Q6. Jacobian of Given Manipulator

1. Direct Differentiation Method

```
% Creating symbolic variables for the manipulator
syms theta1 d2 theta3

% The DH matrix
DH = [[theta1, 10+d2, 0,  0];
      [0,      0,     9,  pi/2];
      [theta3, 0,     5,  0]]
```

DH =

$$\begin{pmatrix} \theta_1 & d_2+10 & 0 & 0 \\ 0 & 0 & 9 & \frac{\pi}{2} \\ \theta_3 & 0 & 5 & 0 \end{pmatrix}$$

```
num_joints = size(DH,1)
```

num_joints = 3

```
for joints = 1:num_joints

    t_i = DH(joints,1); % theta_i
    d_i = DH(joints,2);
    a_i = DH(joints,3);
    alpha_i = DH(joints,4);

    H_mat = [
             [cos(t_i), -sin(t_i)*cos(alpha_i), sin(t_i)*sin(alpha_i),  a_i*cos(t_i)];
             [sin(t_i), cos(t_i)*cos(alpha_i),  -cos(t_i)*sin(alpha_i), a_i*sin(t_i)];
             [0,        sin(alpha_i),           cos(alpha_i),           d_i         ];
             [0,        0,                      0,                      1           ]
            ];
    if joints == 1
        H_1 = H_mat
    elseif joints == 2
        H_2 = H_mat
    else
        H_3 = H_mat
    end
end
```

H_1 =

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_2+10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_2 =

$$\begin{pmatrix} 1 & 0 & 0 & 9 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

H_3 =

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 5\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 5\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% Finding the final transformation matrix

H_end_to_base = H_1*H_2*H_3
```

H_end_to_base =

$$\begin{pmatrix} \cos(\theta_1)\cos(\theta_3) & -\cos(\theta_1)\sin(\theta_3) & \sin(\theta_1) & 9\cos(\theta_1)+5\cos(\theta_1)\cos(\theta_3) \\ \cos(\theta_3)\sin(\theta_1) & -\sin(\theta_1)\sin(\theta_3) & -\cos(\theta_1) & 9\sin(\theta_1)+5\cos(\theta_3)\sin(\theta_1) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & d_2+5\sin(\theta_3)+10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
f_theta = H_end_to_base(1:3,4)
```

f_theta =

$$\begin{pmatrix} 9\cos(\theta_1)+5\cos(\theta_1)\cos(\theta_3) \\ 9\sin(\theta_1)+5\cos(\theta_3)\sin(\theta_1) \\ d_2+5\sin(\theta_3)+10 \end{pmatrix}$$

```
jacob = jacobian(f_theta, [theta1, d2, theta3])
```

jacob =

$$\begin{pmatrix} -9\sin(\theta_1)-5\cos(\theta_3)\sin(\theta_1) & 0 & -5\cos(\theta_1)\sin(\theta_3) \\ 9\cos(\theta_1)+5\cos(\theta_1)\cos(\theta_3) & 0 & -5\sin(\theta_1)\sin(\theta_3) \\ 0 & 1 & 5\cos(\theta_3) \end{pmatrix}$$

2. Column-by-column Method

Building the 1st column of the Jacobian (revolute)

```
% Find R
R_0_to_0 = eye(3);

d_3_to_0 = H_end_to_base(1:3,4);

v_1_to_0 = R_0_to_0 * cross([0;0;1], d_3_to_0);
```

Building the 2nd column of the Jacobian (prismatic)

```
% Find R
R_1_to_0 = H_1(1:3,1:3); % H_1 was found previously

v_2_to_0 = R_1_to_0 * [0;0;1];
```

Building the 3rd column of the Jacobian (prismatic)

```
% Find R
H_2_to_0 = H_1 * H_2;
R_2_to_0 = H_2_to_0(1:3,1:3);

d_3_to_2 = H_3(1:3,4);

v_3_to_0 = R_2_to_0 * cross([0;0;1], d_3_to_2);
```

Combining the Columns

```
% We can see that the second jacobian (column-by-column building method)
% will be the same as the first jacobian (direct differentiation method)
jacob_2 = [v_1_to_0, v_2_to_0, v_3_to_0]
```

jacob_2 =

$$\begin{pmatrix} -9\sin(\theta_1) - 5\cos(\theta_3)\sin(\theta_1) & 0 & -5\cos(\theta_1)\sin(\theta_3) \\ 9\cos(\theta_1) + 5\cos(\theta_1)\cos(\theta_3) & 0 & -5\sin(\theta_1)\sin(\theta_3) \\ 0 & 1 & 5\cos(\theta_3) \end{pmatrix}$$

```
% substituting actual values of theta1 and theta3 into jacobian we get:
theta1 = 0;
theta3 = 0;

% subs(jacob_2)
```

3. Singular Configurations

Singular Configurations can be found by checking for a rank reduction in the Jacobian. This is done by finding the determinant of the Jacobian and setting its value to zero:

```
jacob_det = det(jacob);

eqn = jacob_det == 0
```

eqn =

$$-45\cos(\theta_1)^2\sin(\theta_3) - 45\sin(\theta_1)^2\sin(\theta_3) - 25\cos(\theta_1)^2\cos(\theta_3)\sin(\theta_3) - 25\cos(\theta_3)\sin(\theta_1)^2\sin(\theta_3) = 0$$