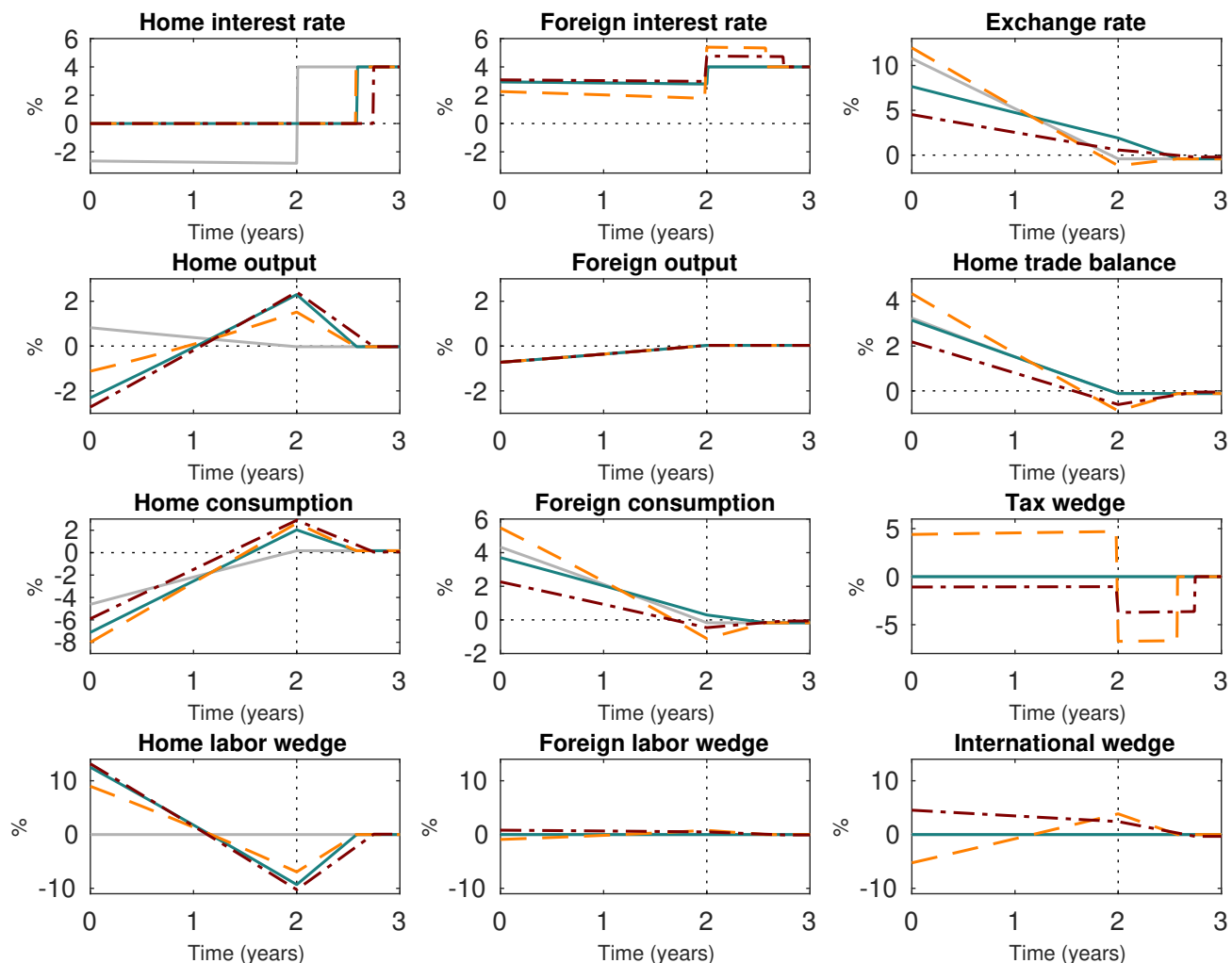


Figure 6: Variable paths under ZLB with free capital mobility (solid dark), ZLB with efficient regime (dashed), ZLB with non-cooperative regime (dashed-dotted), and unconstrained policy (solid light).

⁵⁷The picture is produced using an exact non-linear solution of the game, using the parametrization described in Footnote 30.

Global liquidity trap While our main scenario of interest is one where a single region experiences a liquidity trap, it is easy to extend our analysis to a situation in which the demand shock at Home is large enough to push the entire global economy into a liquidity trap. In this global liquidity trap case, similar forces are at play - dToT management motives still work counter to aggregate demand stabilization. As in the case where only Home hits the ZLB, the Nash equilibrium tax wedge is a weighted average of an aggregate demand stabilization term already present in the efficient regime expression in (26), and a dToT manipulation term. The resulting non-cooperative equilibrium in addition features Foreign output away from the first-best levels.

6 Discussion of potential extensions

Our positive and normative analysis of capital flows in a liquidity trap was conducted in a purposefully stylized general equilibrium model of the world economy. In this section, we briefly discuss the likely robustness of our results with respect to several natural extensions.

Sticky prices For the sake of analytical tractability, we have assumed that prices were fully rigid. A more realistic (and standard) assumption would be one under which prices would instead be sticky. Allowing for sticky prices (e.g., à la Rotemberg, 1982 or Calvo, 1983) would likely not qualitatively change our results, but would substantially reduce tractability. From a positive perspective, increased price stability is known to often be destabilizing in a liquidity trap (see Eggertsson, 2010 and Bhattarai et al., 2014). In our context, sticky rather than rigid prices would allow a stronger deflationary pressure in Home than in Foreign to materialize and thereby create an additional destabilizing force by making the terms-of-trade respond perversely (i.e., shifting expenditure *away* from home goods at the beginning of the liquidity trap).⁵⁸ As such, they would not eliminate – and may even instead reinforce – the benefits of the stabilizing exchange rate movements associated with international capital flows. Our main insights can therefore be expected to apply as long as prices are not fully flexible.

Global liquidity trap While the global events constituting our motivation of the paper led us to focus on a scenario where only Home experiences a liquidity trap, the brief analysis in the concluding paragraph of Section 4 showed that our main insights carry through to a situation where the entire world enters a liquidity trap. Pursuing a detailed analysis in this direction would not compromise our framework’s tractability and may provide a fruitful avenue for future research.

⁵⁸Controlling for the exchange rate, a higher PPI deflation in Home than in Foreign early in the liquidity trap would make the home good cheaper and cheaper over time, relative to the foreign good. Cook and Devereux (2013) refer to this effect as a “perverse response” of the terms-of-trade in a liquidity trap.

Pricing currency The mechanism driving our results on the stabilizing role of capital flows in a liquidity trap crucially relies on the expenditure switching effect brought about by exchange rate movements. As is well known, this effect is at work under the standard producer currency pricing (PCP) assumption we have adopted, but would be absent under the alternative assumption of local currency pricing. Given our motivation stemming from the Great Recession and accordingly, our interpretation of Home as the set of advanced economies and of Foreign as the set of emerging economies, an arguably more realistic pricing assumption would be that of *dominant currency pricing* (see [Gopinath, 2016](#) and [Casas et al., 2016](#)). Under this paradigm, all internationally traded goods would be priced in Home’s currency (i.e., the dominant currency). As a result, expenditure switching would not operate on home consumers but it would still operate on foreign consumers. Under this empirically more plausible pricing currency assumption, capital flows would thus help stabilize aggregate demand in Home by triggering expenditure switching by foreign consumers only (as opposed to by all consumers under PCP). The strength of the forces underlying our mechanism would consequently be somewhat weakened, but our main insights would not change qualitatively.

Non-cooperative monetary policy Throughout the paper, we have assumed that monetary policy was conducted cooperatively. In an earlier version ([Acharya and Bengui, 2015](#)), we analyzed the case where monetary policy is instead conducted non-cooperatively and obtained qualitatively similar results and predictions.

7 Conclusion

We argue that when a large region of the world economy experiences a liquidity trap, global capital flows allow for a reallocation of demand and expenditures and are therefore stabilizing. Owing to aggregate demand externalities operating at the zero lower bound, free capital flows are nonetheless constrained inefficient and result in reallocations that are too small. Global efficiency requires larger flows during and after the liquidity trap, to compensate for monetary policy’s inability to stimulate aggregate demand in the region where the zero bound on interest rates is binding. Despite pointing to inefficient capital flows in a liquidity trap, our analysis does not support the management of capital flows by individual countries. To the contrary, it suggests that the terms-of-trade management objectives underlying such policies may interfere with aggregate demand stabilization and thus hamper, rather than promote, a smooth global macroeconomic adjustment. Consequently, the analysis underscores the importance of international policy coordination.

