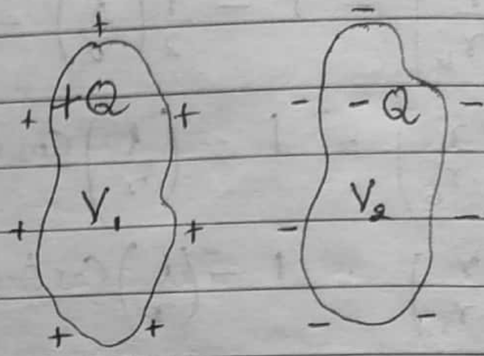
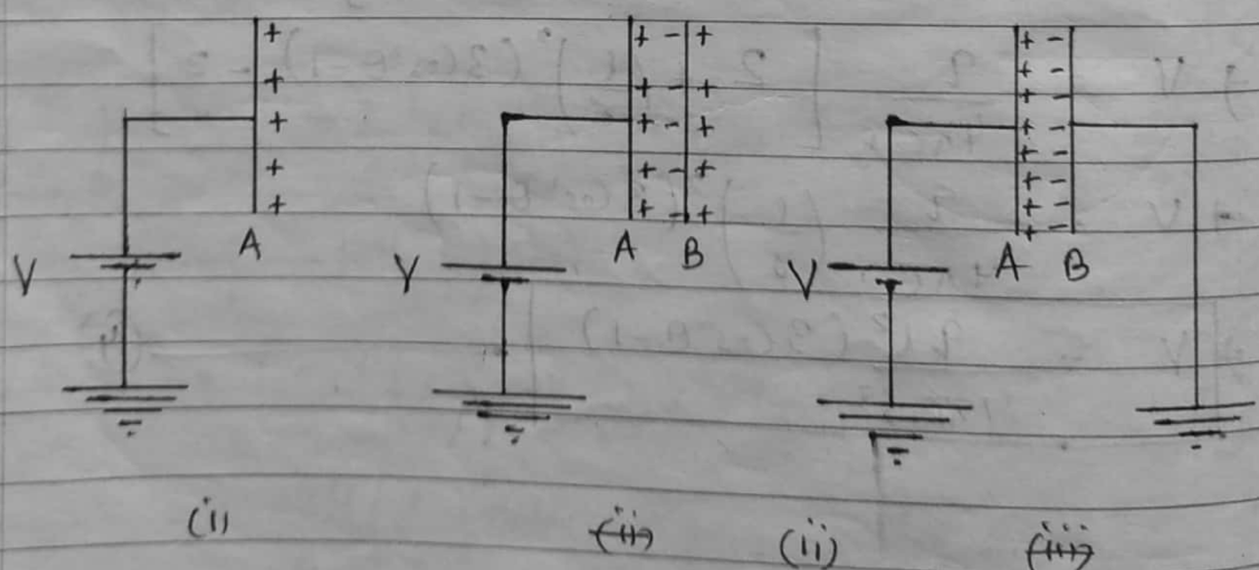
Capacitor

A system of two conductors seperated by an insulator is called a capacitor. The potential difference $V = V_1 - V_2$ between two conductors is called potential of the capacitor. The charge on one of the conductor is called the charge of the capacitor. It must be clear that total charge of the capacitor (i.e, $(+Q) + (-Q)$) is always zero. It is represented by symbol 



Principle of Capacitor

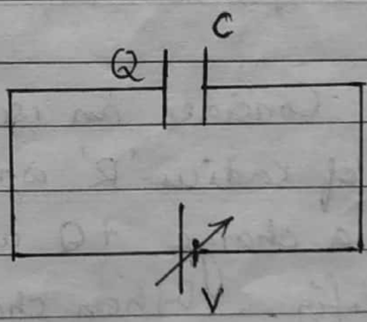


(i) ~~(ii)~~ (ii) ~~(iii)~~

(i) Let us charge plate A positively. The maximum charge accumulates on it when its potential equals the potential of battery.

