Manipulation Estimation and Controls: Assignment 1

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Q1.

Linear system is defined with in the form x = Ax + Bu

Where A and B are defined as

```
A = [[0,1,0];[0,0,1];[1,5,7]];
B = transpose([1,0,0]);
C = [0,1,3];
```

1 a. Stability Criterion for the above system is defined by the eigenvalues of A

```
eig(A)
```

```
ans = 3×1 complex
   7.6690 + 0.0000i
  -0.3345 + 0.1361i
  -0.3345 - 0.1361i
```

Since one of the eigen values is positive, the linear system A is unstable

1 b. Controllability of the system

Matrix Q mentioned below must be of rank n (same rank as of ma

```
Q = ctrb(A,B)
```

```
Q = 3×3
     1     0     0
     0     0     1
     0     1     7
```

```
rank(Q)
```

```
ans = 3
```

Rank of Q = 3. Therefore the system is controllable

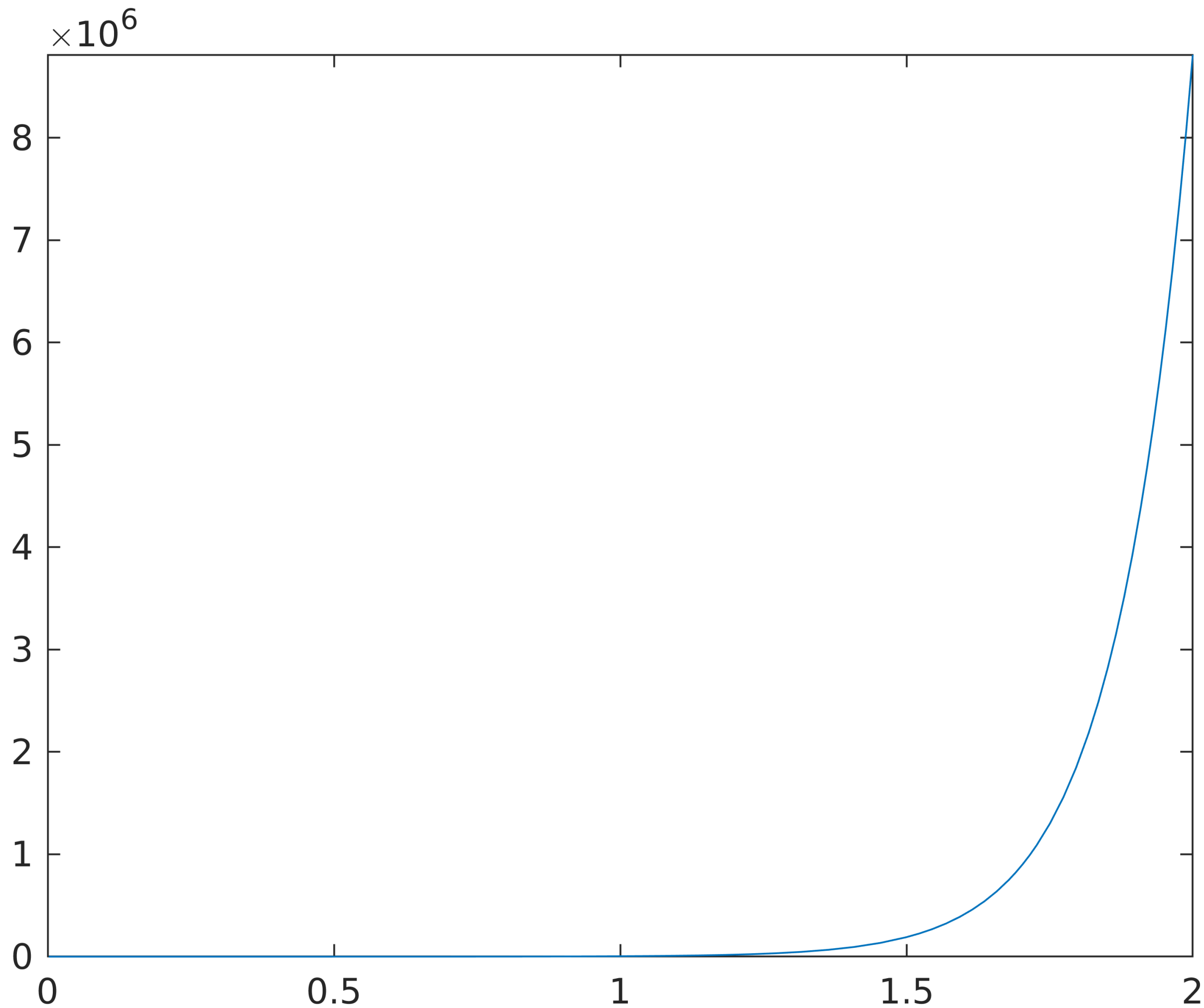
1 c. Initial State Vector is given as:

