Electricity and Magnetism

Electricit Current (I)

The electric current is defined as rate of flow of charge. If 'Q' is the net charge passing through any cross-section in time 't' then the electric current is given by

$$I = \frac{Q}{t} \quad \text{------} \quad (1)$$

Its S.I. unit is Ampere.

$$\therefore 1\,A = \frac{1\,C}{1\,s} = 1\,C s^{-1}$$

A current is said to be one ampere of current if one coulomb of charge passes through it in one second.

Current density (J)

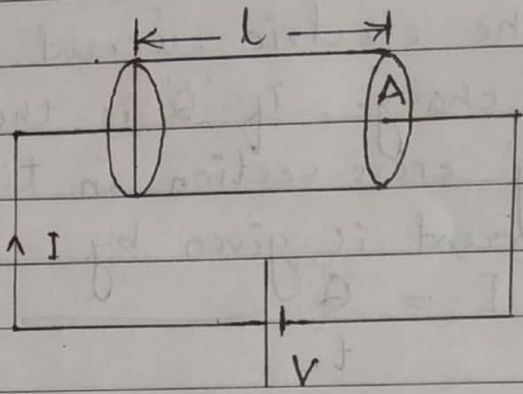
Current density of a conductor carrying current is defined as the rate of flow of charge through unit area of cross-section of the conductor.

If 'I' is the current flowing in a conductor having cross-sectional area 'A' then the current density is given by

$$J = \frac{I}{A} \quad \text{------} \quad (1)$$

Its S.I. unit is $A\,m^{-2}$.

Current density and drift velocity



The small velocity imposed on the random motion of electrons in a conductor on applying electric field to it is called drift velocity (v_d).

Let 'l' be the length and 'A' be the area of cross-section of the conductor.

$\therefore$ volume of the conductor $= Al$

If 'n' is the number of free electrons per unit volume and 'e' is the charge of electron, then total number of charge (free) in the conductor

$= nAl$

$\therefore$ total free charge in the conductor $= neAl$

When battery is connected let 'Q' charge drift through the conductor in time 't' then the current flowing through the conductor

$$I = \frac{Q}{t}$$

$$\Rightarrow I = \frac{neAl}{t}$$

$$\Rightarrow I = neAv_d \quad \text{———} \quad (1)$$

where $v_d = \frac{l}{t}$, called drift velocity.

$$\Rightarrow \frac{I}{A} = n e v_d$$

$$\Rightarrow J = n e v_d \quad \text{------} \quad (2)$$

Ohm's law

According to Ohm's law, "At constant temperature, the current flowing through a conductor is directly proportional to the potential difference across its ends".

Mathematically,

$$V \propto I$$

$$\Rightarrow V = RI$$

$$\Rightarrow V = IR \quad \text{------} \quad (1)$$

where, R is a constant of proportionality called resistance of a conductor.

Resistivity

Resistance of a conductor is

(i) directly proportional to the length of the conductor

i.e., $R \propto l$ ------ (1)

(ii) inversly proportional to the cross-sectional area of the conductor

i.e., $R \propto \frac{1}{A}$ ------ (2)

combining ① and ②, we get

$$R \propto \frac{l}{A}$$

$$\Rightarrow R = \rho \frac{l}{A} \quad \text{———— (3)}$$

where, 'ρ' is a constant of proportionality called resistivity of the material.

from equation ③,

$$\rho = R \frac{A}{l}$$

If $l = 1$ and $A = 1$, then

$$\rho = R$$

∴ the resistivity of the material of a conductor is the resistance offered by a wire of this material of unit length and unit area of cross-section.

Conductance and Conductivity

The reciprocal of resistance of a conductor is called its conductance.

∴ Conductance, $G = \frac{1}{R}$ ———— ①

Its S.I. unit is ohm^{-1} (Ω^{-1}) or mho ($\mho$) or siemens (S).

The reciprocal of the resistivity of the material of a conductor is called its conductivity. It is denoted by σ.