(ii) To increase the collection of charge on plate A another plate B is brought near it. Due to induction negative and positive charges appear on its two faces. If plate B is grounded only negative charges will remain on it. This reduces the potential of plate A. Due to decrease in potential some more charges now flows to the plate A until it attains the maximum value of potential.

Capacitance of a Capacitor



Let V be the potential of the capacitor. The charge Q stored on the capacitor increases when the potential difference across the capacitor (and hence the potential of the capacitor) increases.

∴ charge on capacitor $\propto$ potential of the capacitor

$\Rightarrow \quad Q \propto V$

$\Rightarrow \quad Q = CV$

$\Rightarrow \quad Q = CV$

$\Rightarrow \quad C = \dfrac{Q}{V}$ ——— (1)

, where, C is a constant of

proportionality called capacitance of the capacitor.
The capacitance of a capacitor may also be defined as the ability of capacitor to store charge.
In S.I. unit,

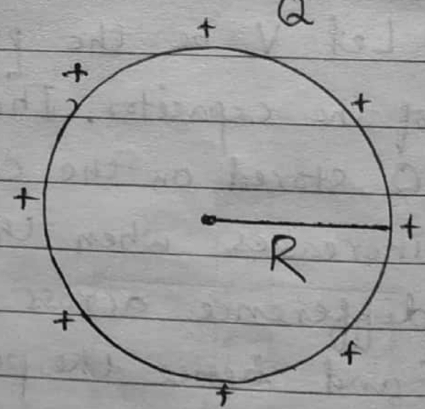
$$[C] = \frac{\text{coulomb}}{\text{volt}} = CV^{-1}$$

This unit of capacitance is also called farad (F).

$$\therefore \; 1\,F = 1\,CV^{-1}$$

The capacitance of a capacitor is said to be one farad (1F) if one coulomb of charge raises potential of capacitor through one volt.

Capacitance of isolated Sphere



Consider an isolated sphere of radius 'R' which is given a charge +Q as shown in fig.. When charge is given to the sphere the potential rises. The potential at any point on its surface is given by

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$$

$$\Rightarrow \frac{V}{Q} = \frac{1}{4\pi\epsilon_0 R} \Rightarrow \frac{Q}{V} = 4\pi\epsilon_0 R \quad \text{——— (1)}$$

Date
Page

If C is the capacitance of the isolated sphere then

$$C = \frac{Q}{V} \quad \text{———} \quad (2)$$

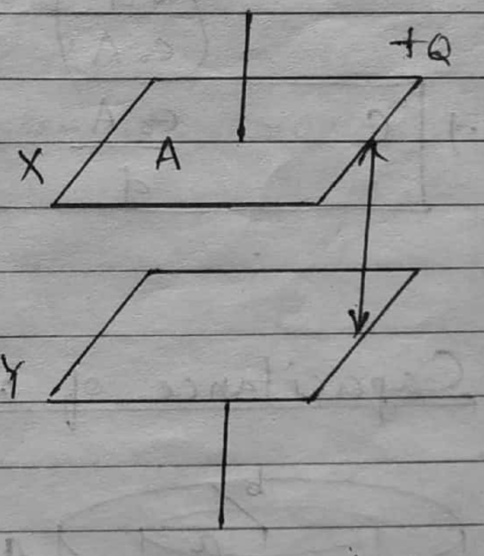
from eq? (1) and (2), we get,

$$C = 4\pi\epsilon_0 R \quad \text{———} \quad (3)$$

i.e. $C \propto R$

or the larger the sphere the more charge it can store.