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A Optimal policy appendix

A.1 First-Best Allocation

The socially optimal allocation, which we refer to as *first-best*, is the allocation that solves an unconstrained social planning problem. Imposing symmetric consumption of the differentiated goods produced within a country,⁵⁹ the planning problem amounts to a sequence of static problems of the form

$$\max_{C_t, C_t^*, C_{H,t}, C_{F,t}, C_{H,t}^*, C_{F,t}^*, N_t, N_t^*} \Xi_t \left[\ln(C_t) - \frac{(N_t)^{1+\phi}}{1+\phi} \right] + \left[\ln(C_t^*) - \frac{(N_t^*)^{1+\phi}}{1+\phi} \right]$$

subject to the constraints:

$$C_t = \frac{(C_{H,t})^{(1-\alpha)} (C_{F,t})^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha},$$

$$C_t^* = \frac{(C_{H,t}^*)^{(1-\alpha)} (C_{F,t}^*)^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha},$$

$$C_{H,t} + C_{H,t}^* = N_t, \quad (\text{A.1})$$

$$C_{F,t} + C_{F,t}^* = N_t^*, \quad (\text{A.2})$$

where $\Xi_t \equiv \Xi e^{-\int_0^t (\zeta_s - \zeta_s^*) ds}$ is a time-varying Pareto weight assigned by the planner to Home.⁶⁰ The first-order conditions of this problem lead to first-best employment/output

$$N_t^{\text{fb}} = Y_t^{\text{fb}} = [\alpha \Xi_t^{-1} + 1 - \alpha]^{\frac{1}{1+\phi}} \quad \text{and} \quad N_t^{*\text{fb}} = Y_t^{*\text{fb}} = [\alpha \Xi_t + 1 - \alpha]^{\frac{1}{1+\phi}}, \quad (\text{A.3})$$

and aggregate consumption

$$C_t^{\text{fb}} = \Xi_t^\alpha \left[(Y_t^{\text{fb}})^{1-\alpha} (Y_t^{*\text{fb}})^\alpha \right]^{-\phi} \quad \text{and} \quad C_t^{*\text{fb}} = \Xi_t^{-\alpha} \left[(Y_t^{\text{fb}})^\alpha (Y_t^{*\text{fb}})^{1-\alpha} \right]^{-\phi}. \quad (\text{A.4})$$

Consumption of home and foreign goods are accordingly given by $C_{H,t}^{\text{fb}} = (1-\alpha) (Y_t^{\text{fb}})^{-\phi}$, $C_{F,t}^{\text{fb}} = \alpha \Xi_t (Y_t^{*\text{fb}})^{-\phi}$, $C_{H,t}^{*\text{fb}} = (1-\alpha) (Y_t^{*\text{fb}})^{-\phi}$ and $C_{F,t}^{*\text{fb}} = \alpha \Xi_t^{-1} (Y_t^{\text{fb}})^{-\phi}$.

Furthermore, the multipliers on the home and foreign resource constraints, (A.1) and (A.2), are given by $\vartheta_{H,t} = \Xi_t (Y_t^{\text{fb}})^\phi$ and $\vartheta_{F,t} = (Y_t^{*\text{fb}})^\phi$. Accordingly, the planner's *shadow terms of trade* is given by

$$\vartheta_t \equiv \frac{\vartheta_{F,t}}{\vartheta_{H,t}} = \frac{Y_t^{\text{fb}}}{Y_t^{*\text{fb}}} \times \frac{\alpha \Xi_t + 1 - \alpha}{\alpha + (1-\alpha) \Xi_t}. \quad (\text{A.5})$$

⁵⁹Such a symmetry is trivially optimal, given the assumed preferences and technologies.

⁶⁰ Ξ is the Pareto weight assigned by the planner to Home at date 0. Note that due to differences in discounting, the weight giving both countries equal importance, which we refer to as the *symmetric weight*, is given by $\Xi = \int_0^\infty e^{-\int_0^s (\rho + \zeta_s^*) ds} dt / \int_0^\infty e^{-\int_0^s (\rho + \zeta_s) ds} dt$.

A.2 Optimal monetary policy

For given paths of $\Theta_t, \tau_t, \tau_t^*$, the optimal monetary policy problem is an optimal control problem with control variables i_t, i_t^* , and state variables Y_t, Y_t^* :

$$\max_{\{i_t, i_t^*\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (Y_t^*)^{\alpha \Xi_t + 1 - \alpha} \right] - \frac{1}{1 + \phi} \left[\Xi_t (Y_t)^{1 + \phi} + (Y_t^*)^{1 + \phi} \right] \right\}$$

subject to

$$\frac{\dot{Y}_t}{Y_t} = i_t - (\rho + \zeta_t) - \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*), \quad (\text{A.6})$$

$$\frac{\dot{Y}_t^*}{Y_t^*} = i_t^* - (\rho + \zeta_t^*) + \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*), \quad (\text{A.7})$$

$$i_t \geq 0 \quad (\text{A.8})$$

$$i_t^* \geq 0 \quad (\text{A.9})$$

The associated present value Hamiltonian is given by

$$\begin{aligned} \mathcal{H} = & e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (Y_t^*)^{\alpha \Xi_t + 1 - \alpha} \right] - \frac{1}{1 + \phi} \left[\Xi_t (Y_t)^{1 + \phi} + (Y_t^*)^{1 + \phi} \right] \right\} \\ & + \lambda_t Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t i_t \\ & + \lambda_t^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t^* i_t^*, \end{aligned}$$

where λ_t, λ_t^* are the co-state variables associated with Y_t, Y_t^* , and ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates.

The planner's optimal choice is characterized by the conditions

$$\lambda_t i_t = 0, \quad i_t \geq 0, \quad \lambda_t \geq 0, \quad (\text{A.10})$$

$$\lambda_t^* i_t^* = 0, \quad i_t^* \geq 0, \quad \lambda_t^* \geq 0, \quad (\text{A.11})$$

the laws of motion for the co-state variables

$$\dot{\lambda}_t = -e^{-\int_0^t (\rho + \zeta_s) ds} \left[\left(Y_t^{\text{fb}} \right)^{1 + \phi} - (Y_t)^{1 + \phi} \right] \frac{1}{Y_t} - \lambda_t \frac{\dot{Y}_t}{Y_t} \quad (\text{A.12})$$

$$\dot{\lambda}_t^* = -e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[\left(Y_t^{*\text{fb}} \right)^{1 + \phi} - (Y_t^*)^{1 + \phi} \right] \frac{1}{Y_t^*} - \lambda_t^* \frac{\dot{Y}_t^*}{Y_t^*}, \quad (\text{A.13})$$

initial conditions $\lambda_0 = \lambda_0^*$ for the co-state variables, and transversality conditions $\lim_{t \rightarrow \infty} \lambda_t Y_t = 0$ and $\lim_{t \rightarrow \infty} \lambda_t^* Y_t^* = 0$.

Integrating (A.12) and (A.13) from 0 to ∞ , and using the initial conditions and transversality

conditions, yields

$$0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s) ds} \left[\left(Y_t^{\text{fb}} \right)^{1+\phi} - (Y_t)^{1+\phi} \right] dt \quad (\text{A.14})$$

$$0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[\left(Y_t^{*\text{fb}} \right)^{1+\phi} - (Y_t^*)^{1+\phi} \right] dt, \quad (\text{A.15})$$

The planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \lambda_t, \lambda_t^*, i_t, i_t^*$ consisting of (A.6), (A.7), (A.8), (A.9), (A.10), (A.11), (A.12) and (A.13) with boundary conditions (A.14), (A.15), $\lambda_0 = 0$ and $\lambda_0^* = 0$.

A.2.1 Symmetric steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = 1$, $\lambda_t = \lambda_t^* = 0$ and $i_t = i_t^* = \rho$. The associated steady-state labor wedges are hence equal to zero: $\omega_t = \omega_t^* = 0$.

