Math201_Calculus_I
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Introduction:

The Newton-Raphson technique, also referred to as Newton's Strategy, is a powerful mathematical method used to approximate condition roots. This approach, which was pioneered in the seventeenth century by Isaac Newton and Joseph Raphson, has become crucial to mathematical analysis in all fields of science, design, and mathematics. The goal of this report is to provide a comprehensive understanding of Newton's Technique. It describes the method, provides a bit-by-bit analysis, and focuses on using Newton's strategy to approximate the negative foundation of $e^x = 4 - x^2$.The importance of diagram plotting is highlighted in the report, which also looks into condition reformulation, subsidiary calculation, and the role of programming in producing data for Succeed plots. It concludes by outlining the suitability of Newton's approach, fundamental insights, and sources for further research.

We will also discuss the importance of plotting the graphs, reformulating the equation, computing the derivative, and applying Newton's method iteration. Additionally, we will highlight the need for a program in a computing language to generate data for the Excel plot. Finally, we will conclude by emphasizing the effectiveness of Newton's method for approximating the negative root and provide critical thinking, conclusion and references for further reading.

Definition of Newton's Method:

This is an iterative process of finding root of an given equation f(x) = 0 using Newton's Method. It iteratively refines the initial guess x_0 until convergence upon the true root of the equation. The idea is similar to using tangent lines to get an estimate of the root by iteratively updating the guess according to the formula

Newton's law:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Where, x_n is current guess, $f(x_1)$ is the value of the equation at x_n, and f'(x_n) is the derivative of the equation evaluated at x_n.

For given function,

To solve this problem, we must start with an initial guess and find the derivatives of the function.

Given the function, $e^x = 4 - x^2$

$$\Rightarrow e^x + x^2 - 4 = 0$$

So, f(x) = $e^{x} + x^{2} - 4$

Now, let's try to find the derivative of this function.

The first derivative will be $f'(x) = \frac{d}{dx}(e^{x}) + \frac{d}{dx}(x^{2})$

$\Rightarrow f'(x) = e^{x} + 2x$ [We know, $\frac{d}{dx}(e^{x}) = e^{x}$ and $\frac{d}{dx}(x^{2}) = 2x$]

Methodologies:

We use the newton's formula to calculate the value upto six decimal points as shown below. The equation $e^{x} = 4 - x^{2}$ represents the intersection of two functions: $y = e^{x}$ and $y = 4 - x^{2}$. Our objective is to find the negative root of this equation, which is the value of x where the two functions intersect in the negative x-axis.

We have, $e^{x} = 4 - x^{2}$; we equate it to 0

$4 - x^{2} = 0$

$x = \pm 2$

So, the negative root is -2.

Since, the negative root is -2, we have the f(x) and f'(x). Now using Newton's method of approximation,

Initial guess $x_0 = -2$:

1st iteration:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -2 - \frac{f(-2)}{f'(-2)} = -2 - \frac{e^{-2} + (-2)^2 - 4}{e^{-2} + 2(-2)} = \mathbf{-1.96498136}$$

2nd iteration:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1.96498136 - \frac{f(-1.96498136)}{f'(-1.96498136)}$$

$$= -1.96498136 - \frac{e^{-1.96498136} + (-1.96498136)^2 - 4}{e^{-1.96498136} + 2(-1.96498136)}$$

$= -1.96463563$

3rd iteration:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.964635 - \frac{f(-1.964635)}{f'(-1.964635)}$$

$$= -1.964635 - \frac{e^{-1.964635} + (-1.964635)^2 - 4}{e^{-1.964635} + 2(-1.964635)}$$

$$= -1.96463560$$

Since, Up till the 6th decimal places, X2 = X3

Therefore, the approximated value of $e^x = 4 - x^2$ to 6 decimal places is **–1.964636**

