Superconducting phase crystals and Majorana flat bands with inhomogeneous magnetic fields

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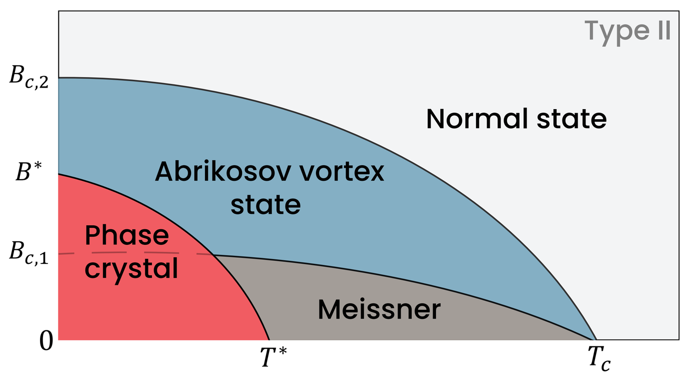
1 What are phase crystals?

Phase crystal[1] is a low-temperature superconducting ground state $\Delta(\mathbf{R}) \propto |\Delta| e^{i\chi(\mathbf{R})}$ with spatially periodic phase modulation, accompanied by superflow patterns $p_s(\mathbf{R}) = (\hbar/2)\nabla\chi(\mathbf{R}) - (e/c)\mathbf{A}$ and circulating currents.



- Phase oscillations break
  ↳ continuous translational invariance
- Circulating currents break
  ↳ time-reversal symmetry

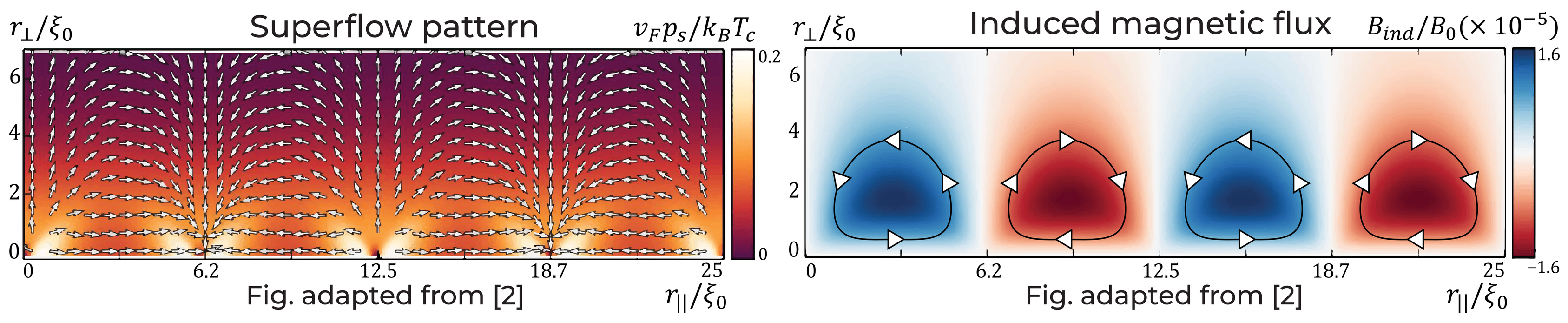
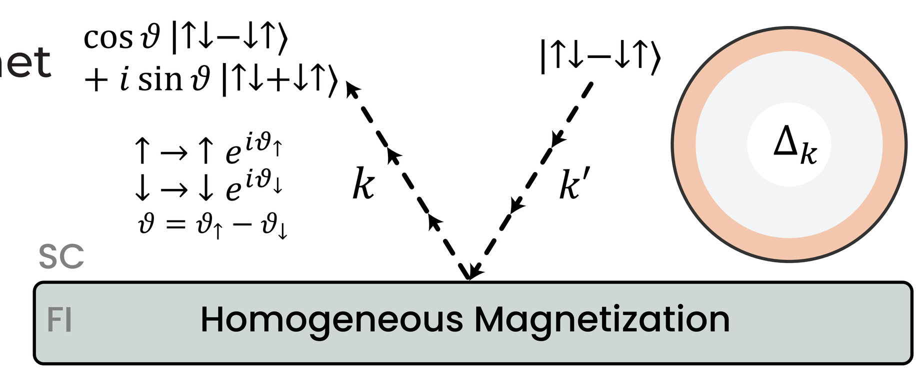


Fig. adapted from [2]   Fig. adapted from [2]

