A Note regarding the Unusual Ridges Intruding from the South Wall of Moretus

by A. P. Lenham

As will be seen on the map there are several, rather curious ridges, that run down the south wall and onto the floor of Moretus. These ridges are linked by chains of hillocks and the areas between the ridges are depressed below the level of the wall on either side of them. From the general appearance of these features it is likely that they are the remnants of secondary craters on the inner slope of Moretus that have been partly flooded and destroyed by the 'magma' that fills the floor of Moretus.

It appears extremely unlikely that these partially submerged secondary features could have been formed after the flooding of the crater floor, so there is good evidence that in the case of Moretus the final flooding occurred after the formation of the parasite craters.

Several theories (tidal, etc.) assume that the flooding was contemporary with the formation of the crater, but the observations and interpretation make this assumption questionable. On the other hand, support is lent to such discussions as make the flooding one of the final stages in crater history.

The Prediction of Occultations of Stars by Minor Planets

By Gordon E. Taylor

1. *Introduction.*—This preliminary paper offers two simple methods for determining the tracks of occultations of stars by minor planets across the Earth's surface. A semi-graphical and a mathematical method, both based on the same fundamental principle, are employed in an example of an occultation of a star by a minor planet.

Both in the standard methods of prediction using Besselian geometry and in the methods outlined below, Besselian elements are calculated as a preliminary step. The essential difference between them is that the former various positions of the observers are assumed and greater accuracy is eventually obtainable by separate computations for each station, whereas with the methods now given predictions of the whole track may readily be made and conditions at any number of stations on the track obtained with a normal accuracy of 1 to 2 minutes. In particular it is possible to define the northern and southern limits of the track.

2. *Preliminary.*—As a preliminary step the ephemeris of the minor planet must be compared with a star catalogue and any close approaches noted. It may be remarked here that the great majority of these close approaches will eventually be rejected; they will prove to be merely appulses and not occultations. The ephemerides of Ceres, Pallas, Juno, and Vesta in the *Nautical Almanac* give apparent positions, but the position of a star in a star catalogue will be referred to a fixed equinox, say 1875·0 or 1950·0. This position must therefore be corrected to bring it up to the equinox of the year in question (e.g. 1954·0) by the addition of precession.

A rough idea of the time of conjunction of the two bodies will already have been formed, and the star position is then rigorously reduced from its mean place (e.g. 1954·0) to its apparent place on the particular date in question

(e.g. 1954 April 7) by means of the Independent Day Numbers and including proper motion if appreciable.*

Having obtained the exact position of the star on the given date, the precise time of conjunction (T_0) may be derived and the declination of the minor planet obtained. For an occultation to be visible from some point on the Earth the sum of the horizontal parallax (H.P.) and semi-diameter of the occulting body (s.d.) must be greater than the difference in declination of the two bodies.† Assuming this condition is satisfied, the calculation of the circumstances of the occultation may proceed. The Greenwich Hour Angle (G.H.A.) of the two bodies at T_0 is readily found from the equation

$$\text{G.H.A.} = \text{G.S.T.} - \alpha \tag{1}$$

where G.S.T. is the Greenwich Sidereal Time and α is the right ascension.

The angle ρ between the north point and the direction of motion of the minor planet is given by

$$\begin{aligned} n \sin \rho &= 15 \cos \delta \,.\, \Delta\alpha \\ n \cos \rho &= \Delta\delta \end{aligned} \tag{2}$$

where $\Delta\alpha$ and $\Delta\delta$ represent the minor planet's motion per minute in right ascension and declination in seconds of time and seconds of arc respectively. These values are interpolated from the ephemeris. The total motion in seconds of arc per minute, n, provides a time scale.

The horizontal parallax (H.P.) is taken from the *Nautical Almanac*; the geocentric distance is interpolated and the angular semi-diameter calculated from the real diameter in miles.

At this stage it is possible to obtain a rough mental picture of the area of the Earth over which the shadow will pass, and it is thus advisable to examine the situation and reject that portion of the path which will pass over sunlit parts of the Earth.

