🧑‍💻 Correspondence Analysis (CA)

- Correspondence Analysis (CA) is a multivariate statistical technique used to analyze and visualize the **relationships between categorical variables in a contingency table**.
- It reduces the dimensionality of the data, representing the associations between rows and columns in a low-dimensional space typically two dimensions—for easier interpretation.

In [3]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import warnings
warnings.filterwarnings('ignore')
```

In [4]:
```
url = "https://raw.githubusercontent.com/selva86/datasets/master/USArrests.csv"
df = pd.read_csv(url)
```

In [7]:
```
df.head()
```

Out[7]:

	Murder	Assault	UrbanPop	Rape	State
0	13.2	236	58	21.2	Alabama
1	10.0	263	48	44.5	Alaska
2	8.1	294	80	31.0	Arizona
3	8.8	190	50	19.5	Arkansas
4	9.0	276	91	40.6	California

In [6]:
```
df.shape
```

Out[6]:
```
(50, 5)
```

In [9]:
```
# Keep only the numeric columns used in classical demos
X = df[['Murder','Assault','UrbanPop','Rape']].copy()
states = df['Unnamed: 0'].rename('State') if 'Unnamed: 0' in df.columns else df.ind
```

In [11]:
```
X.describe().T
```

Out[11]:

	count	mean	std	min	25%	50%	75%	max
Murder	50.0	7.788	4.355510	0.8	4.075	7.25	11.250	17.4
Assault	50.0	170.760	83.337661	45.0	109.000	159.00	249.000	337.0
UrbanPop	50.0	65.540	14.474763	32.0	54.500	66.00	77.750	91.0
Rape	50.0	21.232	9.366385	7.3	15.075	20.10	26.175	46.0

In [12]:
```
# Matrix of nonnegative values
A = X.to_numpy(dtype=float)

# Convert to relative frequencies (P) so CA works on profiles
grand_total = A.sum()
P = A / grand_total

# Row and column masses (marginals)
r = P.sum(axis=1, keepdims=True)        # (n_rows, 1)
c = P.sum(axis=0, keepdims=True)        # (1, n_cols)
```

In [13]:
```
# Expected under independence
rcT = r @ c   # outer product

# Diagonal mass matrices (as sqrt-inverses)
Dr_inv_sqrt = np.diag((r.flatten())**-0.5)
Dc_inv_sqrt = np.diag((c.flatten())**-0.5)

# Standardized residuals
S = Dr_inv_sqrt @ (P - rcT) @ Dc_inv_sqrt
```

In [14]:
```
U, s, Vt = np.linalg.svd(S, full_matrices=False)
eigvals = s**2                            # eigenvalues (principal inertias)
explained = eigvals / eigvals.sum()   # proportion of inertia

pd.DataFrame({
    'eigenvalue': eigvals,
    'explained_inertia_%': 100*explained
}).head(4)
```

Out[14]:

	eigenvalue	explained_inertia_%
0	4.501357e-02	8.353547e+01
1	6.065461e-03	1.125619e+01
2	2.806548e-03	5.208347e+00
3	1.293934e-32	2.401262e-29

In [15]:
```
# Row principal coordinates F and column principal coordinates G
F = Dr_inv_sqrt @ U @ np.diag(s)    # rows (states)
G = Dc_inv_sqrt @ Vt.T @ np.diag(s) # columns (crime types)

# Keep first two dimensions for plotting
F2 = F[:, :2]
G2 = G[:, :2]

row_coords = pd.DataFrame(F2, columns=['Dim1','Dim2'])
row_coords.insert(0, 'State', list(states))
col_coords = pd.DataFrame(G2, columns=['Dim1','Dim2'])
col_coords.insert(0, 'Variable', X.columns)