Capacitance of a parallel plate capacitor



Consider a parallel plate capacitor. Let X and Y be the two plates of the capacitor seperated by distance 'd' and having area A of each plate. If Q is the charge on the plate X then the surface charge density of the plate is

$$\sigma = \frac{Q}{A} \quad \text{———} \quad (1)$$

The electric field between the plate is given by

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \text{———} \quad (2)$$

Since electric field is numerically equal to potential gradient

$$\therefore \quad E = \frac{V}{d}$$

$$\Rightarrow \quad V = E \cdot d$$

$$\Rightarrow \quad V = \frac{Q \cdot d}{\epsilon_0 A} \quad \text{--- (3)}$$

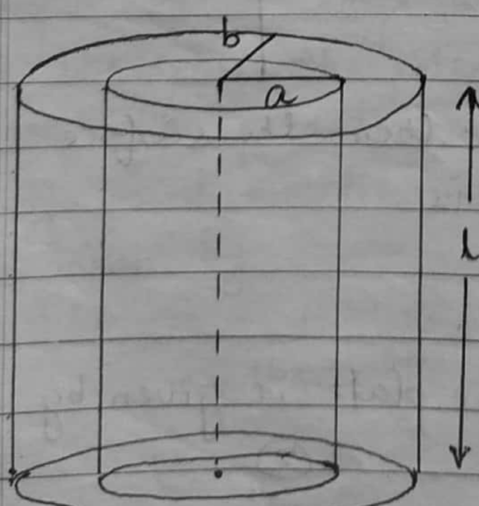
If C is the capacitance of the capacitor then

$$C = \frac{Q}{V}$$

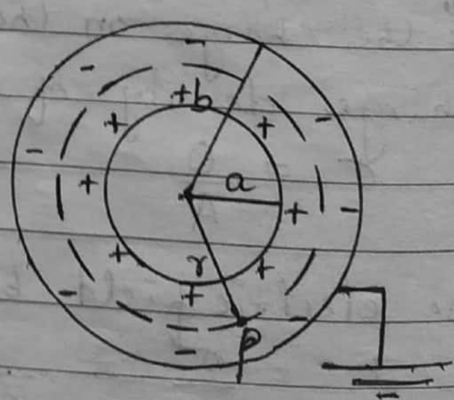
$$\Rightarrow \quad C = \frac{Q}{\left(\frac{Qd}{\epsilon_0 A}\right)}$$

$$\Rightarrow \quad \boxed{C = \frac{\epsilon_0 A}{d}} \quad \text{--- (4)}$$

Capacitance of a Cylinderical Capacitor



Side-view



top-view

Consider a cylinderical capacitor having length 'l' and coaxial cylinder with radius 'a' and 'b' respectively such that b > a. Let us consider a cylinderical gaussian surface of radius 'r' and length 'l'. Let 'E' be the electric field intensity at point P on the gaussian surface.

By Gauss law,

$$E.A = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 A}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 (2\pi r l)} = \frac{q}{2\pi\epsilon_0 r l} \quad \text{——— (1)}$$

The potential difference between the co-axial cylinders is given by

$$V = \int_a^b E \cdot dr$$

$$\Rightarrow V = \int_a^b \frac{q}{2\pi\epsilon_0 r l} dr$$

$$\Rightarrow V = \frac{q}{2\pi\epsilon_0 l} \int_a^b \frac{1}{r} dr$$

$$\Rightarrow V = \frac{q}{2\pi\epsilon_0 l} \left[\ln r \right]_a^b$$

$$\Rightarrow V = \frac{q}{2\pi\epsilon_0 l} \left[\ln(b) - \ln(a) \right]$$

$$\Rightarrow V = \frac{q}{2\pi\epsilon_0 l} \ln\left(\frac{b}{a}\right) \quad \text{——— (2)}$$

Date ______
Page ______

Scanned with CamScanner

If 'C' is the capacitance of the cylinderical capacitor then

$$C = \frac{q}{V}$$

$$\Rightarrow C = \frac{q}{\left\{\frac{q}{2\pi\epsilon_0 l} \ln\left(\frac{b}{a}\right)\right\}}$$

$$\Rightarrow \boxed{C = \frac{2\pi\epsilon_0 l}{\ln\left(\frac{b}{a}\right)}} \quad \text{——— (3)}$$

Dielectrics

The solid or liquid insulators are called dielectrics. Depending on the types of molecules they are divided into:

(i) polar dielectrics:

In the molecule of a dielectric if the center of gravity of positive and negative charges do not coincide with each other then the dielectric is called polar dielectric. For eg, N_2O, H_2O, NH_3, etc..

