# Anyon Colliders: a time-dependent quantum Hall particle collider to reveal fractional statistics in the Laughlin sequence

Sushanth Varada

varada@chalmers.se

linkedin.com/in/sushant-varada

University of Jyväskylä

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Supervisors: Christian Spånslätt, Matteo Acciai

Promotor: Janine Splettstößer

Co-Supervisor: George Simion

Co-Promotor: Kristiaan De Greve

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# Classification of Elementary Particles

**Bosons**

- Force Mediators: photons, gluons, W and Z bosons
- Explains superconductors, Bose-Einstein condensation
- Bose-Einstein Statistics
- Symmetric wavefunction under exchange

*Tend to occupy the same quantum state*

**Fermions**

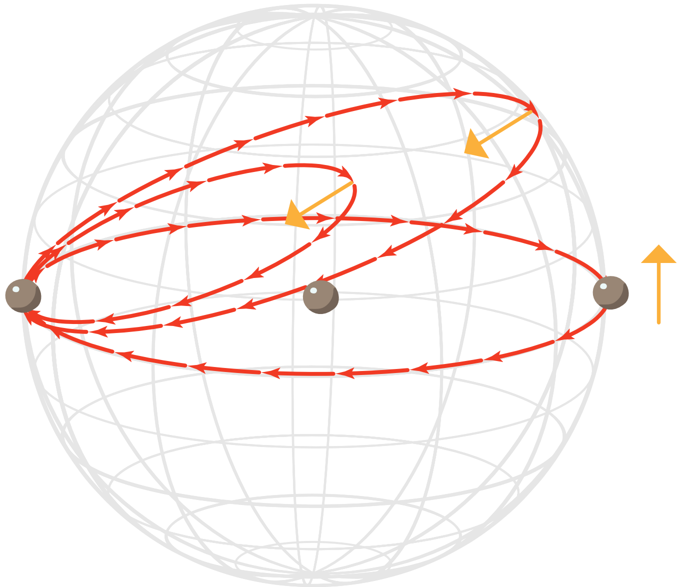
- Matter Particles: electrons, protons, quarks
- Explains metals, insulators, semiconductors
- Fermi-Dirac Statistics
- Anti-symmetric wavefunction under exchange

*Pauli exclusion principle*

*Exchange Statistics distinguishes bosons from fermions*

# Exchange Statistics and Topological Equivalence

J. M. Leinaas et al.: Nuovo Cim. B 37, 1 (1977); G.A. Goldin et al. J. Math. Phys. 21, 650 (1980); F. Wilczek, Phys. Rev. Lett. 49, 957 (1982)



3 Dimensions

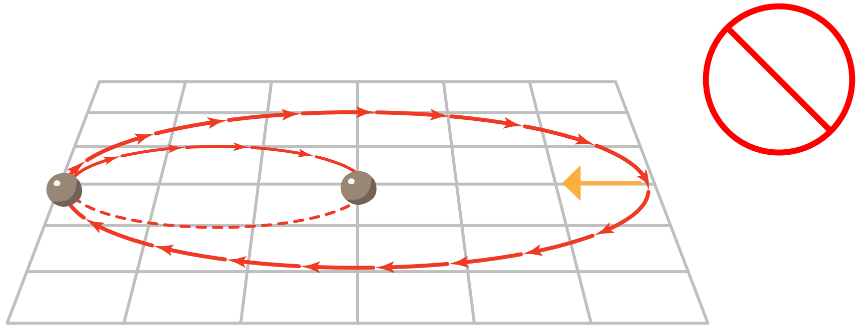
$$\psi(x_1, x_2) \to e^{i\Phi}\psi(x_2, x_1) \to e^{2i\Phi}\psi(x_1, x_2)$$

Deform the loop to a point = Particle did not move

Winding of identical particles is ambiguous

$e^{2i\Phi} = 1 \Rightarrow \Phi = 0 \rightarrow$ *Bosons and* $\Phi = \pi \rightarrow$ *Fermions*

2 Dimensions



Winding of identical particles is well defined!

$$\psi(x_1, x_2) \to e^{im\Phi}\psi(x_1, x_2)$$

$m \rightarrow$ *number of windings* *Any* $\Phi \rightarrow$ ***Anyons***

Topological interaction of particles is "braiding"

$m\Phi$ can be take any fraction of $\pi$

*Fractional Statistics*

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| q | f | l | h | o | i | o | y | v | m | j | s | e | c |
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# Why are anyons important?

*Fundamental:*

- Exotic particles that are neither *bosons* nor *fermions*

  J. M. Leinaas et al.: Nuovo Cim. B 37, 1 (1977)

- They reveal topological ordered states of matter

  X. G. Wen, Int. J. Mod. Phys B 04, 239 (1990)

*Applications:*

- Topological quantum computing based on braiding of anyons

  A. Kitaev, Ann. Phys. 303, 2 (2003)

- Statistical anyons as a resource in quantum thermodynamics?

  Nathan M. Myers et al., PRX Quantum 2, 4 (2021), N. Yunger Halpern et al., npj Quantum Inf. 8, 1 (2022)

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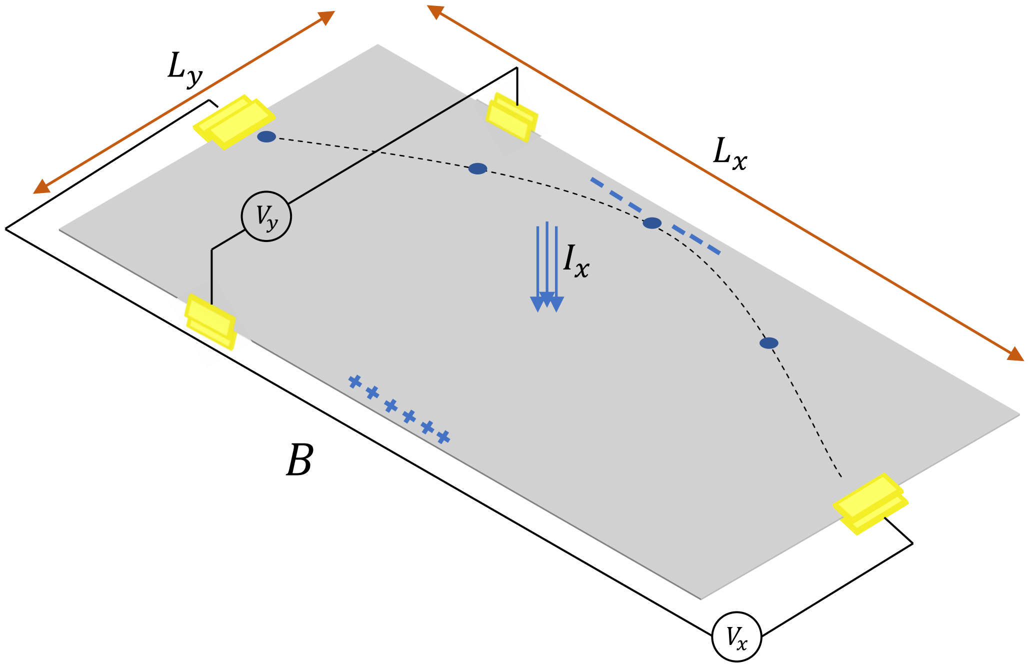
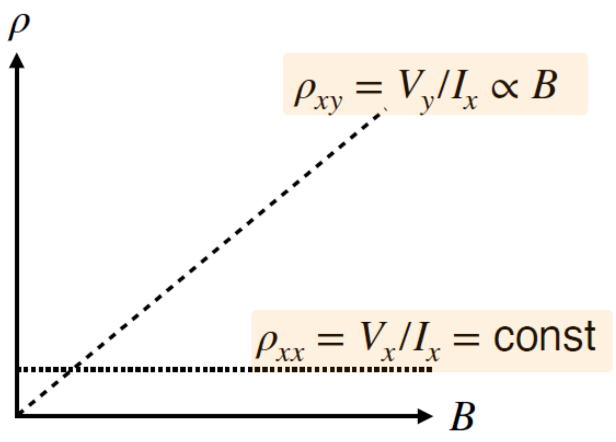
# Quantum Hall Systems as a testbed for anyons