A.3 Efficient capital flows

The optimal policy problem is an optimal control problem with control variables i_t, i_t^* , and state variables Y_t, Y_t^*, Θ_t :

$$\begin{aligned} \max_{\{i_t, i_t^*, \tau_t\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} & \left\{ \ln \left[(Y_t)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (Y_t^*)^{\alpha \Xi_t + 1 - \alpha} \right] - \frac{1}{1 + \phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right. \\ & \left. - \ln \left[\Theta_t^{\alpha(1-\Xi_t)} (\alpha \Theta_t^{-1} + 1 - \alpha)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (\alpha \Theta_t + 1 - \alpha)^{\alpha \Xi_t + 1 - \alpha} \right] \right\} dt \end{aligned}$$

subject to (A.6)-(A.9) (with $\tau_t^* = 0$) and

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta_t^* - \zeta_t + \tau_t \quad (\text{A.16})$$

The associated present value Hamiltonian is given by

$$\begin{aligned} \mathcal{H} = & e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (Y_t^*)^{\alpha \Xi_t + 1 - \alpha} \right] - \frac{1}{1 + \phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right. \\ & \left. - \ln \left[\Theta_t^{\alpha(1-\Xi_t)} (\alpha \Theta_t^{-1} + 1 - \alpha)^{\Xi_t (\alpha \Xi_t^{-1} + 1 - \alpha)} (\alpha \Theta_t + 1 - \alpha)^{\alpha \Xi_t + 1 - \alpha} \right] \right\} \\ & + \lambda_t Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t) \right] + \nu_t i_t \\ & + \lambda_t^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t) \right] + \nu_t^* i_t^* \\ & + \mu_t \Theta_t (\zeta_t^* - \zeta_t + \tau_t), \end{aligned}$$

where $\lambda_t, \lambda_t^*, \mu_t$ are the co-state variables associated with Y_t, Y_t^*, Θ_t , and ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates.

The planner's optimal choice is characterized by the complementary slackness conditions (A.10), (A.11), the first-order condition for τ_t

$$\mu_t \Theta_t - \lambda_t Y_t \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} + \lambda_t^* Y_t^* \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} = 0, \quad (\text{A.17})$$

the laws of motion for the co-state variables (A.12), (A.13) and

$$\begin{aligned} \dot{\mu}_t = & -e^{-\int_0^t (\rho + \zeta_s^*) ds} \Theta_t^{-1} \left[-\alpha (1 - \Xi_t) + \frac{\Xi_t (Y_t^{\text{fb}})^{1+\phi} \alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{(Y_t^{*\text{fb}})^{1+\phi} \alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} \right] \\ & - \lambda_t Y_t \frac{\alpha (1 - \alpha) \Theta_t^{-2}}{(\alpha \Theta_t^{-1} + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda_t^* Y_t^* \frac{\alpha (1 - \alpha)}{(\alpha \Theta_t + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_t \frac{\dot{\Theta}_t}{\Theta_t} \end{aligned} \quad (\text{A.18})$$

non-negativity conditions $\lambda_t \geq 0, \lambda_t^* \geq 0, \mu_t \geq 0$, initial conditions $\lambda_0 = \lambda_0^*, \mu_0 = 0$ for the co-state variables, and transversality conditions $\lim_{t \rightarrow \infty} \lambda_t Y_t = 0, \lim_{t \rightarrow \infty} \lambda_t^* Y_t^* = 0$, and $\lim_{t \rightarrow \infty} \mu_t \Theta_t = 0$.

Differentiating (A.17) with respect to time, substituting the co-state laws of motion (A.12), (A.13) and (A.18), and using the wedge expressions in (15), yields:

$$1 - e^{-\omega_t - \varpi_t} = \Xi_t \left(1 - e^{-\omega_t^* + \varpi_t} \right) \quad (\text{A.19})$$

Differentiating this equation with respect to time yields

$$\begin{aligned} 0 = & e^{-\omega_t - \varpi_t} \left((1 + \phi) \frac{\dot{Y}_t}{Y_t} - \frac{1 - \alpha}{\alpha \Theta_t^{-1} + 1 - \alpha} \frac{\dot{\Theta}_t}{\Theta_t} \right) - \Xi_t e^{-\omega_t^* + \varpi_t} \left((1 + \phi) \frac{\dot{Y}_t^*}{Y_t^*} + \frac{1 - \alpha}{\alpha \Theta_t + 1 - \alpha} \frac{\dot{\Theta}_t}{\Theta_t} \right) \\ & + \left(1 + \Xi_t e^{-\omega_t^* + \varpi_t} \right) (\zeta_t^* - \zeta_t) \end{aligned} \quad (\text{A.20})$$

Integrating (A.12), (A.13) and (A.18) from 0 to ∞ , and using the initial conditions and transversality conditions, yields (A.14), (A.15) and⁶¹

$$0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[\Xi_t \frac{\Theta_t^{-1} (Y_t)^{1+\phi}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{\Theta_t (Y_t^*)^{1+\phi}}{\alpha \Theta_t + 1 - \alpha} \right] dt. \quad (\text{A.21})$$

The planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \Theta_t, \lambda_t, \lambda_t^*, i_t, i_t^*$ consisting of (A.6), (A.7) (both with $\tau_t^* = 0$), (A.8), (A.9), (A.10), (A.11), (A.12), (A.13) and (A.20) with boundary conditions (A.14), (A.15), (A.21), $\lambda_0 = 0$ and $\lambda_0^* = 0$. The path of τ_t then follows from (A.16), and that of μ_t solves (A.18) with initial condition $\mu_0 = 0$.

⁶¹Integration by parts is required to obtain (A.21).

A.3.1 Symmetric steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = \Theta_t = 1$, $\lambda_t = \lambda_t^* = \mu_t = 0$, $i_t = i_t^* = \rho$ and $\tau_t = 0$. The associated steady-state wedges are hence equal to zero: $\omega_t = \omega_t^* = \varpi_t = 0$.

A.4 Noncooperative capital flow management

A.4.1 Global planner's problem

The global planner's problem is an optimal control problem with control variables i_t, i_t^* and b_0 (a date 0 transfer), and state variables Y_t, Y_t^*, Θ_t :

$$\max_{\{i_t, i_t^*\}, b_0} \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^{\Xi_t(\alpha\Xi_t^{-1}+1-\alpha)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right. \\ \left. - \ln \left[\Theta_t^{\alpha(1-\Xi_t)} (\alpha\Theta_t^{-1} + 1 - \alpha)^{\Xi_t(\alpha\Xi_t^{-1}+1-\alpha)} (\alpha\Theta_t + 1 - \alpha)^{\alpha\Xi_t+1-\alpha} \right] \right\}$$

subject to (A.6)-(A.9) and

$$\frac{\dot{\Theta}_t}{\Theta_t} = \zeta_t^* - \zeta_t + \tau_t - \tau_t^* \quad (\text{A.22})$$

$$b_0 = \alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (\Theta_t - 1) dt \quad (\text{A.23})$$

The associated present value Hamiltonian is given by

$$\mathcal{H}_G = e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^{\Xi_t(\alpha\Xi_t^{-1}+1-\alpha)} (Y_t^*)^{\alpha\Xi_t+1-\alpha} \right] - \frac{1}{1+\phi} \left[\Xi_t (Y_t)^{1+\phi} + (Y_t^*)^{1+\phi} \right] \right. \\ \left. - \ln \left[\Theta_t^{\alpha(1-\Xi_t)} (\alpha\Theta_t^{-1} + 1 - \alpha)^{\Xi_t(\alpha\Xi_t^{-1}+1-\alpha)} (\alpha\Theta_t + 1 - \alpha)^{\alpha\Xi_t+1-\alpha} \right] \right\} \\ + \lambda_{G,t} Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t i_t \\ + \lambda_{G,t}^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] + \nu_t^* i_t^* \\ + \mu_{G,t} \Theta_t (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) + \Gamma_G \left[\alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (\Theta_t - 1) dt - b_0 \right],$$

where $\lambda_{G,t}, \lambda_{G,t}^*, \mu_{G,t}$ are the co-state variables associated with Y_t, Y_t^*, Θ_t ; ν_t, ν_t^* are multipliers on the non-negativity constraints for interest rates and Γ_G is the multiplier on the home lifetime budget constraint.