3. *Basis.*—The diagram (Fig. 1) is a projection on the Besselian fundamental plane and shows the track of the minor planet relative to the Earth as seen from the star. The circle has a radius proportional to the H.P. and the width of the track is proportional to the planet's angular diameter. It is therefore convenient to use these actual measurements. Linear distances on the diagram represent angular displacements of the minor planet. The linear distance SM is merely the difference in declination of the centres of the two bodies at the time of conjunction. The minor planet's path is shown as AB, the movement being from A to B if the motion is direct and from B to A if retrograde. O is any point on the track; in practice a number of points may be taken on the centre and edges of this track. The time scale mentioned above allows the measurement of time at any point on the track.

Fig. 2 shows a spherical triangle, PSO, on the surface of the Earth, P being the pole and S and O corresponding to the points of Fig. 1, the sides being as shown. The angle $S\hat{P}O$ is the local hour angle, h, and $P\hat{S}O = \theta$ and is projected

* It may be noted that in the case of the minor planet ephemerides in the *Nautical Almanac* the correction from the date in question to the equinox of 1950·0 is tabulated in the column headed 'Astr.–App.' (i.e. Astrometric minus Apparent). This correction may be applied to the ephemeris which can then be used with the star's 1950·0 position to derive the time of conjunction. Proper motion, if appreciable, must, however, still be applied.

† Strictly speaking, the criterion H.P. + s.d. > $(\delta_p - \delta_s) \sin \rho$ should be used where $(\delta_p - \delta_s)$ is the difference in declination and ρ is found from equation (2) below.

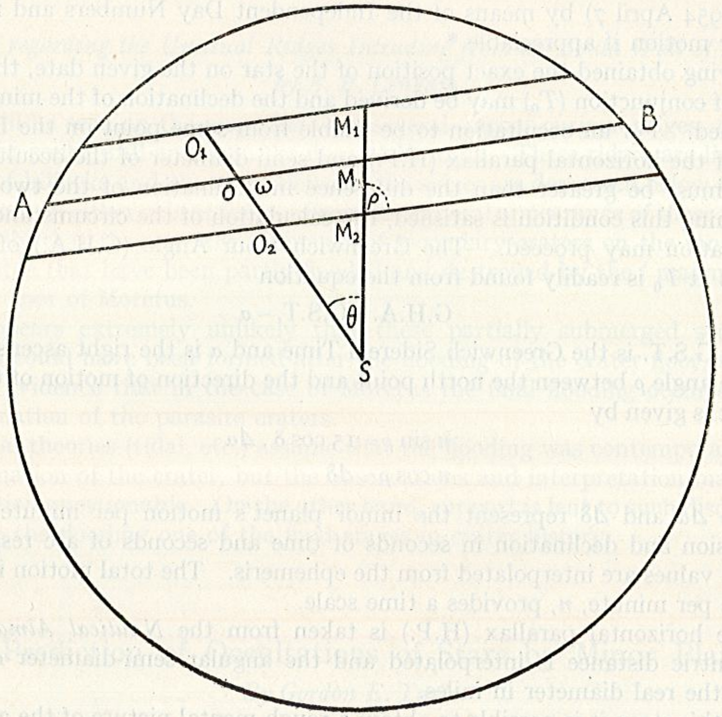

Fig. 1

without change on the fundamental plane as shown in Fig. 1. This fact is of primary importance in the method. The angle $P\hat{O}S$, which is the azimuth, is not required.

4. *Method.*—In order to determine the geographical co-ordinates of the point O it is necessary to find first the altitude of the star at O and the angle θ, and then to use these to solve the spherical triangle PSO.

Having fixed the point O (Fig. 1), the distance SO is measured and the angular distance (90° − alt.) derived from the simple relationship

$$\sin(90^\circ - \text{alt.}) = SO/\text{H.P.} \tag{3}$$

The angle θ is also measured from the diagram; an alternative method is to select an angle θ and then mark off the point O on the track.

This simple graphical method gives all the required quantities, but a more

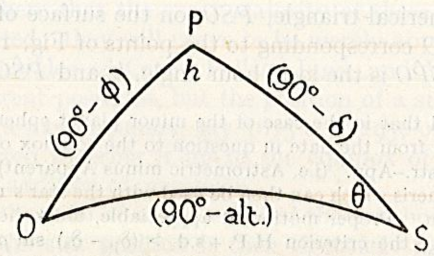

Fig. 2

accurate method of determining the altitude is to solve the plane triangle SMO. This is done, without drawing, by selecting suitable values of θ and deriving ω from the relationship

$$\omega + \theta = 180^\circ - \rho \tag{4}$$

where ρ is derived from (2).