K. v. Klitzing et al., Phys. Rev. Lett. 45, 494 (1980);

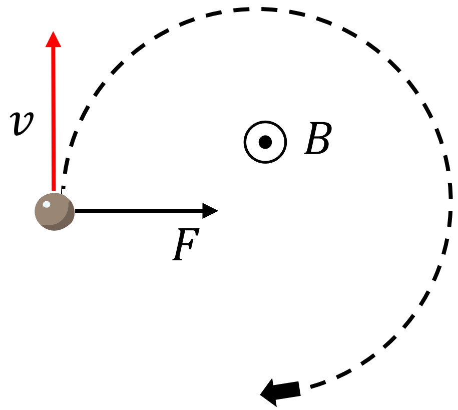
- Classical Hall Effect




*Resistivity Tensor*



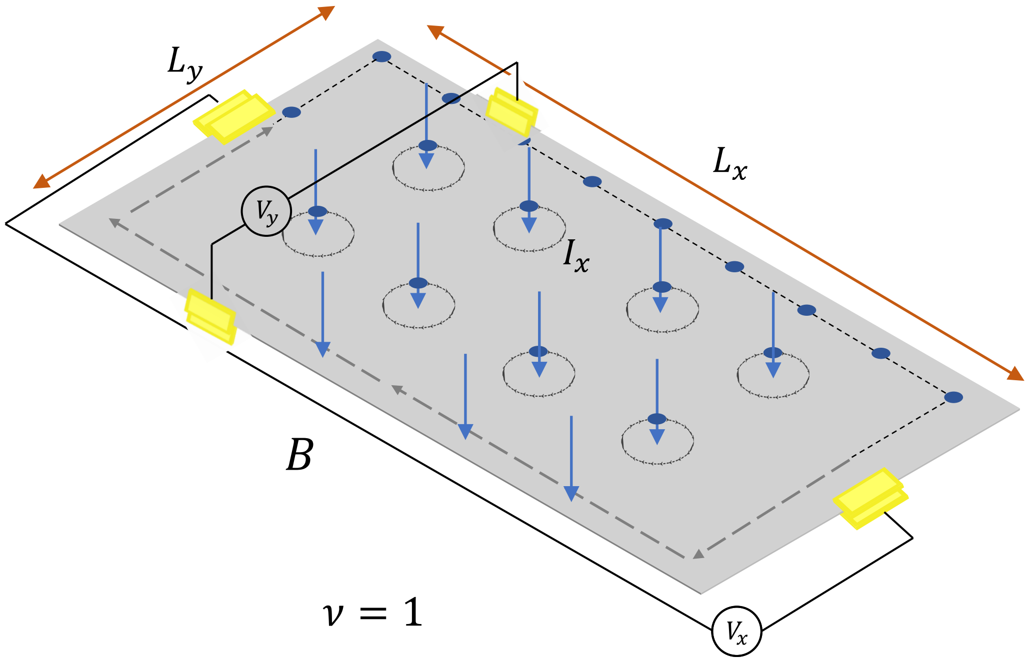

*Lorentz Force*



# Quantum Hall Systems as a testbed for anyons

K. v. Klitzing et al., Phys. Rev. Lett. 45, 494 (1980);

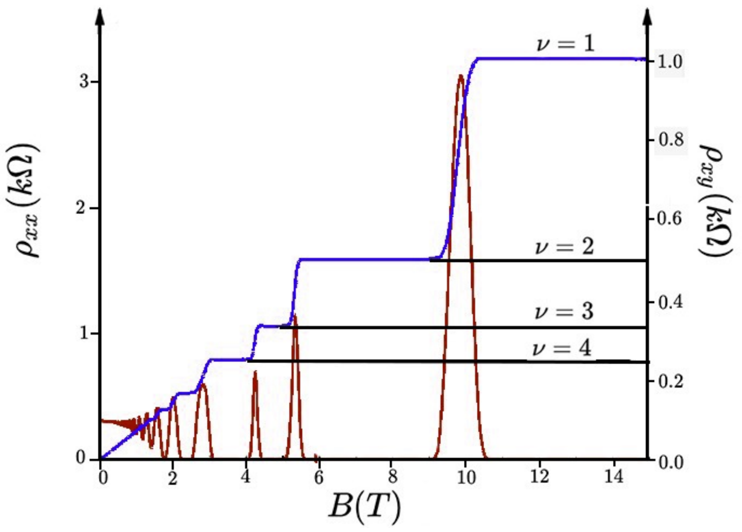
- Integer Quantum Hall Effect




$\nu = 1$

Phase Transition

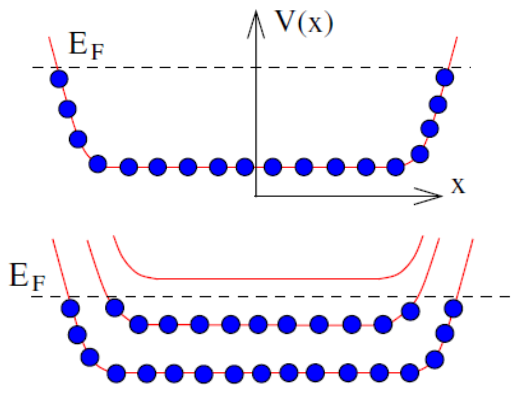
*Quantization of transverse resistivity*




$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu} \qquad \rho_{xx} = 0$$

$\nu = 1, 2, 3\ \ldots$

*Landau Levels*



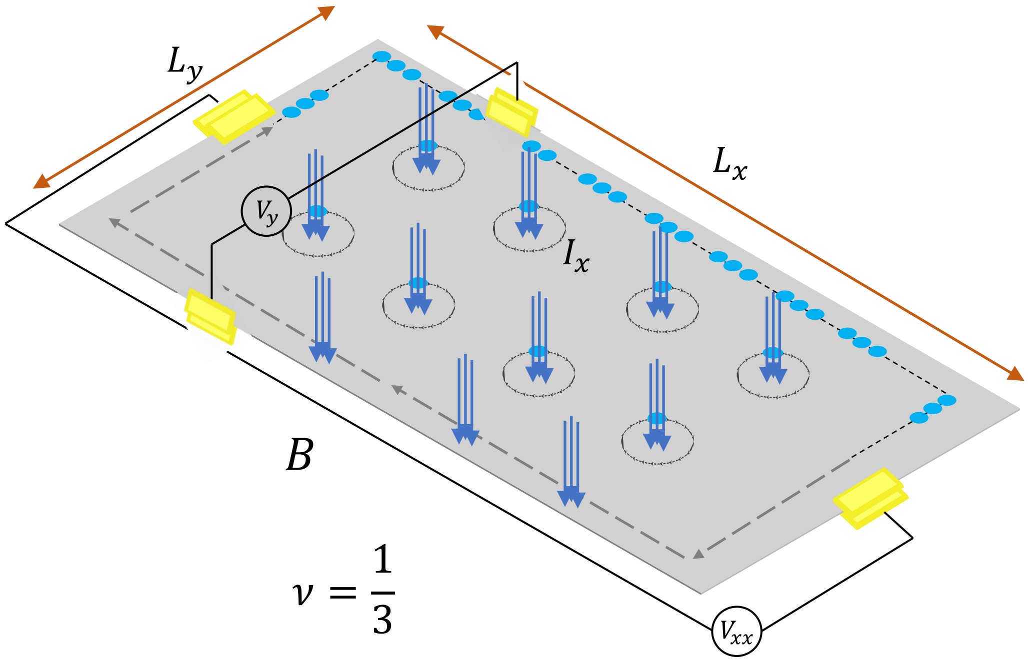

Bending of Landau levels on edges forms 1D conductors

*$\nu$ filled Landau levels*
=
*$\nu$ chiral edge modes*

# Quantum Hall Systems as a testbed for anyons

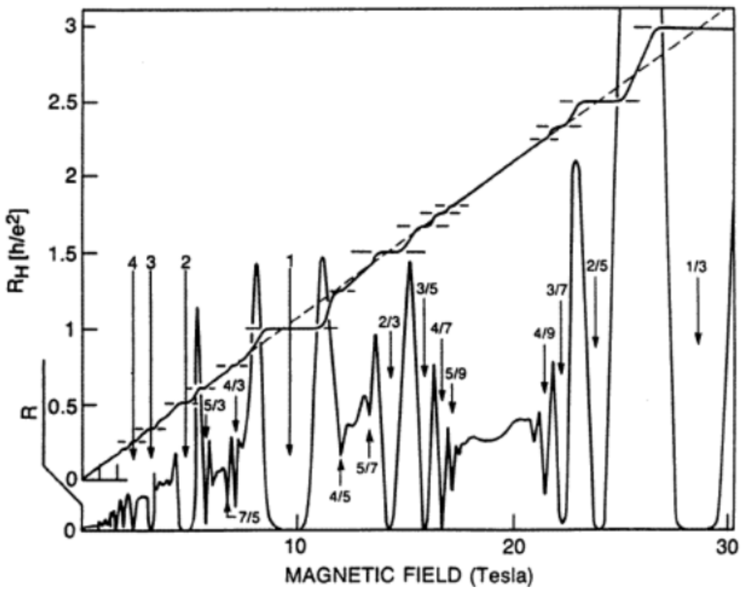
R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983); R. Willett et al., Phys. Rev. Lett. 59, 1776 (1987)