(ii) Non-polar dielectrics:

In the molecule of a dielectric if the center of gravity of positive charge and negative charges coincide with each other then the dielectric is called non polar dielectric. For eg, H_2, N_2, O_2, etc.

Polarization:

A non-polar molecule can be polarized (i.e., made polar) by placing it in the electric field. The process of seperation of center of gravity of positive and negative charges of a molecule by application of an electric field is called polarization.

Dielectric Constant (K)

The ratio of capacitance of a capacitor with and without dielectric between the plates is called dielectric constant. It is denoted by K.

If C_{med} and C_{air} are capacitance of capacitor with and without dielectric then

$$C_{med} = \frac{\epsilon A}{d}$$

and $$C_{air} = \frac{\epsilon_0 A}{d}$$

where, ϵ_0 = absolute permittivity of air

ϵ = absolute permittivity of dielectric

$\therefore$ dielectric constant is

$$K = \frac{C_{med}}{C_{air}} = \frac{\epsilon A/d}{\epsilon_0 A/d} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \quad \text{——} (1)$$

Therefore, the dielectric constant is numerically equal to relative permittivity of the dielectric.

Let 'Q' be the be the charge on the capacitor and E_{med} and E_{air} be the electric field between the plates of capacitor with and without dielectric then

$$C_{med} = \frac{Q}{V_{med}} \Rightarrow C_{med} = \frac{Q}{E_{med} \cdot d}$$

and $$C_{air} = \frac{Q}{E_{air} \cdot d}$$

Now, dielectric constant is

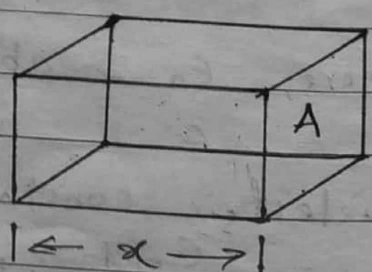
$$k = \frac{C_{med}}{C_{air}}$$

$$\Rightarrow k = \frac{Q / E_{med} \cdot d}{Q / E_{air} \cdot d}$$

$$\Rightarrow k = \frac{E_{air}}{E_{med}} \quad \text{———} \quad (2)$$

Polarization density or Polarization vector ($\vec{P}$)

The polarization density is defined as the dipole moment per unit volume. It is also called polarization vector ($\vec{P}$).



$\therefore \; P = \dfrac{\text{dipole moment}}{\text{volume}}$ ——— (i)

If charge 'q' is induced on opposit faces of a dielectric then

$$P = \frac{\text{dipole moment}}{\text{volume}}$$

$$\Rightarrow P = \frac{q \cdot x}{A \cdot x}$$

$$\Rightarrow P = \frac{q}{A} = \frac{\text{induced charge}}{\text{area}} \quad \text{——— (2)}$$

Displacement Vector ($\vec{D}$)

Displacement vector is related to free charge and is equal to the surface charge density due to free charge.

$$\therefore D = \frac{\text{free charge (on plate of capacitor)}}{\text{area}}$$