In practice it is preferable to make θ an integral number of degrees.

In Fig. 1 there will be three plane triangles requiring solution to define the northern and southern limits (indicated by the subscripts 1 and 2) and the central line. The distance SM is defined as before, and then

$$
\begin{aligned}
MM_1 = MM_2 &= (\text{s.d.}) \operatorname{cosec} \rho \\
SO &= SM \sin\rho \operatorname{cosec}\omega \\
SO_1 &= SM_2 \sin\rho \operatorname{cosec}\omega \\
SO_2 &= SM_2 \sin\rho \operatorname{cosec}\omega
\end{aligned} \tag{5}
$$

It should be noted that for any particular occultation, $SM \sin\rho$, $SM_1 \sin\rho$ and $SM_2 \sin\rho$ are constants; this considerably simplifies the computation. Thus

$$\cos \text{alt.} = SM \sin\rho \operatorname{cosec}\omega/\text{H.P.} \tag{6}$$

and taking for example the central line of an occultation the calculation of cos alt. becomes a simple multiplication of a constant factor ($SM \sin\rho$/H.P.) by cosec ω.

The spherical triangle, in which δ, θ, and alt. are now known, may be solved by logarithms or a calculating machine using the standard formulae. However, there are in existence a number of tables and spherical co-ordinate converters, designed primarily for navigational purposes,* which quickly yield the required solution. Normally the computer commences with the latitude, ϕ, hour angle, h, and declination, δ, and reads off the altitude and azimuth. For the purposes of occultation predictions δ must be substituted for ϕ, θ for h and alt. for δ in order to read off latitude and hour angle. Then the longitude, λ, of the point O is found by

$$\lambda = \text{G.H.A.} - h \tag{7}$$

From the values of latitude, ϕ, and longitude, λ, the path of the shadow across the Earth's surface may be plotted on an ordinary map. Using the calculated motion, times and approximate position angles at any required stations may be found by inspection.

5. *Correction for Earth's rotation.*—Although the Earth's rotation will not be taken into account in the example, it should be noted that it is actually a simple matter to correct for this rotation by an inspection of the diagram. The distance OM (Fig. 1) is taken as a measure of time and the longitude of the point O is altered at the rate of 1° for every four minutes of time. If a diagram is not used the computer may employ the formulae:

$$t = \frac{SM}{n} \sin\theta \operatorname{cosec}\omega \tag{8}$$

$$\Delta\lambda = \tfrac{1}{4}t$$

* Most navigational tables do not cater for *all* possibilities, e.g. no solutions are given in H.O. 214 when the altitude is less than 5°. The author has found that the spherical co-ordinate converter which employs an equatorial stereographic projection offers the quickest solutions. Using this particular type, all points on the track may be marked on the converter together. The converter is set to the required declination and it is then possible to read off all the latitudes and hour angles from this one setting. The general accuracy of this method is about half a degree.

where t is the interval in minutes between the time at O and the time at M; n, the total motion is given in seconds of arc per minute, and $\Delta\lambda$ is the correction to be applied to the longitude. If the planet's motion is direct, then t takes the sign of the hour angle at O, and if the motion is retrograde, then t has the opposite sign to the hour angle.

6. *Example.*—Owing to the very small diameters of the minor planets, the tracks of any occultations will be correspondingly narrow. As even the ephemerides of the 'Big Four' minor planets tabulated in the *Nautical Almanac* may disagree with observation by a few seconds of arc, it is impossible to give an

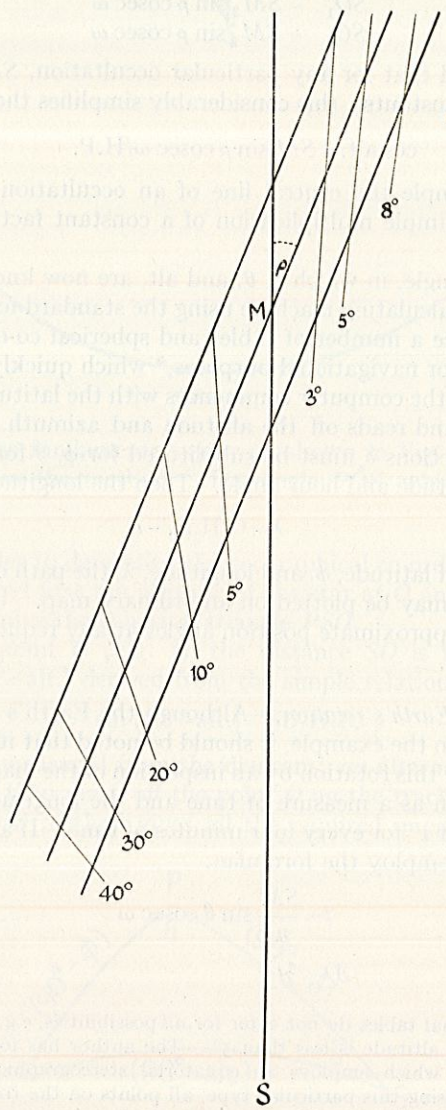

FIG. 3

accurate path. Thus the following example is given primarily to illustrate the method and no exact times are given.

On 1954 April $7^d\ 2^h\ 27^m$ the minor planet Pallas (7^m) is in conjunction with the star BD + 5° 2171 ($8^m \cdot 1$). The horizontal parallax is $5''\cdot 90$ and the difference in declination (SM) of the two bodies is $3''\cdot 5$, Pallas being north of the star. The other elements required for the prediction are found at T_0 to be as follows:

α_*	$9^h\ 22^m\ 03^s\cdot 36$	$\Delta\alpha$	$+\ 0^s\cdot 02110$
δ_*	$+\ 4°\ 57'\ 34''\cdot 7$	$\Delta\delta$	$+\ 0''\cdot 7927$
G.S.T.	$15^h\ 26^m$	n	$0''\cdot 85$
G.H.A.	$6^h\ 04^m = +\ 91°\cdot 0$	ρ	$+\ 21°\cdot 7$

The G.H.A. is found from equation (1) and n and ρ from equation (2).

An atlas shows that the longitude + 91°·0 passes through America, and noting also that the minor planet is well north of the star, it may be deduced that the occultation will be visible from the northern hemisphere. Thus the probable area of visibility is found to be North America. Note that if ρ is large the track will cover a large range of longitude, but if ρ is small (as in this case) the track will cover a large range of latitude and may extend in this case into the southern hemisphere.

The semi-diameter of Pallas at unit distance is taken as $0''\cdot 34$ and therefore its semi-diameter at T_0 (at a distance of 1·49 A.U.) is $0''\cdot 23$.

On a sheet of graph paper, using as large a scale as possible, the positions of the star S and the point M at the time of conjunction are plotted (Fig. 3) as explained in paragraph 3. Using the calculated value of ρ, the path of the minor planet is now drawn in. With the star S as origin, various values of θ are used to mark off points along the central line and the two limit lines at reasonable intervals. The distances of these points from S are also measured and the altitudes thereby obtained. Alternatively, the more accurate method previously described in paragraph 4 may be used. This is the method actually shown in Tables 1 and 2, which are self-explanatory.

TABLE 1

θ	40°	30°	20°	10°	5°	3°	5°	7°	8°
ω	118°·3	128°·3	138°·3	148°·3	153°·3	18°·7	16°·7	14°·7	13°·7
cosec ω	1·136	1·274	1·503	1·903	2·226	3·119	3·480	3·941	4·222

TABLE 2*a*

Northern Limit where cos. alt. = 0·258 cosec ω

cos alt.	alt.	θ	h	λ	ϕ
0·293	73°·0	40°	$-\ 11\frac{1}{2}°$	$+\ 102\frac{1}{2}°$	$+\ 18°$
329	70·8	30	$10\frac{1}{2}$	$101\frac{1}{2}$	$21\frac{1}{2}$
388	67·2	20	$8\frac{1}{2}$	$99\frac{1}{2}$	$26\frac{1}{2}$
491	60·6	10	6	97	34
574	55·0	5	$-\ 3\frac{1}{2}$	$94\frac{1}{2}$	40
698	45·7	0	0	91	$49\frac{1}{2}$
0·805	36·4	3	$+\ 4\frac{1}{2}$	$+\ 86\frac{1}{2}$	$+\ 58\frac{1}{2}$