- Fractional Quantum Hall Effect




Phase Transition
+
Strongly correlated state of $e^-$ liquid

*Fractional Quantization of transverse resistivity*




$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu} \qquad \rho_{xx} = 0$$

$$\nu = \frac{1}{3}, \frac{2}{3}, \frac{3}{5} \dots$$

*Partially filled Landau Levels*

*Stabilized by $e^- \leftrightarrow e^-$ interactions*

$$e^* = \frac{e}{3} \ @\ \nu = \frac{1}{3}$$

Collective excitation of system with fractional charge = quasiparticles

*Strong theoretical arguments that the quasiparticles are anyons*

F. Wilczek, Phys. Rev. Lett. 49, 957 (1990)

*Fractionally charged quasiparticles experimentally detected*

L. Saminadayar et al. Phys. Rev. Lett. 79, (1997)

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| q | v | s | u | b | s | o | v | i | t | i | o | q | j |
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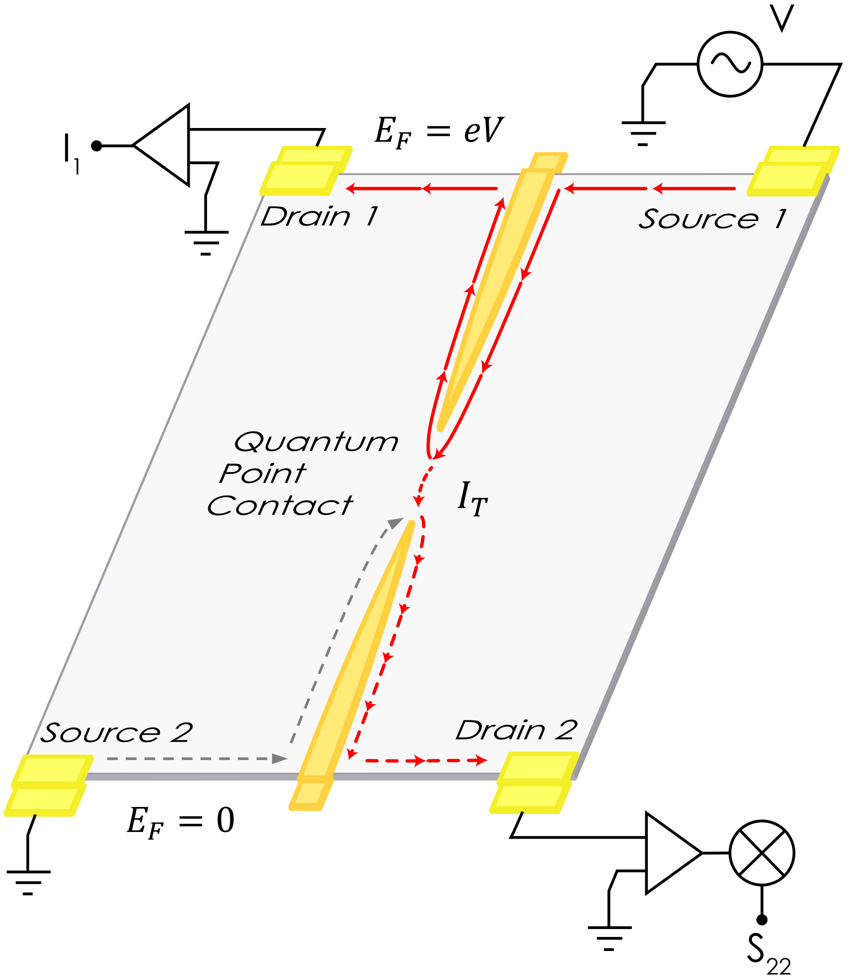
| q | f | l | h | o | i | o | y | v | m | j | s | e | c |
| g | i | w | r | c | v | o | s | d | b | p | u | v | i |
| m | a | q | v | e | a | k | l | w | q | t | k | h | m |

# Probing fractional charge of quasiparticles in a FQH system

L. Saminadayar et al. Phys. Rev. Lett. 79, (1997)




- FQH system with filling factor $\nu = \frac{1}{2n+1}$ [Laughlin Sequence]

*QPC*: Narrow constriction in a 2D electron gas

- Charge can flow through it only one at a time

Conditions

- Tunneling amplitude $\Lambda \ll 1$ (weak backscattering regime)

*Stochastic tunneling of quasiparticles with charge $q^* = \nu q$*

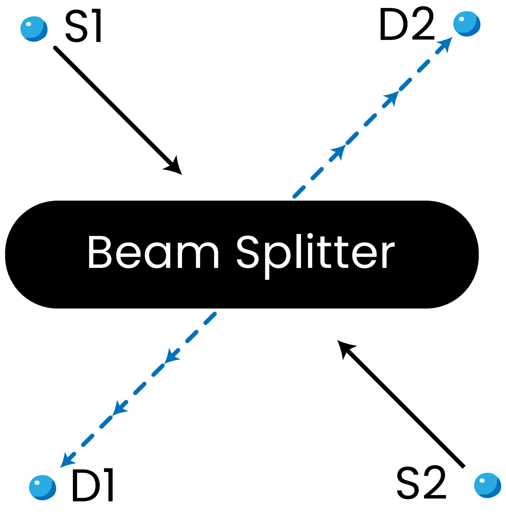
Shot Noise - Current measurements probe information about $q^*$

$S_{22} = 2q^* \langle I_T \rangle$, in the limit $V \gg k_B \theta$

where $\theta$ is temperature, $k_B$ is Boltzmann constant

# Hong-Ou-Mandel (HOM) effect and quantum statistics

Hong, Ou, Mandel, PRL 59, 2044 (1987)

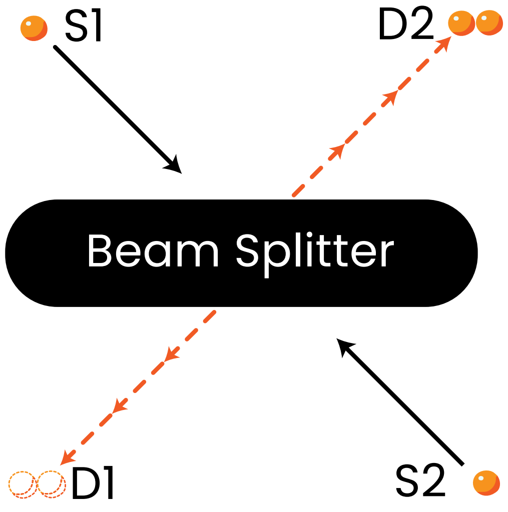



S → Source

D → Detector

Simultaneous arrival

$\Gamma, 1-\Gamma \rightarrow 50\%$



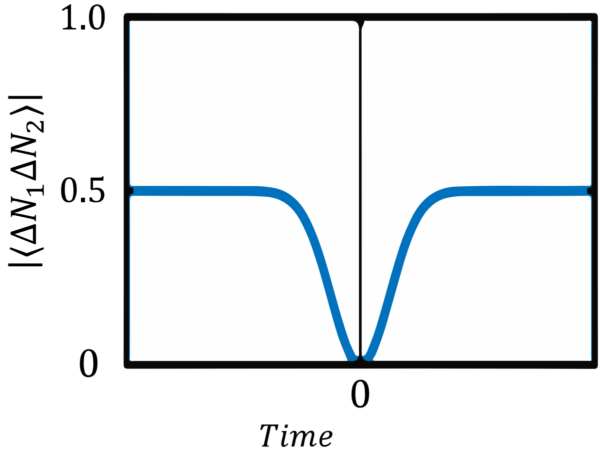

*Anti-Bunching*

Fermions

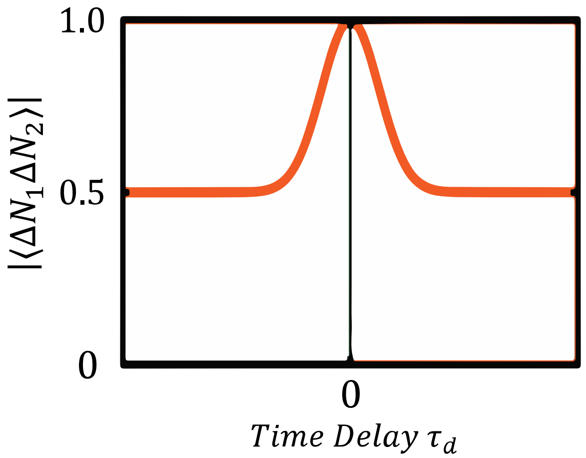
Can HOM be carried over to anyons?

