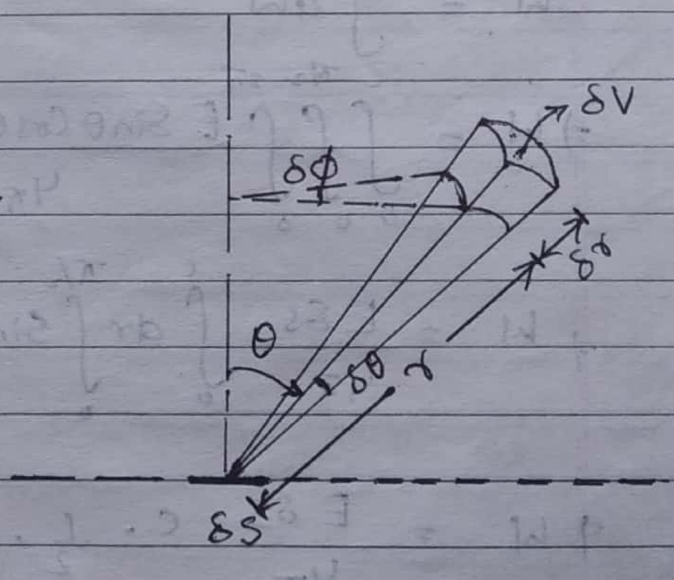
Acoustics

Reverberation of sound

When sound is contineously produced in a closed hall it is absorbed by the wall. But in actual case, there is always some reflection from the walls. Due to this, the energy density in the enclosure (closed hall) gradually builds up to a state (steady) value when the energy produced by source per second equals the rate of loss due to absorption. If the source of sound is made suddenly off, still the sound persists for some time due to one or multiple reflection from the reflecting surfaces of the enclosure. This phenomenon is known as reverberation.

Theoretical treatment of reverberation



Consider an enclosure having uniform distribution of energy density (E) through-out the enclosure. Let 'E' be the energy density at any instant of time and 'δS' be the element of area forming the enclosure.

Let us consider within enclosure an element of volume $\delta V = r^2 \sin\theta \, dr \, d\theta \, d\phi$ (in spherical coordinate) at a point P distance 'r' away from 'δS', 'r' making an angle 'θ' with the normal to 'δS'.

Since the energy stream is same in all directions, the amount of energy reaching 'δS' from 'δV' is given by

$$\delta W = \frac{E \, \delta V}{4\pi} \delta \omega$$

where, '$\delta \omega$' is the small solid angle substended by 'δS' at P.

$$\Rightarrow \delta W = \frac{E r^2 \sin\theta \, dr \, d\theta \, d\phi}{4\pi} \cdot \frac{\delta S \cos\theta}{r^2}$$

$$\Rightarrow \delta W = \frac{E \sin\theta \cos\theta \, dr \, d\theta \, d\phi \, \delta S}{4\pi} \quad \text{——— (1)}$$

∴ Energy passing per second through 'δS' from the front side is given by

$$W = \int dW$$

$$\Rightarrow W = \int_0^c \int_0^{\pi/2} \int_0^{2\pi} \frac{E \sin\theta \cos\theta \, dr \, d\theta \, d\phi \, \delta S}{4\pi}$$

$$\Rightarrow W = \frac{E \delta S}{4\pi} \int_0^c dr \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta \int_0^{2\pi} d\phi$$

$$\Rightarrow W = \frac{E \delta S}{4\pi} \cdot c \cdot \frac{1}{2} \cdot 2\pi$$

$$\Rightarrow W = \frac{1}{4} E c \, \delta S \quad \text{——— (2)}$$

Date
Page

If 'a' is the absorption coefficient (energy absorbed per unit area per second) of the enclosure then the energy absorbed per second by the enclosure is given by

$$\frac{1}{4} EC \sum a\delta S = \frac{1}{4} ECA \quad \text{——— (3)}$$

where,

$$A = \sum a.\delta S,$$

and A is total absorption by the wall of enclosure.