TABLE 2*b*

Central Line where cos alt. = 0·219 cosec ω

cos alt.	alt.	θ	h	λ	ϕ
0·249	75°·6	40°	$-9\frac{1}{2}$°	$+100\frac{1}{2}$°	+16°
279	73·8	30	$8\frac{1}{2}$	$99\frac{1}{2}$	19
329	70·8	20	7	98	23
417	65·4	10	5	96	29
487	60·9	5	−3	94	34
593	53·6	0	0	91	$41\frac{1}{2}$
683	46·9	3	+3	88	48
0·762	40·4	5	$+6\frac{1}{2}$	$+84\frac{1}{2}$	$+54\frac{1}{2}$

TABLE 2*c*

Southern Limit where cos alt. = 0·181 cosec ω

cos alt.	alt.	θ	h	λ	ϕ
0·206	78°·1	40°	−8°	+99°	+14°
231	76·6	30	7	98	$16\frac{1}{2}$
272	74·2	20	6	97	20
344	69·9	10	$3\frac{1}{2}$	$94\frac{1}{2}$	25
403	66·2	5	$-2\frac{1}{2}$	$93\frac{1}{2}$	$28\frac{1}{2}$
488	60·8	0	0	91	34
565	55·6	3	+2	89	$39\frac{1}{2}$
630	50·9	5	$4\frac{1}{2}$	$86\frac{1}{2}$	44
713	44·5	7	8	83	50
0·764	40·2	8	$+10\frac{1}{2}$	$+80\frac{1}{2}$	+54

The track of the occultation can now be plotted on a map. The motion of Pallas is found to be 0″·85 per minute and therefore near the south-west coast of North America the time of occultation will be only about three minutes earlier than the time of conjunction.

7. *Acknowledgements*.—This paper is published with the permission of the Astronomer Royal. The author acknowledges with gratitude the assistance of Mr D. H. Sadler and Dr J. G. Porter in the presentation of the paper, and of Mr M. P. Candy who drew the diagrams.

ASTRONOMICAL REMINDERS

All references are to the *Handbook*

February

Mars and Jupiter are evening stars; Venus and Saturn are morning stars.
Full Moon February 7
Occultations (Gr. and Ed.) February 1, 2, 3, 4, 10, 28 (page 14)

February 12 Mercury at inferior conjunction (page 18)
17 Pluto at opposition (page 37)
28 Occultation of stars in the Pleiades (Cape and Johannesburg) (page 15)

March

Mars and Jupiter are evening stars; Mercury, Venus, and Saturn are morning stars.
Full Moon March 8
Occultations (Gr. and Ed.) March 1, 2, 10, 28, 29, 31 (page 14)

March 8 Venus occults π Cap (page 17)
9 Occultation of minor planet Hebe (page 40)
11 Mercury at greatest western elongation (page 18)
Favourable elongation of Hyperion (page 31)
12 Mars occults a star (page 17)
19 Iapetus at western elongation (page 34)
26 Minor planet Massalia at opposition (page 40)

April

Mars and Jupiter are evening stars; Venus and Saturn are morning stars.
Full Moon April 7.
Occultations (Gr. and Ed.) April 2, 4, 30 (page 14)

April 2 Minor planet Hygeia at opposition (page 40)
4 Jupiter occults an 8th mag. star (page 17)
17 Neptune at opposition (pages 37–8)
20–22 Lyrid meteor shower (page 44)
25 Mars in conjunction (0°·7 S.) with the Moon (page 20)
25–6 Minor planet Euterpe in Pleiades cluster (page 40)

May

Mercury, Mars, Jupiter, and Saturn are evening stars; Venus is a morning star.
Full Moon May 6
Occultation (Gr. and Ed.) May 1 (page 14)

May 3 Occultation of minor planet Hygeia (page 40)
5 Minor planet Juno at opposition (page 39)
9 Saturn at opposition (page 31)
10 Jupiter 1′ south of Uranus (page 21)
13 Minor planet Irene at opposition (page 40)
14 Favourable elongation of Hyperion (page 31)
21 Mercury at greatest eastern elongation (page 18)
23 Mars 1′ north of the Moon (page 20)
25 Ceres occults a star (page 17)
27 Jupiter occults a star (page 17)
31 Occultation of minor planet Hygeia (page 40)