Bosons

*Bunching*




Direct evidence of underlying exchange statistics

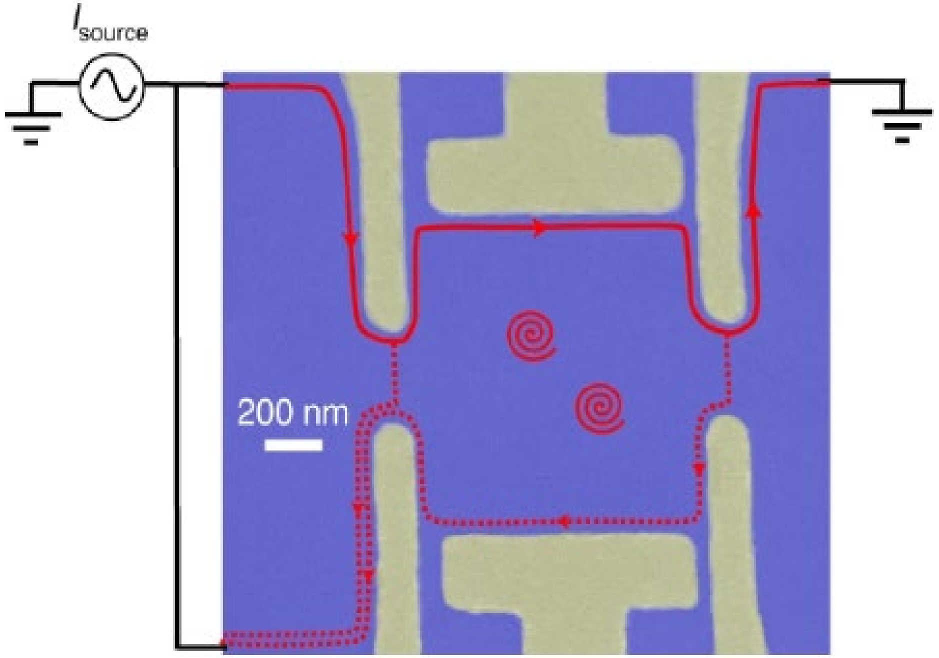


# Experimental Observation of Fractional Statistics

J. Nakamura et al., Nat. Phys. 16, 931 (2020), Bartolomei et al. Science 368, 173-177 (2020)

**Direct observation of anyonic braiding statistics**

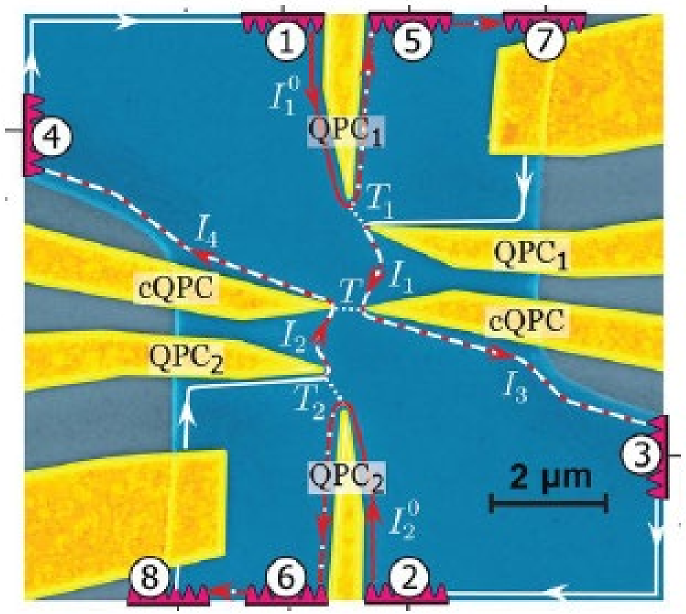
J. Nakamura[1,2], S. Liang[1,2], G. C. Gardner[2,3] and M. J. Manfra[1,2,3,4,5]




Fabry-Perot Interferometer

**Fractional statistics in anyon collisions**

H. Bartolomei[1]*, M. Kumar[1]*†, R. Bisognin[1], A. Marguerite[1]‡, J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1]§



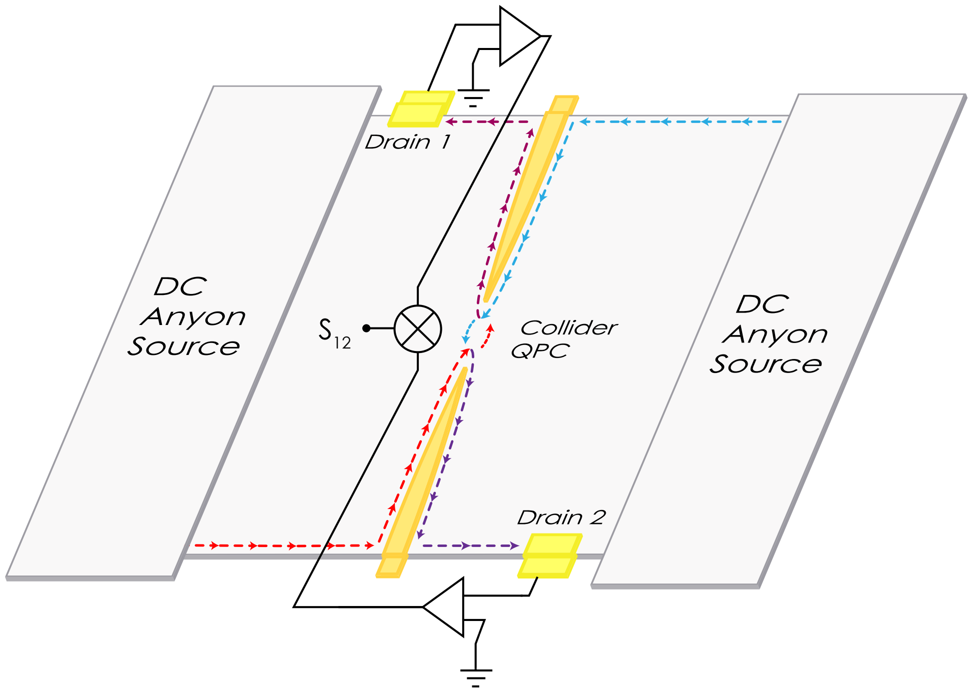

Anyon Collider

Abelian fractional statistics were detected in two seminal experiments conducted in 2020