(i) Growth of energy :

Let 'P' be the rate of supply (or production) of energy by source then, we have,

$$P - \frac{ECA}{4} = \frac{d}{dt}(EV)$$

$$\Rightarrow P - \frac{ECA}{4} = V\frac{dE}{dt} \quad \text{——— (4)}$$

$$\Rightarrow dt = \frac{V}{\left(P - \frac{ECA}{4}\right)} dE \quad \text{——— (5)}$$

Let $x = P - \frac{ECA}{4}$

then

$$dx = -\frac{CA}{4} dE$$

$$\Rightarrow dE = -\frac{4}{CA} dx$$

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∴ substituting this value in equation (5), we get,

$$dt = -\frac{4V}{CA}\frac{dx}{x}$$

Integrating both sides,

$$\int_0^t dt = \int_P^{\left(P-\frac{ECA}{4}\right)} -\frac{4V}{CA}\frac{dx}{x}$$

$$\Rightarrow t = -\frac{4V}{CA}\left[\log x\right]_P^{\left(P-\frac{ECA}{4}\right)}$$

$$\Rightarrow t = -\frac{4V}{CA}\left[\log\left(P-\frac{ECA}{4}\right) \log P\right]$$

$$\Rightarrow -\frac{CA}{4V}t = \log\left[\frac{\left(P-\frac{ECA}{4}\right)}{P}\right]$$

$$\Rightarrow -\frac{CA}{4V}t = \log\left(1-\frac{ECA}{4P}\right)$$

$$\Rightarrow e^{-\frac{CA}{4V}t} = 1-\frac{ECA}{4P}$$

$$\Rightarrow \frac{ECA}{4P} = \left(1-e^{-\frac{CA}{4V}t}\right)$$

$$\Rightarrow E = \frac{4P}{CA}\left(1-e^{-\frac{CA}{4V}t}\right) \quad (6)$$

In steady state,

$E = E_0$ (maximum value of E)

$$\therefore \frac{dE}{dt} = 0$$

∴ from eqⁿ ④,

$$P - \frac{E_0 CA}{4} = 0$$

$$\Rightarrow \frac{E_0 CA}{4} = P$$

$$\Rightarrow E_0 = \frac{4P}{CA} \quad \text{——— ⑦}$$

∴ from equations ⑥ and ⑦, we get,

$$\boxed{E = E_0\left(1 - e^{-\frac{CA}{4V}t}\right)} \quad \text{——— ⑧}$$

(ii) **Decay of energy:**

When source cease to supply sound (i.e, when $P = 0$) the decay of energy starts.

We have, for decay of energy density,

$$-\frac{ECA}{4} = V\frac{dE}{dt}$$

$$\Rightarrow \frac{dE}{E} = -\frac{CA}{4V}dt$$

Integrating both sides, we get,

$$\int_{E_0}^{E} \frac{dE}{E} = \int_0^t -\frac{CA}{4V}dt$$

$$\Rightarrow \left[\log E\right]_{E_0}^{E} = -\frac{CA}{4V}t$$

$$\Rightarrow \log E - \log E_0 = -\frac{CA}{4V}t$$

$$\Rightarrow \log\left(\frac{E}{E_0}\right) = -\frac{CA}{4V}t$$

$$\Rightarrow \frac{E}{E_0} = e^{-\frac{CA}{4V}t}$$

$$\Rightarrow \boxed{E = E_0\, e^{-\frac{CA}{4V}t}} \quad \text{——— (9)}$$

Reverberation time is defined as the time in which the energy density just falls to audible value from an initial energy density which is 10^6 times as large

i.e. $\frac{E_0}{E} = 10^6$

$\therefore$ from eq^n (9),

$$\frac{E}{E_0} = e^{-\frac{CA}{4V}t}.$$

$$\Rightarrow \frac{E_0}{E} = e^{\frac{CA}{4V}t}$$

When $t = T$, reverberation time then $\frac{E_0}{E} = 10^6$

$$\therefore 10^6 = e^{\frac{CA}{4V}T}$$

$$\Rightarrow \log(10^6) = \frac{CA}{4V}T$$

$$\Rightarrow T = \frac{4\log(10^6)V}{CA}$$

$$\Rightarrow T = \frac{4\log(10^6)}{C}\cdot\frac{V}{A}$$

$$\Rightarrow \boxed{T = \frac{0.165V}{A}} \quad \text{——— (10)}$$

This equation (10) is called Sabine's formula.

