Manipulation Estimation and Controls: Assignment 2

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Q1. Given open loop transfer function

```
G = tf(200, [1 22 141 2])
```

```
G =

          200
  ------------------------
  s^3 + 22 s^2 + 141 s + 2

Continuous-time transfer function.
```

a. Closed loop transfer function with unity feedback (assumig negative feedback)

```
sys1 = feedback(G,1)
```

```
sys1 =

           200
  --------------------------
  s^3 + 22 s^2 + 141 s + 202

Continuous-time transfer function.
```

b. Poles and Zeros of closed loop transfer function

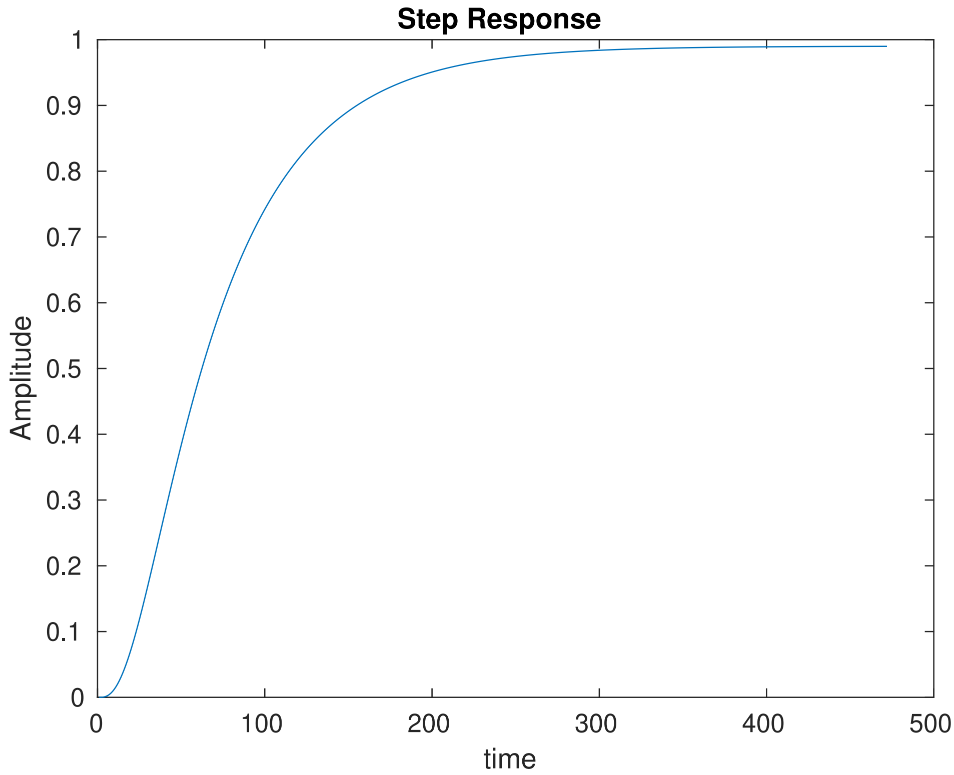
The poles are shown below, but the system has no zeros

```
[zeros, poles, gains] = tf2zp(200,[1 22 141 202])
```

```
zeros =

  0x1 empty double column vector
poles = 3x1 complex
 -10.0000 + 1.0000i
 -10.0000 - 1.0000i
  -2.0000 + 0.0000i

gains = 200
```

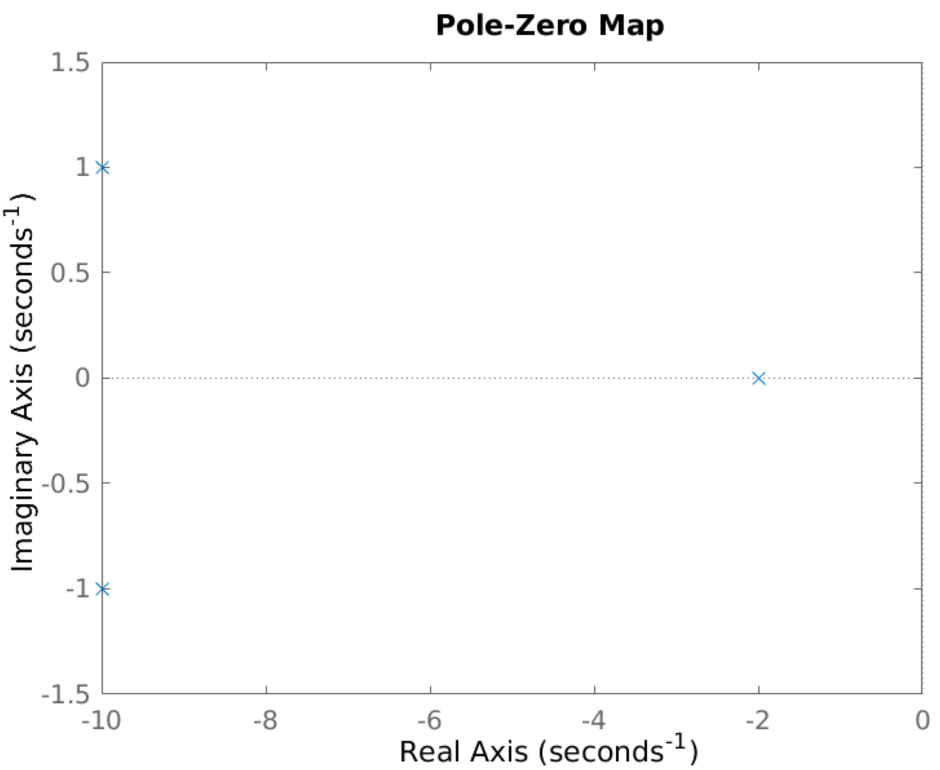
c. Plot y(t) using step function

```
y = step(sys1);
plot(y)
title('Step Response')
ylabel('Amplitude')
xlabel('time')
```



