Exchange statistics in time-resolved anyon collisions in the fractional quantum Hall effect

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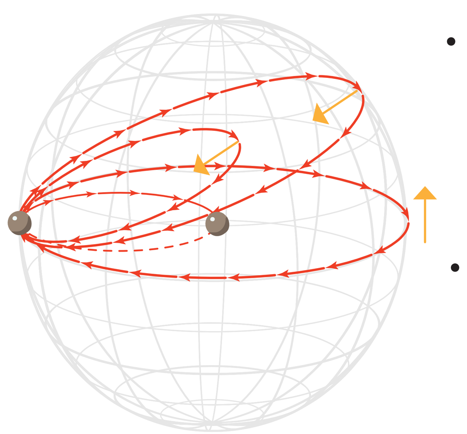
1 What are anyons?

Anyons are quasiparticles that exist in 2+1 dimensions and obey exchange statistics intermediate between bosons and fermions.

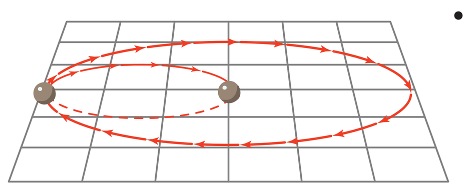


- Exchanging two identical particles twice is equivalent to encircling one another

$$\psi(x_1, x_2) \to e^{i\theta}\psi(x_2, x_1) \to e^{2i\theta}\psi(x_1, x_2)$$

- Winding loop can be topologically deformed to a point in 3D as if particles never moved

$\theta = 0$ → Bosons $\quad \theta = \pi$ → Fermions

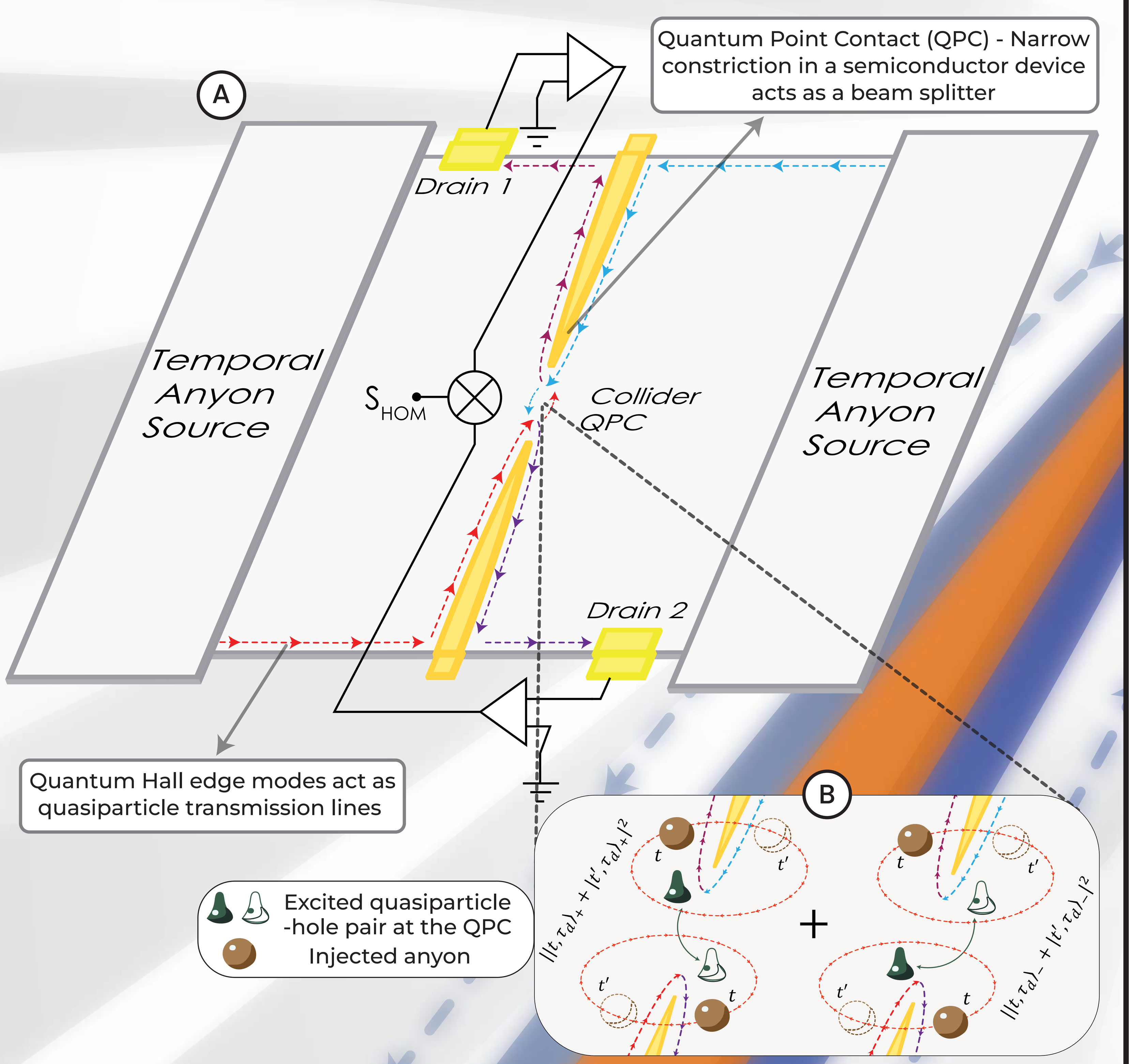


- Interchanging two particles is non-trivial in 2D[1]

$\psi(x_1, x_2) \to e^{2im\theta}\psi(x_1, x_2) \quad m \to$ *number of windings*

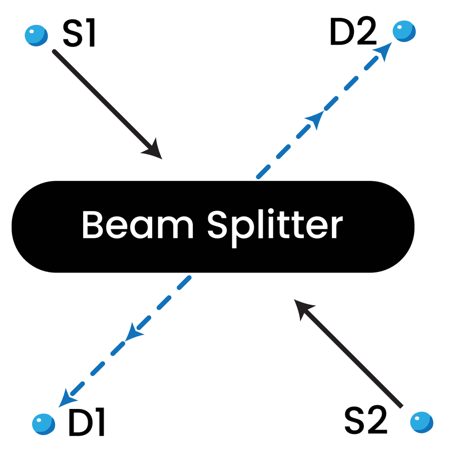
Any θ → Anyons[2] obeying Fractional Statistics

Fractional quantum Hall (FQH) is a phase of matter hosting anyons[3].



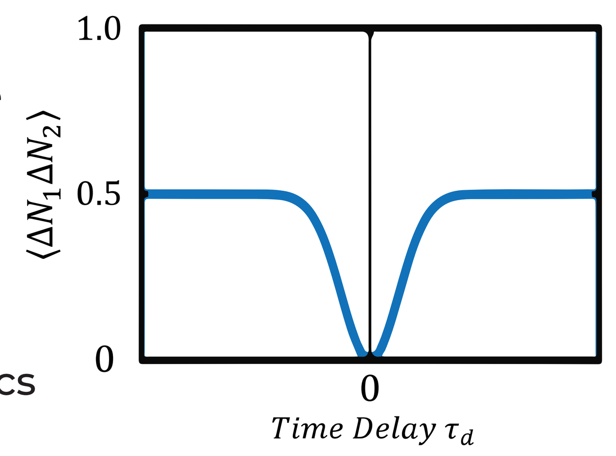
2 Hong-Ou-Mandel (HOM) effect

Partition or Shot noise $\langle \Delta N_1 \Delta N_2 \rangle$, arises from the random distribution of a stream of indistinguishable particles into transmitted and reflected signals[4].

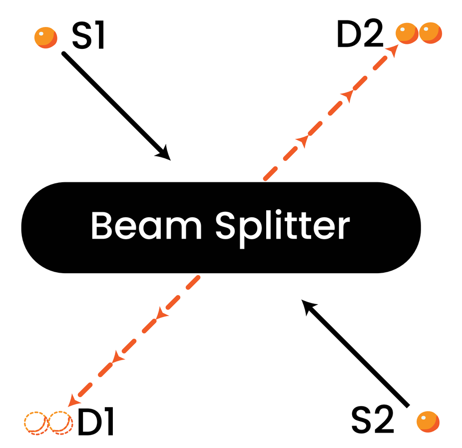


Fermions

- Detectors always measure a particle at $\tau_d = 0$, suppressing fluctuations $\Delta N_{1,2} = 0$, due to Pauli Exclusion
- $\langle \Delta N_1 \Delta N_2 \rangle = 0$ → Uncorrelation
- Antibunching → Fermi-Dirac Statistics

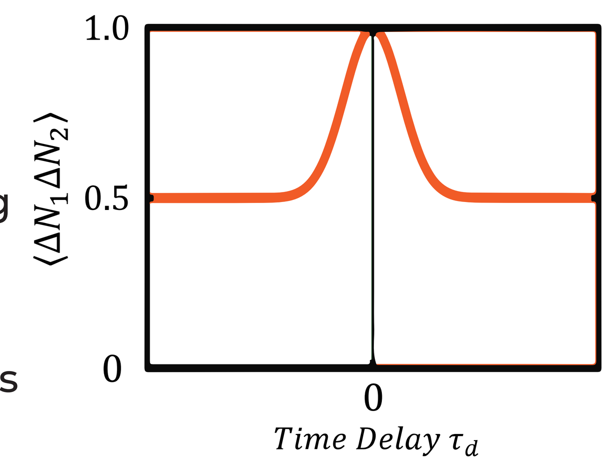


S= Sources, D = Detectors



Bosons

- Detectors measure 0 or 2 particles at $\tau_d = 0$, enhancing fluctuations $\Delta N_{1/2} = \pm 1$, due to bosonic bunching
- $\langle \Delta N_1 \Delta N_2 \rangle = -1$ → Anticorrelation
- Bunching → Bose-Einstein Statistics



Can HOM for anyons probe Fractional Statistics?

3 Exchange phase erasure in anyonic HOM

We analyze an FQH setup[A] in the Laughlin sequence with filling factor $\nu = \frac{1}{2n+1}, n \in \mathbb{Z}^+$

- Time-resolved anyon sources are modeled using an auxiliary state with quasiparticle creation operators acting on the system's ground state $|\varphi\rangle_{HOM} = \psi_u^\dagger(t_u)\,\psi_d^\dagger(t_d)|0\rangle$
- The injection of anyons generates a tunneling current of quasiparticles at the QPC:

$$I_\mathrm{T}(t, \tau_d) = 4q\nu\gamma \sin(2\pi\delta)\sin(2\theta)\,\Theta(t)\left[\mathcal{B}\left(e^{-2\pi \mathrm{T}(t)}, 2\delta, 1-4\delta\right) - \Theta(t - |\tau_d|)\mathcal{B}\left(e^{-2\pi\mathrm{T}(t-|\tau_d|)}, 2\delta, 1-4\delta\right)\right]$$

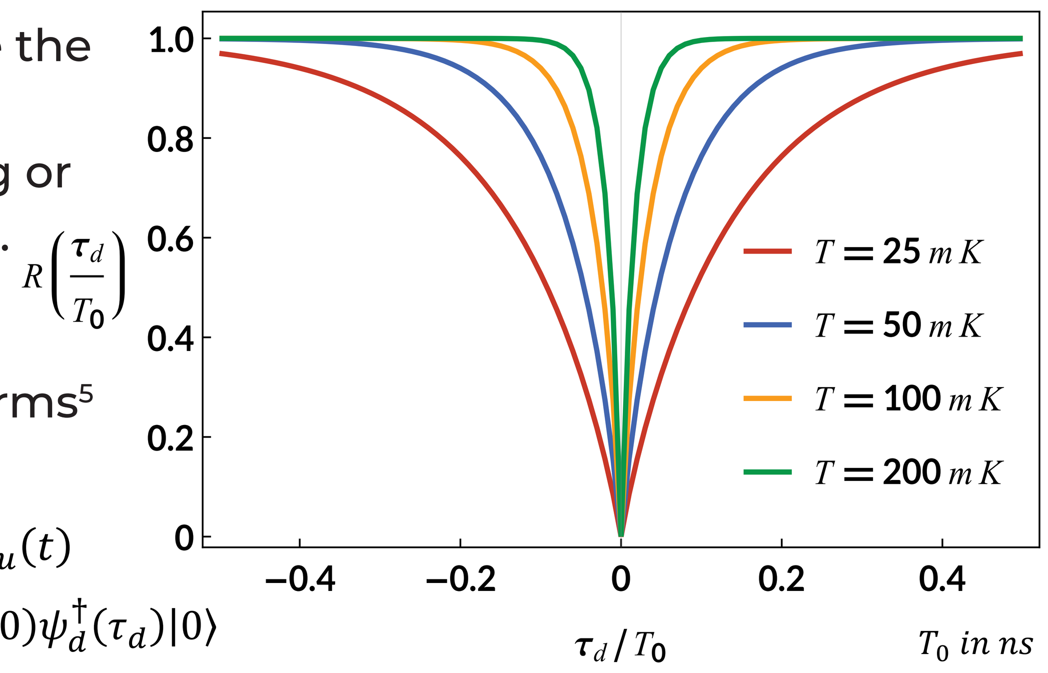
θ → Statistical exchange/braiding phase, $\tau_d = t_d - t_u$ → Injection time delay, T → Temperature, δ → Scaling dimension of quasiparticles excited at the QPC, $\mathcal{B}(x, a, b)$ → Incomplete Beta function, q → Charge of electron, γ → Prefactor dependent on energy cut-off, tunneling amplitude, T, δ

- We compute the zero-frequency HOM noise and normalize it with Hanbury Brown -Twiss (HBT) noise, which refers to fluctuations observed for a single particle injection

$$R\left(\frac{\tau_d}{T_0}\right) = \frac{S^{HOM} - S^{eq}}{S_R^{HBT} + S_L^{HBT} - 2S^{eq}} = \left(\frac{\cancel{\cos(2\theta)} - 1}{\cancel{\cos(2\theta)} - 1}\right)\frac{1}{2}\left[1 + \frac{\int_0^{|\tau_d|} dt\, \mathcal{B}\left(e^{2\pi\mathrm{T}(t-|\tau_d|)}, 2\delta, 1-4\delta\right) - \int_{-\infty}^{0} dt\, \mathcal{B}\left(e^{2\pi\mathrm{T}(t-|\tau_d|)}, 2\delta, 1-4\delta\right)}{\int_{-\infty}^{0} dt\, \mathcal{B}\left(e^{2\pi\mathrm{T}(t)}, 2\delta, 1-4\delta\right)}\right]$$

$R\left(\frac{\tau_d}{T_0}\right) = \mathcal{J}(\delta, \tau_d, T)$ *Information about the exchange phase θ is erased from the HOM noise!!*

The HOM noise dip's characteristics probe the non-universal scaling dimension δ, which governs the time correlations of tunneling or thermal quasiparticles excited at the QPC.



Interpretation:

Rewrite the HOM noise as interference terms[5] by introducing the tunneling operator $\hat{A}$:

$\hat{A}(t) = \psi_u^\dagger(t)\psi_d(t) \qquad \hat{A}^\dagger(t) = \psi_d^\dagger(t)\psi_u(t)$

$|t, \tau_d\rangle_+ = \hat{A}^\dagger(t)\psi_u^\dagger(0)\psi_d^\dagger(\tau_d)|0\rangle \qquad |t, \tau_d\rangle_- = \hat{A}(t)\psi_u^\dagger(0)\psi_d^\dagger(\tau_d)|0\rangle$

$$S_\mathrm{HOM}(t, t') = 4q^2\nu^2\gamma \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \left(\left||t, \tau_d\rangle_+ + |t', \tau_d\rangle_+\right|^2 + \left||t, \tau_d\rangle_- + |t', \tau_d\rangle_-\right|^2\right); \quad {}_\pm\langle t, 0\,|t', 0\,\rangle_\mp = S_{eq}$$

$\left||t, \tau_d\rangle_+ + |t', \tau_d\rangle_+\right|^2$ → Braiding in the upper edge, $\left||t, \tau_d\rangle_- + |t', \tau_d\rangle_-\right|^2$ → Braiding in the lower edge

Indicates dominant braiding subprocesses[B] that negate each other erasing θ!!

4 Conclusions

- The conventional zero-frequency HOM noise measurements do not capture the universal exchange phase θ of anyons
- Braiding subprocesses between injected and QPC-excited anyons dominate over the direct collision between injected anyons, making information about θ inaccessible

Outlook:

- Exploring finite frequency noise offers a potential avenue to access information about the exchange phase
- Alternative two-particle interferometers[6,7] provide a promising approach to investigating fractional statistics

References

[1] J. M. Leinaas, J. Myrheim: Nuovo Cim. B 37, 1 (1977)
[2] F. Wilczek, Phys. Rev. Lett. 49, 957 (1982)
[3] L. Saminadayar et al. Phys. Rev. Lett. 79, (1997)
[4] Hong, Ou, Mandel, PRL 59, 2044 (1987)
[5] H.S. Sim et al. Phys. Rev. Lett. 123, (2019)
[6] J. Nakamura et al., Nat. Phys. 16, 931 (2020)
[7] Bartolomei et al., Science 368, 173–177 (2020)