Ultrasonic waves

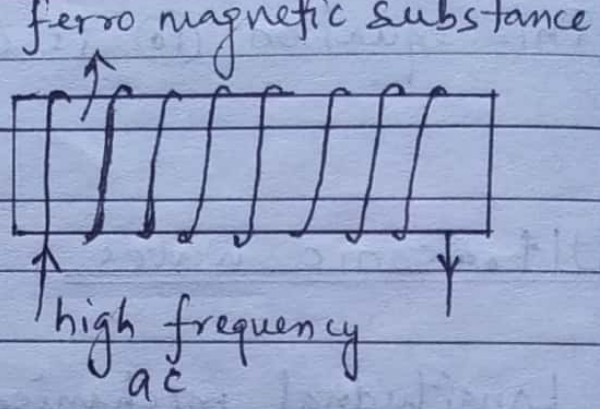
Longitudnal mechanical waves whose frequency are below the audible range (i.e., 20 Hz) are called infrasonic waves. A longitudnal mechanical wave whose frequency is above audible range (i.e. 20kHz) is called ultrasonic wave or ultrasound.

Production of ultrasonic waves

The mechanical generator such as tuning fork is capable of generating ultrasonics of frequency upto 10000 Hz. But the most common methods used to generate ultrasonics are:
(i) magnetostriction method, and
(ii) piezoelectric method.

(i) Magnetostriction method.

Wherever a rod of ferromagnetic material like iron or nickle is magnetized, it undergoes a change in length. This phenomenon of change in length of ferromagnetic materials on application of magnetic



field is called magnetostriction.
When the rod is placed in a coil through which an (high frequency) alternating cycle is passed, the length of rod changes. The change in length of rod due to magnetostriction is very minute. But when the frequency of magnetising field is the same as that of one of the natural mode of vibration of the rod, specially the fundamental one, the amplitude becomes very large due to resonance, and the rod is then powerful source of ultrasonic waves.

For a rod of length 'l', the frequency of fundamental vibration is given by

$$f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$$

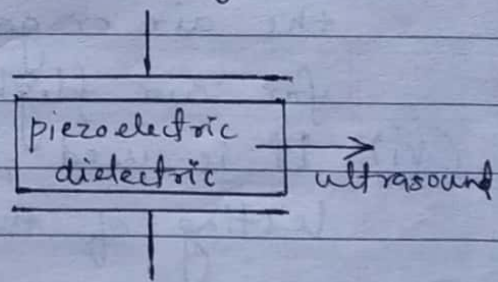
Therefore, the frequency of wave can be selected by suitable choice of the length of rod.

This method is suitable for producing ultrasonic waves upto 60000 Hz. For frequency above this piezoelectric effect method are suitable.

(ii) Piezo-electric method

When suitably cut piece of a crystal is placed under a mechanical strain such a compression or expansion, an emf will be developed across the opposit faces of that piece. This phenomenon is called piezo-electric effect. Conversly, when an emf is applied across such crystal the mechanical deformation of the crystal take place. The quartz crystal is an example of piezo-electric crystal.

On application of an alternating voltage across a quartz plate it vibrates with small amplitude at the frequency of the applied alternating voltage. If however the frequency of applied alternating voltage equals a frequency of mechanical resonance of the crystal then the amplitude of vibration becomes very large. This generates ultrasonic waves of high frequency.



The frequency of vibration of crystal depends upon (i) crystal dimension, (ii) orientation of crystal plates cut from natural crystal (iii) elastic constants. Therefore, the frequency of ultrasonic wave can be selected by suitable choice of crystal plates.

Date ____
Page ____

Application of ultrasonic waves:

Following are applications of ultrasonic waves:

(i) It is used in underwater signalling and detecting. In other words, the presence of icebergs and submarines can be detected by using ultrasonic waves. It can also be used to determine the depth of sea.

(ii) It is used to clean parts of watches and costly woolen and silken cloths.

(iii) Intense ultrasonic waves transforms immiscible liquids like water and oil into stable emulsion.

(iv) Many micro-organisms are destroyed in the ultrasonic field and so it is used to control them.

(v) Ultrasonic waves coagulates dust particle from the air or gas. So, it is used to remove mist for air flights.

(vi) It is used in the non-distructive mechanical testing of materials.