Plotting the poles of the system

```
pzplot(sys1)
```



The pole at (-2 ± 0i) is dominant since it's closer to the imaginary axis. Also the poles at (-10 ± i) do not seem to be causing any oscillations and are therefore not dominant.

d. Steady state value using final value theorem

It is found below that the steady state value = 0.99

1d.) The closed loop transfer function was :

$$T(s) = \frac{Y(s)}{U(s)} = \frac{200}{s^3 + 22s^2 + 141s + 202}$$

According to final value theorem:

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s) \quad — (1)$$

for step input: $Y(s) = \frac{H(s)}{s} \quad — (2)$

Substituting eq (2) in (1)

$$\therefore y_s = \lim_{s \to 0} H(s)$$

$$y_s = \frac{200}{202}$$

$$\boxed{y_s = 0.99}$$

Q2. Implementation of PID Controller

Given Transfer function:

2.) The transfer function is given as:

$$G(s) = \frac{s+10}{s^4 + 71s^3 + 1070s^2 + 1000s}$$

Let $G(s) = \frac{B}{A}$

$\therefore$ adding the K_p, K_d & K_i feedback we get:

$$T_{cl} = \frac{\left(K_p + sK_d + \frac{K_i}{s}\right) B/A}{1 + \left(K_p + sK_d + K_i/s\right) B/A}$$

$$= \frac{\left(K_p + sK_d + \frac{K_i}{s}\right) . B}{A + \left(K_p + sK_d + \frac{K_i}{s}\right) B}$$

$$= \frac{\left(K_p + sK_d + K_i/s\right)(s+10)}{\left(s^4 + 71s^3 + 1070s^2 + 1000s\right) + \left(K_p + sK_d + K_i/s\right)(s+10)}$$

$$T_{ce} = \frac{s\left(K_p + 10K_d\right) + s^2K_d + K_i + 10\left(K_p + K_i/s\right)}{\left[s^4 + 71s^3 + s^2\left(1070 + K_d\right) + s\left(1000 + 10K_d + K_p\right) + K_i + 10\left(K_p + K_i/s\right)\right]}$$

Multiplying numerator and denominator by s, we get:

$$\boxed{T_{ce} = \frac{s^2\left(K_p + 10K_d\right) + s^3K_d + s\left(K_i + 10K_p\right) + 10K_i}{\left[s^5 + 71s^4 + s^3\left(1070 + K_d\right) + s^2\left(1000 + 10K_d + K_p\right) + s\left(K_i + 10K_p\right) + 10K_i\right]}}$$

The PID controller was tuned using the below mentioned gains.

- Rise Time = 0.397s
- Maximum Percent Overshoot = 0.07

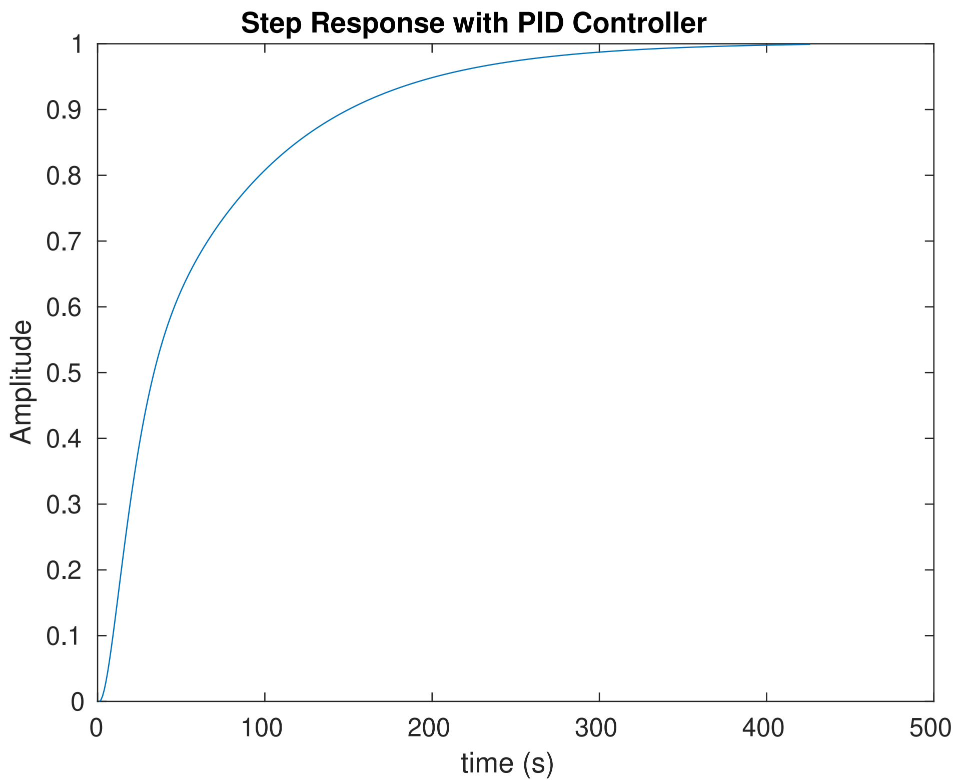
```
Kp = 600;
Kd = 590;
Ki = 4.7;
```

Plotting the step response and calculating the steady-state error:

```
G2 = tf([Kd, Kp+10*Kd, Ki+10*Kp, 10*Ki],[1, 71, 1070+Kd, 1000+10*Kd+Kp, Ki+10*Kp, 10*Ki])
```

```
G2 =

          590 s^3 + 6500 s^2 + 6005 s + 47
  ------------------------------------------------
  s^5 + 71 s^4 + 1660 s^3 + 7500 s^2 + 6005 s + 47

Continuous-time transfer function.
```

```
y2 = step(G2);
plot(y2)
title('Step Response with PID Controller')
ylabel('Amplitude')
xlabel('time (s)')
```



```
stepinfo(G2)
```

```
ans =
         RiseTime: 0.3965
    TransientTime: 0.7538
     SettlingTime: 0.7538
      SettlingMin: 0.9013
      SettlingMax: 1.0007
        Overshoot: 0.0737
       Undershoot: 0
             Peak: 1.0007
         PeakTime: 1.4050
```

```
% Find the steady state error
% expected step output is 1
y2(end)
```

