Question.1

Yes, the right-hand side of the equation is a rational number. To see this, we can use the formula for the sum of an infinite geometric series:

1 + r + r^2 + r^3 + ... + r^n = (r^(n+1) - 1) / (r - 1)

If we set r = 1/2 and n = 2^n, we get:

1 + 1/2 + 1/4 + 1/8 + ... + 1/2^(2^n) = (1/2^(2^(n+1)) - 1) / (1/2 - 1)

This will give,

2^(2^(n+1)) / (2 - 1) = 2^(2^(n+1))

Therefore, the right-hand side of the equation is equal to 2^(2^(n+1)), which is a ratio of two integers (namely, 1 and 2^(2^(n+1))).

Question no.4

Let M be the number of mathematics students and C be the number of computer science students at the university. We are given that:

2/3 \* M = 3/5 \* C

Multiplying both sides of the equation by 15 gives us:

10 \* M = 9 \* C

Dividing both sides by 3 gives us:

(10/3) \* M = (9/3) \* C

This simplifies to:

M = (9/10) \* C

Since M is at least 100, we have C >= (10/9) \* 100 = 111.11111...

The least possible values for M and C are 110 and 120, respectively.

Question.no 3

No, the customer will not win $100 because none of the numbers on the card add up to 100. To see this, we can simply check all possible combinations of two or three numbers from the list and verify that none of them sum to 100.

For example, we could check the pairs (72, 28), (21, 79), (15, 85), (36, 64), (69, 31), (81, 19), (9, 91), (27, 73), (42, 58), and (63, 37). We could also check the triplets (72, 21, 27), (72, 21, 42), (72, 15, 63), (72, 36, 42), and so on. In all cases, the sum will be less than 100.

Therefore, the customer will not win $100.

Question no.2

To prove this statement using induction, we need to first establish the base case and the induction step.

Base case: Let c be a real number that satisfies the equation 𝑟3𝑥^3 + 𝑟2𝑥^2 + 𝑟1𝑥+ 𝑟0 = 0, where 𝑟0, 𝑟1, 𝑟2, and 𝑟3 are rational numbers. We want to show that c satisfies an equation of the form 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0 = 0, where 𝑛0, 𝑛1, 𝑛2, and 𝑛3 are integers.

In the base case, we can take 𝑛3 = 𝑟3, 𝑛2 = 𝑟2, 𝑛1 = 𝑟1, and 𝑛0 = 𝑟0. This means that c satisfies the equation 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0 = 0, where 𝑛0, 𝑛1, 𝑛2, and 𝑛3 are integers.

Now suppose that c satisfies an equation of the form 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0 = 0, where 𝑛0, 𝑛1, 𝑛2, and 𝑛3 are integers. We want to show that c also satisfies an equation of the form 𝑚3𝑥^3 + 𝑚2𝑥^2 + 𝑚1𝑥+ 𝑚0 = 0, where 𝑚0, 𝑚1, 𝑚2, and 𝑚3 are integers.

Since c satisfies the equation 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0 = 0, we have 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0 - (𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0) = 0, which simplifies to 0 = 0.

This means that c satisfies the equation 𝑚3𝑥^3 + 𝑚2𝑥^2 + 𝑚1𝑥+ 𝑚0 - (𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+ 𝑛0) = 0, or 𝑚3𝑥^3 + 𝑚2𝑥^2 + 𝑚1𝑥+ 𝑚0 = 𝑛3𝑥^3 + 𝑛2𝑥^2 + 𝑛1𝑥+