The global planner's optimal choice is characterized by the complementary slackness conditions

$$\lambda_{G,t} i_t = 0, \quad i_t \geq 0, \quad \lambda_{G,t} \geq 0 \quad (\text{A.24})$$

$$\lambda_{G,t}^* i_t^* = 0, \quad i_t^* \geq 0, \quad \lambda_{G,t}^* \geq 0 \quad (\text{A.25})$$

the first-order condition for the transfer $\Gamma_G = 0$, the laws of motion for the co-state variables

$$\dot{\lambda}_{G,t} = -e^{-\int_0^t (\rho + \zeta_s) ds} \left[\left(Y_t^{\text{fb}} \right)^{1+\phi} - (Y_t)^{1+\phi} \right] \frac{1}{Y_t} - \lambda_{G,t} \frac{\dot{Y}_t}{Y_t} \quad (\text{A.26})$$

$$\dot{\lambda}_{G,t}^* = -e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[\left(Y_t^{*\text{fb}} \right)^{1+\phi} - (Y_t^*)^{1+\phi} \right] \frac{1}{Y_t^*} - \lambda_{G,t}^* \frac{\dot{Y}_t^*}{Y_t^*}, \quad (\text{A.27})$$

$$\begin{aligned} \dot{\mu}_{G,t} = & -e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[-\frac{\alpha(1-\Xi_t)}{\Theta_t} + \frac{\Xi_t (Y_t^{\text{fb}})^{1+\phi} \alpha \Theta_t^{-2}}{\alpha \Theta_t^{-1} + 1 - \alpha} - \frac{(Y_t^{*\text{fb}})^{1+\phi} \alpha}{\alpha \Theta_t + 1 - \alpha} \right] \\ & - \lambda_{G,t} Y_t \frac{\alpha(1-\alpha) \Theta_t^{-2}}{(\alpha \Theta_t^{-1} + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda_{G,t}^* Y_t^* \frac{\alpha(1-\alpha)}{(\alpha \Theta_t + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_{G,t} \frac{\dot{\Theta}_t}{\Theta_t} \end{aligned} \quad (\text{A.28})$$

non-negativity conditions $\lambda_{G,t} \geq 0$, $\lambda_{G,t}^* \geq 0$, $\mu_{G,t} \geq 0$, initial conditions $\lambda_{G,0} = \lambda_{G,0}^* = \mu_{G,0} = 0$ for the co-state variables, and transversality conditions $\lim_{t \rightarrow \infty} \lambda_{G,t} Y_t = 0$, $\lim_{t \rightarrow \infty} \lambda_{G,t}^* Y_t^* = 0$, and $\lim_{t \rightarrow \infty} \mu_{G,t} \Theta_t = 0$.

Integrating (A.26), (A.27) and (A.28) from 0 to ∞ , and using the initial and transversality conditions, yields (A.14), (A.15) and (A.21). For given paths for τ_t, τ_t^* , the global planner's optimal plan can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \Theta_t, \lambda_{G,t}, \lambda_{G,t}^*, i_t, i_t^*$ consisting of (A.6), (A.7), (A.8), (A.9), (A.22), (A.24), (A.25), (A.26) and (A.27) with boundary conditions (A.14), (A.15), (A.21), $\lambda_{G,0} = 0$ and $\lambda_{G,0}^* = 0$. The optimal transfer is then given by (A.23), and the path of $\mu_{G,t}$ solves (A.28) with initial condition $\mu_{G,0} = 0$.

A.4.2 Home planner's problem

The home planner's problem is an optimal control problem with control variables τ_t , and state variables Y_t, Y_t^*, Θ_t :

$$\max_{\{\tau_t\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_s) ds} \left\{ \ln \left[(Y_t)^{1-\alpha} (Y_t^*)^\alpha \right] - \frac{(Y_t)^{1+\phi}}{1+\phi} - \ln \left[\Theta_t^{-\alpha} (\alpha \Theta_t^{-1} + 1 - \alpha)^{1-\alpha} (\alpha \Theta_t + 1 - \alpha)^\alpha \right] \right\}$$

subject to (A.6)-(A.9), (A.22), (A.23).

The associated present value Hamiltonian is given by

$$\begin{aligned}\mathcal{H}_H = & e^{-\int_0^t(\rho+\zeta_s)ds} \left\{ \ln \left[(Y_t)^{1-\alpha} (Y_t^*)^\alpha \right] - \frac{(Y_t)^{1+\phi}}{1+\phi} - \ln \left[\Theta_t^{-\alpha} (\alpha\Theta_t^{-1} + 1 - \alpha)^{1-\alpha} (\alpha\Theta_t + 1 - \alpha)^\alpha \right] \right\} \\ & + \lambda_{H,t} Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] \\ & + \lambda_{H,t}^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) \right] \\ & + \mu_{H,t} \Theta_t (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) + \Gamma_H \left[\alpha \int_0^\infty e^{-\int_0^t(\rho+\zeta_s^*-\tau_s^*)ds} (\Theta_t - 1) dt - b_0 \right],\end{aligned}$$

where $\lambda_{H,t}, \lambda_{H,t}^*, \mu_{H,t}$ are the co-state variables associated with Y_t, Y_t^*, Θ_t ; Γ_H is home planner's multiplier on the home lifetime budget constraint.

The home planner's optimal choice is characterized by the first-order condition

$$\mu_{H,t} \Theta_t - \lambda_{H,t} Y_t \frac{\alpha\Theta_t^{-1}}{\alpha\Theta_t^{-1} + 1 - \alpha} + \lambda_{H,t}^* Y_t^* \frac{\alpha\Theta_t}{\alpha\Theta_t + 1 - \alpha} = 0, \quad (\text{A.29})$$

the laws of motion of the co-state variables

$$\dot{\lambda}_{H,t} = -e^{-\int_0^t(\rho+\zeta_s)ds} \left[(1-\alpha) - \left(\frac{Y_t}{A} \right)^{1+\phi} \right] \frac{1}{Y_t} - \lambda_{H,t} \frac{\dot{Y}_t}{Y_t} \quad (\text{A.30})$$

$$\dot{\lambda}_{H,t}^* = -e^{-\int_0^t(\rho+\zeta_s)ds} \alpha \frac{1}{Y_t^*} - \lambda_{H,t}^* \frac{\dot{Y}_t^*}{Y_t^*} \quad (\text{A.31})$$

$$\begin{aligned}\dot{\mu}_{H,t} = & -e^{-\int_0^t(\rho+\zeta_s)ds} \left[\frac{\alpha}{\Theta_t} + \frac{(1-\alpha)\alpha\Theta_t^{-2}}{\alpha\Theta_t^{-1} + 1 - \alpha} - \frac{\alpha^2}{\alpha\Theta_t + 1 - \alpha} \right] + \alpha\Gamma_H e^{-\int_0^t(\rho+\zeta_s^*-\tau_s^*)ds} \\ & - \lambda_{H,t} Y_t \frac{\alpha(1-\alpha)\Theta_t^{-2}}{(\alpha\Theta_t^{-1} + 1 - \alpha)^2 \Theta_t} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda_{H,t}^* Y_t^* \frac{\alpha(1-\alpha)}{(\alpha\Theta_t + 1 - \alpha)^2 \Theta_t} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_{H,t} \frac{\dot{\Theta}_t}{\Theta_t}\end{aligned} \quad (\text{A.32})$$

non-negativity conditions $\lambda_{H,t} \geq 0, \lambda_{H,t}^* \geq 0, \mu_{H,t} \geq 0$, initial conditions $\lambda_{H,0} = \lambda_{H,0}^* = 0$ for the co-state variables associated with Y_t, Y_t^* , and transversality conditions $\lim_{t \rightarrow \infty} \lambda_{H,t} Y_t = 0, \lim_{t \rightarrow \infty} \lambda_{H,t}^* Y_t^* = 0$, and $\lim_{t \rightarrow \infty} \mu_{H,t} \Theta_t = 0$.

Differentiating (A.29) with respect to time, and substituting the co-state laws of motion (A.30), (A.31), (A.32), and using the wedge expressions in (15), yield:

$$\Gamma_H \Theta_t e^{\int_0^t(\zeta_s - \zeta_s^* + \tau_s^*)ds} = 1 + \Theta_t^{-1} e^{-\omega t}. \quad (\text{A.33})$$

Taking natural logarithms and differentiating with respect to time, and re-arranging yields

$$\left(1 + \frac{e^{-\omega t}}{\Theta_t + e^{-\omega t}} \times \frac{1-\alpha}{\alpha\Theta_t^{-1} + 1 - \alpha} \right) \frac{\dot{\Theta}_t}{\Theta_t} + (\zeta_t - \zeta_t^* + \tau_t^*) = (1+\phi) \frac{e^{-\omega t}}{\Theta_t + e^{-\omega t}} \frac{\dot{Y}_t}{Y_t}. \quad (\text{A.34})$$

Integrating (A.30) and (A.31) from 0 to ∞ , and using the initial and transversality conditions for λ_t, λ_t^* yields (A.14) and (A.15). For given paths for i_t, i_t^*, τ_t^* and a given transfer b_0 , the home planner's

optimal plan can be described as the particular solution to the system of first-order differential equations in Y_t, Θ_t consisting of (A.6) and (A.34), with boundary conditions (A.14) and (A.23). The path of τ_t then follows from (A.22); that of Y_t^* solves (A.7) with boundary condition (A.15); those of $\lambda_{H,t}, \lambda_{H,t}^*$ solve (A.30), (A.31) with boundary conditions $\lambda_{H,0} = \lambda_{H,0}^* = 0$; and that of $\mu_{H,t}$ follows from (A.29). Finally, the value of Γ_H can then be backed out from (A.32).

