Electric field due to dipole
(not lying along equitorial/axial line)

We have,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2} \quad \text{——— (1)}$$

The radial component of electric field,

$$E_r = -\frac{\partial V}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot p\cos\theta \frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)$$

$$= -\frac{1}{4\pi\epsilon_0} \cdot p\cos\theta \left(-\frac{2}{r^3}\right)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{2p\cos\theta}{r^3} \quad \text{——— (2)}$$

The transverse component of electric field is

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$= -\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{r} \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2} \frac{\partial}{\partial \theta}(\cos\theta)$$

$$= \frac{-1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}(-\sin\theta)$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{p\sin\theta}{r^3} \quad \text{——— (3)}$$

The net electric field is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$E = \sqrt{\left(\frac{1}{4\pi\epsilon_0} \frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{p\sin\theta}{r^3}\right)^2}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \sqrt{(2\cos\theta)^2 + (\sin\theta)^2}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \sqrt{3\cos^2\theta + \cos^2\theta + \sin^2\theta}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \sqrt{3\cos^2\theta + 1}} \quad \text{---} \quad (4)$$

Electric field due to quadrupole

Electric field:

The radial component of electric field

$$E_r = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left[\frac{q l^2}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 1)\right]$$

$$= -\frac{q l^2}{4\pi\epsilon_0}(3\cos^2\theta - 1)\frac{\partial}{\partial r}\left(\frac{1}{r^3}\right)$$

$$= -\frac{q l^2}{4\pi\epsilon_0}(3\cos^2\theta - 1)\left(\frac{-3}{r^4}\right)$$

$$= \frac{3 \cdot q l^2 (3\cos^2\theta - 1)}{4\pi\epsilon_0 r^4} \quad \text{———} \quad (1)$$

The transverse (radial) component of electric field is

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = -\frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{q l^2}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 1)\right]$$

$$= -\frac{1}{r}\left[\frac{q l^2}{4\pi\epsilon_0 r^3}\frac{\partial}{\partial \theta}(3\cos^2\theta - 1)\right]$$

$$= \frac{q l^2}{4\pi\epsilon_0 r^4}\left[3\frac{\partial}{\partial \theta}(\cos^2\theta) - 0\right]$$

$$= \frac{q l^2}{4\pi\epsilon_0 r^4}\left[3 \cdot 2 \cdot \cos\theta \sin\theta\right]$$

$$= \frac{6 \cdot q l^2}{4\pi\epsilon_0 r^4} \cdot \sin\theta\cos\theta \quad \text{———} \quad (2)$$

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The net electric field is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$\Rightarrow E = \sqrt{\left(\frac{3ql^2}{4\pi\epsilon_0 r^4}(3\cos^2\theta - 1)\right)^2 + \left(\frac{6ql^2}{4\pi\epsilon_0 r^4}\sin\theta\cos\theta\right)^2}$$

$$\Rightarrow E = \frac{3ql^2}{4\pi\epsilon_0 r^4}\sqrt{(3\cos^2\theta - 1)^2 + (2\sin\theta\cos\theta)^2}$$

$$\Rightarrow E = \frac{3ql^2}{4\pi\epsilon_0 r^4}\sqrt{(9\cos^4\theta + 1 - 6\cos^2\theta) + 4\sin^2\theta\cos^2\theta}$$

$$\Rightarrow E = \frac{3ql^2}{4\pi\epsilon_0 r^4}\sqrt{9\cos^4\theta + 1 - 6\cos^2\theta + 4(1 - \cos^2\theta)\cos^2\theta}$$

$$\Rightarrow E = \frac{3ql^2}{4\pi\epsilon_0 r^4}\sqrt{9\cos^4\theta + 1 - 6\cos^2\theta + 4\cos^2\theta - 4\cos^4\theta}$$

$$\boxed{E = \frac{3ql^2}{4\pi\epsilon_0 r^4}\sqrt{5\cos^4\theta - 2\cos^2\theta + 1}} \quad \text{--- } (3)$$

Eq. (3) is required expression for E