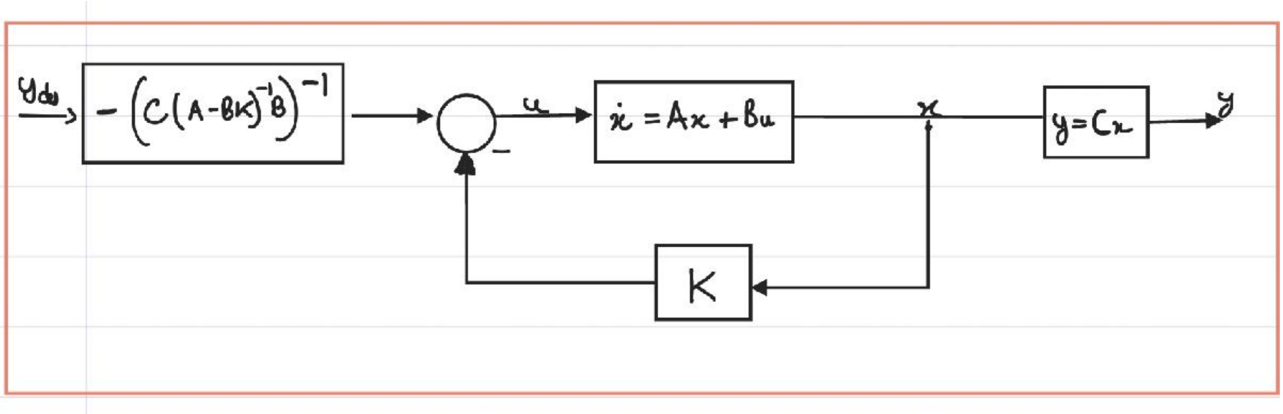
```
ans = 0.9990
```

```
std_state_error = 1 - y2(end)
```

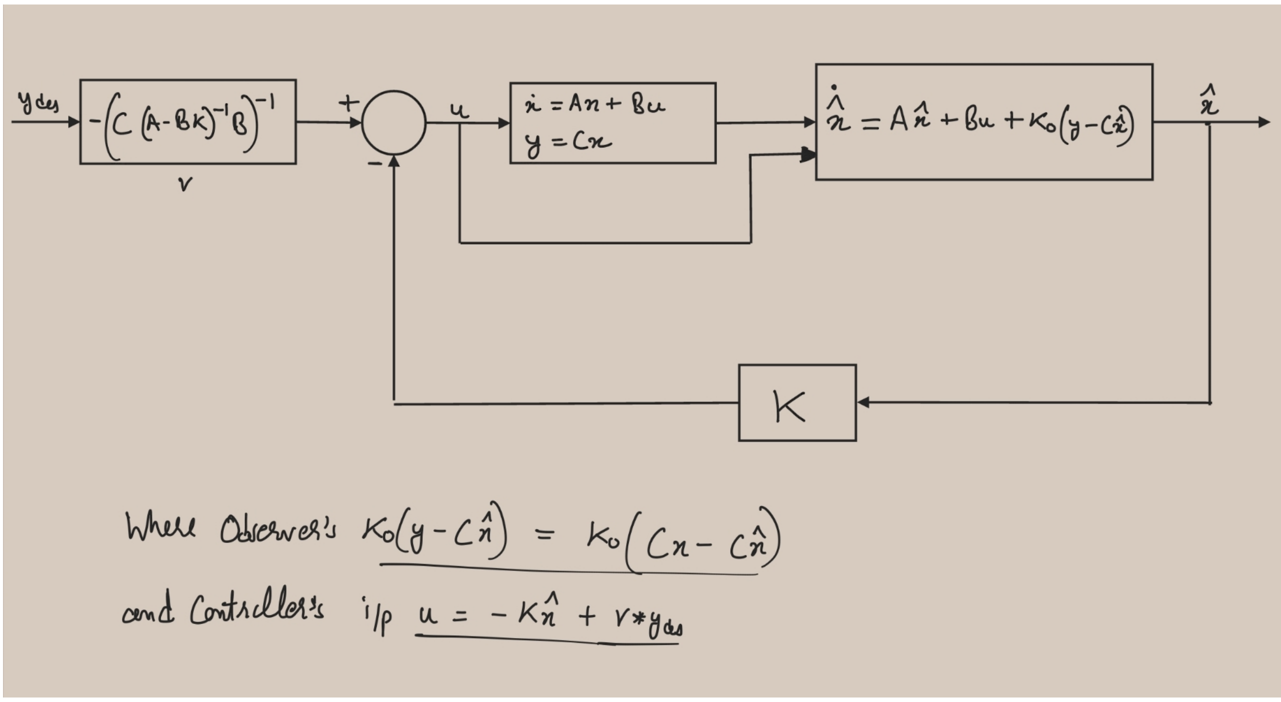
```
std_state_error = 9.5398e-04
```

Q3. Observer for Cart Pendulum System

The tracking controller in Problem 2(h) in Problem Set 1 was shown as:



Now, along with an observer, the system state is shown below



The system variables for the inverted cart pendulum model were

```
% Define the system variables:
A = [[0,0,1,0];[0,0,0,1];[0,1,-3,0];[0,2,-3,0]]
```

```
A = 4x4
     0     0     1     0
     0     0     0     1
     0     1    -3     0
     0     2    -3     0
```

```
B = [0;0;1;1]
```

```
B = 4x1
     0
     0
     1
     1
```

```
C = [39.3701, 0, 0, 0]
```

```
C = 1x4
   39.3701         0         0         0
```

```
% Find the optimal feedback gain
Q = [[20,0,0,0];[0,5,0,0];[0,0,1,0];[0,0,0,5]];
R = 20;
K = lqr(A, B, Q, R)
```

```
K = 1x4
   -1.0000   12.0469   -8.3985   11.0015
```

We need to keep the observer poles far left of the controller poles, i.e. around 4x further than the most dominant pole of the controller (in this case is -0.26)

```
% Find the K0 values for observer
controller_poles = eig(A-B*K)
```

```
controller_poles = 4x1
   -3.3702
   -0.2621
   -1.1245
   -0.7799
```

```
% Based on the position of controller poles, the observer poles are places
% as shown
observer_poles = [-4, -4.6, -5, -5.5]
```

```
observer_poles = 1x4
   -3.0000   -2.2000   -2.0000   -2.5000
```

```
% Calculating the error term K_0
K_tmp = place(transpose(A), transpose(C), observer_poles);
K_0 = transpose(K_tmp)
```

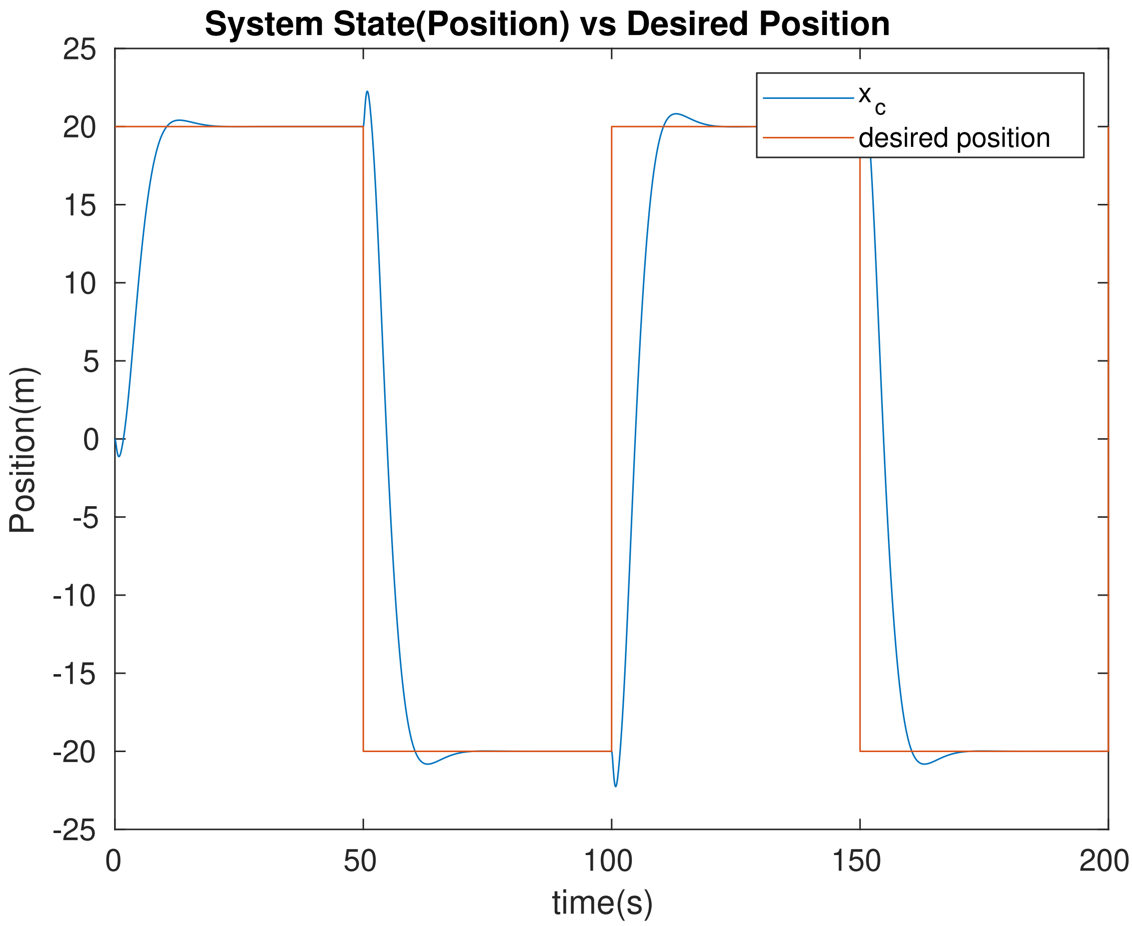
```
K_0 = 4x1
    0.1702
    1.8313
    0.4293
    2.2073
```

```
% Defining the timespan for the whole system
t_1 = 0 : 0.01 : 200;

% define parameters for y_des only for plotting
freq=0.01;
offset=0;
amp=20;
duty=50;

% define y_des as a square wave function
y_des = offset+amp*square(2*pi*freq.*t_1,duty);

v = -1 * inv(C*inv((A-B*K))*B);

controller_tracker = [];
observer_tracker = [];

count = 0;
for timespan=0:0.01:200
    if count == 0
        x_1 = [0; 0; 0; 0];
        x_hat = [0; 0; 0; 0];
    else
        x_1 = carry_over_1_last;
        x_hat = carry_over_2_last;
    end
    % controller
    [t1,x1] = ode45(@(t1,x1)(tracking_controller(x1, -K*x_hat, v, timespan)), [0,0.01], x_1);
    carry_over_1 = transpose(x1);
    carry_over_1_last = carry_over_1(:,end);
    controller_tracker = vertcat(controller_tracker, transpose(carry_over_1_last));

    if count == 0
        x_2 = [0; 0; 0; 0];
    else
        x_2 = carry_over_2_last;
    end

    % observer
    [t2,x2] = ode45(@(t1,x2)(tracking_controller_observer(x2, K, v, timespan, K_0, carry_over_1_last,
    carry_over_2 = transpose(x2);
    carry_over_2_last = carry_over_2(:,end);
    observer_tracker = vertcat(observer_tracker, transpose(carry_over_2_last));
    count = count + 1;
end
```