# Mesoscopic Anyon Colliders

B. Rosenow et al. Phys. Rev. Lett. 116, (2016), Bartolomei et al. Science 368, 173-177 (2020)

DC biased anyon sources emit quasiparticles *randomly*

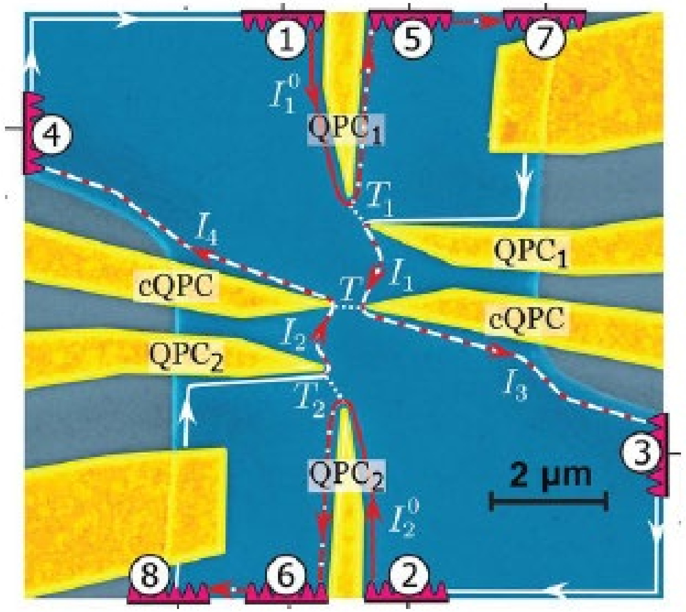



Laughlin anyons with $\nu = \frac{1}{2n+1}$ exhibit *intermediate bunching*

No control over the emission times of the anyons


Fractional statistics in anyon collisions

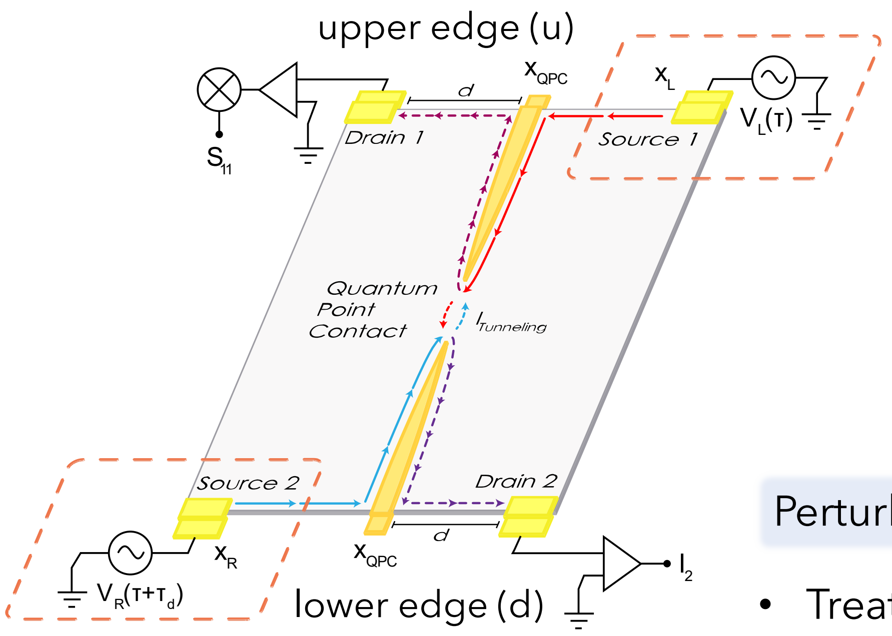
H. Bartolomei[1*], M. Kumar[1*†], R. Bisognin[1], A. Marguerite[1‡], J.-M. Berroir[1], E. Bocquillon[1], B. Plaçais[1], A. Cavanna[2], Q. Dong[2], U. Gennser[2], Y. Jin[2], G. Fève[1§]




Anyon Collider

Time-dependent anyon emission is required to demonstrate Hong-Ou-Mandel effect

# Analysis of collider with time-dependent sources




## Bosonization

- Describe 1D edge modes with compact bosonic field $\phi(x,t)$
- Quasiparticle creation and annihilation operators $\psi_{u,d}(x,t) \sim e^{-i\phi_{u,d}(x,t)}$
- Quasiparticle operators are the anyonic operators that acquire the fractional exchange phase $\vartheta$ upon exchange

$$\psi(x,t)\psi(y,t) = \psi(y,t)\psi(x,t)\, e^{i\vartheta\,\mathrm{sgn}(x-y)}$$

## Perturbation Theory

- Treat tunnelling of quasiparticles as a weak perturbation to the system

$H_\Lambda = \Lambda\, A(t) + \Lambda^\dagger A^\dagger(t)$, where $A(t)$ is the tunneling operator $= \psi_u^\dagger(x_{QPC},t)\psi(x_{QPC},t) + \mathrm{h.c.}$

## Green's functions

- Building block to calculate tunneling current and zero-frequency shot noise

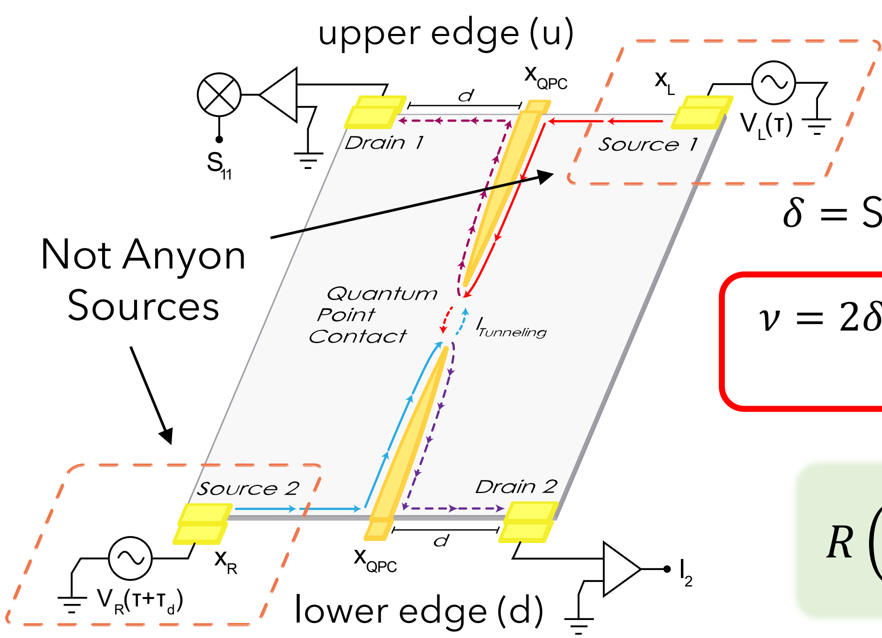
$$\langle I_T(t)\rangle = 2iq\nu|\Lambda|^2 \int_{-\infty}^{t} dt''[G_-^2(t-t'') - G_+^2(t-t'')]\sin\left(q\nu\int_{t''}^{t-d/v} d\tau\, \Delta V(\tau)\right)$$

$$\Delta V(\tau) = V_R(\tau) - V_L(\tau)$$

$$S_{\mathrm{HOM}} = (2q\nu|\Lambda|)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} ds\, G_+^2(s)\cos\left(q\nu\int_t^{t+s} d\tau\, \Delta V(\tau)\right)$$

$G_\pm(t) = \langle \psi_{u,d}^\dagger(t)\psi_{u,d}(0)\rangle$, $s$ = delay between the fluctuations

# Analysis of collider with time-dependent sources




Green's functions $G_{\pm}^{2}(t) = \left(\frac{1}{1 \pm \omega_c t} \frac{\pi k_B \theta t}{\sinh[\pi k_B \theta t]}\right)^{2\delta}$ $\omega_C$ = Energy cut-off

$\delta$ = Scaling dimension of quasi-particle-hole pairs created at the quantum point contact.

$\nu = 2\delta$ in the theoretical model only under specific and ideal conditions such as absence of 1/f noise, edge interactions, and neutral modes.

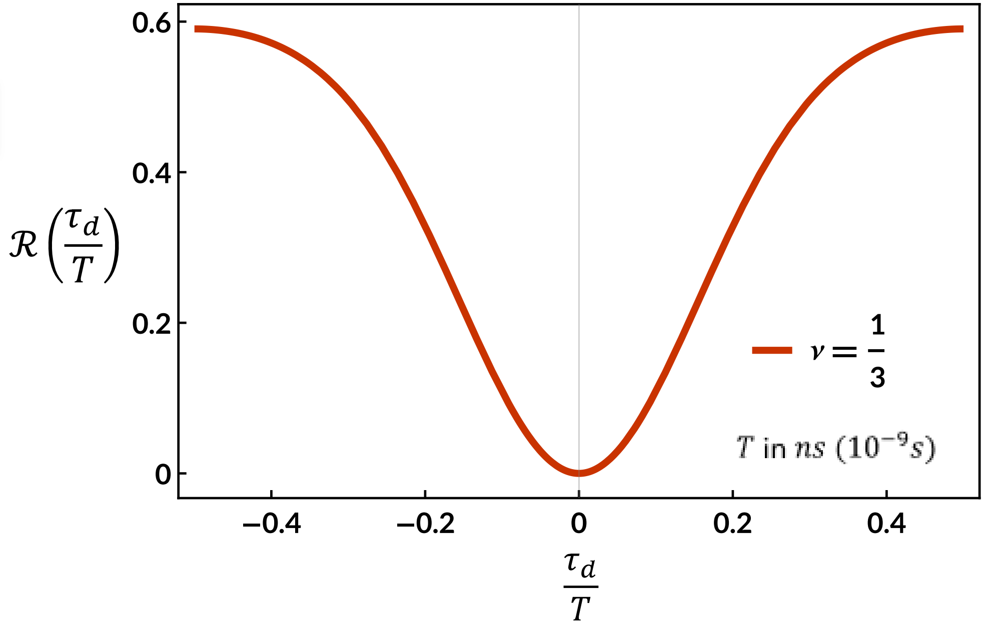
$$R\left(\frac{\tau_d}{T}\right) = \frac{S^{HOM} - S^{eq}}{S_R^{HBT} + S_L^{HBT} - 2S^{eq}}$$