$\therefore$ conductivity, $\sigma = \frac{1}{\rho}$ ——— (2)

Its S.I. unit is $\Omega^{-1} m^{-1}$.

Drift velocity (v_d) and relaxation time (τ)

At any temperature, when there is no external field, electrons are likely to move in all direction and there is no overall drift. In the presence of external field E, each electron experiences an acceleration of $\left(\frac{eE}{m}\right)$ opposit to the field direction. But, this acceleration is momentary, since electrons are contineously making random collisions with viberating atoms or ions or other electrons of metal. After a collision, each electron makes a fresh start, accelerating only to be deflected randomly again.

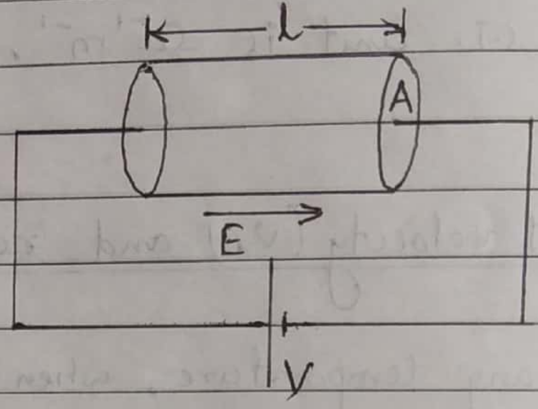
If 'τ' is the average time between the collisions, the average drift speed or average drift velocity of electron is given by

$$v_d = \frac{|e|E}{m}\tau \quad \text{——— (1)}$$

The average time 'τ' between collisions is called the relaxation time because it is a measure of time for the system to relax back to thermal equilibrium through collision.

Atomic view of resistivity

Consider a conductor of length 'l' and cross-sectional area 'A' across which potential difference 'V' is applied. If 'n' is the number of free electrons per unit volume and 'v_d' is the drift velocity of free electrons then the current density is given by



$$J = nev_d \quad \text{——} \quad (1)$$

Let 'E' be the electric field experienced by drifting electrons. The drift velocity of free electrons is given by

$$v_d = \frac{eE}{m}\tau \quad \text{——} \quad (2)$$

where, τ is relaxation time

from eq.ⁿ (1) and eq.ⁿ (2), we get,

$$J = ne\left(\frac{eE}{m}\tau\right)$$

$$\Rightarrow J = \frac{ne^2}{m}\tau E \quad \text{——} \quad (3)$$

If 'I' is the current flowing in the conductor then

$$I = JA$$

$$\Rightarrow I = \frac{ne^2\tau}{m} E \cdot A$$

$\because E = \frac{V}{l}$

$\therefore I = \frac{ne^2\tau}{m} \cdot \frac{V}{l} A$

$\Rightarrow V = \frac{m}{ne^2\tau} \cdot \frac{l}{A} I$ ——— (4)

Comparing eqⁿ (4) with Ohm's law,

$V = IR$,

we get,

$R = \frac{m}{ne^2\tau} \frac{l}{A}$ ——— (5)

Comparing eqⁿ (5) with equation

$R = \rho \frac{l}{A}$

we get,

$\boxed{\rho = \frac{m}{ne^2\tau}}$ ——— (6)

Mobility (μ)

The mobility of charge carriers is defined as the magnitude of drift velocity per unit electric field. It is denoted by μ.

$\therefore \mu = \frac{|v_d|}{E}$ ——— (1)

Its value is positive for both positive as well as negative charge carriers.

We have,

$$v_d = \frac{qE}{m}\tau \quad \text{---} \quad (2)$$

$\therefore$ from eq. (1) and eq. (2),

$$\mu = \frac{qE}{m}\tau \Big/ E \quad \text{---} \quad (\because e = q)$$

$$\Rightarrow \mu = \frac{q\tau}{m} \quad \text{---} \quad (3)$$

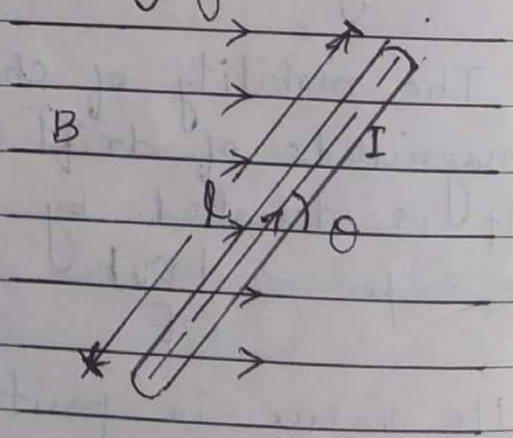
Its S.I. unit is $m^2V^{-1}s^{-1}$.

Magnetic field

A magnetic field can be defined as a region where magnetic force is experienced by magnetic particles.

Magnetic force on current carrying conductor

Consider a conductor of length 'l' carrying current 'I' placed at an angle of 'θ' with field of strength 'B'. As the current flows through



the conductor the free electron moves with drift velocity 'v' opposit to the direction of current. Due to the motion of these free electrons inside the magnetic field, these free electrons experience lorentz force as a result conductor also experience a force.

Let 'N' be the number of free electrons on the conductor then the force experienced by the conductor is given by

$F = N \times$ (force experienced by one electron)

$\Rightarrow F = N \times (Bev \sin\theta)$

$\Rightarrow F = BNev \sin\theta$ ——— ①

We have,

$I = neAv$

where,

n = no. of free electrons per unit volume

A = area of cross section

$\Rightarrow I = \left(\frac{N}{V}\right) eAv$ ——— $\left(n = \frac{N}{V}\right)$

$\Rightarrow I = \frac{N}{Al} eAv$

$\Rightarrow I = \frac{Nev}{l}$

$\Rightarrow Nev = Il$ ——— ②

∴ from eqⁿ ① and ②, we get,

$\boxed{F = BIl \sin\theta}$ ——— ③

In vector form,

$$\vec{F} = I(\vec{l} \times \vec{B}) \quad \text{——— (9)}$$

The direction of $\vec{F}$ is given by Fleming left hand rule.

Special Cases:

(i) If $\theta = 0$ or $180°$, i.e., conductor is parallel to the field.

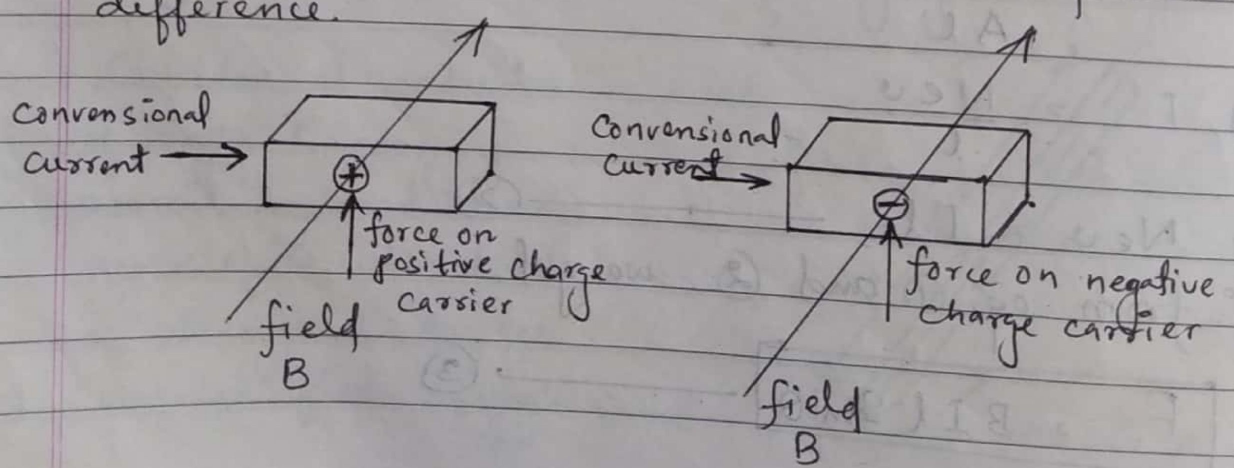
$F = 0$, i.e., minimum force is experienced

(ii) If $\theta = 90°$, i.e, if conductor is perpendicular to field.

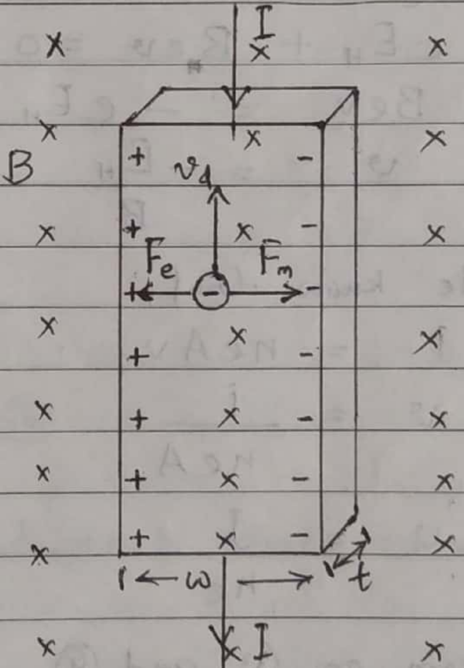
$F = BIl$, i.e., maximum force is experienced.

Hall effect

A current-carrying conductor in a magnetic field has a small potential difference across its sides, in a direction at right angles to the field. The phenomenon is called Hall effect. The potential difference is called Hall potential difference.



The Hall effect can be attributed to the forces experienced by charge carriers in the conductor. The force acts at right angle to directions of the magnetic field and the current (as given by Fleming's left-hand rule) and causes the charge carriers to be pushed sideways, increasing their concentration towards one side of the conductor. As a result potential difference is produced across the conductor. Hall potential difference therefore revels the sign of charge carriers (more exactly majority charge carriers) in the conductor.



Let a field of magnetic induction 'B' is set up at right angle to the strip by placing the strip between the ~~plat~~ poles faces of an electromagnet. This field exerts a deflecting force F_m given by

$$F_m = Bev \quad \text{——} \quad (1)$$

Due to this force the charge carriers (electrons in this case) accumulates toward right face of the metal strip producing transverse Hall potential difference V_H. Due to these accumulated charges carriers an electric field called Hall electric field E_H is set up which acts inside the conductor which oppose the sideways drift of the carriers. The opposing force on the charge carriers due to E_H is given by

$$F_e = eE_H \quad \text{——— (2)}$$

After sometime equilibrium is reached ~~to~~ in which the sideways magnetic deflecting force on charge carrier is just cancelled by the oppositely directed electric force due to E_H (Hall electric field).

∴ At equilibrium,

$$F_e + F_m = 0$$

$$\Rightarrow eE_H + Bev = 0$$

$$\Rightarrow Bev = -eE_H$$

$$\Rightarrow v = -\frac{E_H}{B} \quad \text{——— (3)}$$

We know that,

$$I = neAv$$

$$\Rightarrow v = \frac{I}{neA}$$

$$\Rightarrow v = \frac{J}{ne} \quad \text{——— (4)} \quad \left(\because J = \frac{I}{A}\right)$$

from eq. (3) and (4),

$$\frac{J}{ne} = -\frac{E_H}{B}$$

$$\Rightarrow \frac{E_H}{JB} = -\frac{1}{ne}$$

$$\Rightarrow R_H = -\frac{1}{ne} \quad \text{——— (5)}$$

Here, $R_H \left(= \frac{E_H}{JB} \right)$ is called Hall coefficient and n is the number of charge carriers per unit volume.