$$\Rightarrow D = \frac{q}{A} \quad \text{——— (1)}$$

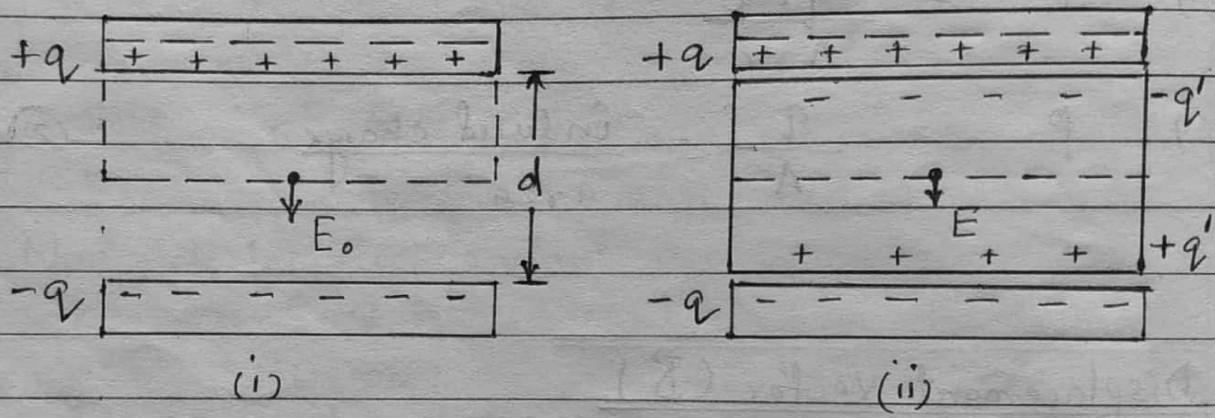
Three Electric vectors ($\vec{D}$, $\vec{E}$ and $\vec{P}$)

Let E_0 be the electric field inside capacitor when no dielectric is present.

Applying Gauss law, we get,

$$E_0 A = \frac{q}{\epsilon_0}$$

$$\Rightarrow E_0 = \frac{q}{\epsilon_0 A} \quad \text{--- (1)}$$



When dielectric is inserted between the plates let q' be the charge induced on the surface of the dielectric. Applying Gauss law,

$$E A = \frac{q - q'}{\epsilon_0}$$

$$\Rightarrow E A = \frac{q}{\epsilon_0} - \frac{q'}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \quad \text{--- (2)}$$

Here, E_0 and E are electric field between the plates without and with dielectrics, so dielectric constant is given by

$$k = \frac{E_0}{E}$$

Scanned with CamScanner

$\Rightarrow E = \frac{E_0}{K}$

$\Rightarrow E = \frac{q}{K\epsilon_0 A}$ ——— (3)

$\therefore$ from eq^n (2) and (3), we get,

$\frac{q}{K\epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$ ——— (4)

$\Rightarrow \frac{q'}{\epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q}{K\epsilon_0 A}$

$\Rightarrow q' = q - \frac{q}{K}$

$\Rightarrow q' = q\left(1 - \frac{1}{K}\right)$ ——— (5)

$\because K > 1$

$\therefore q' < q$

i.e., the induced charge q' is always less than free charge q.

Now, from eq^n (4),

$\frac{q}{\epsilon_0 A} = \frac{q}{K\epsilon_0 A} + \frac{q'}{\epsilon_0 A}$

$\Rightarrow \frac{q}{A} = \epsilon_0\left(\frac{q}{K\epsilon_0 A}\right) + \frac{q'}{A}$

$\Rightarrow \boxed{D = \epsilon_0 E + P}$ ——— (6)

In vector form,

$\boxed{\vec{D} = \epsilon_0 \vec{E} + \vec{P}}$ ——— (7)

Eq. (6) and (7) are relationship between three electric vectors.

Energy Stored in a charged capacitor

At any instant of charging process, suppose the charge on the plate be 'q' and potential difference between the plates be 'v'. If 'dW' is the amount of work done in charging capacitor by additional charge 'dq' such that the potential difference across the plate does not change considerably, then

$$dW = v\,dq \qquad \text{(1)}$$

∴ total work done by battery to charge capacitor from 0 to Q is

$$W = \int dW$$

$$\Rightarrow W = \int_0^Q v\,dq \qquad \text{(2)}$$

If C is the capacitance of capacitor then

$$q = Cv$$

$$\Rightarrow v = \frac{q}{C} \qquad \text{(3)}$$

∴ from eqⁿ (2) and (3), we get,

$$W = \int_0^Q \frac{q}{C}\,dq$$

$$\Rightarrow W = \frac{1}{C}\int_0^Q q\,dq$$

$$\Rightarrow W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

Date ____
Page ____

$$\Rightarrow W = \frac{1}{2C}\left[q\right]_0^Q$$

$$\Rightarrow W = \frac{Q^2}{2C} \quad \text{———— (4)}$$

This work done is stored in the capacitor as its potential energy. Thus, the energy stored in the capacitor is