A.4.3 Foreign planner's problem

The foreign planner's problem is an optimal control problem with control variables τ_t^* , and state variables Y_t, Y_t^*, Θ_t :

$$\max_{\{\tau_t^*\}} \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[(Y_t)^\alpha (Y_t^*)^{(1-\alpha)} \right] - \frac{(Y_t^*)^{1+\phi}}{1+\phi} - \ln \left[\Theta_t^\alpha (\alpha \Theta_t^{-1} + 1 - \alpha)^\alpha (\alpha \Theta_t + 1 - \alpha)^{1-\alpha} \right] \right\}$$

subject to (A.6)-(A.9), (A.22), and

$$\tilde{b}_0 = \alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_h - \tau_h) dh} (\Theta_t^{-1} - 1) dt \quad (\text{A.35})$$

where \tilde{b}_0 is the transfer between Foreign and Home set by the global planner, but expressed in terms of date 0 home (rather than foreign) marginal utility.⁶²

The associated present value Hamiltonian is given by

$$\begin{aligned} \mathcal{H}_F = & e^{-\int_0^t (\rho + \zeta_s^*) ds} \left\{ \ln \left[Y_t^\alpha (Y_t^*)^{(1-\alpha)} \right] - \frac{(Y_t^*)^{1+\phi}}{1+\phi} - \ln \left[\Theta_t^\alpha (\alpha \Theta_t^{-1} + 1 - \alpha)^\alpha (\alpha \Theta_t + 1 - \alpha)^{1-\alpha} \right] \right\} \\ & + \lambda_{F,t} Y_t \left[i_t - (\rho + \zeta_t) - \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} (\tau_t - \tau_t^* + \zeta_t^* - \zeta_t) \right] \\ & + \lambda_{F,t}^* Y_t^* \left[i_t^* - (\rho + \zeta_t^*) + \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} (\tau_t - \tau_t^* + \zeta_t^* - \zeta_t) \right] \\ & + \mu_{F,t} \Theta_t (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) + \Gamma_F \left[\tilde{b}_0 - \alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_h - \tau_h) dh} (\Theta_t^{-1} - 1) dt \right], \end{aligned}$$

where $\lambda_{F,t}, \lambda_{F,t}^*, \mu_{F,t}$ are the co-state variables associated with Y_t, Y_t^*, Θ_t ; Γ_F is foreign planner's multiplier on the foreign lifetime budget constraint.

The foreign planner's optimal choice is characterized by the first-order condition

$$\mu_{F,t} \Theta_t - \lambda_{F,t} Y_t \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} + \lambda_{F,t}^* Y_t^* \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} = 0, \quad (\text{A.36})$$

⁶²It is analytically more convenient to express the foreign country's lifetime budget constraint in terms of home marginal utility. See online Appendix C.5.2 for details.

the laws of motion of the co-state variables

$$\dot{\lambda}_{F,t} = -e^{-\int_0^t (\rho + \zeta_s^*) ds} \alpha \frac{1}{Y_t} - \lambda_{F,t} \frac{\dot{Y}_t}{Y_t} \quad (\text{A.37})$$

$$\dot{\lambda}_{F,t}^* = -e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[(1 - \alpha) - (Y_t^*)^{1+\phi} \right] \frac{1}{Y_t^*} - \lambda_{F,t}^* \frac{\dot{Y}_t^*}{Y_t^*} \quad (\text{A.38})$$

$$\begin{aligned} \dot{\mu}_{F,t} = & -e^{-\int_0^t (\rho + \zeta_s^*) ds} \left[-\frac{\alpha}{\Theta_t} + \alpha \frac{\alpha \Theta_t^{-2}}{\alpha \Theta_t^{-1} + 1 - \alpha} - (1 - \alpha) \frac{\alpha}{\alpha \Theta_t + 1 - \alpha} \right] - \alpha \Gamma_F e^{-\int_0^t (\rho + \zeta_s - \tau_s) ds} \Theta_t^{-2} \\ & - \lambda_{F,t} Y_t \frac{\alpha (1 - \alpha) \Theta_t^{-2}}{(\alpha \Theta_t^{-1} + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \lambda_{F,t}^* Y_t^* \frac{\alpha (1 - \alpha)}{(\alpha \Theta_t + 1 - \alpha)^2} \frac{\dot{\Theta}_t}{\Theta_t} - \mu_{F,t} \frac{\dot{\Theta}_t}{\Theta_t} \end{aligned} \quad (\text{A.39})$$

non-negativity conditions $\lambda_{F,t} \geq 0$, $\lambda_{F,t}^* \geq 0$, $\mu_{F,t} \geq 0$, initial conditions $\lambda_{F,0} = \lambda_{F,0}^* = 0$ for the co-state variables associated with Y_t, Y_t^* , and transversality conditions $\lim_{t \rightarrow \infty} \lambda_{F,t} Y_t = 0$, $\lim_{t \rightarrow \infty} \lambda_{F,t}^* Y_t^* = 0$, and $\lim_{t \rightarrow \infty} \mu_{F,t} \Theta_t = 0$.

Differentiating (A.36) with respect to time, and substituting the co-state laws of motion (A.37), (A.38), (A.39), and using the wedge expressions in (15), yield:

$$\Gamma_F \Theta_t^{-1} e^{\int_0^t (-\zeta_s + \zeta_s^* + \tau_s) ds} = 1 + \Theta_t e^{-\omega_t^*}. \quad (\text{A.40})$$

Taking natural logarithms and differentiating with respect to time, and re-arranging yields

$$-\left(1 + \frac{e^{-\omega_t^*}}{\Theta_t^{-1} + e^{-\omega_t^*}} \times \frac{1 - \alpha}{\alpha \Theta_t + 1 - \alpha}\right) \frac{\dot{\Theta}_t}{\Theta_t} + (\zeta_t^* - \zeta_t + \tau_t) = (1 + \phi) \frac{e^{-\omega_t^*}}{\Theta_t^{-1} + e^{-\omega_t^*}} \frac{\dot{Y}_t^*}{Y_t^*}. \quad (\text{A.41})$$

Integrating (A.30) and (A.31) from 0 to ∞ , and using the initial and transversality conditions for λ_t, λ_t^* yields (A.14) and (A.15). For given paths for i_t, i_t^*, τ_t^* and a given transfer \tilde{b}_0 , the home planner's optimal plan can be described as the particular solution to the system of first-order differential equations in Y_t^*, Θ_t consisting of (A.7) and (A.41), with boundary conditions (A.15) and (A.35). The path of τ_t^* then follows from (A.22); that of Y_t solves (A.6) with boundary condition (A.14); those of $\lambda_{F,t}, \lambda_{F,t}^*$ solve (A.37) and (A.38) with boundary conditions (A.14) and $\lambda_{F,0} = \lambda_{F,0}^* = 0$; and that of $\mu_{F,t}$ follows from (A.36). Finally, the value of Γ_F can then be backed out from (A.39).

A.4.4 Nash equilibrium

A Nash equilibrium of the game is a set of policy actions by the three planners $\{i_t, i_t^*, \tau_t, \tau_t^*\}_{t \geq 0}, b_0$ and associated allocations $\{Y_t, Y_t^*, \Theta_t\}_{t \geq 0}$ such that:

1. Taking $\{\tau_t, \tau_t^*\}_{t \geq 0}$ as given, the actions $\{i_t, i_t^*\}_{t \geq 0}, b_0$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t \geq 0}$ solve the global planner's problem.
2. Taking $\{i_t, i_t^*, \tau_t^*\}_{t \geq 0}$ and b_0 as given, the actions $\{\tau_t\}_{t \geq 0}$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t \geq 0}$ solve the home planner's problem.
3. Taking $\{i_t, i_t^*, \tau_t\}_{t \geq 0}$ and b_0 as given, the actions $\{\tau_t^*\}_{t \geq 0}$ and allocations $\{Y_t, Y_t^*, \Theta_t\}_{t \geq 0}$ solve the foreign planner's problem.