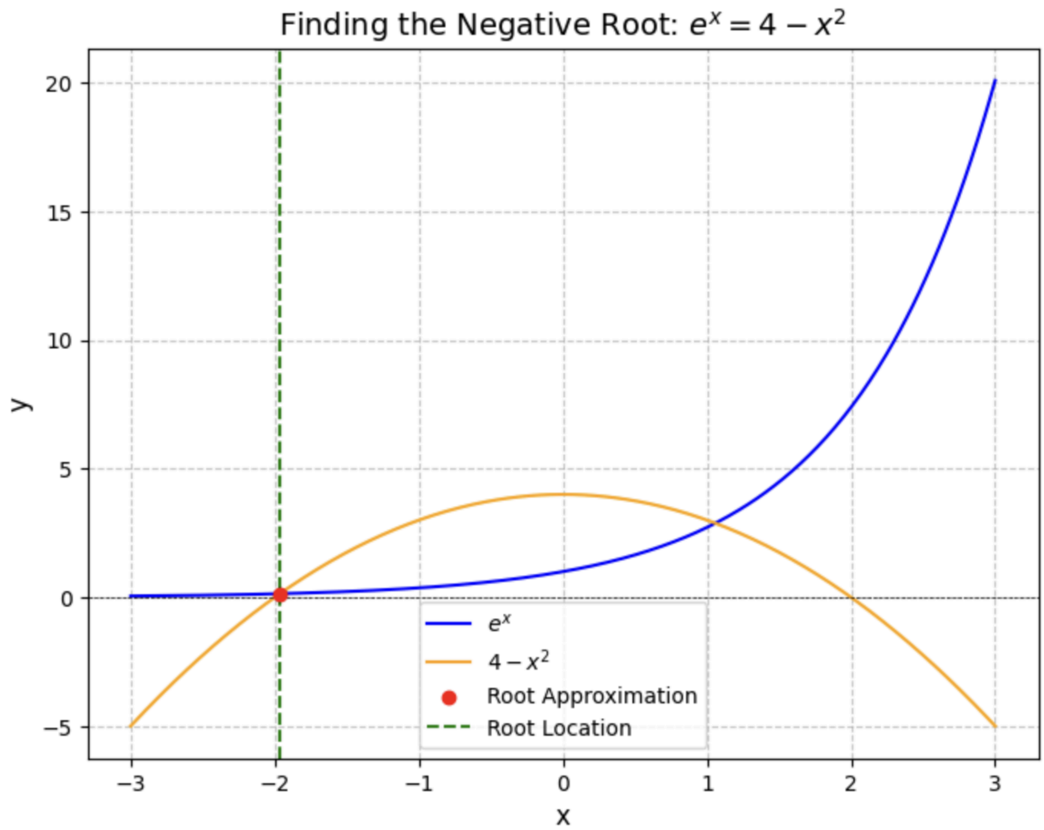
Using the excel sheet to verify all the calculations are correct.

Plotting values in the Excel sheet:

Xn	f(Xn) = e^x + x² - 4	f'(Xn) = e^x + 2x	Xn+1 = Xn - (f(Xn) / f'(Xn))
-2	0.135335283	-3.864664717	-1.964981365
-1.96498	0.001310263	-3.789804231	-1.964635631
-1.96464	1.28E-07	-3.789064298	-1.964635597
-1.96464	0	-3.789064226	-1.964635597

Every time the value of xn changes, we can observe it closely in the Excel sheet. In our mathematical calculation, the first row represents the first iteration, where $xn = x0 = -2$. The value of $x1$ after the calculation seems to be -1.964635. The value of the previous xn-1 value is passed on to the new xn in each iteration.

Additionally, a Python program has been created to display the equation's unfavorable approximate root.



This Python script measures the negative foundation of ex = 4 − x2 using Newton's Strategy.

illustrating the estimation line and the iterative cycle. With the underlying assumption, orange guess line, and reference lines for x-hub and y=0, the diagram illustrates the methodology's union. The plot outwardly addresses the special exchange between the two capabilities, with the orange bend following the descending opening parabola of ex = 4 − x2 and the blue bend depicting the remarkable idea of An. The iterative outcome of Newton's technique, which indicates the point where the left and right sides of the condition converge, is represented by the red point.

Critical Thinking:

Although Newton's method is useful for approximating roots, it doesn't always combine them precisely. The combination may fail or lead to a different path if the underlying hypothesis is not carefully chosen. To avoid this, it is essential to test various initial hypotheses in order to understand the behavior of cycle focuses and ensure union to the ideal root. This iterative approach strengthens the technique's unwavering quality and protects against potential errors.

Conclusion:

In conclusion, approximating the negative root of the equation e^x = 4 − x^2 using Newton's method involves reformulating the equation, computing the derivative, and applying the iteration formula. Visualizing the convergence through a plot in Excel provides a helpful visual representation. Newton's method is an effective approach for approximating roots, but caution must be exercised in choosing the initial guess. Further reading on Newton's method and numerical methods can provide additional insights into this topic.

References:

https://perhuaman.files.wordpress.com/2014/07/metodos-numericos.pdf

https://brilliant.org/wiki/newton-raphson-method/