$S_{HBT}$ = Noise obtained for a single voltage source

$S_{eq}$ = Equilibrium noise or background fluctuations

$V_R(\tau) = VSin(\Omega[\tau + \tau_d])$ $V_L(\tau) = VSin(\Omega\tau)$ $T$ = Time period of one cycle

- Vanishing Hong-Ou-Mandel noise ratio at filling factor $\nu = \frac{1}{3}$ should not be interpreted as stemming from statistics of anyons
- Conventional voltage sources cannot excite a fractionally charged quasiparticle using any of Lorentzian, sinusoidal, or square drives



# Anyon Sources?

- Applying random $\delta(\tau)$ voltage pulses = single QPC Poissonian anyon source

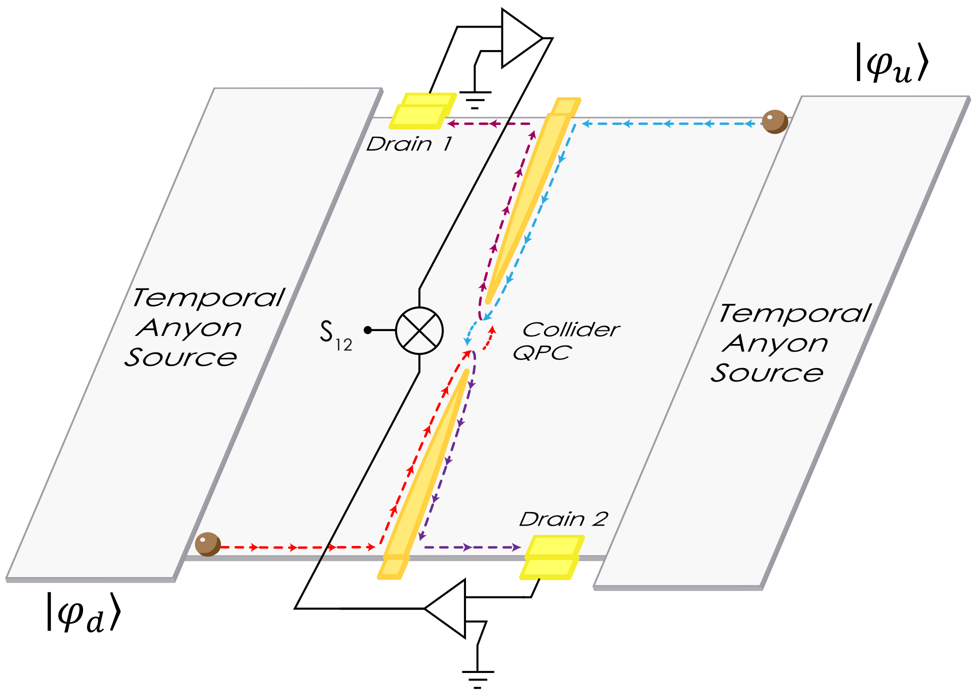
  C. Mora arXiv 2212.05123, (2022)

- Applying $V(\tau) = (2\pi/e)\delta(\tau)$ pulse at electrodes creates a single anyon

  T. Jonckheere et al. Phys. Rev. Lett. 130, (2023)

- Model Anyon Sources as a time-resolved auxiliary state $|\varphi_{u,d}\rangle = \psi^{\dagger}_{u,d}(t_{u,d})|0\rangle$

- It creates a point-like anyon on the ground state of the system




Double injection of anyons in the upper edge (u) and the lower edge (d) to demonstrate the Hong-Ou-Mandel effect

$$|\varphi\rangle = |\varphi_u\rangle \otimes |\varphi_d\rangle = \psi^{\dagger}_{u}(t_u)\, \psi^{\dagger}_{d}(t_d)|0\rangle$$

Consider $\tau_d = t_d - t_u$ that corresponds to the delay between the arrival of the injected anyons at the collider quantum point contact

$$|\varphi\rangle = \psi^{\dagger}_{u}(0)\, \psi^{\dagger}_{d}(\tau_d)|0\rangle$$

# Exchange phase erasure in anyonic Hong-Ou-Mandel effect

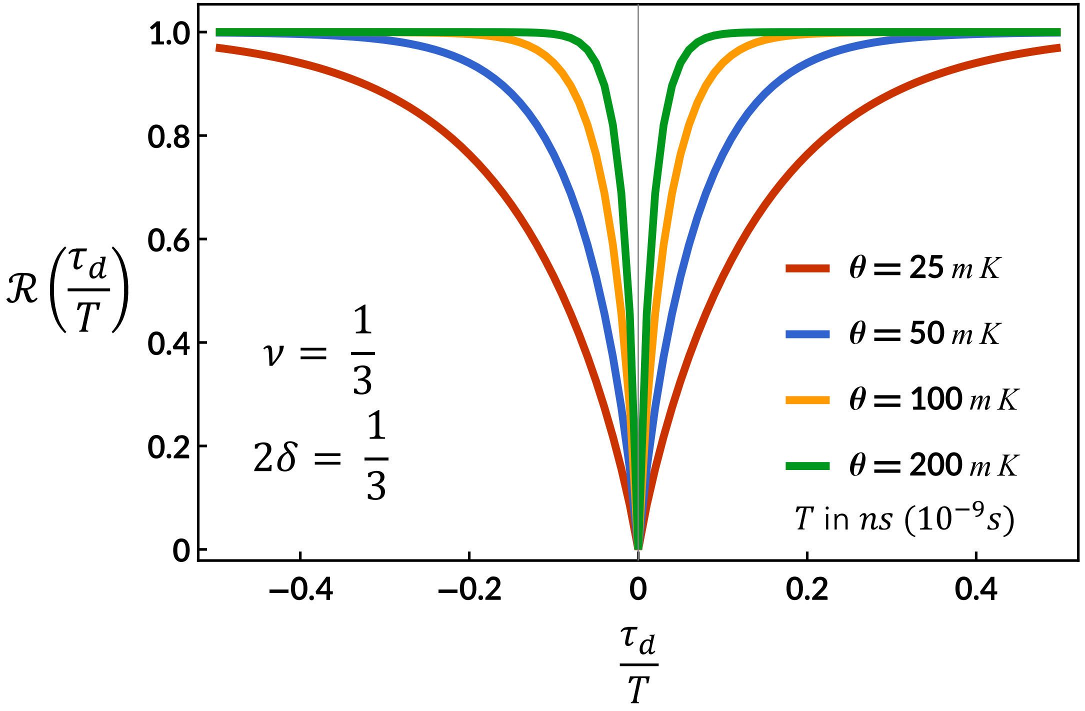
$$I_{\mathrm{T}}(t) = 4qv\Lambda^2(2\pi T_0)^{4\delta-1}\alpha^{4\delta}\boxed{\sin(2\pi\delta)\sin(2\vartheta)}\Theta(t)[\mathcal{B}(e^{-2\pi k_B\theta t}, 2\delta, 1-4\delta) - \Theta(t-|\tau_d|)\mathcal{B}(e^{-2\pi k_B\theta(t-\tau_d)}, 2\delta, 1-4\delta)]$$

