III Canonical Correlation Analysis (CCA)

- Canonical Correlation Analysis (CCA) is an advanced statistical technique used to probe the relationships between **two sets of multivariate variables** on the **same subjects**.
- It is particularly applicable in circumstances where multiple regression would be appropriate, but there are multiple intercorrelated outcome variables.
- CCA identifies and quantifies the associations among these two variable groups.

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import warnings
warnings.filterwarnings('ignore')
```

Read the dataset

```
In [3]: mm = pd.read_stata('mmreg.dta')
    mm
```

Out[3]:		id	locus_of_control	self_concept	motivation	read	write	math	!
	0	303.0	-0.84	-0.24	1.00	54.799999	64.500000	44.500000	52.
	1	404.0	-0.38	-0.47	0.67	62.700001	43.700001	44.700001	52.
	2	225.0	0.89	0.59	0.67	60.599998	56.700001	70.500000	58.
	3	553.0	0.71	0.28	0.67	62.700001	56.700001	54.700001	58.
	4	433.0	-0.64	0.03	1.00	41.599998	46.299999	38.400002	36.
	•••								
	595	464.0	0.94	-0.30	1.00	60.099998	67.099998	52.400002	55.
	596	291.0	0.23	0.03	1.00	65.400002	56.700001	65.400002	58.
	597	348.0	0.46	0.03	1.00	65.400002	51.500000	61.400002	60.
	598	193.0	0.51	0.03	1.00	54.799999	54.099998	66.400002	41.
	599	380.0	0.25	0.03	0.67	49.500000	51.500000	55.500000	44.

600 rows × 9 columns

```
In [4]: mm.isna().sum()
```

 out[4]:
 0

 id
 0

 locus_of_control
 0

 self_concept
 0

 motivation
 0

 read
 0

 write
 0

 science
 0

 female
 0

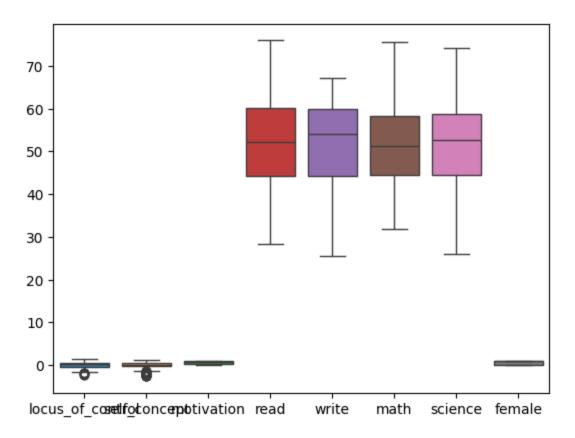
dtype: int64

In [5]: mm.describe().T

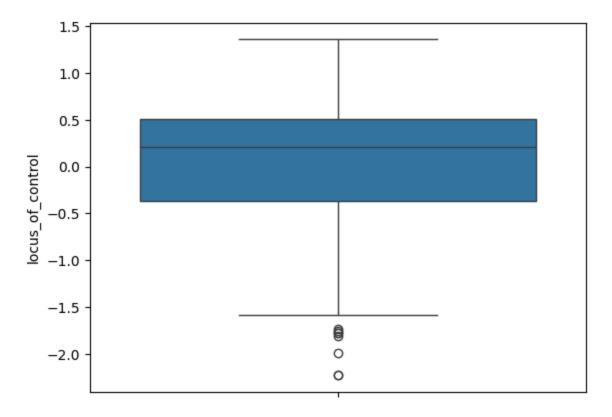
Out[5]: min 25% 50% count mean std 7 300.500000 300.500000 id 600.0 173.348816 1.000000 150.750000 450.250 locus_of_control 600.0 0.096533 -2.230000 0.210000 0.510 0.670280 -0.372500 600.0 self_concept 0.004917 0.705512 -2.620000 -0.300000 0.030000 0.440 motivation 600.0 0.660833 0.342729 0.000000 0.330000 0.670000 1.000 600.0 read 51.901833 10.102986 28.299999 44.200001 52.099998 60.099 600.0 write 52.384834 9.726455 25.500000 44.299999 54.099998 59.900 44.500000 math 600.0 51.848999 9.414738 31.799999 58.374 51.299999 600.0 science 51.763332 9.706173 26.000000 44.400002 52.599998 58.650 female 600.0 0.000000 0.000000 1.000 0.545000 0.498387 1.000000



In [6]: sns.boxplot(data=mm.drop('id', axis=1))
 plt.show()

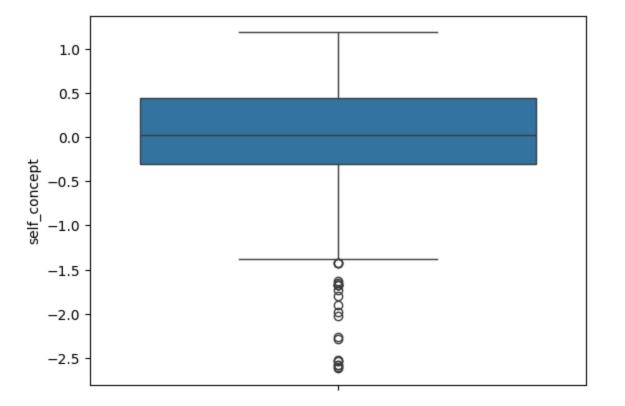