row_coords.head(), col_coords
```

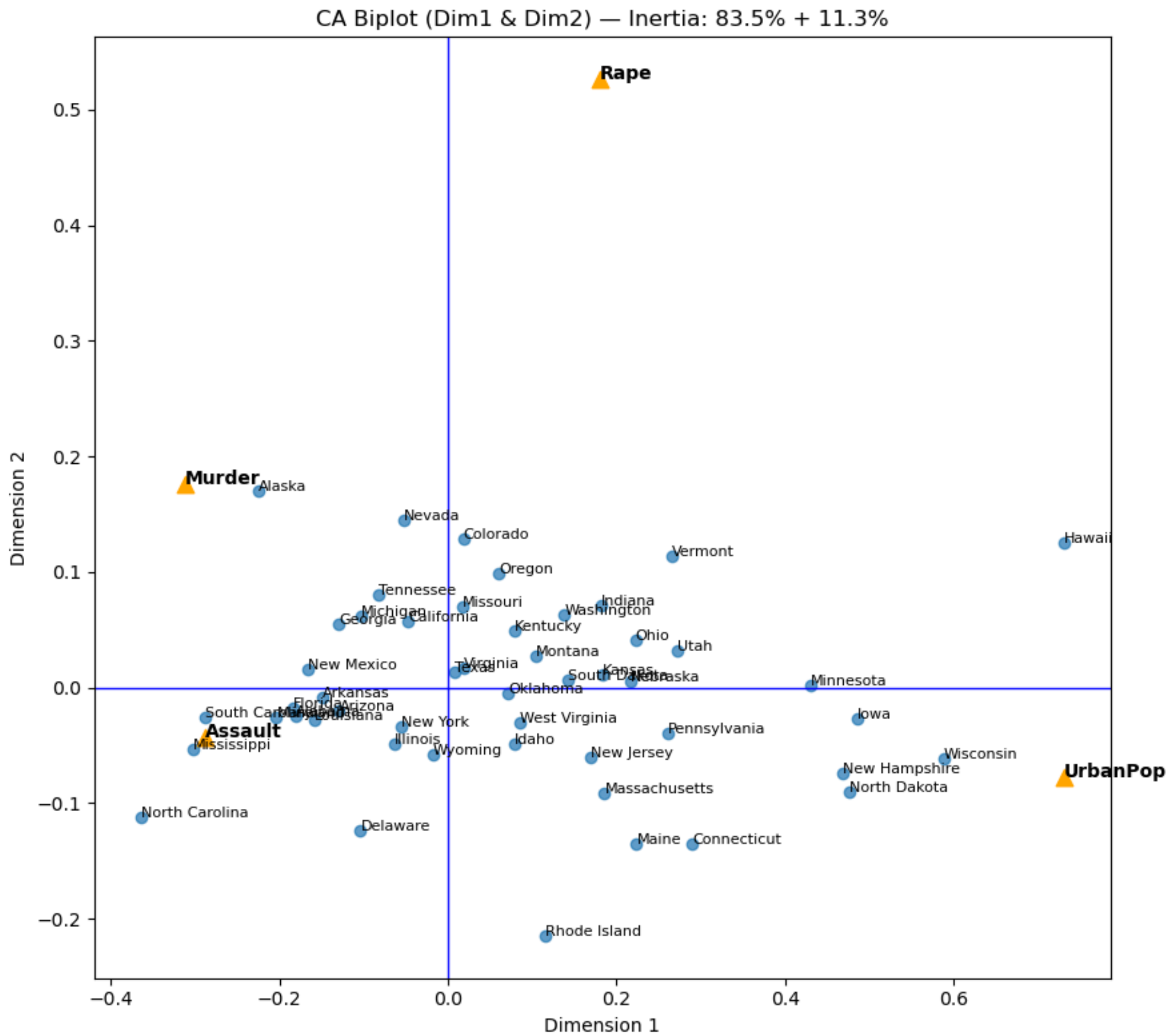
Out[15]:
```
(   State      Dim1      Dim2
 0      0 -0.181415 -0.024962
 1      1 -0.224989  0.170053
 2      2 -0.129295 -0.019609
 3      3 -0.149569 -0.008356
 4      4 -0.047561  0.057339,
    Variable      Dim1      Dim2
 0    Murder -0.151595  0.085575
 1   Assault -0.140128 -0.021127
 2  UrbanPop  0.354758 -0.037788
 3      Rape  0.087510  0.255170)
```

In [16]:
```
# Row contributions to axis k: ctr_{ik} = r_i * F_{ik}^2 / λ_k
ctr_rows = (r * (F**2)) / eigvals
ctr_rows = pd.DataFrame(ctr_rows[:, :2], columns=['CTR_Dim1','CTR_Dim2'])
ctr_rows.insert(0, 'State', list(states))

# Column contributions to axis k: ctr_{jk} = c_j * G_{jk}^2 / λ_k
ctr_cols = (c.T * (G**2)) / eigvals
ctr_cols = pd.DataFrame(ctr_cols[:, :2], columns=['CTR_Dim1','CTR_Dim2'])
ctr_cols.insert(0, 'Variable', X.columns)

