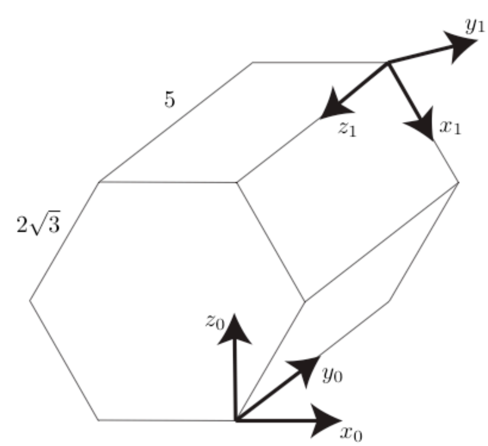
Fall 2022—16-642 Manipulation, Estimation and Control
Problem Set 4

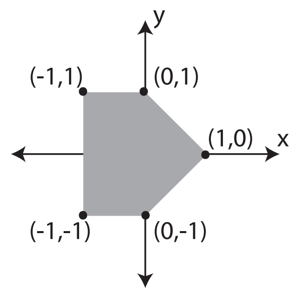
Due: 5 December 2022

- **Submit write up:** You must *neatly* write up (preferably type up) your solutions and submit it electronically via canvas.
- **Collaboration Policy** You are encouraged to work with other students in the class, however you must turn in your own *unique* solution. If you work with others, you must list their names on your submission.
- **Time and late Policy**: Submit via canvas by the start of the lecture on the due date. If you do not turn your problem set in on time, you can turn it in up to 48 hours later but you will lose half of the points. After 48 hours, you will receive a zero.

1. (10 points) Consider the 3D shape pictured below. Find H_1^0. The front and back faces are regular hexagons, the length of each side is $2\sqrt{3}$. The six side faces are all identical rectangles that have two sides with length $2\sqrt{3}$ and two sides with length 5. The interior angles of a regular hexagon are 120^o.



2. (15 points) **Fun With SE(2):** Consider the planar rigid body defined below. It is possible to "move" this body by transforming all of the corner points according to a 2D homogeneous transformation matrix and plotting the result.



Consider the two transformation matrices:

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.866 & 0.500 & 0 \\ -0.500 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Move the rigid body according to the motions described below and plot the results. (It's probably best to write some sort of program to do this – we recommend using MATLAB, but you can use any language you want. Please include the source file of your program with your solution.)

(a) A, relative to the fixed frame.

(b) A, relative to the fixed frame, followed by B, relative to the current frame.

(c) A, relative to the fixed frame, followed by B, relative to the fixed frame.

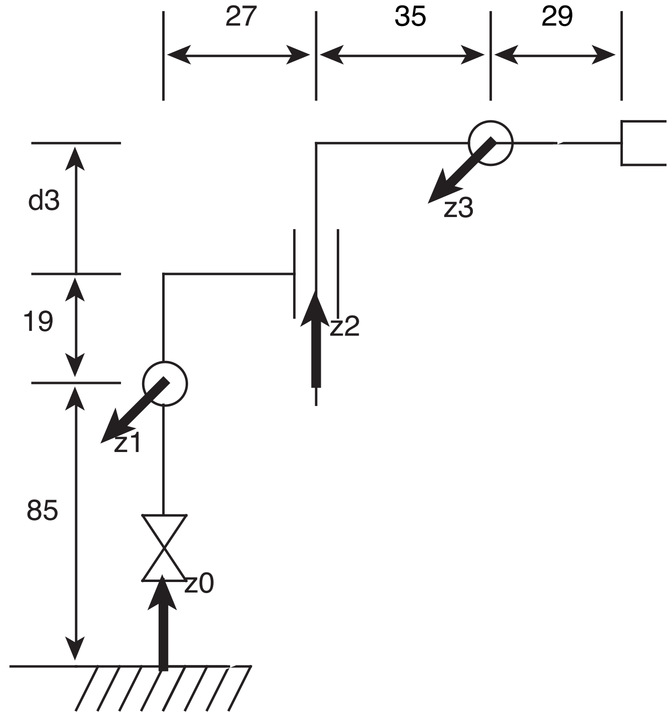
(d) B, relative to the fixed frame.

(e) B, relative to the fixed frame, followed by A, relative to the fixed frame.

(f) B, relative to the fixed frame, followed by A, relative to the current frame.

3. (10 points) A multipurpose robot arm will be picking up a screwdriver and then trying to use it to reach a screw. Sensors on the robot's wrist will determine the pose of the tip of the screwdriver in the wrist's frame H_t^w. A camera mounted on the ceiling will determine the pose of the slot in the top of a screw in the camera's frame H_s^c. The location of the camera in the robot arm base frame is a known quantity H_c^b. The robot arm's wrist postition can be programmed directly by specifying the homogeneous transform that describes the desired wrist postition in the base frame H_w^b. What should H_w^b be for the tip of the screwdriver to meet the slot in the screw? (assume that the frames on the screwdriver and screw are defined so that they are colocated when the tip of the screwdriver is inserted in the slot).

4. (20 points) For the manipulator drawn below, draw the location of the DH frames and create a table of DH parameters. The positive direction of each joint is depicted by the z axis associated for that joint, which has conveniently been included for you. For each frame, explain whether the frame was uniquely defined by the DH convention. If it was not, describe the choices you made in defining it. (and don't forget to include the last frame!)

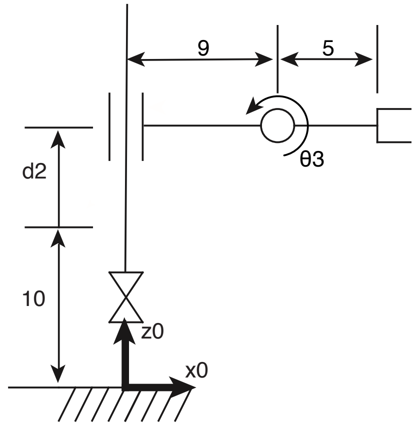


5. (15 points) Given the DH parameters in the table below, draw the manipulator that they describe. Assume that frame 0 is oriented with the z-axis pointing up, x-axis pointing right, and y-axis pointing into the page.

i	θ_i	d_i	a_i	α_i
1	θ_1	5	7	90^o
2	θ_2	0	2	-90^o
3	0	ℓ_3	0	90^o
4	θ_4	0	3	0

hint: First use the parameters to determine where all of the frames should be, then determine where the joints should go, then draw in the links connecting them.

6. Consider the manipulator drawn below in a configuration where $\theta_1 = \theta_3 = 0$ and the task space is assumed to be the position only of the end effector (i.e., $\mathbb{X} = \mathbb{R}^3$). The positive direction of the first joint is given by the z_0 axis.



- (10 points) Find the Jacobian $J(\Theta)$ using the direct differentiation method.
- (10 points) Find the Jacobian $J(\Theta)$ using the column-by-column building method. Make sure to explain your answer.
- (10 points) Are there any singular configurations? If so list them. You can use whatever method you want to find them, but make sure to explain your answer.