```
In [7]: # Function to detect outliers using IQR
        def detect_outliers_iqr(mm, column):
            q1 = mm[column].quantile(0.25)
            q3 = mm[column].quantile(0.75)
            iqr = q3 - q1
            lower_bound = q1 - 1.5 * iqr
            upper_bound = q3 + 1.5 * iqr
            outliers = mm[(mm[column] < lower_bound) | (mm[column] > upper bound)]
            return outliers
        # Detect outliers in numerical columns
        numerical_cols = ['locus_of_control', 'self_concept', 'motivation', 'read', 'write'
        for col in numerical_cols:
            outliers = detect_outliers_iqr(mm, col)
            if not outliers.empty:
                print(f"Outliers in '{col}':")
            else:
                print(f"No outliers found in '{col}'")
       Outliers in 'locus_of_control':
       Outliers in 'self concept':
       No outliers found in 'motivation'
       No outliers found in 'read'
       No outliers found in 'write'
       No outliers found in 'math'
       No outliers found in 'science'
In [8]: sns.boxplot(mm['locus_of_control'])
Out[8]: <Axes: ylabel='locus_of_control'>
```



In [10]: sns.boxplot(mm['self_concept'])

Out[10]: <Axes: ylabel='self_concept'>

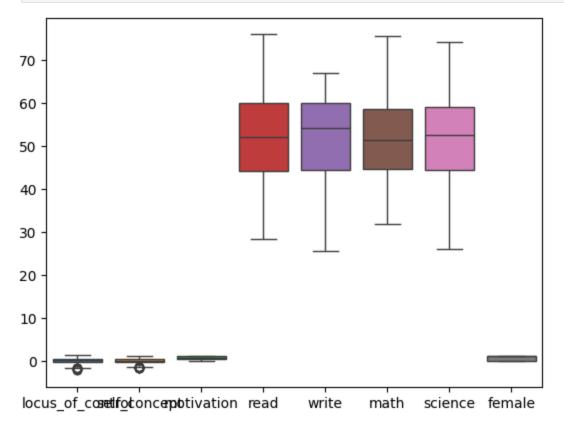


In this locus_of_control & self_concept have outliers. So, we need to remove this outliers.

```
In [11]: df = mm.copy()
In [12]: # Step 1: Calculate Q1 (25th percentile) and Q3 (75th percentile)
          Q1 = df['locus_of_control'].quantile(0.25)
          Q3 = df['locus_of_control'].quantile(0.75)
          # Step 2: Calculate IQR
          IQR = Q3 - Q1
          # Step 3: Define Lower and upper bounds
          lower\_bound = Q1 - 1.5 * IQR
          upper bound = Q3 + 1.5 * IQR
In [13]: # Remove the outliers and make a new dataframe
          df no outliers = df[(df['locus of control'] >= lower bound) & (df['locus of control
         df_no_outliers = df[(df['self_concept'] >= lower_bound) & (df['self_concept'] <= up</pre>
In [14]:
In [15]: new df = df no outliers.copy()
In [17]:
          new df
Out[17]:
                  id locus of control self concept motivation
                                                                    read
                                                                              write
                                                                                        math
            0 303.0
                                -0.84
                                             -0.24
                                                               54.799999 64.500000 44.500000
                                                                                              52.
            1 404.0
                                -0.38
                                             -0.47
                                                          0.67 62.700001 43.700001 44.700001 52.
            2 225.0
                                 0.89
                                              0.59
                                                          0.67 60.599998
                                                                         56.700001
                                                                                    70.500000
                                                                                               58.
            3 553.0
                                 0.71
                                              0.28
                                                          0.67
                                                               62.700001
                                                                         56.700001
                                                                                    54.700001
                                                                                               58.
            4 433.0
                                -0.64
                                              0.03
                                                              41.599998
                                                                         46.299999
                                                                                    38.400002
                                                                                              36.
          595 464.0
                                 0.94
                                             -0.30
                                                          1.00
                                                               60.099998 67.099998
                                                                                    52.400002
                                                                                              55.
                                 0.23
                                              0.03
          596 291.0
                                                               65.400002 56.700001 65.400002 58.
          597 348.0
                                 0.46
                                              0.03
                                                          1.00 65.400002 51.500000 61.400002 60.
                                              0.03
          598 193.0
                                 0.51
                                                          1.00 54.799999
                                                                         54.099998
                                                                                    66.400002 41.
          599 380.0
                                 0.25
                                              0.03
                                                          0.67 49.500000 51.500000 55.500000 44.
         586 rows × 9 columns
```

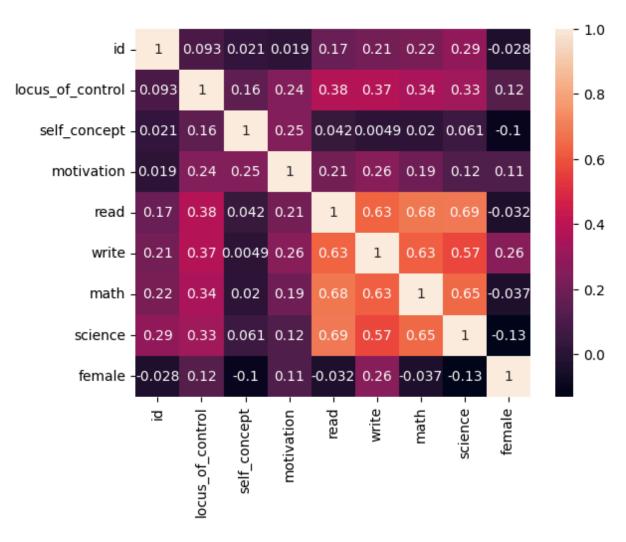
After remove outliers here we have 586 rows, that means 14 rows are removed that have outliers.