# COS² for rows/cols: share of a point's inertia carried by each axis
row_dist2 = (F**2).sum(axis=1, keepdims=True)
col_dist2 = (G**2).sum(axis=1, keepdims=True)
cos2_rows = (F2**2) / row_dist2
cos2_cols = (G2**2) / col_dist2

cos2_rows = pd.DataFrame(cos2_rows, columns=['COS2_Dim1','COS2_Dim2'])
cos2_rows.insert(0, 'State', list(states))
cos2_cols = pd.DataFrame(cos2_cols, columns=['COS2_Dim1','COS2_Dim2'])
cos2_cols.insert(0, 'Variable', X.columns)

ctr_rows.sort_values('CTR_Dim1', ascending=False).head(10), \
ctr_cols, \
cos2_rows.sort_values('COS2_Dim1', ascending=False).head(10), \
cos2_cols
```

Out[16]:
```
(    State  CTR_Dim1  CTR_Dim2
 10     10  0.138273  0.029829
 32     32  0.091632  0.064574
 48     48  0.076853  0.006248
 23     23  0.051762  0.011832
 39     39  0.050754  0.003038
 14     14  0.049927  0.001140
 22     22  0.048342  0.000004
 28     28  0.045851  0.008443
 33     33  0.036955  0.009820
 1       1  0.030983  0.131357,
    Variable  CTR_Dim1  CTR_Dim2
 0    Murder  0.014986  0.035440
 1   Assault  0.280751  0.047361
 2  UrbanPop  0.690649  0.058154
 3      Rape  0.013614  0.859046,
     State  COS2_Dim1  COS2_Dim2
 22     22   0.998177   0.000012
 26     26   0.997005   0.000664
 14     14   0.996802   0.003068
 40     40   0.996705   0.002039
 3       3   0.996338   0.003110
 35     35   0.990549   0.004465
 8       8   0.990266   0.009261
 48     48   0.986939   0.010811
 15     15   0.983697   0.003851
 39     39   0.981176   0.007915,
    Variable  COS2_Dim1  COS2_Dim2
 0    Murder   0.194279   0.061909
 1   Assault   0.971805   0.022090
 2  UrbanPop   0.988413   0.011215
 3      Rape   0.102889   0.874815)
```

In [25]:
```python
plt.figure(figsize=(9,8))

# Scale columns so both clouds fit nicely
scale = (np.abs(F2).max() / np.abs(G2).max()) if np.abs(G2).max() > 0 else 1.0
G2_plot = G2 * scale

# Ensure 'states' contains the actual names from the dataset
states = df['State']  # Column with state names

# Plot states (rows) with names instead of numbers
plt.scatter(F2[:,0], F2[:,1], alpha=0.7)
for i, name in enumerate(states):
    plt.text(F2[i,0], F2[i,1], name, fontsize=8)

# Plot variables (columns)
plt.scatter(G2_plot[:,0], G2_plot[:,1], marker='^', s=80, color="orange")
for j, var in enumerate(X.columns):
    plt.text(G2_plot[j,0], G2_plot[j,1], var, fontsize=10, fontweight='bold')

# Reference lines
plt.axhline(0, linewidth=1, color='blue')
plt.axvline(0, linewidth=1, color='blue')
```

```
# Titles and labels
plt.title(f"CA Biplot (Dim1 & Dim2) — Inertia: {explained[0]*100:.1f}% + {explained
plt.xlabel("Dimension 1")
plt.ylabel("Dimension 2")
plt.tight_layout()
plt.show()
```



Correspondence Analysis (CA) Biplot Interpretation

This plot is a **Correspondence Analysis (CA) biplot** that visualizes the relationship between different U.S. states and crime statistics — specifically **Murder**, **Assault**, **Rape**, and **UrbanPop** (urban population percentage).

1. Axes (Dimensions)

- **Dimension 1 (X-axis)** and **Dimension 2 (Y-axis)** are the first two principal dimensions from correspondence analysis.

- Together, they explain **83.5% + 11.3% = 94.8%** of the variation in the data, meaning this plot captures most of the information.

2. Points on the Plot

- **Blue points** → U.S. states.
- **Orange triangles** → Variables: *Murder*, *Assault*, *Rape*, *UrbanPop*.

3. Interpretation of Distances

- **Closer points** indicate stronger similarity or association.
- **States near a crime variable** tend to have relatively higher values for that variable.
- **States far from a variable** have lower influence or relation to that variable.

4. Reading the Relationships

- **UrbanPop (bottom right)**
  States like **Hawaii, Wisconsin, Iowa, New Hampshire, North Dakota** are close to it, suggesting these states have higher urban population percentages.

- **Rape (top center-right)**
  No state is extremely close, but it is in the general direction of **Vermont, Indiana, Ohio, Utah** — suggesting slightly higher rates of rape relative to other crimes.

- **Murder (upper left)**
  Alaska is nearest, indicating relatively higher murder rates there.

- **Assault (bottom left)**
  Mississippi, South Carolina are closer, indicating higher assault rates.

5. Neutral/Central States

- States near the center (like **Texas, Missouri, Georgia**) have average or mixed crime statistics — no strong association with a specific variable.

6. Quadrant Insights

- **Top-right quadrant** → States leaning toward higher rape rates.
- **Bottom-right quadrant** → States with high urban population.
- **Top-left quadrant** → States with high murder rates.
- **Bottom-left quadrant** → States with high assault rates.