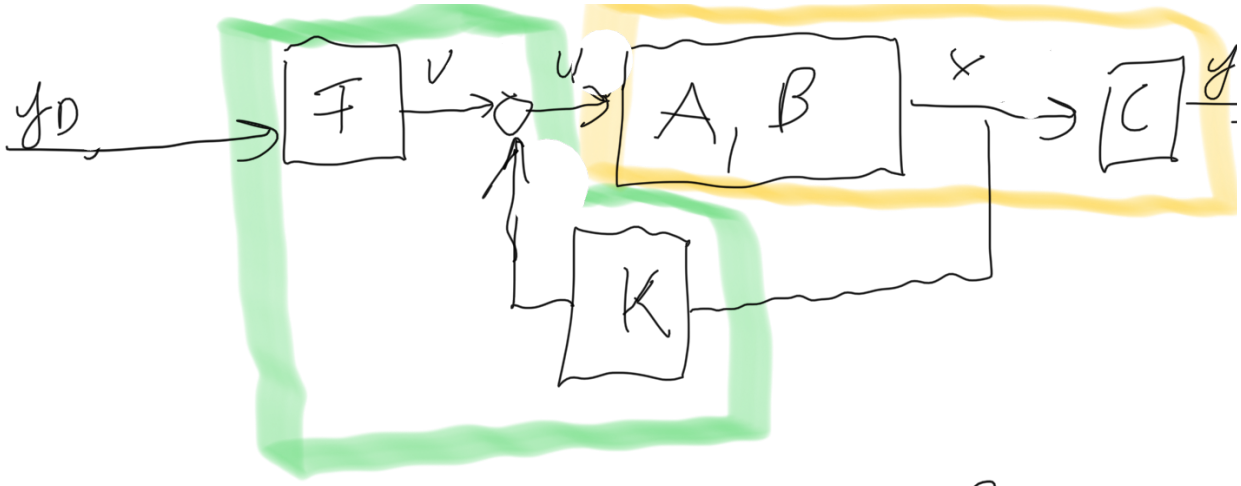
Lecture 4 Handwritten Notes



F? $\dot{x} = 0 = Ax_e - BKx_e + Bv$

$0 = (A - BK)x_e + Bv$

$-(A - BK)x_e = Bv \Rightarrow x_e = -(A - BK)^{-1}Bv$

$y_e = Cx_e = -C(A - BK)^{-1}Bv$

$v = \underbrace{(-C(A - BK)^{-1}B)^{-1}}_{F} y_D$

$y_D = r$

What if we have disturbances?

⇒ Add an integrator

How can we add that to our system?

$$z = \int_0^t (y - r)\, dt$$

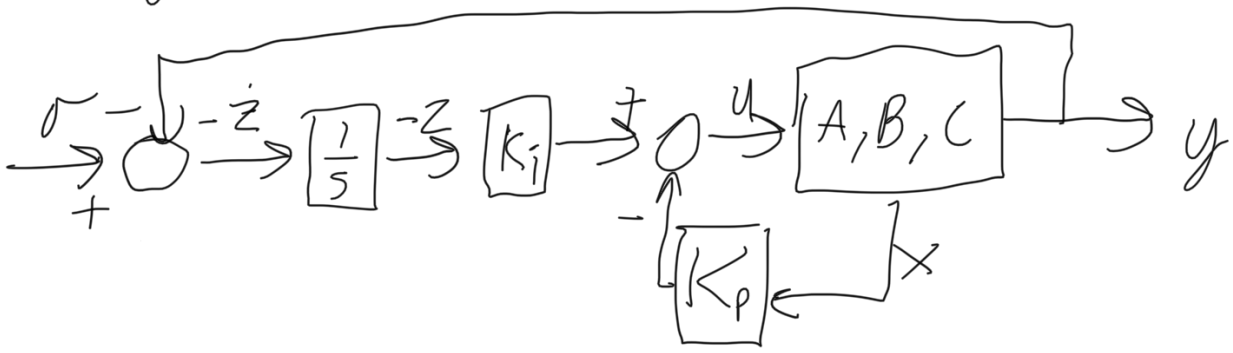
Plant:

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx \quad (2)$$

Integral state:

$$\dot{z} = y - r \overset{(2)}{=} Cx - r \quad (3)$$



Control:

$$u = -K_p x - K_i z = \begin{bmatrix} K_p & K_i \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}$$

$$= -K x_a \quad (4)$$

$$\Rightarrow K = \begin{bmatrix} K_p & K_i \end{bmatrix} \wedge x_a = \begin{bmatrix} x \\ z \end{bmatrix}$$

New System Dynamics:

(1), (4) ⇒

$$\dot{x} = Ax + B(-K_p x - K_i z)$$

$$= (A - Bk_p)x - Bk_i z$$

③ $\Rightarrow$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \underbrace{\begin{bmatrix} A - Bk_p & \vdots & -Bk_i \\ C & \vdots & 0 \end{bmatrix}}_{\text{New } A} \begin{bmatrix} x \\ z \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}_{\text{New } B}$$

$\Downarrow$

Now can apply LQR etc...