$\mathcal{B}(x,a,b)$ = Incomplete Beta function, $\delta$ = scaling dimension of quasiparticles excited at the QPC, $\vartheta$ = braiding phase

$$R\left(\frac{\tau_d}{T}\right) = \frac{\cancel{\cos(2\vartheta)-1}}{\cancel{\cos(2\vartheta)-1}}\,\frac{1}{2}\left(1+\frac{\int_0^{|\tau_d|} dt\,\mathcal{B}\left(e^{2\pi k_B\theta(t-|\tau_d|)},2\delta,1-4\delta\right)-\int_{-\infty}^{0} dt\,\mathcal{B}\left(e^{2\pi k_B\theta(t-|\tau_d|)},2\delta,1-4\delta\right)}{\int_{-\infty}^{0} dt\,\mathcal{B}\left(e^{2\pi \mathrm{T}(t)},2\delta,1-4\delta\right)}\right)$$

- The universal braiding phase is erased from the noise ratio
- The noise ratio probes the non-universal scaling dimension
- Width of the anyonic HOM dip is governed by temperature

  This temperature dependence of anyon HOM curves was also shown in T. Jonckheere et al. Phys. Rev. Lett. 130, (2023)
- Starkly contrasts with Hong-Ou-Mandel effect for electrons where width of noise suppression is only dependent on the temporal extension of input electronic excitations



# Interpretation: Time Domain Braiding

H.S. Sim et al. Phys. Rev. Lett. 123, (2019)

Express shot noise as an interference pattern

$$S_{\mathrm{HOM}}(t,t') \sim \int_{-\infty}^{\infty} \frac{dt}{T} \int_{-\infty}^{\infty} dt' \sum_{k=+,-} \langle t,\tau_d | t',\tau_d \rangle_k + \langle t',\tau_d | t,\tau_d \rangle_k$$

$$|t,\tau_d\rangle_- = A(t)|\varphi\rangle,\ |t,\tau_d\rangle_+ = A^\dagger(t)|\varphi\rangle,$$
$$_-\langle t,\tau_d| = \langle\varphi|A^\dagger(t),\ _+\langle t,\tau_d| = \langle\varphi|A(t)$$

$A(t)$ creates a quasiparticle in the upper edge (u) and a quasihole in the lower edge (d)

$A^\dagger(t)$ creates a quasiparticle in the lower edge (d) and a quasihole in the upper edge (u)

injected anyons

quasi-particle-hole pair

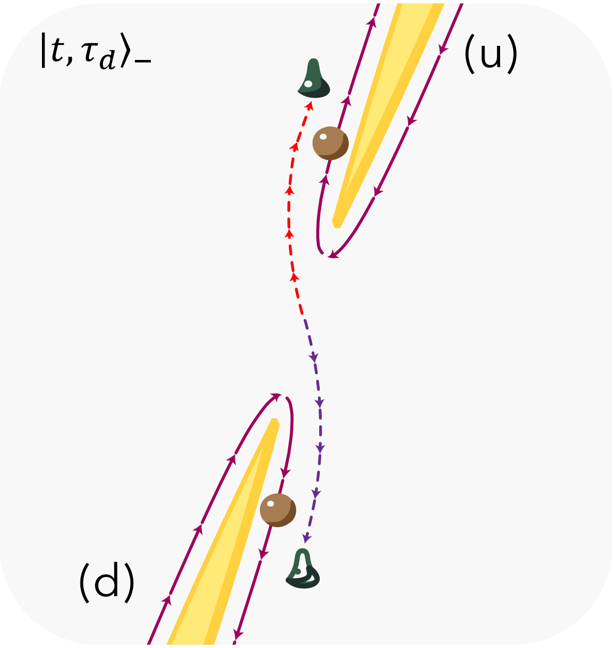
$k = - \Rightarrow |\,.\rangle_- \Rightarrow A(t) \rightarrow$ in (u) and in (d)

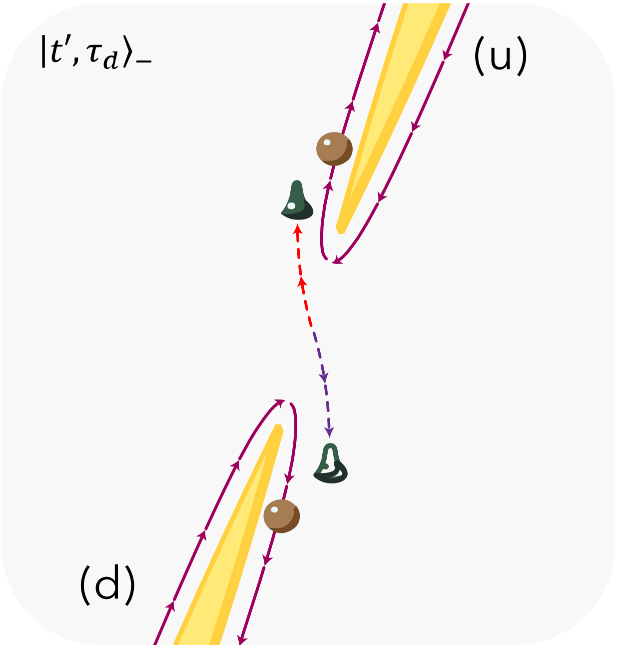
$k = + \Rightarrow |\,.\rangle_+ \Rightarrow A^\dagger(t) \rightarrow$ in (u) and in (d)

- Anyon injection times $(t_u, t_d)$ fall within the time window $(t', t)$ of quasi-particle-hole pair creation and
- Assume: $t' > (t_u, t_d) > t \ \Rightarrow$ events at time $t'$ are the last

$|t,\tau_d\rangle_\pm \rightarrow$ Creates quasi-particle-hole pair *before* the arrival of injected anyons

$|t',\tau_d\rangle_\pm \rightarrow$ Creates quasi-particle-hole *after* the arrival of injected anyons

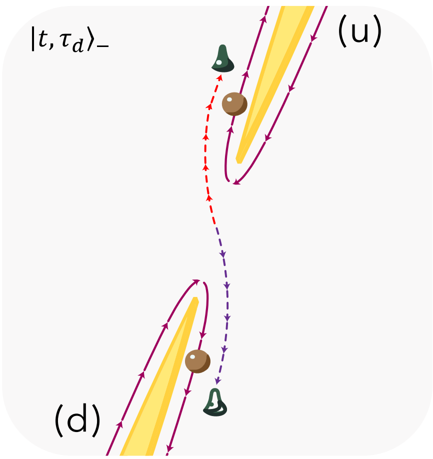


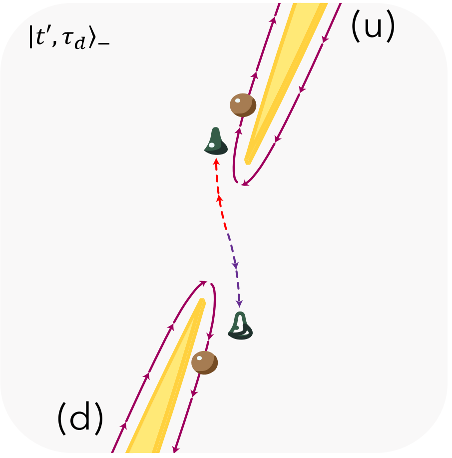



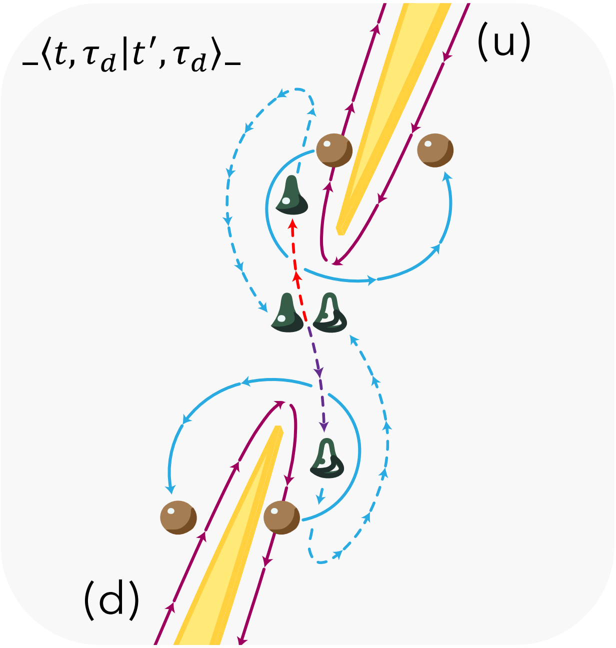
# The Braid Rewind

$$S_{\mathrm{HOM}}(t,t') \sim \int_{-\infty}^{\infty} \frac{dt}{T} \int_{-\infty}^{\infty} dt' \sum_{k=+,-} \langle t,\tau_d|t',\tau_d\rangle_k + \langle t',\tau_d|t,\tau_d\rangle_k$$