$$E = \frac{Q^2}{2C} \quad \text{———— (5)}$$

$$\Rightarrow E = \frac{1}{2}\left(\frac{Q}{C}\right)Q = \frac{1}{2}QV \quad \text{———— (6)}$$

$$\Rightarrow E = \frac{1}{2}(CV)V = \frac{1}{2}CV^2 \quad \text{———— (7)}$$

22.14 Charging of a capacitor through a Resistor

Suppose, the charging of a capacitor of capacitance 'C' is done by connecting a battery of emf 'E', a resistor of resistance 'R' as shown in figure.

Initially, there is no charge on the capacitor and so no P.d across it. So, the P.d across R is equal to the applied emf, E. Initially the charging current is maximum and it gradually decreases to zero when capacitor gets fully charged.

Let at any instant't', the current flowing in the circuit is I, the p.d across the capacitor is V_C and the p.d across the resistor is V_R. Then,

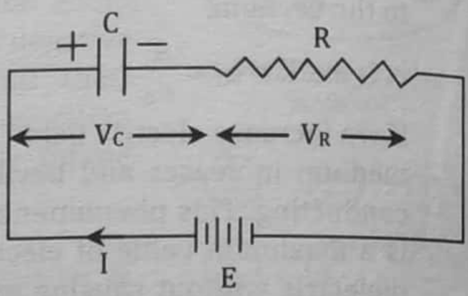


Fig:22.11 Charging of a capacitor

$$E = V_C + V_R = \frac{q}{C} + IR$$

$$= \frac{q}{C} + \frac{dq}{dt}R \;...... (22.25) \left(\because I = \frac{dq}{dt}\right)$$

if q_o be the maximum charge stored in the capacitor when fully charged, then

$$q_o = CE \quad \text{or, } E = \frac{q_o}{C}$$

Substituting this value of E into equation (22.25), we get

$$\frac{q_o}{C} = \frac{q}{C} + \frac{dq}{dt}R$$

or, $$\frac{q_o}{C} - \frac{q}{C} = \frac{dq}{dt}R$$

or, $$\frac{q_o - q_o}{C} = \frac{dq}{dt}R$$

or, $$\frac{dq}{q_o - q} = \frac{1}{RC}dt$$

Integrating both sides we get,

$$\frac{1}{RC}\int_0^t dt = \int_0^q \frac{dq}{q_o - q}$$

or, $$\frac{t}{RC} = [-\ln(q_o - q)]^{q_o} \text{ or, } \frac{t}{RC} = -[\ln(q_o - q) - \ln(q_o - o)]$$

or, $$\frac{t}{RC} = -\ln\left\{\frac{q_o - q}{q_o}\right\}$$

or, $$\ln\left(1 - \frac{q}{q_o}\right) = -\frac{t}{RC}$$

taking anti-natural log on both sides, we get

$$1 - \frac{q}{q_o} = e^{-\frac{t}{RC}}$$

or, $$\frac{q}{q_o} = 1 - e^{-\frac{t}{RC}}$$

or, $$q = q_o\left(1 - e^{-\frac{t}{RC}}\right) .... (22.26)$$

This is the amount of charge stored in the capacitor at any instant during charging. which shows that, during charging, the charge increases in the capacitor exponentially. That is, charge grows intially very rapidly & then slowly afterwards.

