calculus

SUSHANT HUMAGAIN

April 2025

1

(a) $\int_4^9 3\sqrt{x}\,dx$

First, find the indefinite integral:

$$\int 3\sqrt{x}\,dx = 3\int x^{1/2}dx = 3\cdot\frac{2}{3}x^{3/2} = 2x^{3/2} + C$$

Now, apply the limits:

$$\int_4^9 3\sqrt{x}dx = \left[2x^{3/2}\right]_4^9 = 2(9)^{3/2} - 2(4)^{3/2}$$

Calculate: - $(9)^{3/2} = (9^1)^{3/2} = (3^2)^{3/2} = 3^3 = 27$ - $(4)^{3/2} = (2^2)^{3/2} = 2^3 = 8$

$$= 2(27) - 2(8) = 54 - 16 = 38$$

—

(b) $\int_1^e \ln(x)dx$

First, find the indefinite integral:

$$\int \ln(x)dx = x\ln(x) - x + C$$

Now, apply the limits:

$$\int_1^e \ln(x)dx = [x\ln(x) - x]_1^e$$

$$= (e\ln e - e) - (1\ln 1 - 1) = (e\cdot 1 - e) - (0 - 1) = (e - e) - (-1) = 0 + 1 = 1$$

—

(c) $\int_0^1 \cos^{-1}(x)dx$

Let's use integration by parts: Let $u = \cos^{-1}x,\ dv = dx$ Then $du = -\frac{1}{\sqrt{1-x^2}}dx,\ v = x$

$$\int u\,dv = uv - \int v\,du$$

$$= x\cos^{-1}x\Big|_0^1 + \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$$

Let's compute $\int \frac{x}{\sqrt{1-x^2}}dx$:

Let $w = 1 - x^2 dw = -2xdx - \frac{1}{2}dw = xdx$

$$\int \frac{x}{\sqrt{1-x^2}}dx = \int \frac{1}{\sqrt{w}}\left(-\frac{1}{2}dw\right) = -\frac{1}{2}\int w^{-1/2}dw = -w^{1/2}+C = -\sqrt{1-x^2}+C$$

So,

$$\int_0^1 \frac{x}{\sqrt{1-x^2}}dx = -\sqrt{1-x^2}\Big|_0^1 = -[\sqrt{1-1^2} - \sqrt{1-0^2}] = -[0-1] = 1$$

Now, $x\cos^{-1}x\Big|_0^1 = 1\cdot\cos^{-1}1 - 0\cdot\cos^{-1}0 = 0 - 0 = 0$

Thus, the answer is: 1

—

(d) $\int_{-1}^{1} \pi\cos\left(\frac{\pi x}{2}\right)dx$

Factor out the constant:

$$= \pi\int_{-1}^{1}\cos\left(\frac{\pi x}{2}\right)dx$$

Let's integrate: Let $u = \frac{\pi x}{2} du = \frac{\pi}{2}dxdx = \frac{2}{\pi}du$

When $x = -1, u = -\frac{\pi}{2}$ When $x = 1, u = \frac{\pi}{2}$

So,

$$\int_{-1}^{1}\cos\left(\frac{\pi x}{2}\right)dx = \int_{-\pi/2}^{\pi/2}\cos(u)\cdot\frac{2}{\pi}du = \frac{2}{\pi}\int_{-\pi/2}^{\pi/2}\cos(u)du$$

$$= \frac{2}{\pi}[\sin(u)]_{-\pi/2}^{\pi/2} = \frac{2}{\pi}[\sin(\pi/2) - \sin(-\pi/2)] = \frac{2}{\pi}[1-(-1)] = \frac{2}{\pi}\cdot 2 = \frac{4}{\pi}$$

Thus,

$$\pi\cdot\frac{4}{\pi} = 4$$

question no. 2

a. $\int x^2\cos(x^3)\,dx$

Let's use substitution:

Let $u = x^3 du = 3x^2dxx^2dx = \frac{1}{3}du$

So,

$$\int x^2\cos(x^3)dx = \int\cos(u)\cdot\frac{1}{3}du = \frac{1}{3}\int\cos(u)du = \frac{1}{3}\sin(u) + C$$

Substitute back $u = x^3$:

$$\int x^2 \cos(x^3)\,dx = \frac{1}{3}\sin(x^3) + C$$

—

b. $\int \frac{\cos(3t)}{1+\sin(3t)} dt$

Let's use substitution:

Let $u = 1 + \sin(3t) du = 3\cos(3t)dt \cos(3t)dt = \frac{1}{3}du$

So,

$$\int \frac{\cos(3t)}{1+\sin(3t)} dt = \int \frac{1}{u} \cdot \frac{1}{3} du = \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln|u| + C$$

Substitute back $u = 1 + \sin(3t)$:

$$\int \frac{\cos(3t)}{1+\sin(3t)} dt = \frac{1}{3} \ln|1 + \sin(3t)| + C$$

question no. 3

2. Average Value of $f(x)$ on $[0, 4]$

The average value of a function $f(x)$ over an interval $[a, b]$ is:

$$Averagevalue = \frac{1}{b-a} \int_a^b f(x)\,dx$$

For $f(x) = x^3 - 5x^2 + 30$, $a = 0$, $b = 4$:

$$Averagevalue = \frac{1}{4-0} \int_0^4 (x^3 - 5x^2 + 30) dx$$

Let's compute the integral:

$$\int_0^4 (x^3 - 5x^2 + 30)\,dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 30x\right]_0^4$$

Plugging in $x = 4$:

- $\frac{4^4}{4} = \frac{256}{4} = 64$ - $\frac{5 \times 4^3}{3} = \frac{5 \times 64}{3} = \frac{320}{3}$ - $30 \times 4 = 120$

So,

$$\int_0^4 f(x)\,dx = \left(64 - \frac{320}{3} + 120\right) - (0 - 0 + 0)$$

$$= 64 + 120 - \frac{320}{3}$$

$$= 184 - \frac{320}{3}$$

$$= \frac{552 - 320}{3}$$

$$= \frac{232}{3}$$

Now average value:

$$Averagevalue = \frac{1}{4} \times \frac{232}{3} = \frac{232}{12} = \frac{58}{3} \approx 19.33$$

graph: