Diffusion and conduction equation

Whenever there is a concentration gradient of particle there is a net diffusional motion of particle in the direction of decreasing concentration. To quantify particle flow, we define the particle flux (Γ) just like current, as the number of particles crossing unit area per unit time.

Let $\Delta N =$ Number of particle

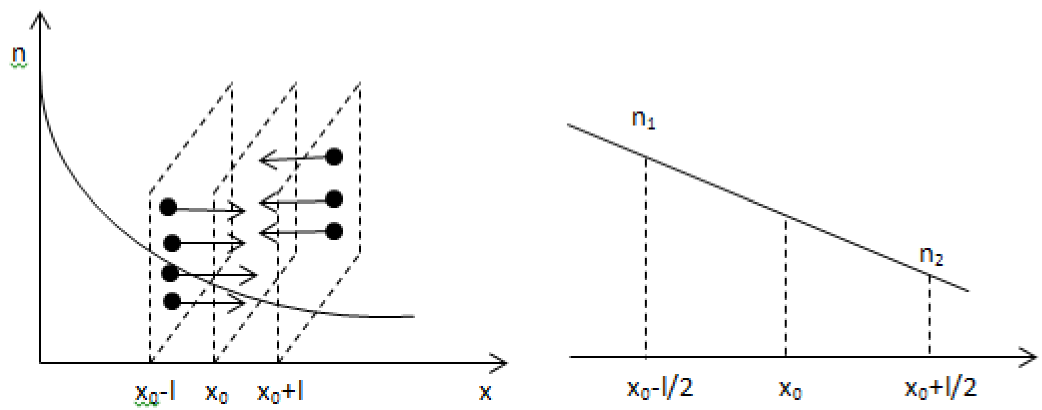
$A =$ Area

$\Delta t =$ Time

Then $\Gamma = \dfrac{\Delta N}{A\Delta t}$...............(1)

If the particle is charged with a charge Q (-e for electrons and +e for holes) then the electric current density $'J'$ which is basically a charge flux is related to particle flux Γ by

$J = \Gamma Q$...............(2)



Let us assume that the electron concentration changes only in X-direction. We know that in the absence of an electric field the electron motion is random and involve scattering from lattice vibration and impurities. Let "l" is the mean free path in the X-direction and "τ" is the mean free time between scattering events. The electron moves a mean distance "l" in the $\pm X-$direction and then it is scattered and changes directions. The mean speed along X is $v_x = l/\tau$.

Half of the electron in $(x_0 - l)$ would be moving towards "x_0" and other half away from "x_0" and in time "τ" half of them will reach "x_0" and cross as shown in figure above.

If "n_1"is the concentration of electron at $\left(x_0 - l/2\right)$ then the number of electrons moving towards right to cross "x_0" is $\frac{n_1}{2}Al$. Where $Al =$Volume of the segment. Similarly number of electrons moving towards left to cross "x_0" is $\frac{n_2}{2}Al$. Where "n_2" is the concentration of electron at $\left(x_0 + l/2\right)$.

The net number of electrons crossing "x_0" per unit time per unit area in the positive X-direction is Γ_e.

$$\Gamma_e = \frac{\frac{n_1}{2}Al - \frac{n_2}{2}Al}{A\tau} = -\frac{l}{2\tau}(n_2 - n_1)$$

$$(n_2 - n_1) \simeq \left(\frac{dn}{dx}\right)\Delta x = \left(\frac{dn}{dx}\right).l$$

$$\Gamma_e = -\frac{l^2}{2\tau}\left(\frac{dn}{dx}\right)$$

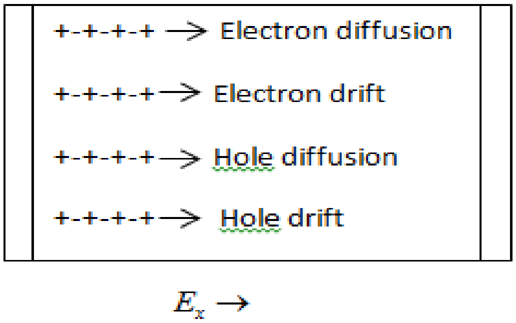
$\Gamma_e = -D_e\left(\frac{dn}{dx}\right)$ this is called **FICK's Law.**

Where "D_e" is diffusion coefficient of electrons. The unit is $\left(\frac{m^2}{Sec}\right)$.

Diffusion coefficient is the measure of how readily the particle diffuses in the medium.

Now from equation (2), $J_e = -e\Gamma_e = eD_e\frac{dn}{dx}$

Similarly for $J_h = -e\Gamma_h = -eD_h\frac{dp}{dx}$



Suppose that there is also a positive electric field E_x acting along positive X-direction as shown in figure above.

Then the total current density due to the electron drifting and diffusing.

$$J_e = ne\mu_e E_x + eD_e \frac{dn}{dx}$$

Similarly $J_p = ne\mu_h E_x - eD_h \frac{dp}{dx}$

$$D = \frac{l^2}{\tau}$$