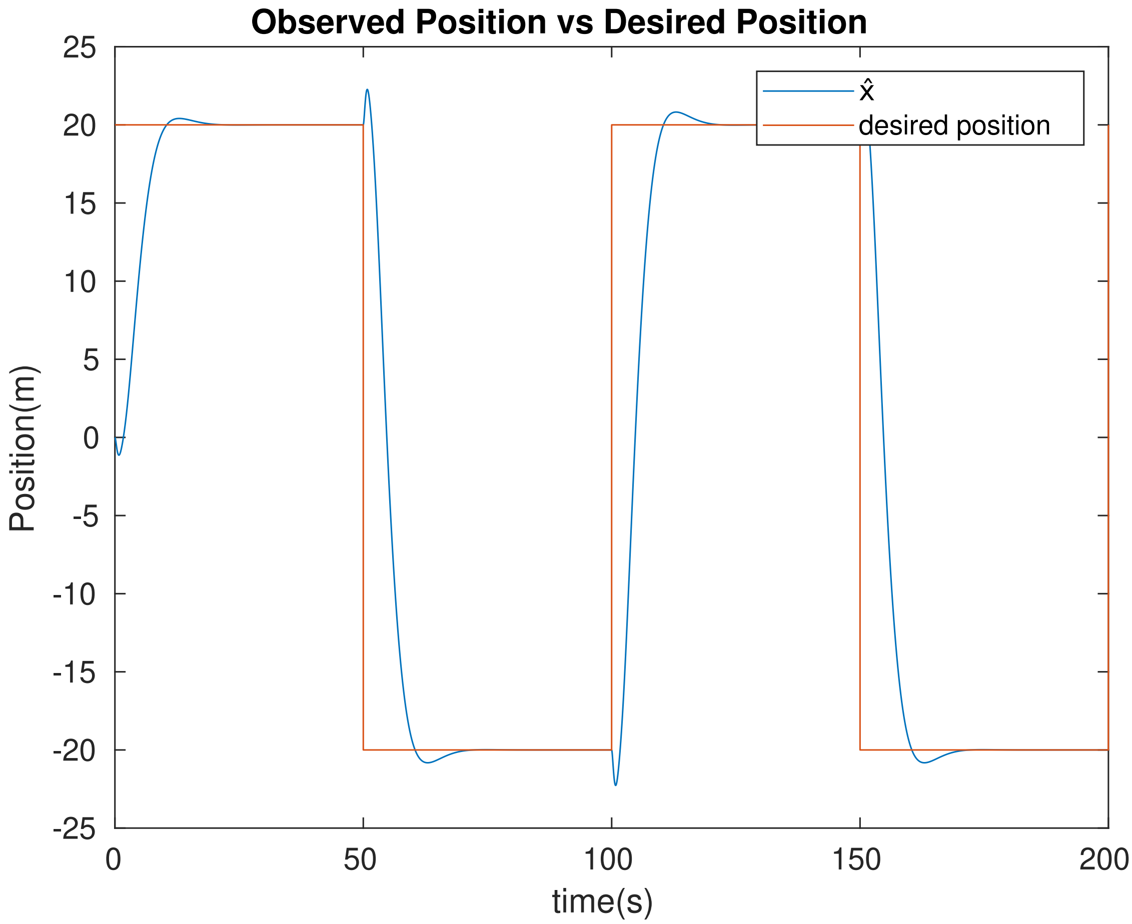
Plot only the position state w.r.t time

```
controller_tracker_position = controller_tracker(:,1)*39.3701;
% Plot of controller output against desired output
plot(t_1, controller_tracker_position)
hold on
plot(t_1,y_des)
hold off
title('System State(Position) vs Desired Position')
ylabel('Position(m)')
```

```
xlabel('time(s)')
legend({'x_c', 'desired position'})
```