Now,

$$E_H = \frac{V_H}{w} \quad \text{——— (6)}$$

and

$$J = \frac{I}{A} = \frac{I}{wt} \quad \text{——— (7)}$$

from eq. (5),

$$\frac{E_H}{JB} = -\frac{1}{ne}$$

$$\Rightarrow n = -\frac{JB}{eE_H}$$

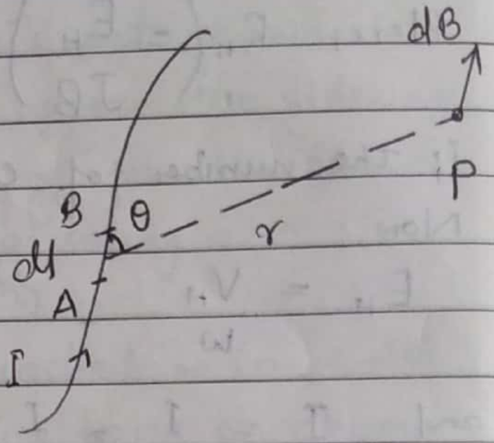
$$\Rightarrow n = -\frac{\left(\frac{I}{\cancel{w}t}\right) B}{e\left(\frac{V_H}{\cancel{w}}\right)}$$

$$\Rightarrow n = -\frac{IB}{etV_H} \quad \text{——— (8)}$$

Here, negative sign represents that charge carriers are free electrons.

Biot - Savart Law

This law gives quantative measurement of the magnetic field due to current carrying element.



Consider a small current carrying element AB of length 'dl' carrying current 'I'. Let P be a point at a distance 'r' away from current element at which magnetic field is to be determined and 'θ' be the angle between the current element 'dl' and 'r'.

According to Biot-Savart law, the magnetic field (dB) at point P due to current carrying element 'dl' is

(i) directly proportional to magnitude of current passing through it

i.e., $dB \propto I$ ——— (1)

(ii) directly proportional to length of the element dl,

i.e., $dB \propto dl$ ——— (2)

(iii) directly proportional to the sine of angle 'θ' between 'dl' and 'r'.

i.e., $dB \propto \sin\theta$ ——— (3)

(iv) inversly proportional to the square of the distance between element length and the point P,

i.e., $dB \propto \frac{1}{r^2}$ ——— (4)

Combining eqns. (1), (2), (3) and (4), we get,

$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

Scanned with CamScanner

$\Rightarrow \quad dB = k \dfrac{I\,dl \sin\theta}{r^2}$

where, k is a constant of proportionality and its value depends upon the unit we take.
In S.I. unit,

$$k = \frac{\mu_0}{4\pi}$$

where, $\mu_0 = 4\pi \times 10^{-7}$ H/m, is permeablity of free space.

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2} \quad \text{———} \quad (5)$$

In vector form,

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\, r \sin\theta}{r^3}$$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi} \frac{I\,(\vec{dl} \times \vec{r})}{r^3} \quad \text{———} \quad (6)$$

The direction of dB is perpendicular to plane containing 'dl' and 'r' and is given by right hand grip rule.
In c.g.s. unit, $k = 1$.

and $\quad dB = \dfrac{I\,dl \sin\theta}{r^2} \quad \text{———} \quad (7)$

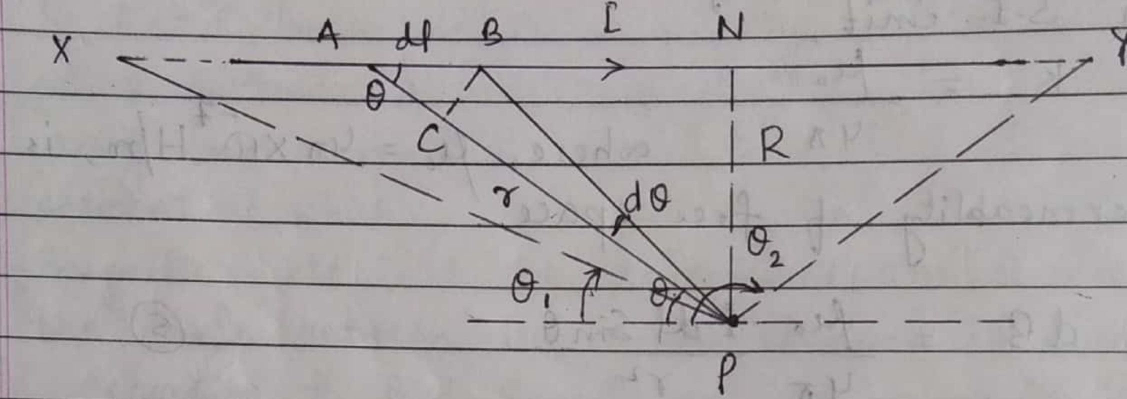
For magnetic field due to whole length,

$$B = \int dB$$

$$\Rightarrow \quad B = \int \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2} \quad \text{———} \quad (8)$$

Application of Biot-Savart law

(i) Magnetic field due to straight conductor



Consider a straight conductor XY carrying current 'I'. Let P be a point at perpendicular distance 'R' from the conductor. Let a small length element AB, at a distance 'r' away from point P, substand angle $d\theta$ at point P. Draw perpendicular BC to AP.

Here, since AB is small so $d\theta$ is also small

$\therefore \; AP \approx BP \approx r$

The magnetic field at point P due to length element $AB = dl$ is given by

$$dB = \frac{\mu_0}{4\pi} \frac{I \, dl \sin\theta}{r^2} \qquad (1)$$

where, θ is the angle between dl and r.

In ΔABC,

$$\sin\theta = \frac{BC}{AB}$$

$$\Rightarrow BC = AB \sin\theta$$

Scanned with CamScanner

Date ______
Page ______

$\Rightarrow BC = dl \sin\theta$ ——— (2)

In ΔBCP,

$$\sin(d\theta) = \frac{BC}{BP}$$

$\Rightarrow d\theta = \frac{BC}{r}$ - - - - ($\because d\theta$ is small $\therefore \sin(d\theta) \approx d\theta$)

$\Rightarrow BC = r\,d\theta$ ——— (3)

from eqn (2) and (3), we get,

$dl \sin\theta = r\,d\theta$ ——— (4)

$\therefore$ from eqn (1) and (4), we get,

$$dB = \frac{\mu_0}{4\pi} \frac{I\,r\,d\theta}{r^2}$$

$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\,d\theta}{r}$ ——— (5)

In ΔAPN,

$\sin\theta = \frac{PN}{AP} \Rightarrow \sin\theta = \frac{R}{r}$

$\Rightarrow \frac{1}{r} = \frac{\sin\theta}{R}$ ——— (6)

$\therefore$ from eqn (5) and (6), we get,

$$dB = \frac{\mu_0}{4\pi} \frac{I}{R} \sin\theta\, d\theta$$

$\therefore$ Magnetic field due to whole length of conductor at point P is given by

$$B = \int dB$$

$$\Rightarrow B = \int_{\theta_1}^{\theta_2} \frac{\mu_0}{4\pi} \frac{I}{R} \sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} \left[-\cos\theta \right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{4\pi R} (\cos\theta_1 - \cos\theta_2)} \quad \text{——— (7)}$$

If conductor is long, then
$\theta_1 \approx 0^\circ$ and $\theta_2 \approx 180^\circ$

$$\therefore B = \frac{\mu_0 I}{4\pi R} (\cos 0^\circ - \cos 180^\circ)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} (1 - (-1))$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi R} \cdot 2$$