- Origin: Flat bands of zero-energy edge states hinder superconductivity by increasing the free energy Ω
- $\chi(\mathbf{R})$ modulates along the edge, creating superflow patterns lowering Ω by Doppler shifting the zero-energy in-gap states for all $T < T^*$
- Eg. Superconductor-ferromagnet with spin-mixing angle ϑ yield bound states with energies $\epsilon_b = \pm\Delta\cos\vartheta/2$ of Yu[3]-Shiba[4]-Rusinov[5] (YSR) origin

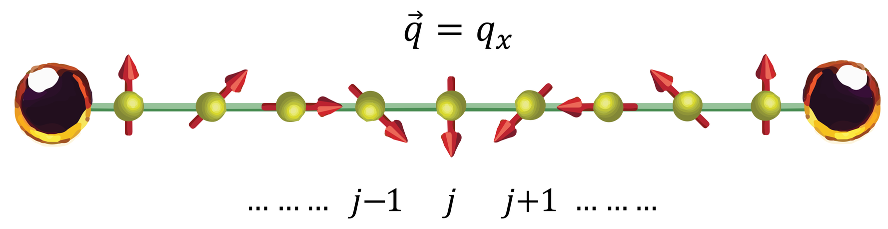


$\vartheta = \pi$ results in degenerate zero-energy YSR states, leading to phase crystal instability

What if these zero-energy states were Majorana states?

2 Majorana states to flat bands

- Majorana fermion is a particle that is its own antiparticle: $\gamma = \gamma^\dagger$
- Realizable from particle-hole symmetry of Bogoliubovs in effectively spinless superconductors: $\gamma(E) = \gamma^\dagger(-E) \rightarrow E = 0 \rightarrow \gamma(0) = \gamma^\dagger(0)$
- Magnetic inhomogeneity of YSR states with suitable pitch $\vec{q}$ can create a 1D topological superconductor hosting Majorana states at its ends[6]

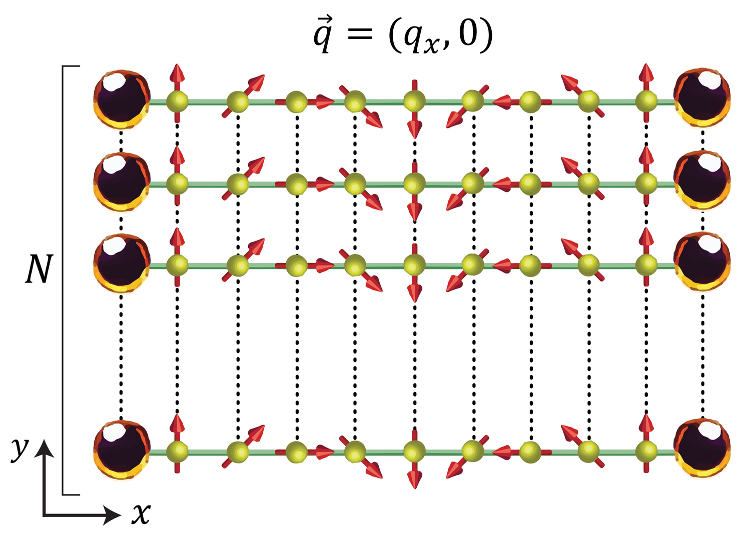


spin orientation at site $j: \sigma_j = \vec{n_j}.\vec{\sigma}$
$n_j = (\cos\varphi_j \sin\theta_j, \sin\varphi_j \cos\theta_j, \cos\theta_j)$
out-of-plane angle: $\theta_j = \theta_0$
in-plane angle: $\varphi_j = 2\pi\vec{q}.\vec{r_j}$

- Gauge transformation aligning spin quantization axis to local spin σ_j

$$H = \underbrace{-\sum_j \Psi_j^\dagger[\mu\tau_z + \Delta\tau_x]\Psi_j - \frac{t}{2}\sum_{\langle i,j\rangle}\Psi_i^\dagger \cos[\pi\vec{q}.\vec{r}_{ij}]\tau_z\Psi_j}_{chemical\ potential\ +\ superconductivity\ +\ hopping} + \underbrace{B\sum_j \Psi_j^\dagger\sigma_z\Psi_j}_{Zeeman} + \underbrace{\frac{t}{2}\sum_{\langle i,j\rangle}\Psi_i^\dagger i\sigma_x \sin[\pi\vec{q}.\vec{r}_{ij}]\tau_z\Psi_j}_{eff.\ spin\ orbit\ coupling}$$

- Longitudinal stacking of 'N' 1D wires:
  ↳ Effective 2D lattice hosting Majorana flat bands[7] (MFB)
- Max. Majorana states $= 2 \times (N\ wires)$
  ↳ protected by 'weak' topology i.e., 1D wires remain independent



