7

=

1

7

1

5

12

12

(0*.*5)

+

(0*.*5)

*L*(0*.*5) =

2

*·*

5

This is *L*(0*.*5):

Percentage of total income received by the bottom 50% of households

1.

12

12

*x*:

+

*x*

(b) Given *L*(*x*) =

2

7

+

1

48

24

:

as

Write

14

7

24

48

5

=

7

5

2

12

4

12

*·*

+

0

=

*x*

*x dx* =

*B* =

2

1

1

1

1

Z

- The area under the line *y* = *x* from 0 to 1 is:

[*x − L*(*x*)] *dx*

*A* =

1

Z

*y* = *L*(*x*) is:

- The area between the line *y* = *x* and the Lorenz curve

Recall definitions:

Lorenz curve and the line *y* = *x*

inequality is twice the area between the

(a) Show that the coefficient of

1

question no.

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SUSHANT HUMAGAIN

1

2

0

*B*

1

[*x − L*(*x*)] *dx*

= 2

=

*coef f icientof inequality* =

0

*A*

[*x − L*(*x*)] *dx*

calculusI, Assignment 7

1

Z

R

Substituting the expressions above:

*Areaundery* = *x*

*coef f icientof inequality* =

*Areabetweeny* = *xandy* = *L*(*x*)

- The coefficient of inequality is:

0

0

2

2

Z

*xdx* =

*x*

=

2

2

0

0

2

1

1

1

1

2

3

*x*

*dx* =

*x*

=

Compute

2

[*x − L*(*x*)]*dx* =

(*x − x*

)*dx*

12

0

0

R

1

2

(*x − x*

)*dx*:

0

Z

1

1

1

1

*·*

3

6

6

0

So,

Z

1

5

1

5

[*x − L*(*x*)]*dx* =

2

=

12

6

72

0

Thus,

2

1

3

3

0

0

Z

1

1

1

3 *−* 2

5

2

(*x − x*

)*dx* =

*−*

=

=

5

We use:

Z

1

*coef f icientof inequality* = 2

[*x − L*(*x*)] *dx*

0

Compute *x − L*(*x*):

5

7

Find the coefficient of inequality

7

2

2

*x − L*(*x*) = *x −*

*x*

+

*x*

= *x −*

*x*

*−*

So, as a percentage:

14

19

=

+

=

48

48

48

*x*

19

*≈* 0*.*3958 *≈* 39*.*6%

48

The bottom 50 percent of

households receive about 39*.*6% of

the total

in-

come.

2.

2

12

12

12

12

12

5

*x*

=

(*x − x*

)

12

Now integrate:

Z

Z

1

1

7

12

12

12

12

12

5

5

5

5

2

2

=

*x −*

*x*

*−*

*x* =

*x −*

7*π*

6

2

2

0

0

Z

7*π*

2

2

2

3

3

*x*

3

(7*π/*6)

3

49*π*

147*π*

7*π*

6

*√*

*√*

Z

7*π*

7*π*

7*π*

3

3

6

6

6

sin *x dx* = *−* cos *x*

= *−* cos

+ cos(0) = *−*

*−*

+ 1 =

+ 1

2

504

24

0

0

So, total area:

*√*

3

7*π*

*A* =

+ 1 +

72

24

Approximate value:

1*.*732

7 *·* 3*.*1416

*A ≈*

+ 1 +

*≈* 0*.*866 + 1 + 0*.*9167 *≈* 2*.*783

2

24

3

*x dx* =

*·*

=

*·*

=

*·*

=

=

!

7*π*

7*π*

2

7*π*

2

7*π*

the line using the slope-intercept form:

1.

Integral definition, and 2.

Monte Carlo simulation in Python.

7*π*

1

The line passes through (0*,* 0) and

*, −*

, so we compute the equation of

6

2

1

*−*

*−* 0

*−*1

6

*−*3

2

*m* =

=

*·*

=

question no.

10

5

*coef f icientof inequality* = 2 *·*

=

=

*≈* 0*.*1389

72

72

36

7*π*

2

== solution:

We are asked to find the area between the curve *y* = sin *x* and the line

7*π*

1

segment joining the points (0*,* 0) and

*, −*

using:

6

2

*x*

Z

Z

7*π*

7*π*

3

3

6

6

*A* =

sin *x −*

*−*

*dx* =

sin *x* +

*x*

*dx*

7*π*

7*π*

0

0

Now evaluate it:

*y* = *−*

2

7*π*

7*π*

*−* 0

6

So the line is:

3

5

*x*

7*π*

7*π*

We compute the area between the curve and the line from *x* = 0 to *x* =

:

6