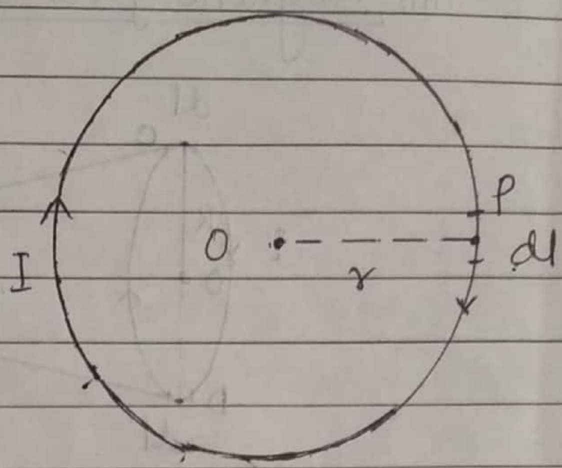
$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R}} \quad \text{——— (8)}$$

The direction of magnetic field is given by right hand grip rule.

(ii) Magnetic field at the center of a circular loop:

Let us consider a circular coil of radius 'r' carrying current 'I'. To find magnetic field at the center of the coil, let us consider an element length 'dl' on the coil at point P. Since the element length 'dl' is tangent to the radius 'r' so angle between dl and r is 90°.

According to Biot-Savart law the magnetic field at O due to current in element length dl is.



$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\,\sin(90^\circ)}{r^2}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$\therefore$ magnetic field at the center of the coil due to whole circular coil is

$$B = \int dB$$

$$\Rightarrow B = \int_0^{2\pi r} \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

for a given coil r and I are constant.

$$\therefore B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_0^{2\pi r} dl$$

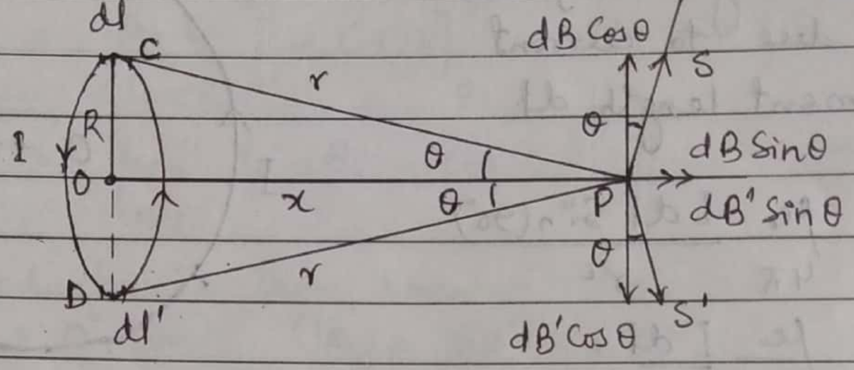
$$\Rightarrow B = \frac{\mu_0 I}{4\pi r^2}(2\pi r)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2r}} \quad \text{—— (1)}$$

If there are 'N' no. of turns then

$$B = \frac{\mu_0 N I}{2r} \quad \text{—— (2)}$$

(iii) Magnetic field on the axis of the circular coil



Let us consider a circular coil of radius 'R' carrying current 'I' in anticlockwise direction. Let P be a point along the axis passing through center O of the coil at a distance 'x' from center O. Consider an elementry length 'dl' at point C on the coil. If we join point C and point P the line joining them makes 90° with the elementary length 'dl'. Let 'r' be the distance between 'dl' and point P.

According to Biot-Savart law, magnetic field dB at point P due to 'dl' is

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin(90^\circ)}{r^2}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2} \quad \text{---} (1)$$

This field is perpendicular to plane containing 'dl' and 'r', i.e. directed along PS as shown in fig. Let us consider similar current element 'dl'' at point D, diametrically opposit to C. The magnetic field at point P due to this elementary length dl' is

$$dB' = \frac{\mu_0}{4\pi} \frac{I\, dl' \sin(90^\circ)}{r^2}$$

$$\Rightarrow dB' = \frac{\mu_0}{4\pi} \frac{I\, dl'}{r^2} \quad (2)$$

The direction of field is along PS'. We resolve dB and dB' into two perpendicular components, one along axis and another perpendicular to the axis. Since the magnitude $dB\cos\theta$ and $dB'\cos\theta$ is same but opposit in direction they cancle each other. But components $dB\sin\theta$ and $dB'\sin\theta$ are along same direction and hence adds up to give resultant magnetic field at P