The output of the unforced system plotted below:

```
x_0 = [0;1;0]
```

```
x_0 = 3×1
     0
     1
     0
```

```
syms t
fplot(C*expm(A*t)*x_0, [0,2])
```



1 d. Find the feedback gain K to make the system stable

```
eig_values = [-1 + 1i, -1 - 1i, -2]
```

```
eig_values = 1×3 complex
  -1.0000 + 1.0000i  -1.0000 - 1.0000i  -2.0000 + 0.0000i
```

```
K = place(A,B,eig_values)
```

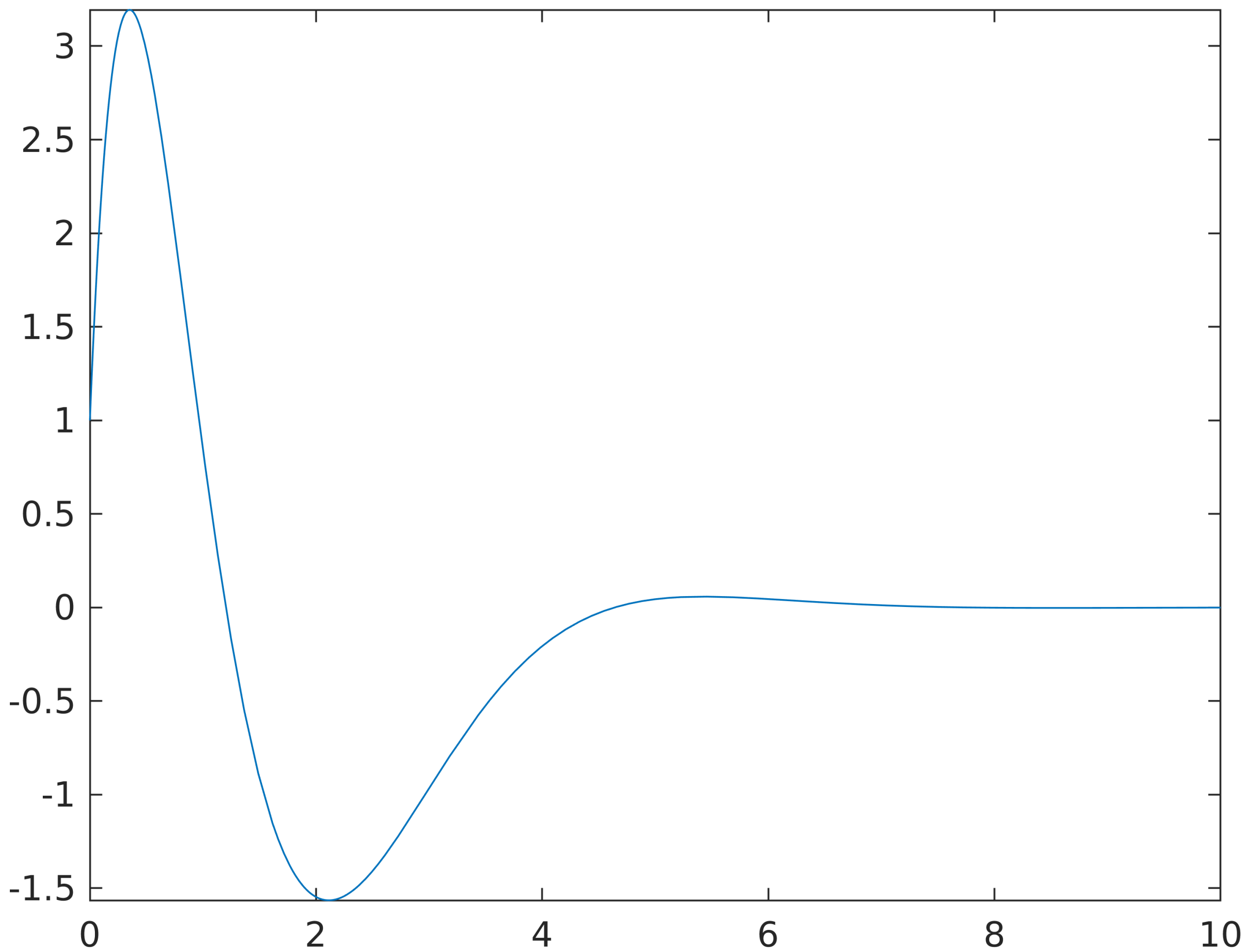
```
K = 1×3
   11.0000   60.0000   88.0000
```