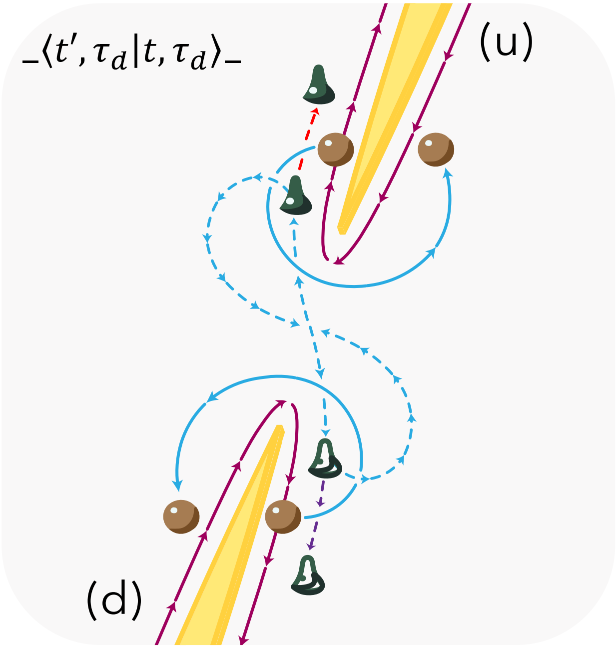
The conjugate of the ket states $|.\rangle_{\pm}^{\dagger} \rightarrow {}_{\pm}\langle .|$ reverses the path traced by particles



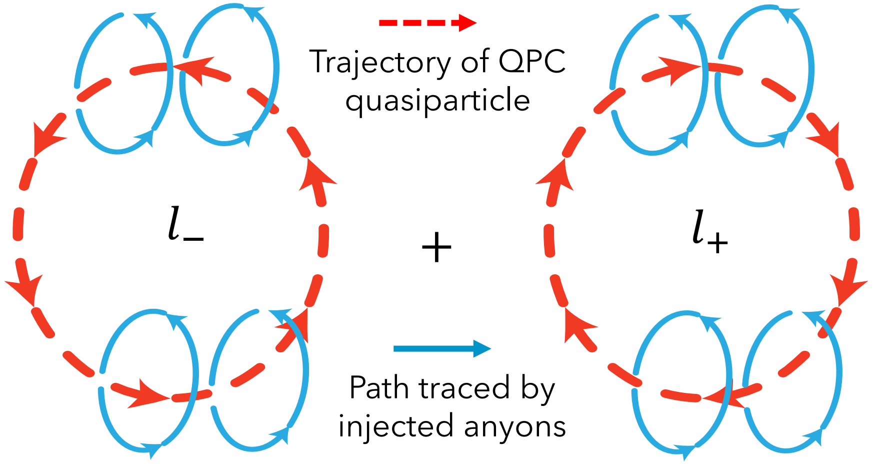




Quasi-particle-hole pair excited at $t$ rewind to form interference loop $l_-$




Quasi-particle-hole pair excited at $t'$ rewind to form interference loop $l_-$

Braiding in the upper edge (u)




Braiding in the lower edge (d)

Counteracting processes cancel the effects of braiding angle $\vartheta$ from the noise

Blue trajectories denote the *rewound* path of particles in time

# Conclusion

- Hong-Ou-Mandel interferometry for anyons does not probe their universal braiding phase.
- Instead, it probes the non-universal scaling dimension of quasiparticle excitations created at the QPC
- Counteracting time domain braiding is a possible interpretation of exchange phase erasure from the noise

# Next Step …

- Would finite frequency noise offer a potential avenue to access information about the exchange phase?
- Auxiliary states beyond point-like anyon injection?

# Thank you

Sushanth Varada
varada@chalmers.se
linkedin.com/in/sushant-varada

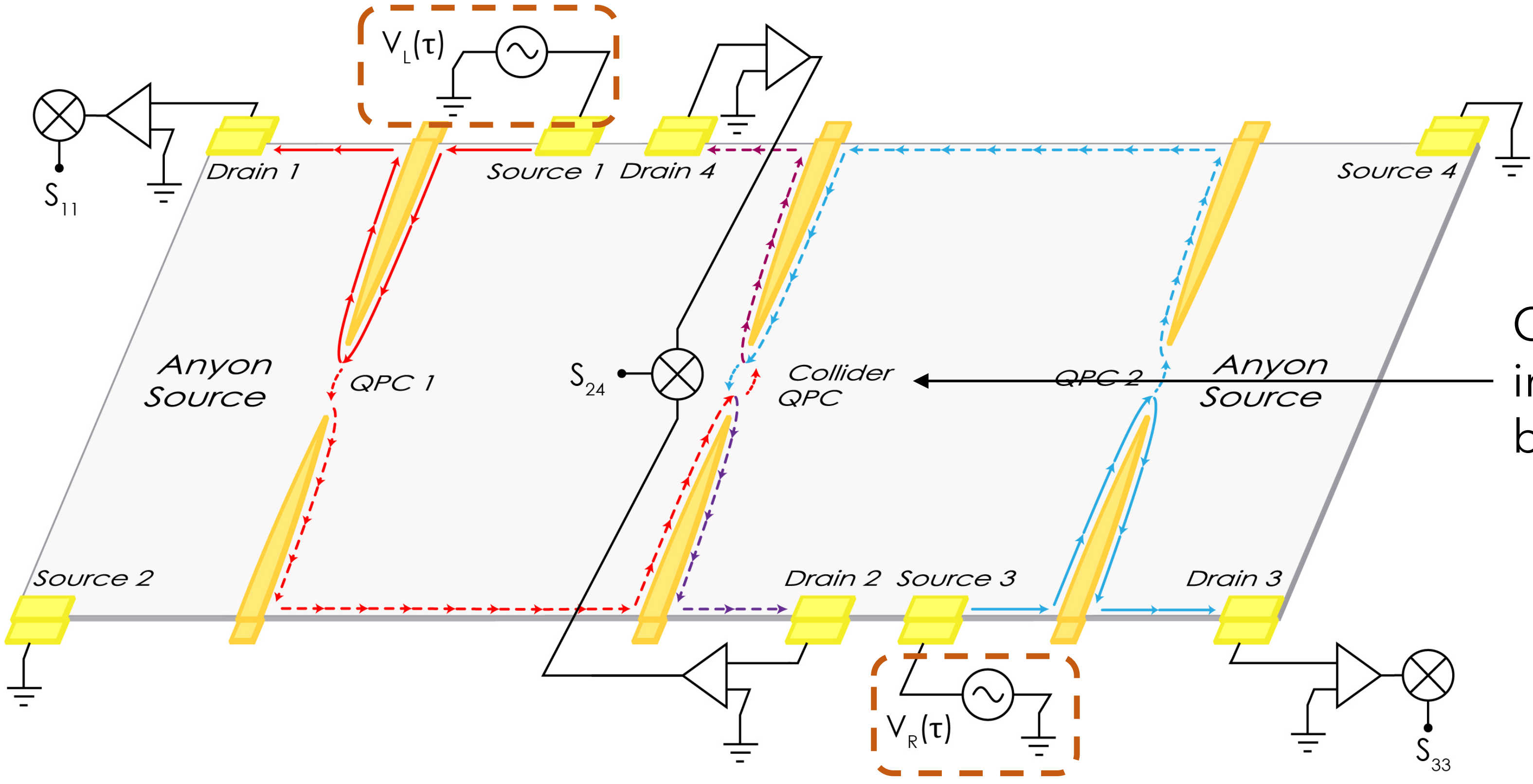
Supervisors: Christian Spånslätt, Matteo Acciai
Promotor: Janine Splettstößer
Co-Supervisor: George Simion
Co-Promotor: Kristiaan De Greve

Erasmus Mundus

KU LEUVEN

CHALMERS
UNIVERSITY OF TECHNOLOGY

# Adding time-dependence to “engineer anyons”




Collisions can be re-interpreted as braiding of anyons

- Probe the properties of anyons with tailored phase and shape
- A step toward on-demand braiding of anyons for quantum computing

J. Rech et al. Phys. Rev. Lett. 118, (2017); M. Kapfer et al., Science 363, (2019)