Question no. 1

⇒ Here, we have two events, let's define two events by A and B.
A: luggage containing prohibited items and,
B: Luggage is identified as containing prohibited item by X-ray machine

Now, acc. To the question we have to find the probability of A given by B, which is denoted by P(A|B).
We, know the formula ,

$$P(A|B) = P(B|A).P(A) / P(B)$$

Where,

P(A | B) is the probability that the luggage contains prohibited items given that it has been identified.

P(B | A) is the probability that the luggage is identified as containing prohibited items given that it actually contains prohibited items.

P(A) is the prior probability that a luggage contains prohibited items.

P(B) is the probability of being identified.

Using Bayes' theorem,
we have:
P(A|B) = P(B|A) * P(A) / P(B)

We know that:
P(B|A) = 1 - 10^-6 = 0.999999
P(A) = 10^-4 P(B) = P(B|A) * P(A) + P(B|not A) * P(not A)

We also know that:

P(B|not A) = 1/10 P(not A) = 1 - P(A) = 1 - 10^-4

Substituting these values,
we get:
P(B) = 0.999999 * 10^-4 + 1/10 * (1 - 10^-4)   = 0.0001999

Therefore, the probability that a luggage actually contains prohibited items given that the X-ray inspection machine has identified it as containing prohibited items is:
P(A|B) = P(B|A) * P(A) / P(B)
= 0.999999 * 10^-4 / 0.0001999
= 0.4999749987499375

Therefore, the probability that a luggage actually contains prohibited items given that the X-ray inspection machine has identified it as containing prohibited items is approximately 0.499975.

Question no. 2

To find the probability distribution of N, we need to calculate the probability of each possible value of N. Since each game is independent, we can use the binomial distribution to calculate the probability of winning a certain number of games. The probability of winning exactly k games out of n games is given by the formula:

P(k) = (n choose k) * p^k * (1-p)^(n-k)

where n is the total number of games, k is the number of games won, and p is the probability of winning a single game. In this case, n = 7, p(A) = 0.6, and p(B) = 0.4. Therefore, the probability of A winning exactly k games out of 7 games is given by:

P(k) = (7 choose k) * 0.6^k * 0.4^(7-k)

Using this formula, we can calculate the probability of each possible value of N:

P(N=4) = P(4) = (7 choose 4) * 0.6^4 * 0.4^3 = 0.18522
P(N=5) = P(5) = (7 choose 5) * 0.6^5 * 0.4^2 = 0.35527199999999996
P(N=6) = P(6) = (7 choose 6) * 0.6^6 * 0.4^1 = 0.302526
P(N=7) = P(7) = (7 choose 7) * 0.6^7 * 0.4^0 = 0.11529600000000002

Therefore, the probability distribution of N is:

N	Probability
4	0.18522 5
5	0.355272 6
6	0.302526 7
7	0.115296

now,
To find the expected value of N, we can use the formula:

E(N) = sum(N * P(N))
Using the probabilities we calculated above, we get:

E(N) = 4 * 0.18522 + 5 * 0.355272 + 6 * 0.302526 + 7 * 0.115296
= 5.04

question no. 3

Given discrete random variable X are 0, 1, 2, 3
And the corresponding probability P(X) is in the diagram above.
Here,
P(X=0) = 0.2
P(X=1) = 0.1* (1+1) = 0.2
P(X=2) = 0.3*(2+1) = 0.9
P(X=3) = 0.2

Now lets calculate the expected value of X i.e E(X)

Here, $E(X) = \sum_{I} x_i \cdot P(X=x_i)$

Where x_i are the possible value of X and $P(X=x_i)$ is the probability of X taking the value x_i.
In our case the value of X are 0,1, 2, 3 .
Now,
E(X) = 0.02 + 1.02 + 2.09 + 3.02
= 0 + 0.2 + 1.8 + 0.6
= 2.6
So, the expected value of X is 2.6

question no. 4

To find the probability distribution of X, we need to calculate the probability of each possible value of X. Since there are 20 female students and 30 male students in the class, there are a total of 50 students. The number of ways to choose 5 students out of 50 is given by the binomial coefficient:

(50 choose 5) = 2,118,760

The probability of selecting exactly k female students out of 5 students is given by the formula

P(X = k) = (20 choose k) * (30 choose 5 - k) / (50 choose 5)
where k is the number of female students selected. Using this formula, we can calculate the probability of each possible value of X:

P(X = 0) = (20 choose 0) * (30 choose 5) / (50 choose 5) = 0.026
P(X = 1) = (20 choose 1) * (30 choose 4) / (50 choose 5) = 0.204
P(X = 2) = (20 choose 2) * (30 choose 3) / (50 choose 5) = 0.411

P(X = 3) = (20 choose 3) * (30 choose 2) / (50 choose 5) = 0.274
P(X = 4) = (20 choose 4) * (30 choose 1) / (50 choose 5) = 0.081
P(X = 5) = (20 choose 5) * (30 choose 0) / (50 choose 5) = 0.004

Therefore, the probability distribution of X is:

X	probability
0	0.026
1	0.204
2	0.411
3	0.274
4	0.081
5	0.004

Question no.5

⇒

A.

If the sampling is done without replacement, the probability distribution of X is given by:

X	Probability
0	0.025
1	0.324
2	0.540
3	0.111

The mean of X is given by: E(X) = np = 3 * 0.7 = 2.1

The variance of X is given by:
Var(X) = np(1-p)(N-n)/(N-1) = 3 * 0.7 * 9/11 = 1.8909

The probability that the entire batch of TV sets can be accepted is the probability that all three selected TVs are good, which is given by: P(all three are good) = (9/12) * (8/11) * (7/10) = 0.3818

B.

If the sampling is done with replacement, the probability distribution of X is given by:

X	Probability
0	0.008

1	0.189
2	0.576
3	0.227

The mean of X is given by:
E(X) = np = 3 * 0.7 = 2.1

The variance of X is given by:
Var(X) = np(1-p) = 3 * 0.7 * 0.3 = 0.63
The probability that the entire batch of TV sets can be accepted is the probability that all three selected TVs are good, which is given by:
P(all three are good) = 0.7^3 = 0.343

C.
If the sampling is done without replacement, the probability that the third one is one defective is given by:

P(third is defective) = (9/12) * (8/11) * (3/10) = 0.1636