calculusI, Assignment 7

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question no. 1

(a) Show that the coefficient of inequality is twice the area between the Lorenz curve and the line $y = x$

Recall definitions: - The area between the line $y = x$ and the Lorenz curve $y = L(x)$ is:

$$A = \int_0^1 [x - L(x)]\, dx$$

- The area under the line $y = x$ from 0 to 1 is:

$$B = \int_0^1 x\, dx = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2}$$

- The coefficient of inequality is:

$$coefficientofinequality = \frac{Areabetweeny = xandy = L(x)}{Areaundery = x}$$

Substituting the expressions above:

$$coefficientofinequality = \frac{A}{B} = \frac{\int_0^1 [x - L(x)]\, dx}{\frac{1}{2}} = 2\int_0^1 [x - L(x)]\, dx$$

(b) Given $L(x) = \frac{5}{12}x^2 + \frac{7}{12}x$:

1. Percentage of total income received by the bottom 50% of households

This is $L(0.5)$:

$$L(0.5) = \frac{5}{12}(0.5)^2 + \frac{7}{12}(0.5)$$

$$= \frac{5}{12} \cdot \frac{1}{4} + \frac{7}{12} \cdot \frac{1}{2}$$

$$= \frac{5}{48} + \frac{7}{24}$$

Write $\frac{7}{24}$ as $\frac{14}{48}$:

$$= \frac{5}{48} + \frac{14}{48} = \frac{19}{48}$$

So, as a percentage:

$$\frac{19}{48} \approx 0.3958 \approx 39.6\%$$

The bottom 50 percent of households receive about 39.6% of the total income.

2. Find the coefficient of inequality

We use:

$$coefficientofinequality = 2 \int_0^1 [x - L(x)] \, dx$$

Compute $x - L(x)$:

$$x - L(x) = x - \left(\frac{5}{12}x^2 + \frac{7}{12}x \right) = x - \frac{5}{12}x^2 - \frac{7}{12}x$$

$$= \frac{12}{12}x - \frac{5}{12}x^2 - \frac{7}{12}x = \frac{5}{12}x - \frac{5}{12}x^2$$

$$= \frac{5}{12}(x - x^2)$$

Now integrate:

$$\int_0^1 [x - L(x)]dx = \frac{5}{12} \int_0^1 (x - x^2)dx$$

Compute $\int_0^1 (x - x^2)dx$:

$$\int_0^1 xdx = \left[\frac{1}{2}x^2 \right]_0^1 = \frac{1}{2}$$

$$\int_0^1 x^2dx = \left[\frac{1}{3}x^3 \right]_0^1 = \frac{1}{3}$$

$$\int_0^1 (x - x^2)dx = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

So,

$$\int_0^1 [x - L(x)]dx = \frac{5}{12} \cdot \frac{1}{6} = \frac{5}{72}$$

Thus,

$$coefficientofinequality = 2 \cdot \frac{5}{72} = \frac{10}{72} = \frac{5}{36} \approx 0.1389$$

question no. 2

== solution:

We are asked to find the area between the curve $y = \sin x$ and the line segment joining the points $(0,0)$ and $\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$ using:

1. Integral definition, and 2. Monte Carlo simulation in Python.

The line passes through $(0,0)$ and $\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$, so we compute the equation of the line using the slope-intercept form:

$$m = \frac{-\frac{1}{2} - 0}{\frac{7\pi}{6} - 0} = \frac{-1}{2} \cdot \frac{6}{7\pi} = \frac{-3}{7\pi}$$

So the line is:

$$y = -\frac{3}{7\pi}x$$

We compute the area between the curve and the line from $x = 0$ to $x = \frac{7\pi}{6}$:

$$A = \int_0^{\frac{7\pi}{6}} \left(\sin x - \left(-\frac{3}{7\pi}x\right)\right) dx = \int_0^{\frac{7\pi}{6}} \left(\sin x + \frac{3}{7\pi}x\right) dx$$

Now evaluate it:

$$\int_0^{\frac{7\pi}{6}} \sin x\, dx = -\cos x\Big|_0^{\frac{7\pi}{6}} = -\cos\left(\frac{7\pi}{6}\right) + \cos(0) = -\left(-\frac{\sqrt{3}}{2}\right) + 1 = \frac{\sqrt{3}}{2} + 1$$

$$\int_0^{\frac{7\pi}{6}} \frac{3}{7\pi}x\, dx = \frac{3}{7\pi} \cdot \frac{x^2}{2}\Big|_0^{\frac{7\pi}{6}} = \frac{3}{7\pi} \cdot \frac{(7\pi/6)^2}{2} = \frac{3}{7\pi} \cdot \frac{49\pi^2}{72} = \frac{147\pi}{504} = \frac{7\pi}{24}$$

So, total area:

$$A = \frac{\sqrt{3}}{2} + 1 + \frac{7\pi}{24}$$

Approximate value:

$$A \approx \frac{1.732}{2} + 1 + \frac{7 \cdot 3.1416}{24} \approx 0.866 + 1 + 0.9167 \approx 2.783$$