Combining (A.22), (A.34) and (A.41) to eliminate τ_t and τ_t^* yields

$$\begin{aligned} & \left(1 + \frac{e^{-\omega_t}}{\Theta_t + e^{-\omega_t}} \times \frac{1 - \alpha}{\alpha\Theta_t^{-1} + 1 - \alpha} + \frac{e^{-\omega_t^*}}{\Theta_t^{-1} + e^{-\omega_t^*}} \times \frac{1 - \alpha}{\alpha\Theta_t + 1 - \alpha} \right) \frac{\dot{\Theta}_t}{\Theta_t} \\ & + (\zeta_t^* - \zeta_t) + (1 + \phi) \left(\frac{e^{-\omega_t}}{\Theta_t + e^{-\omega_t}} \frac{\dot{Y}_t}{Y_t} - \frac{e^{-\omega_t^*}}{\Theta_t^{-1} + e^{-\omega_t^*}} \frac{\dot{Y}_t^*}{Y_t^*} \right). \end{aligned} \quad (\text{A.42})$$

The Nash equilibrium allocations can be described as the particular solution to the system of first-order differential equations in $Y_t, Y_t^*, \Theta_t, \lambda_{G,t}, \lambda_{G,t}^*, i_t, i_t^*$ consisting of (A.6), (A.7), (A.24), (A.25), (A.26), (A.27) and (A.42), with boundary conditions (A.14), (A.15) and (A.21). The paths of τ_t and τ_t^* follow from (A.22) and (A.34); the transfer is given by (A.23); and the home and foreign lifetime budget constraint multipliers Γ_H and Γ_F can be backed up from (A.33) and (A.40).

A.4.5 Symmetric steady-state

The symmetric steady-state associated with $\zeta_t = \zeta_t^* = 0$ for all $t \geq 0$ (and thus $\Xi_t = \Xi = 1$) is the (unique) stationary point of the above described system. It is given by $Y_t = Y_t^* = \Theta_t = 1$, $\lambda_{G,t} = \lambda_{G,t}^* = 0$, $i_t = i_t^* = \rho$ and $\tau_t = \tau_t^* = 0$. The associated steady-state wedges are hence equal to zero: $\omega_t = \omega_t^* = \varpi_t = 0$, and the multipliers are given $\Gamma_H = \Gamma_F = 2$.

B Proofs appendix

B.1 Proof of Lemma 1

The home labor wedge is given by

$$\begin{aligned} \omega_t &= -\ln \left(\frac{MRS_t}{MPL_t} \right) = -\ln \left(S_t^\alpha \frac{(N_t)^\phi C_t}{1} \right) = -\ln \left(S_t^\alpha \frac{(Y_t)^{1+\phi} C_t}{Y_t} \right) \\ &= -\ln \left(S_t^\alpha \frac{(Y_t)^{1+\phi} C_t}{(1 - \alpha) S_t^\alpha C_t + \alpha S_t^\alpha Q_t C_t^*} \right) \\ &= -\ln \left(\frac{(Y_t)^{1+\phi}}{\alpha\Theta_t^{-1} + 1 - \alpha} \right) \end{aligned}$$

where the second line follows from the home aggregate market clearing condition (5), and the third line follows from the international “risk”-sharing condition (3).

The foreign labor wedge is given by

$$\begin{aligned} \omega_t^* &= -\ln \left(\frac{MRS_t^*}{MPL_t^*} \right) = -\ln \left(S_t^{-\alpha} \frac{(N_t^*)^\phi C_t^*}{1} \right) = -\ln \left(S_t^{-\alpha} \frac{(Y_t^*)^{1+\phi} C_t^*}{Y_t^*} \right) \\ &= -\ln \left(S_t^{-\alpha} \frac{(Y_t^*)^{1+\phi} C_t^*}{(1 - \alpha) S_t^{-\alpha} C_t^* + \alpha S_t^{-\alpha} Q_t^{-1} C_t^*} \right) \\ &= -\ln \left(\frac{(Y_t^*)^{1+\phi}}{\alpha\Theta_t + 1 - \alpha} \right) \end{aligned}$$

where the second line follows from the foreign aggregate market clearing condition (6), and the third line follows from the international “risk”-sharing condition (3).

B.2 Proof of Lemma 2

At a point in time where the home ZLB constraint (A.8) does not bind, the home co-state variable λ_t is zero according to (A.10), home output is at its first-best level $Y_t = Y_t^{\text{fb}}$ according to (A.12), and the home nominal rate, which we refer to as the *unconstrained* home nominal interest rate, is given by

$$i_t = \mathcal{I}_t \equiv \rho + \zeta_t + \frac{\alpha \Theta_t^{-1}}{\alpha \Theta_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) - \frac{1}{1 + \phi} \frac{\alpha \Xi_t^{-1}}{\alpha \Xi_t^{-1} + 1 - \alpha} (\zeta_t^* - \zeta_t). \quad (\text{B.1})$$

Similarly, at a point in time where the foreign ZLB constraint (A.9) does not bind, the foreign co-state variable λ_t^* is zero according to (A.11), foreign output is at its first-best level $Y_t^* = Y_t^{*\text{fb}}$ according to (A.13), and the foreign nominal rate, which we refer to as the *unconstrained* foreign nominal interest rate, is given by

$$i_t^* = \mathcal{I}_t^* \equiv \rho + \zeta_t^* - \frac{\alpha \Theta_t}{\alpha \Theta_t + 1 - \alpha} (\zeta_t^* - \zeta_t + \tau_t - \tau_t^*) + \frac{1}{1 + \phi} \frac{\alpha \Xi_t}{\alpha \Xi_t + 1 - \alpha} (\zeta_t^* - \zeta_t). \quad (\text{B.2})$$

Hence, under a regime of free capital flows where $\tau_t = \tau_t^* = 0$ and (as a result) $\Theta_t = \Xi_t$, the expressions reduce to those in (16) (for $\zeta_t^* = 0$). Similarly, under a regime of closed capital accounts where $\tau_t - \tau_t^* = \zeta_t - \zeta_t^*$ and (as a result) $\Theta_t = 1$, the expressions reduce to those in (20) (for $\zeta_t^* = 0$).

B.3 Proof of Lemma 3

The proof is by construction. First, we observe that the system of first-order differential equations described at the end of Appendix A.2 can be split into two separate systems in Y_t, λ_t, i_t on one hand, and $Y_t^*, \lambda_t^*, i_t^*$ on the other hand. Therefore, one can analyze the solution for these two sets of variables separately.

Starting with the foreign country, it is easy to verify that Assumption 1 implies that the unconstrained foreign interest rate is strictly positive for any t , both under the free capital mobility regime (expression in (16)) and under the closed capital account regime (expression in (20)). It follows that the unconstrained policy is feasible and optimal for the foreign country⁶³ The conditions (A.11), (A.13), (A.15) and $\lambda_0^* = 0$ are hence satisfied with $i_t^* = \mathcal{I}_t^*$, $Y_t^* = Y_t^{*\text{fb}}$ and $\lambda_t^* = 0$ for all $t \geq 0$.

Turning to the home country, it is easy to verify that Assumption 1 implies that the unconstrained home interest rate is strictly negative for $t \in [0, T)$ but strictly positive for $t \geq T$, both under the free capital mobility regime (expression in (16)) and under the closed capital account regime (expression in (20)). We conjecture that the optimal plan consists in setting $i_t = 0$ for $t \in [0, \hat{T})$ and $i_t \geq \hat{T}$ provided

⁶³Notice that the planner’s objective is additively separable in Y_t and Y_t^* , and that Y_t is independent of i_t^* .

$\widehat{T} > T$ is chosen to satisfy

$$0 = \int_0^{\widehat{T}} e^{-\int_0^t (\rho + \zeta_s) ds} \left[\left(\frac{Y_t^{\text{fb}}}{Y_T^{\text{fb}}} \right)^{1+\phi} - e^{(1+\phi) \int_t^{\widehat{T}} (\rho + \zeta_s) ds} \left(\frac{\alpha \Theta_t^{-1} + 1 - \alpha}{\alpha \Theta_{\widehat{T}}^{-1} + 1 - \alpha} \right)^{1+\phi} \right] dt. \quad (\text{B.3})$$

The associated values of the state and co-state variables are given as follows: $Y_t = Y_t^{\text{fb}}$ and $\lambda_t = 0$ for $t \geq \widehat{T}$; and

$$Y_t = Y_T^{\text{fb}} e^{\int_t^{\widehat{T}} (\rho + \zeta_s) ds} \frac{\alpha \Theta_t^{-1} + 1 - \alpha}{\alpha \Theta_{\widehat{T}}^{-1} + 1 - \alpha},$$

and

$$\lambda_t = \frac{1}{Y_t} \int_t^{\widehat{T}} e^{-\int_0^h (\rho + \zeta_s^*) ds} \Xi_h \left[\left(Y_h^{\text{fb}} \right)^{1+\phi} - (Y_h)^{1+\phi} \right] dh$$

for $t \in [0, \widehat{T})$. It is straightforward to verify that this plan (by construction) satisfies all relevant conditions (A.10), (A.12), (A.14) and $\lambda_0 = 0$.