The curve obtained by plotting charge against time below shows the exponential charging of the capacitor. The quantity $t = RC$ in the equation (22.26) is called the time constant or the relaxation time of the circuit. If we put, $t = RC$, in the equation (22.26) we get

$$q = q_0(1 - e^{-1})$$

or, $$q = q_0\left(1 - \frac{1}{e}\right) = q_0(1 - 0.368)$$

or, $$q = 0.632q_0$$

or, $$q = 63.2\% \text{ of } q_0 \quad \text{.......(22.27)}$$

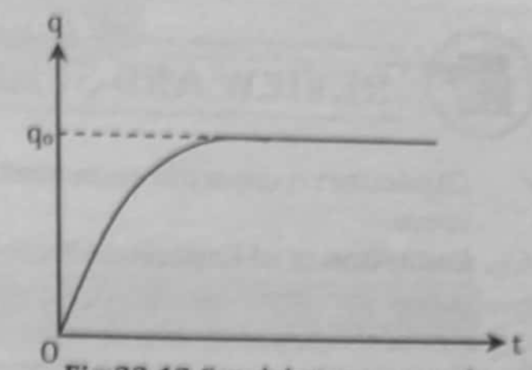


Fig:22.12 Graph between q and t

Hence, the time constant of a charging circuit represent the time during which the charge in the capacitor grows to about 63% of its final value. If t is small, the capacitor charges quickly, and when t is large, the capacitor takes long time for charging.

22.15 Discharging of a Capacitor through a Resistor

Let a fully charged capacitor with charge q_0 is connected to a resistor of resistance R as shown in figure. If V_0 be the potential difference between its plates, then

$$q_0 = CV_0$$

Where C is the capacitance of the capacitor. Suppose, at any instant 't' during discharging, I be the current flowing through the circuit, V be the potential difference and q be the charge left in the capacitor. Then,

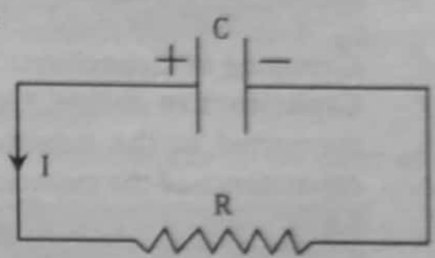


Fig:22.13 Discharging of capacitor

P.d. across capacitor = P.d. across resistor

or, $$V_C = V_R$$

or, $$\frac{q}{C} = IR \quad \text{......... (22.28)}$$

Also $I = -\frac{dq}{dt}$, where – sign indicates that q decreases as t increases.

$\therefore$ from equation (22.28)

$$\frac{q}{C} = -\frac{dq}{dt}R$$

or, $$-\frac{dt}{RC} = \frac{dq}{q}$$

Integrating both sides, we get,

$$-\frac{1}{RC}\int dt = \int_{q_0}^{q} \frac{dq}{q}$$

$$-\frac{t}{RC} = [\ln q]^{q}_{q_0}$$

or, $$-\frac{t}{RC} = [\ln q - \ln q_0]$$

or, $$-\frac{t}{RC} = \ln\frac{q}{q_0}$$

or, $$\frac{q}{q_0} = e^{-\frac{t}{RC}}$$

or, $$q = q_0 e^{-\frac{t}{RC}} \quad \text{.......................(22.29)}$$

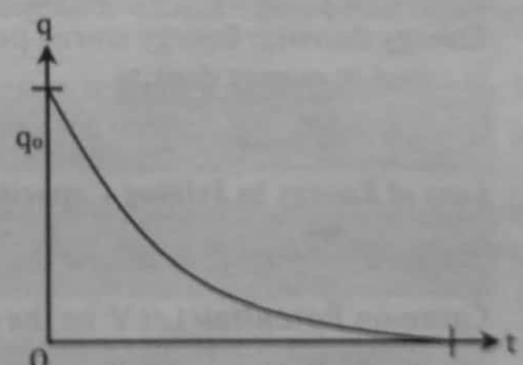


Fig:22.14 Graph between charge and time for discharging process.

This is the equation for discharging. The graph plotted between charge & time during discharging of a capacitor is as shown in Fig:22.14.