3 Phase crystal vs. Majorana flat bands

→ Visualizing MFB in a semi-infinite space

- Fourier Transform 2D lattice along y:
  ↳ N independent wires labeled by $k_n = \frac{2\pi n}{N}$

$$H_{eff} = \sum_n^N H_{k_n} \longrightarrow \text{Hamiltonian of '}n\text{'th wire}$$

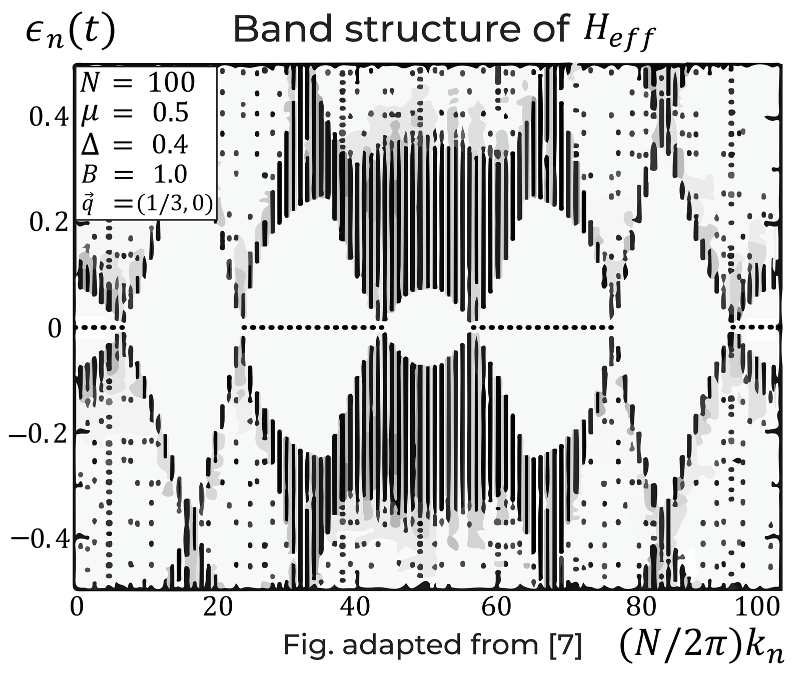


Fig. adapted from [7]

Can a phase crystal emerge from MFB?
If so, can we tune the phase crystal?

→ Tuning the density of Majorana states

$\rho_\gamma = \frac{N_\gamma}{N}$ ⟶ No. of Majorana states along one edge

$\vec{q} = (q_x, q_y) = |\vec{q}|(\cos\eta, \sin\eta)$
η ⟶ orientation of the rotating in-plane magnetic field

- As η increases, k_n-resolved 1D wires transition to trivial phase, reducing MFB density as gap-closing points move closer

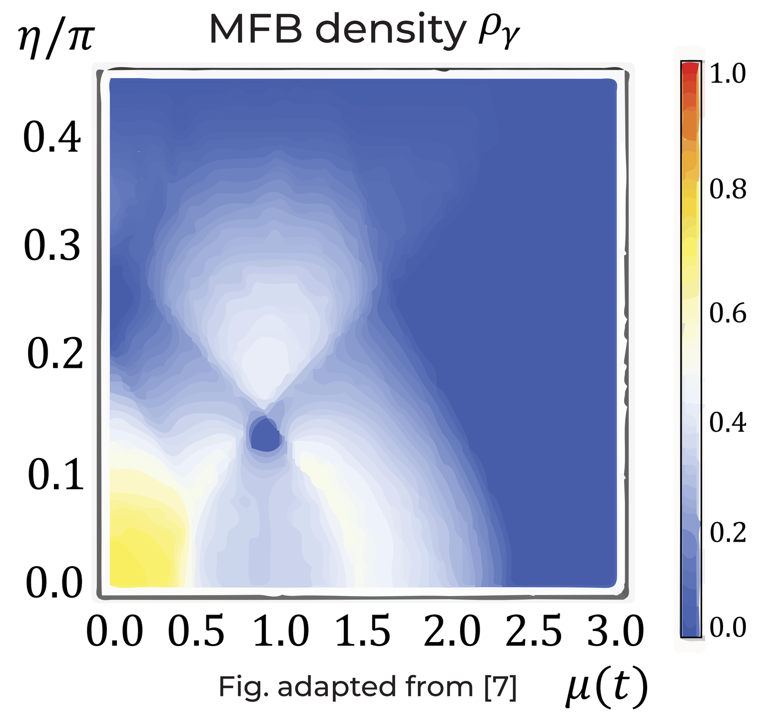


Fig. adapted from [7]

4 Outlook

- Investigate emergence of phase crystal using MFB
- Explore the possibility of tuning phase crystal formation

Future Work:

- Investigate utility of phase crystals for engineering higher-order topological states

References

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