B.4 Proof of Lemma 6

Substituting (A.22) and the definition of the international wedge $\varpi_t \equiv \ln \Theta_t - \ln \Xi_t$ into (A.33) and (A.40) yields (27) and (28), respectively.

B.5 Proof of Proposition 1

Claim 1. Defining the functions

$$\begin{aligned} f_1(z) &\equiv \int_0^T e^{-(\rho - \bar{\zeta})t} \left[\left(\frac{Y_t^{\text{fb}}}{Y_T^{\text{fb}}} \right)^{1+\phi} - \left(\frac{\alpha \Theta_t^{-1} + 1 - \alpha}{\alpha \Theta_T^{-1} + 1 - \alpha} \right)^{1+\phi} e^{(1+\phi) \int_t^T (\rho + \zeta_s) ds} \right] dt, \\ f_2(z) &\equiv -e^{\bar{\zeta}} \int_T^z e^{-\rho t} \left[1 - e^{(1+\phi)\rho(z-t)} \right] dt, \end{aligned}$$

condition (B.3) can be written as

$$f_1(\widehat{T}) = f_2(\widehat{T}). \quad (\text{B.4})$$

The functions satisfy $f_1'(z) < 0$, $f_2'(z) > 0$, with $f_1(T) > 0$, $f_2(T) = 0$, $\lim_{z \rightarrow -\infty} f_1(z) = +\infty$.⁶⁴ (B.4) therefore has a unique solution $\widehat{T} > T$.

Now, observe that under free capital mobility, $\Theta_t^{-1} > \Theta_T^{-1}$ for $t < T$, and $\Theta_t = \Theta_T$ for $t \geq T$, while under closed capital accounts $\Theta_t = 1$ for all $t \geq 0$. As a result, we have $f_1^{\text{free}}(z) < f_1^{\text{closed}}(z)$ and $f_2^{\text{free}} = f_2^{\text{closed}}$ for $z > T$. It must thus be that $\widehat{T}^{\text{free}} < \widehat{T}^{\text{closed}}$.

Claim 2. We observe that under both capital flow regimes under consideration, the growth rate of home output is given by $-\rho$ for $t \in [T, \widehat{T})$. Since $\ln Y_{\widehat{T}^{\text{closed}}}^{\text{closed}} = \ln Y_{\widehat{T}^{\text{free}}}^{\text{free}} = \ln Y_T^{\text{fb}}$, it must be that

⁶⁴A sufficient condition for $f_1(T) > 0$ is that $\dot{Y}_t/Y_t > 0$ for $t \in [0, T)$. Assumption 1 ensures that this holds true under $i_t = 0$ for both the regime of free capital mobility and the regime of closed capital accounts.

$\ln Y_T^{\text{closed}} > \ln Y_T^{\text{free}} > \ln Y_T^{\text{fb}}$. This in turn implies, in light of the higher growth rate of home output under closed capital accounts ($-\rho + \bar{\zeta} > 0$) than under free capital mobility ($-\rho + \bar{\zeta}(1 - \alpha)/(\alpha\Theta_t^{-1} + 1 - \alpha) > 0$), that $\ln Y_0^{\text{closed}} < \ln Y_0^{\text{free}} < \ln Y_0^{\text{fb}}$. Claim #2 follows immediately.

C Model appendix (online appendix)

C.1 Home household

Using the price index definitions, the home household's budget constraint (1) can be expressed as

$$\dot{a}_t = i_t a_t + W_t N_t + T_t + \Pi_t - P_t C_t + \left(i_t^* - i_t + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} \right) \mathcal{E}_t D_{F,t}. \quad (\text{C.1})$$

for net foreign assets $a_t \equiv D_{H,t} + \mathcal{E}_t D_{F,t}$. Expenditure minimization requires $C_{H,t}(l) = \left(\frac{P_{H,t}(l)}{P_{H,t}} \right)^{-\epsilon} C_{H,t}$, $C_{F,t}(l) = \left(\frac{P_{F,t}(l)}{P_{F,t}} \right)^{-\epsilon} C_{F,t} \forall l$, $C_{H,t} = (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t$ and $C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t$. The households' optimality conditions for labor supply, home currency bonds and foreign currency bonds are given by

$$\begin{aligned} \frac{W_t}{P_t} &= N_t^\phi C_t, \\ \frac{\dot{C}_t}{C_t} &= i_t - \pi_t - (\rho + \zeta_t), \end{aligned} \quad (\text{C.2})$$

$$\frac{\dot{C}_t}{C_t} = i_t^* + \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t - (\rho + \zeta_t). \quad (\text{C.3})$$

C.2 Foreign household

Preferences of the foreign households are represented by the utility functional

$$\int_0^\infty e^{-\int_0^t (\rho + \zeta_h^*) dh} \left[\ln C_t^* - \frac{(N_t^*)^{1+\phi}}{1+\phi} \right] dt$$

with $C_t^* \equiv \left(C_{F,t}^* \right)^{1-\alpha} \left(C_{H,t}^* \right)^\alpha / [(1-\alpha)^{1-\alpha} \alpha^\alpha]$, $C_{F,t}^* \equiv \left[\int_0^1 C_{F,t}^*(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$ and $C_{H,t}^* \equiv \left[\int_0^1 C_{H,t}^*(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$. Its budget constraint expressed in its own currency is given by

$$\frac{\dot{D}_{H,t}^*}{\mathcal{E}_t} + \dot{D}_{F,t}^* = (i_t + \tau_t^* - \tau_t) \frac{D_{H,t}^*}{\mathcal{E}_t} + i_t^* D_{F,t}^* + W_t^* N_t^* + T_t^* + \Pi_t^* - \int_0^1 P_{H,t}^*(l) C_{H,t}^*(l) dl - \int_0^1 P_{F,t}^*(l) C_{F,t}^*(l) dl$$

Defining $a_t^* \equiv D_{H,t}^* / \mathcal{E}_t + D_{F,t}^*$ as the foreign household's net assets in foreign currency terms and making use of the price index definitions, the budget constraint can be expressed as

$$a_t^* = i_t^* a_t^* + W_t^* N_t^* + T_t^* + \Pi_t^* - P_t^* C_t^* + \left(i_t - i_t^* + \tau_t^* - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} \right) \frac{D_{H,t}^*}{\mathcal{E}_t}. \quad (\text{C.4})$$

Expenditure minimization requires $C_{F,t}^*(l) = \left(\frac{P_{F,t}^*(l)}{P_{F,t}^*} \right)^{-\epsilon} C_{F,t}^*$, $C_{H,t}^*(l) = \left(\frac{P_{H,t}^*(l)}{P_{H,t}^*} \right)^{-\epsilon} C_{H,t}^* \forall l$, $C_{F,t}^* = (1 - \alpha) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^*$ and $C_{H,t}^* = \alpha \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-1} C_t^*$. The households' optimality conditions for labor supply,

home currency bonds and foreign currency bonds are given by

$$\begin{aligned}\frac{W_t^*}{P_t^*} &= (N_t^*)^\phi C_t^*, \\ \frac{\dot{C}_t^*}{C_t^*} &= i_t + \tau_t^* - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t^* - (\rho + \zeta_t^*),\end{aligned}\tag{C.5}$$

$$\frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* - (\rho + \zeta_t^*).\tag{C.6}$$

C.3 International “risk”-sharing condition

Subtracting Foreign’s Euler equation for the home currency bond (C.5) from Home’s Euler equation for the home currency bond (C.2) yields

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*} = \tau_t - \tau_t^* + \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} + \pi_t^* - \pi_t + \zeta_t^* - \zeta_t,$$

which can be rewritten as

$$\frac{d}{ds} \left[\ln \left(\frac{C_s}{C_s^* Q_s} \right) \right] = \tau_s - \tau_s^* + \zeta_s^* - \zeta_s.$$

Integrating from 0 to t , we obtain the international “risk”-sharing condition (3), or

$$C_t = \Theta_t C_t^* Q_t,\tag{C.7}$$

with $\Theta_t \equiv \Theta_0 \exp \left[\int_0^t (\zeta_s^* - \zeta_s + \tau_s - \tau_s^*) ds \right]$.