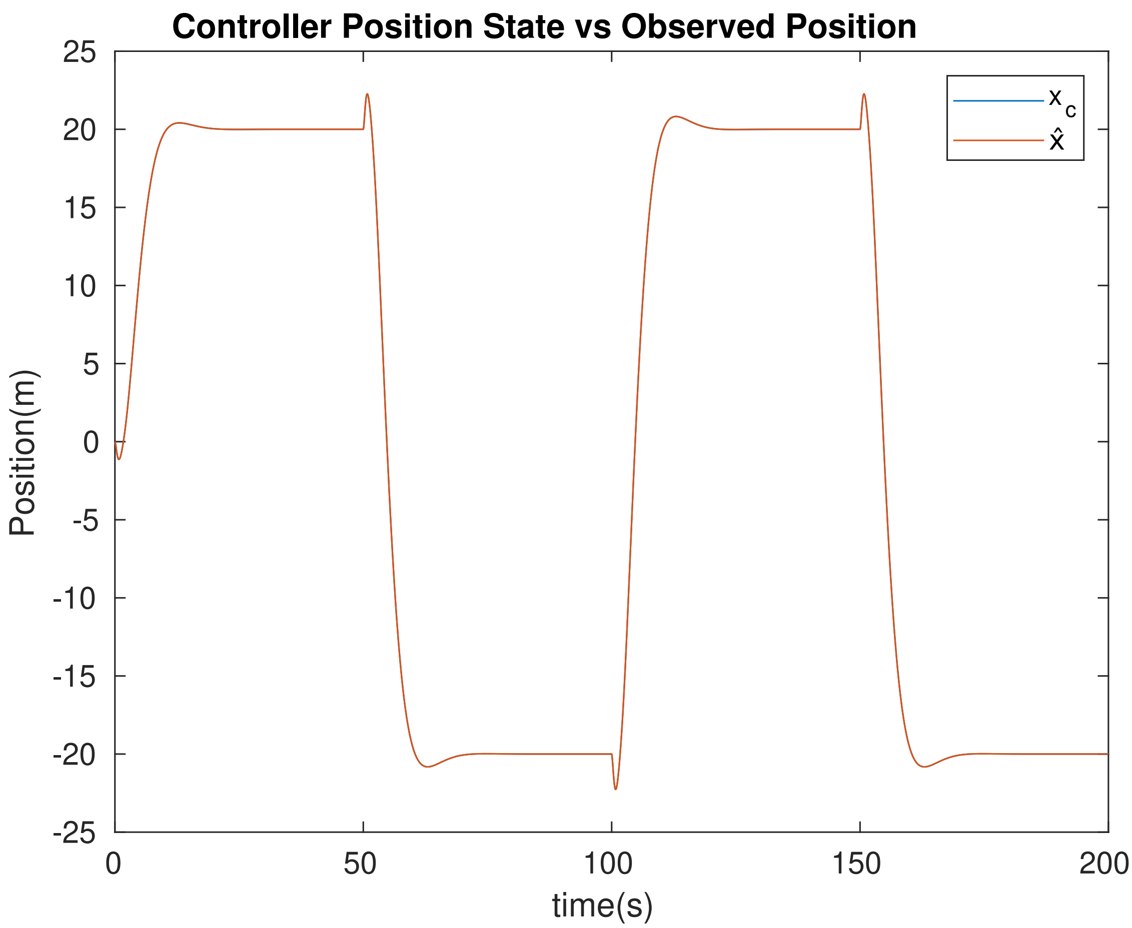



```
observer_tracker_position = observer_tracker(:,1)*39.3701;
% Plot of observer output against desired output
plot(t_1, observer_tracker_position)
hold on
plot(t_1,y_des)
hold off
title('Observed Position vs Desired Position')
ylabel('Position(m)')
xlabel('time(s)')
legend({'x', 'desired position'})
```



Controller System States vs Observer Estimates

```
% Plot of observer output against desired output
plot(t_1, controller_tracker_position)
hold on
plot(t_1, observer_tracker_position)
hold off
title('Controller Position State vs Observed Position')
ylabel('Position(m)')
xlabel('time(s)')
legend({'x_c', 'x'})
```

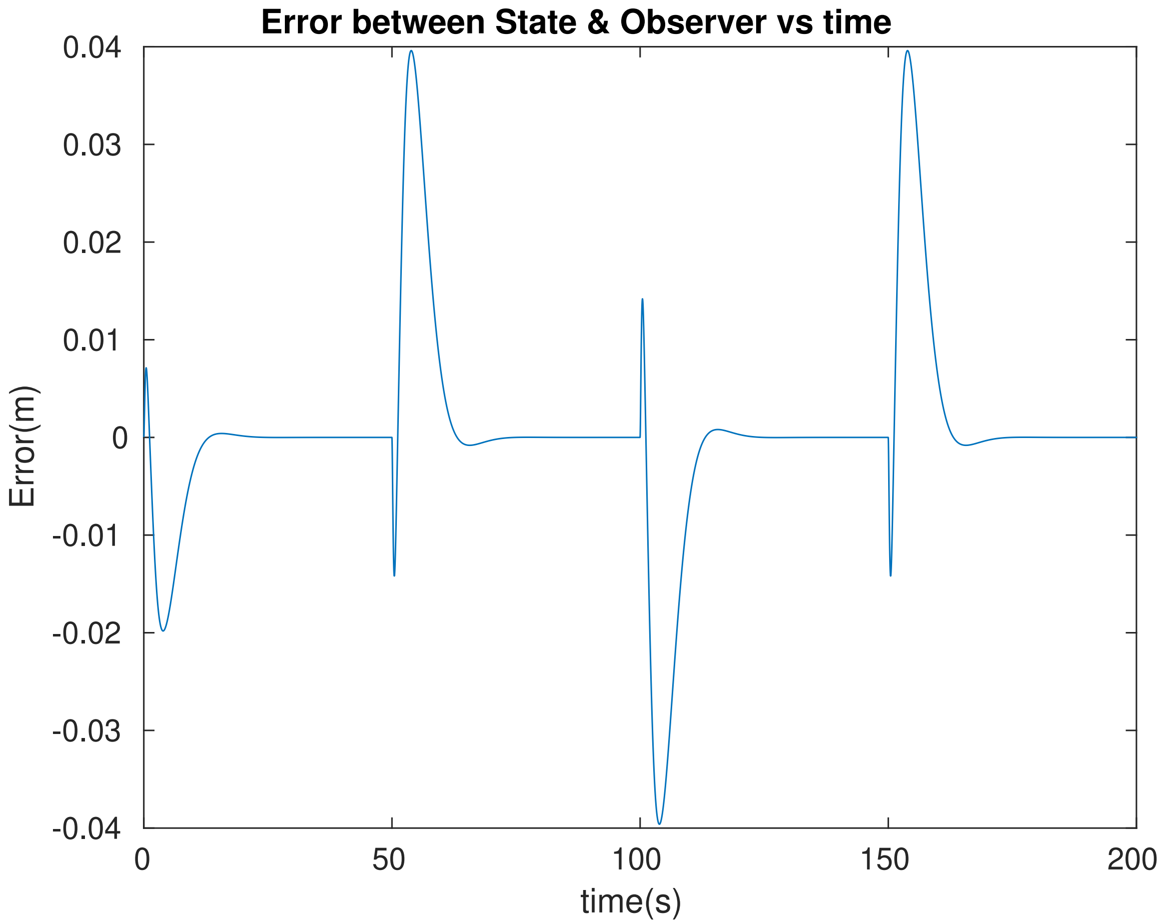


Error Plot: Controller System State (position) - Observer Estimate of Position

```
% Find the max error between the two plots
max_error = max(abs(controller_tracker_position - observer_tracker_position))
```