1 e. Plot output of the system after control input

```
new_mat = A-B*K
```

```
new_mat = 3×3
  -11.0000  -59.0000  -88.0000
         0         0    1.0000
    1.0000    5.0000    7.0000
```

```
fplot(C*expm(new_mat*t)*x_0, [0,10])
```



Q 2.

"Pendulum on a cart" system

2 c. Compute the eigen values of matrix A which represents the linearized system about the equilibrium point at x = 0

```
A = [[0,0,1,0];[0,0,0,1];[0,1,-3,0];[0,2,-3,0]]
```

```
A = 4×4
     0     0     1     0
     0     0     0     1
     0     1    -3     0
```

```
     0     2    -3     0
```

```
B = [0;0;1;1]
```

```
B = 4×1
     0
     0
     1
     1
```

```
eig(A)
```

```
ans = 4×1
         0
   -3.3301
    1.1284
   -0.7984
```

Since one the eigenvalues of A have a real positive value the system is unstable about the state x = 0

This means that the vertical upright pendulum is not in stable equilibrium

2 d. Finding the optimal feedback control gain

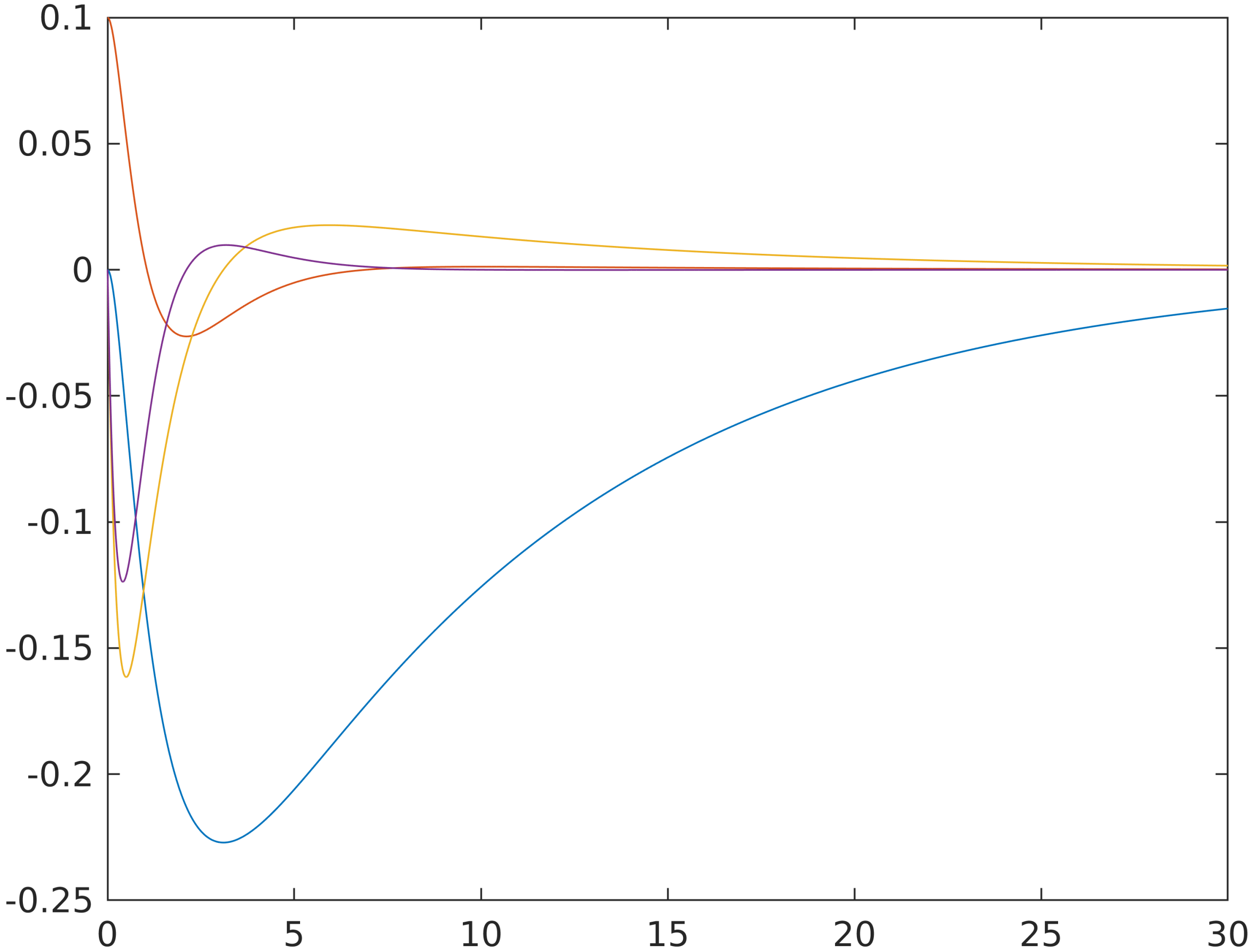
```
Q = [[1,0,0,0];[0,5,0,0];[0,0,1,0];[0,0,0,5]];
R = 10;

K = lqr(A, B, Q, R)
```

```
K = 1×4
   -0.3162   10.2723   -6.7857    9.2183
```

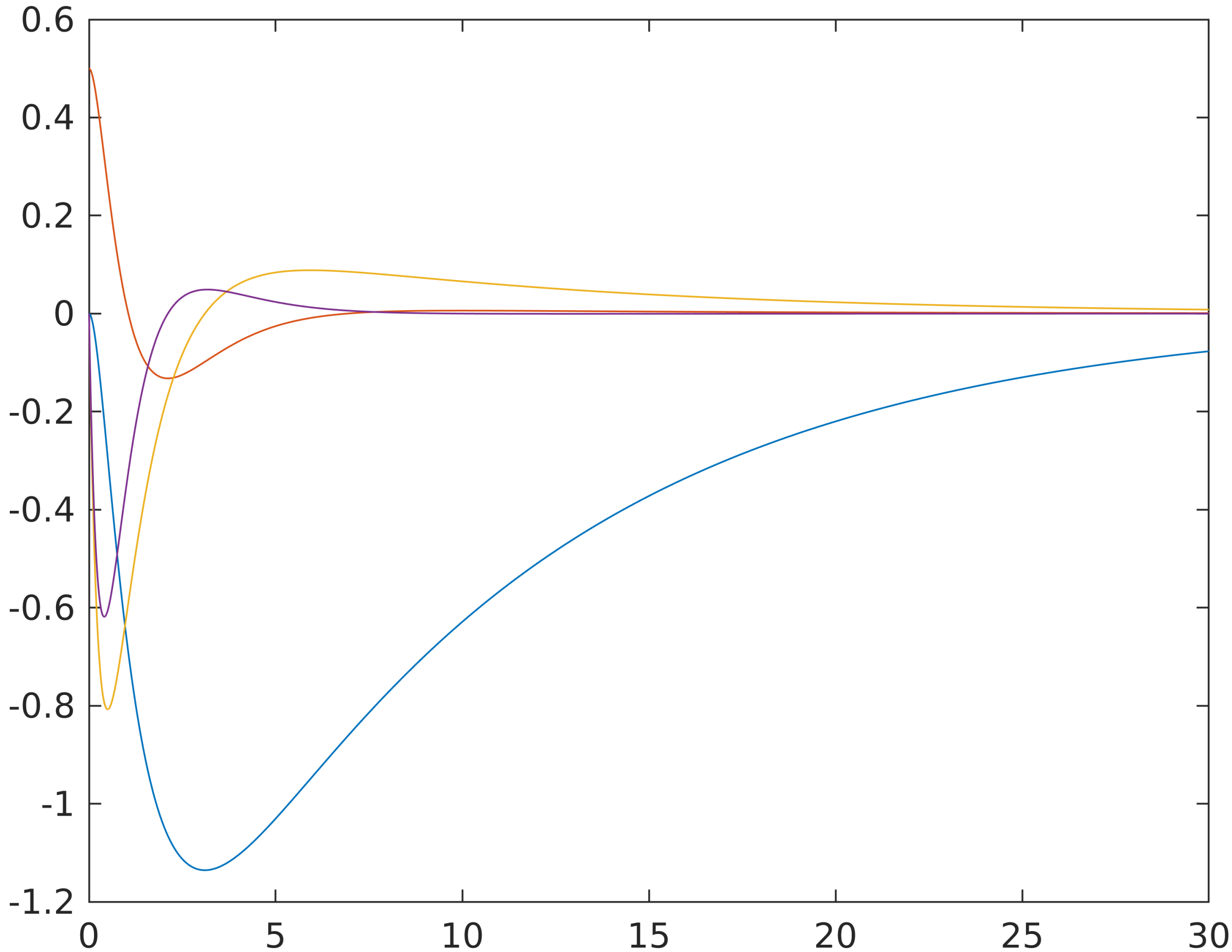
Using the above feedback control to plot the state of the linearized system

```
t = 0 : 0.01 : 30 ;

% define three initial state vectors x_0, x_1, and x_2
x_0 = [0 ; 0.1; 0; 0];
x_1 = [0; 0.5; 0; 0];
x_2 = [0; 1.0886; 0; 0];
x_3 = [0; 1.1; 0; 0];
x_trial = [0;0.01;0;0];

% plotting the state of system beginning at x_0
[t,x0] = ode45(@(t,x)(A-B*K)*x, t, x_0);
```