```
In [18]: sns.boxplot(data=new_df.drop('id', axis=1))
    plt.show()
```



```
In [19]: sns.heatmap(new_df.corr(), annot=True)
```

Out[19]: <Axes: >



Apply Canonical Correlation method

Separate the two variable groups considering one of "Psychological variables" and another is "Academic variables"

```
In [20]: # Psychological variables
X = new_df[['locus_of_control', 'self_concept', 'motivation']]
# Academic variables
Y = new_df[['read', 'write', 'math', 'science']]
```

Apply the Canonical Correlation method

```
Out[31]: 

CCA (n_components=1)
```

```
In [32]: # Transform the data to get the canonical variates (scores)
X_c, Y_c = cca.transform(X, Y)

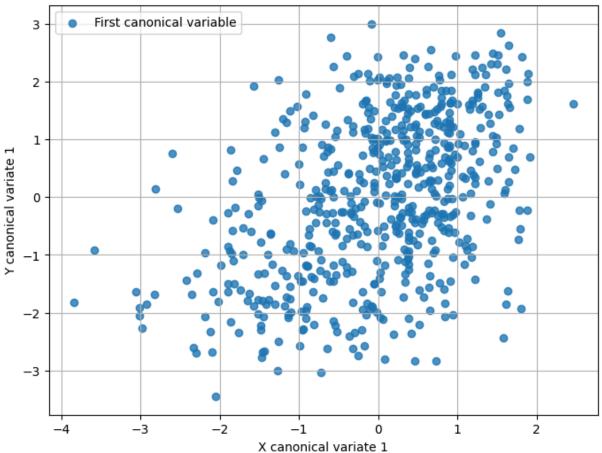
In [33]: # Get the canonical correlation by correlating the transformed data
canonical_correlation = np.corrcoef(X_c[:, 0], Y_c[:, 0])[0, 1]
print(f"Canonical Correlation: {canonical_correlation:.4f}")
```

Canonical Correlation: 0.4588

```
In [34]: #Plot the Canonical Variates

plt.figure(figsize=(8, 6))
plt.scatter(X_c[:, 0], Y_c[:, 0], label="First canonical variable", alpha=0.8)
plt.xlabel("X canonical variate 1")
plt.ylabel("Y canonical variate 1")
plt.title("Canonical Correlation Plot")
plt.legend()
plt.grid(True)
plt.show()
```





```
In [35]: # These show how much each original variable contributes to the canonical variate
         X_weights = cca.x_weights_
         Y_weights = cca.y_weights_
         print("\nLoadings for Psychological Variables:")
         for i, var in enumerate(X.columns):
             print(f" {var}: {X_weights[i][0]:.4f}")
         print("\nLoadings for Academic Variables:")
         for i, var in enumerate(Y.columns):
             print(f" {var}: {Y_weights[i][0]:.4f}")
        Loadings for Psychological Variables:
          locus_of_control: 0.8775
          self_concept: -0.1957
          motivation: 0.4378
        Loadings for Academic Variables:
          read: 0.5929
          write: 0.7631
          math: 0.2486
```

Interpretation: The canonical correlation analysis produced a correlation of 0.4588, indicating a moderate positive relationship between the psychological and academic variable sets.

For the psychological variables, locus of control had the highest positive weight (0.8775), making it the most influential contributor, followed by motivation (0.4378).

In contrast, self-concept showed a small negative weight (-0.1957), suggesting a minimal inverse contribution.

For the academic variables, writing had the strongest positive weight (0.7631), followed by reading (0.5929) and math (0.2486), while science had a very small negative weight (–0.0656), indicating negligible influence.

Overall, the findings suggest that higher **locus of control** and **motivation** are moderately associated with better performance in **writing and reading** skills.

science: -0.0656