C.4 Goods market equilibrium

Clearing on the market for variety l in Home requires

$$\begin{aligned}Y_t(l) &= C_{H,t}(l) + C_{H,t}^*(l) \\ &= \left(\frac{P_{H,t}(l)}{P_{H,t}} \right)^{-\epsilon} \left[(1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t + \alpha \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*} \right)^{-1} C_t^* \right]\end{aligned}$$

Given price symmetry ($P_{H,t}(l) = P_{H,t} \forall l$), substituting this equation into the definition of home aggregate output $Y_t \equiv \left[\int_0^1 Y_t(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$, we obtain

$$\begin{aligned}Y_t &= (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t + \alpha \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*} \right)^{-1} C_t^* \\ &= (1 - \alpha) S_t^\alpha C_t + \alpha S_t^\alpha Q_t C_t^* \\ &= [(1 - \alpha) \Theta_t + \alpha] S_t^\alpha Q_t C_t^*\end{aligned}\tag{C.8}$$

Similarly, market clearing for foreign aggregate output $Y_t^* \equiv \left[\int_0^1 Y_t^*(l)^{\frac{\epsilon-1}{\epsilon}} dl \right]^{\frac{\epsilon}{\epsilon-1}}$ requires

$$\begin{aligned} Y_t^* &= (1-\alpha) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-1} C_t^* + \alpha \left(\frac{P_{F,t}^*}{P_t/\mathcal{E}_t} \right)^{-1} C_t \\ &= (1-\alpha) S_t^{-\alpha} C_t^* + \alpha S_t^{-\alpha} Q_t^{-1} C_t \\ &= [(1-\alpha) + \alpha \Theta_t] S_t^{-\alpha} C_t^* \end{aligned} \tag{C.9}$$

C.5 Intertemporal budget constraints

C.5.1 Home intertemporal budget constraint

The equilibrium lump-sum rebate T_t in Home is the sum of the negative of the subsidy expenses on outflows and the tax proceeds on inflows:

$$\begin{aligned} T_t &= -\tau_t \mathcal{E}_t D_{F,t} + \tau_t D_{H,t}^* \\ &= -\tau_t a_t + \tau_t (D_{H,t} + D_{H,t}^*) \\ &= -\tau_t a_t, \end{aligned} \tag{C.10}$$

where the second line follows from the definition of home net foreign assets $a_t = D_{H,t} + \mathcal{E}_t D_{F,t}$, and the third line follows from the market clearing condition for home currency bond $D_{H,t} + D_{H,t}^* = 0$.

Substituting $\Pi_t = P_{H,t} Y_t - W_t N_t$ and (C.10) into the home household' budget constraint (C.1), while recognizing that the Euler equations (C.2)-(C.3) (or, for that matter, (C.5)-(C.6)) imply a distorted interest parity condition $i_t - i_t^* - \tau_t + \tau_t^* - \dot{\mathcal{E}}_t/\mathcal{E}_t = 0$, we obtain Home's resource constraint:

$$\dot{a}_t = (i_t - \tau_t) a_t + P_{H,t} Y_t - P_t C_t.$$

Expressed in terms of the marginal utility of foreign agents (i.e., normalizing by $P_t^* \mathcal{E}_t C_t^*$), noting that $P_t = \mathcal{E}_t^\alpha$ and $P_t^* = \mathcal{E}_t^{-\alpha}$, the resource constraint is given by

$$\dot{b}_t = \left(i_t - \tau_t - \frac{\dot{\mathcal{E}}_t}{\mathcal{E}_t} - \pi_t^* - \frac{\dot{C}_t^*}{C_t^*} \right) b_t + (C_t^*)^{-1} \left(S_t^{-(1-\alpha)} Y_t - Q_t^{-1} C_t \right),$$

for $b_t \equiv a_t/(P_t^* C_t^* \mathcal{E}_t)$. Substituting Foreign's Euler equation for the home currency bond (C.5) yields a current account equation given by

$$\dot{b}_t = (\rho + \zeta^* - \tau_t^*) b_t - (C_t^*)^{-1} \left(Q_t^{-1} C_t - S_t^{-(1-\alpha)} Y_t \right).$$

Integrating from 0 to ∞ , while imposing a no-Ponzi game condition, yields the intertemporal budget constraint

$$b_0 = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (C_t^*)^{-1} \left(Q_t^{-1} C_t - S_t^{-(1-\alpha)} Y_t \right) dt.$$

Using the home good's market clearing condition (C.8) and the international "risk"-sharing condition

(C.7), this constraint can be expressed as (11).

C.5.2 Foreign intertemporal budget constraint

The derivation is similar to that of the home intertemporal budget constraint. The equilibrium lump-sum rebate T_t^* in Foreign is the sum of minus the subsidy expenses on outflows and the tax proceeds on inflows:

$$\begin{aligned} T_t^* &= -\tau_t^* \frac{D_{H,t}^*}{\mathcal{E}_t} + \tau_t^* D_{F,t} \\ &= -\tau_t^* a_t^* + \tau_t^* (D_{F,t}^* + D_{F,t}) \\ &= -\tau_t^* a_t^*, \end{aligned} \tag{C.11}$$

where, again, the second line follows from the definition of foreign net assets $a_t^* \equiv D_{H,t}^*/\mathcal{E}_t + D_{F,t}^*$, and the third line follows from the market clearing condition for the foreign currency bond $D_{F,t}^* + D_{F,t} = 0$.

Substituting $\Pi_t^* = P_{F,t}^* Y_t^* - W_t^* N_t^*$ and (C.11) into the foreign household' budget constraint (C.4), while recognizing that the Euler equations (C.2)-(C.3) (or, for that matter, (C.5)-(C.6)) imply a distorted interest parity condition $i_t - i_t^* - \tau_t + \tau_t^* - \dot{\mathcal{E}}_t/\mathcal{E}_t = 0$, we obtain Foreign's resource constraint:

$$\dot{a}_t^* = (i_t^* - \tau_t^*) a_t^* + P_{F,t}^* Y_t^* - P_t^* C_t^*. \tag{C.12}$$

Expressed in terms of the marginal utility of foreign agents (i.e., normalizing by $P_t^* C_t^*$), noting that $P_t = \mathcal{E}_t^\alpha$ and $P_t^* = \mathcal{E}_t^{-\alpha}$, the resource constraint is given by

$$\dot{b}_t^* = \left(i_t^* - \tau_t^* - \pi_t^* - \frac{\dot{C}_t^*}{C_t^*} \right) b_t^* - (C_t^*)^{-1} (C_t^* - S_t^{-\alpha} Y_t^*),$$

for $b_t^* \equiv a_t^*/(P_t^* C_t^*)$. Substituting Foreign's Euler equation for the foreign currency bond (C.6) yields a current account equation given by

$$\dot{b}_t^* = (\rho + \zeta_t^* - \tau_t^*) b_t^* - (C_t^*)^{-1} (C_t^* - S_t^{-\alpha} Y_t^*).$$

Integrating from 0 to ∞ , while imposing a no-Ponzi game condition, yields the intertemporal budget constraint

$$b_0^* = \int_0^\infty e^{-\int_0^t (\rho + \zeta_s^* - \tau_s^*) ds} (C_t^*)^{-1} (C_t^* - S_t^{-\alpha} Y_t^*) dt.$$

Using the foreign good's market clearing condition (C.9), this constraint can be expressed as (11), with $-b_0^*$ on the left-hand side instead of b_0 .

Alternatively, expressing (C.12) in terms of the marginal utility of home agents (i.e., normalizing by $P_t C_t/\mathcal{E}$), and substituting the home Euler equation for the home bond (C.2) and the interest parity condition, the current account equation is given by

$$\dot{\tilde{b}}_t^* = (\rho + \zeta_t - \tau_t) \tilde{b}_t^* - C_t^{-1} (Q_t C_t^* - Y_t^* S_t^{1-\alpha}),$$

for $\tilde{b}_t^* \equiv a_t^* \mathcal{E}_t / (P_t C_t)$. Integrating from 0 to ∞ , while imposing a no-Ponzi game condition and using the foreign good's market clearing condition (C.9) yields

$$\tilde{b}_0^* = \alpha \int_0^\infty e^{-\int_0^t (\rho + \zeta_s - \tau_s) ds} (\Theta_t^{-1} - 1) dt.$$

This version of Foreign's lifetime budget constraint does not explicitly feature τ_t^* , and is therefore more convenient to work with in setting up the foreign planner's problem in Appendix A.4.3.