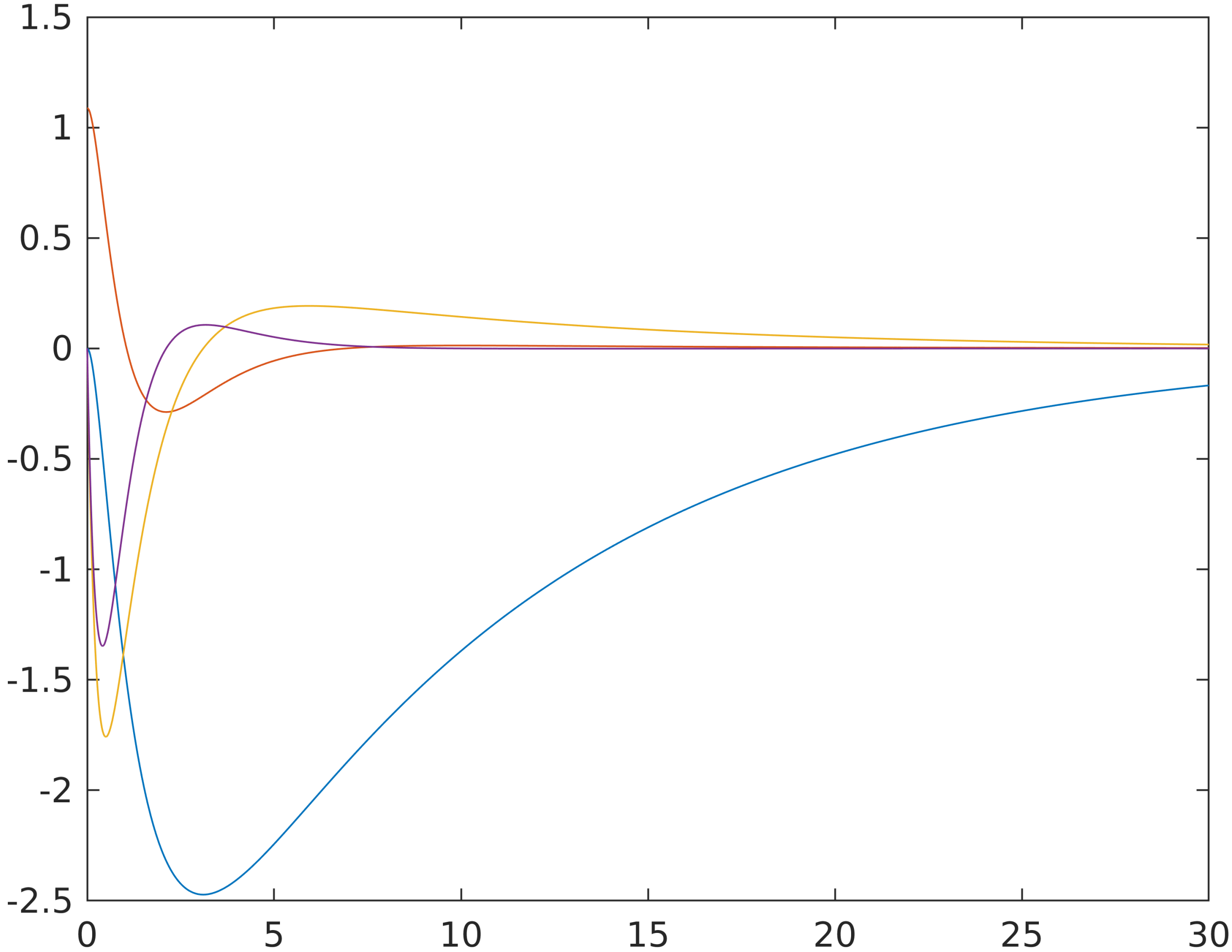
```
plot(t0,x0)
```



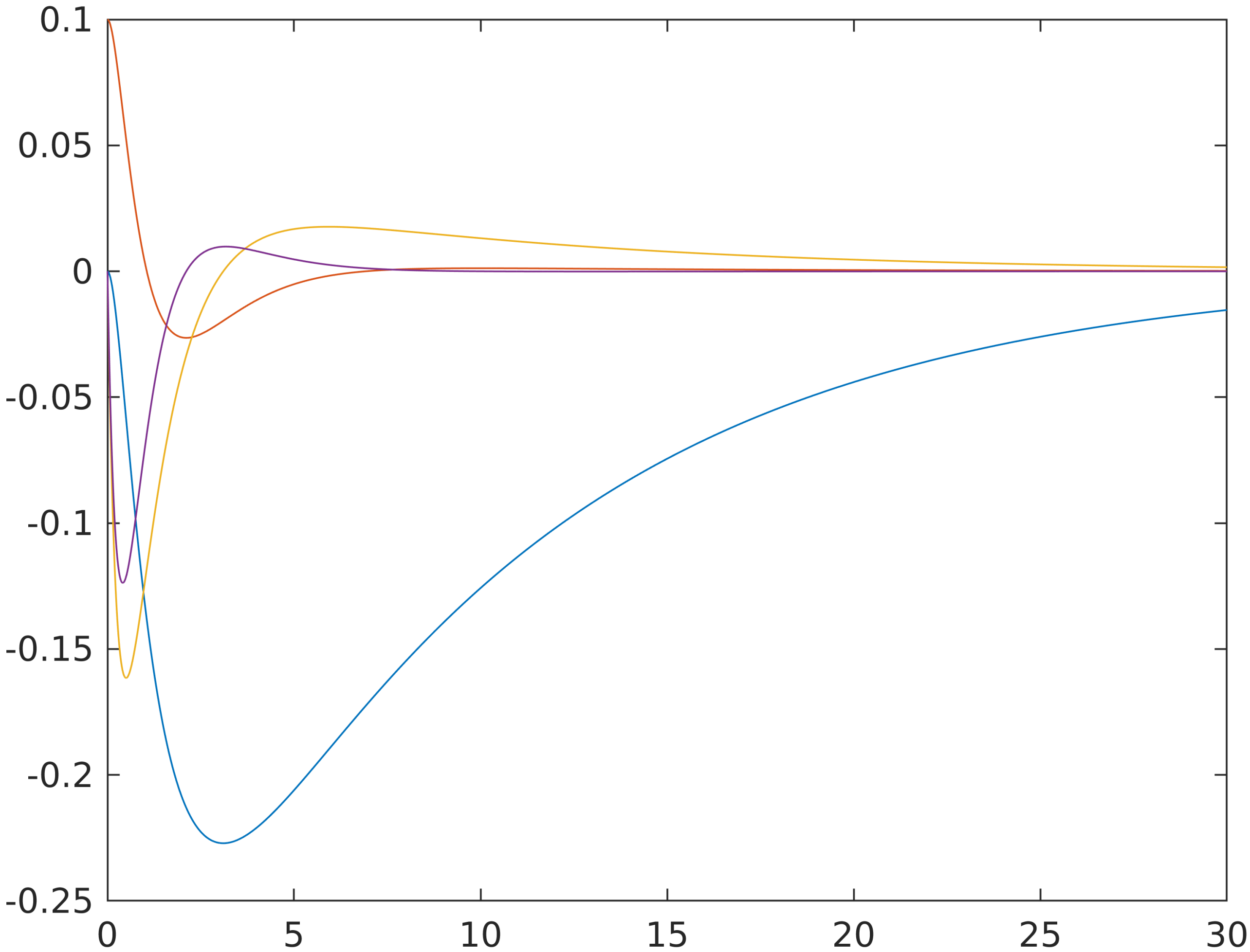

```
% plotting the state of system beginning at x_1
[t,x1] = ode45(@(t,x)(A-B*K)*x, t, x_1);
plot(t,x1)
```




```
% plotting the state of system beginning at x_2
[t,x2] = ode45(@(t,x)(A-B*K)*x, t, x_2);
plot(t,x2)
```




```
% plotting the state of system beginning at x_0
[t,x3] = ode45(@(t,x)(A-B*K)*x, t, x_3);
plot(t,x0)
```