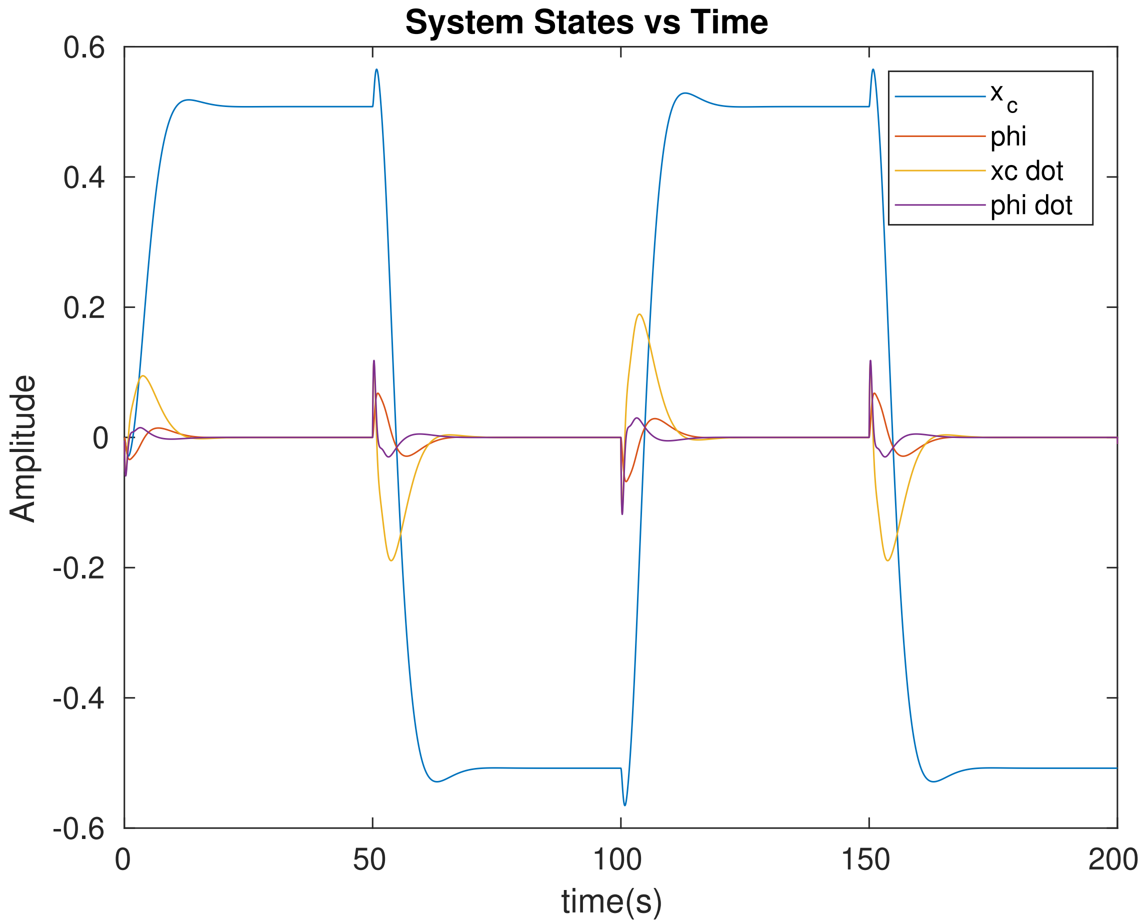
```
max_error = 0.0470
```

```
% Plot the error over time
controller_observer_error = controller_tracker_position - observer_tracker_position;
plot(t_1, controller_observer_error)
title('Error between State & Observer vs time')
ylabel('Error(m)')
xlabel('time(s)')
```



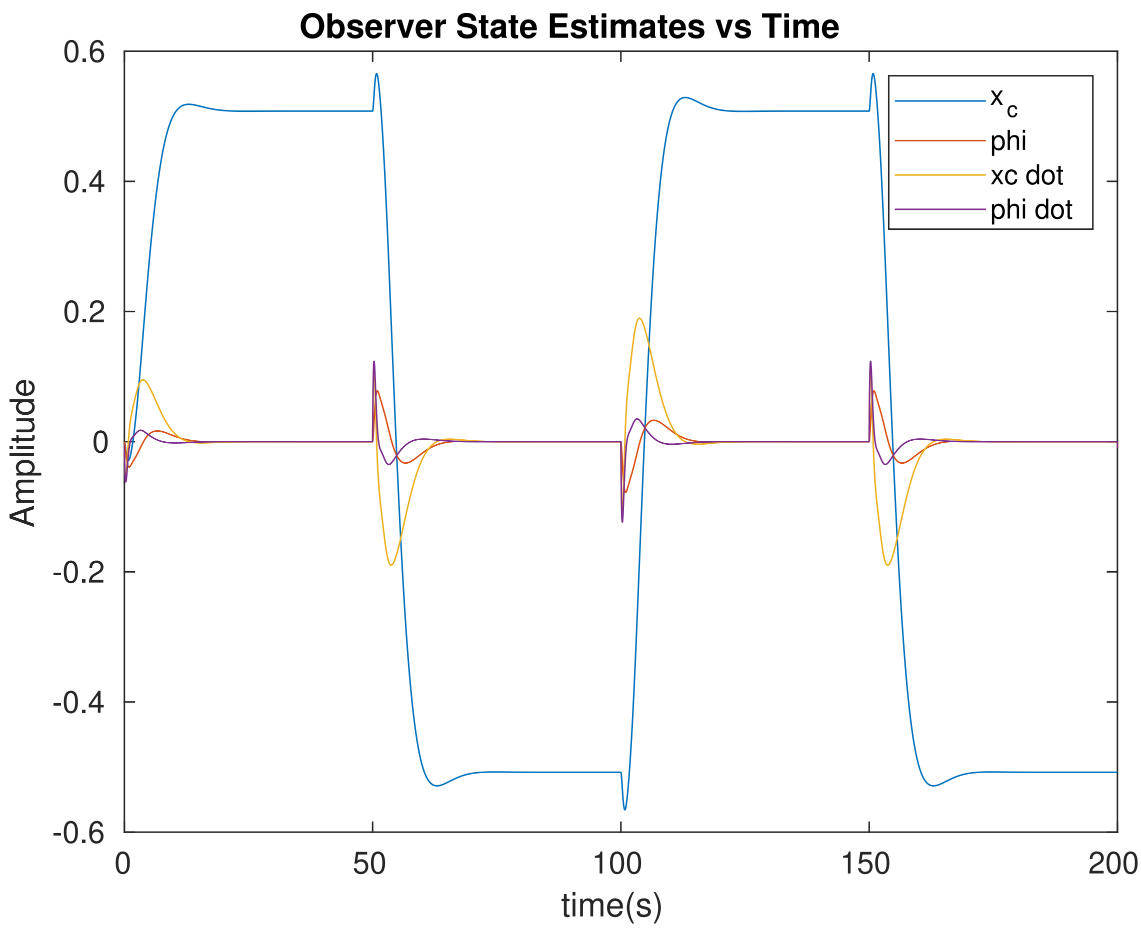
Plotting the controller states vs time

```
controller_tracker(:,1) = controller_tracker(:,1);
% Plot of controller state outputs
plot(t_1, controller_tracker)
title('System States vs Time')
ylabel('Amplitude')
xlabel('time(s)')
legend({'x_c', 'phi','xc dot', 'phi dot'})
```



Plotting the observer state estimates vs time

```
observer_tracker(:,1) = observer_tracker(:,1);
% Plot of controller state outputs
plot(t_1, observer_tracker)
title('Observer State Estimates vs Time')
ylabel('Amplitude')
xlabel('time(s)')
legend({'x_c', 'phi','xc dot', 'phi dot'})
```



Helper function for controller ODE45

```
function x_dot = tracking_controller (x, F, v, t)
xc = x(1);
phi = x(2);
xcdot = x(3);
phidot = x(4);

% constants in the system
gamma = 2;
alpha = 1;
beta = 1;
D = 1;
mu = 3;

% define parameters for y_des
freq=0.01;
offset=0;
amp=20;
duty=50;

% define y_des as a square wave function
y_des = offset+amp*square(2*pi*freq.*t,duty);
u = F + v*y_des;

divisor = ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)))^(-1);

x_dot = [xcdot;
```

```
        phidot;
        divisor*((u*alpha) - (beta*phidot*phidot*sin(phi)*alpha) - (alpha*mu*xcdot) + (beta*D*sin(phi
        divisor*((u*beta*cos(phi)) - (beta*beta*phidot*phidot*sin(phi)*cos(phi)) - (beta*cos(phi)*mu*


% divisor_1 = alpha / ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)));
% divisor_2 = (beta*cos(phi)) / ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)));

% x_dot = [xcdot;
%         phidot;
%         divisor_1*(u - (beta*phidot*phidot*sin(phi)) - (mu*xcdot) + ((beta*D*cos(phi)*sin(phi))/alph
%         divisor_2*(u - (beta*phidot*phidot*sin(phi)) - (mu*xcdot) + ((gamma*D*sin(phi)) / (beta*cos(

end
```

Helper function for observer ODE45

```
function x_dot = tracking_controller_observer (x, K, v, t, K_0, controller_op, C)
xc = x(1);
phi = x(2);
xcdot = x(3);
phidot = x(4);

% constants in the system
gamma = 2;
alpha = 1;
beta = 1;
D = 1;
mu = 3;

% define parameters for y_des
freq=0.01;
offset=0;
amp=20;
duty=50;

% define y_des as a square wave function
y_des = offset+amp*square(2*pi*freq.*t,duty);
u = -K*x + v*y_des;

divisor = ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)))^(-1);

x_dot = [xcdot;
        phidot;
        divisor*((u*alpha) - (beta*phidot*phidot*sin(phi)*alpha) - (alpha*mu*xcdot) + (beta*D*sin(phi
        divisor*((u*beta*cos(phi)) - (beta*beta*phidot*phidot*sin(phi)*cos(phi)) - (beta*cos(phi)*mu*

error = K_0*(C*controller_op - C*x);
x_dot = x_dot + error;
% divisor_1 = alpha / ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)));
% divisor_2 = (beta*cos(phi)) / ((gamma*alpha) - (beta*beta*cos(phi)*cos(phi)));

% x_dot = [xcdot;
%         phidot;
%         divisor_1*(u - (beta*phidot*phidot*sin(phi)) - (mu*xcdot) + ((beta*D*cos(phi)*sin(phi))/alph
%         divisor_2*(u - (beta*phidot*phidot*sin(phi)) - (mu*xcdot) + ((gamma*D*sin(phi)) / (beta*cos(

end
```