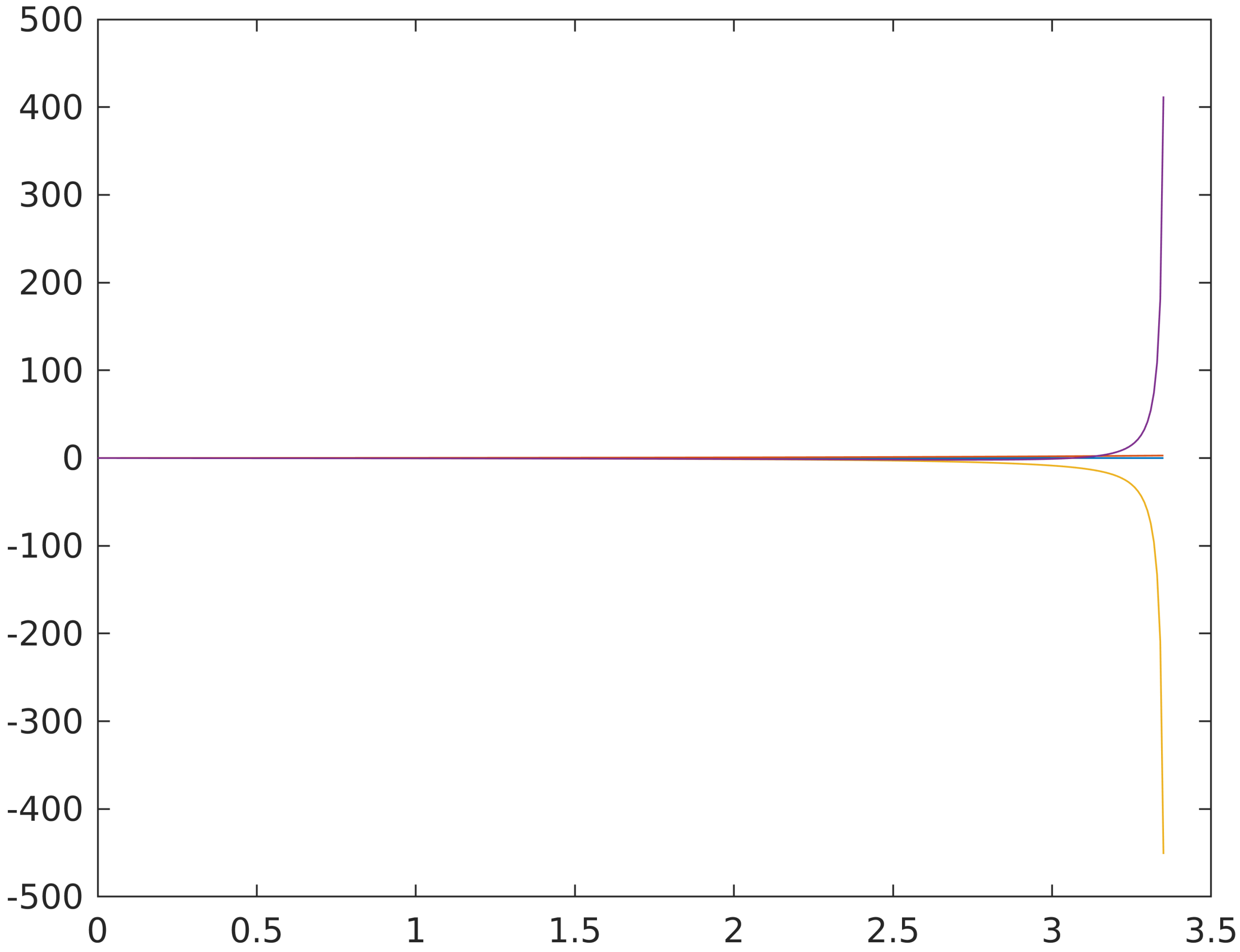
2 e. Repeat 2 d. using non-linearized state equations

The non linear system is computed below:

```
[t,x4] = ode45(@(t,x)(non_linear_fun(x, -K*x)), t, x_0);
```

```
Warning: Failure at t=3.359354e+00.  Unable to meet integration tolerances without reducing the
step size below the smallest value allowed (7.105427e-15) at time t.
```

```
plot(t, x4)
```




```
% Defining function for x_dot
```

```
function x_dot = non_linear_fun (x, u)
xc = x(1);
phi = x(2);
xcdot = x(3);
phidot = x(4);

gamma = 2;
alpha = 1;
beta = 1;
D = 1;
mu = 3;


end
```