$\therefore$ the magnetic field at P due to coil is

$$B = \int dB \sin\theta$$

$$\Rightarrow B = \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

$$\Rightarrow B = \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{I\, dl}{r^2} \frac{R}{r}$$

$$\Rightarrow B = \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{IR\, dl}{r^3}$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{IR}{r^3} \int_0^{2\pi R} dl$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{IR}{r^3} (2\pi R)$$

$$\Rightarrow B = \frac{\mu_0 I R^2}{2 r^3} \quad (3)$$

Scanned with CamScanner

Also, In $\triangle$ COP,

$$r^2 = R^2 + x^2$$

$$\Rightarrow r = (R^2 + x^2)^{1/2}$$

$\therefore$ from eq. (3),

$$B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \quad \text{———} (4)$$

If coil has 'N' number of turns then

$$B = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}} \quad \text{———} (5)$$

The direction of B is given by Right hand grip rule.

Special cases:

(i) When $x=0$, i.e, at the center of the coil

$$B = \frac{\mu_0 N I R^2}{2(R^2)^{3/2}} = \frac{\mu_0 N I}{2R} \quad \text{———} (6)$$

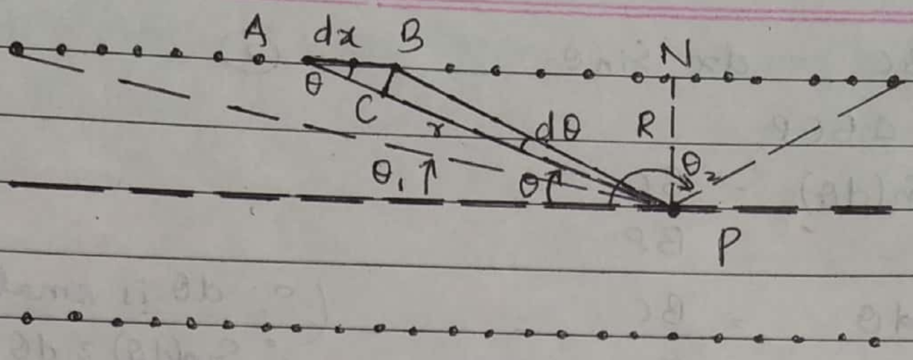
(ii) When $x >> R$, i.e, when point P lies very far from the coil,

$$B = \frac{\mu_0 N I R^2}{2(x^2)^{3/2}} = \frac{\mu_0 N I R^2}{2x^3} \quad \text{———} (7)$$

(iv) <u>Magnetic field inside the solenoid</u>

A solenoid is a long cylinderical coil consisting of a large number of circular turns.

Consider a solenoid of radius 'R' and carrying current I in the clockwise direction. Consider a small coil of length 'dx' (i.e, AB) at a distance 'r'



from point P where magnetic field is to be determined.
Let small coil 'dx' (i.e., AB) substand angle 'dθ' at point P. Draw perpendicular BC to AP.
Here, since AB is small so 'dθ' is also small

$\therefore$ $AP \approx BP = r$

If 'n' is the no. of turns in per unit length then no. of turns in the coil (AB), $N = ndx$

$\therefore$ The magnetic field at point P due to the coil of turn N (= ndx) is given by

$$dB = \frac{\mu_0 I R^2}{2r^3} N$$

$$\Rightarrow dB = \frac{\mu_0 I R^2}{2r^3} ndx$$

$$\Rightarrow dB = \frac{\mu_0 I R}{2r^2} \cdot \frac{R}{r} \cdot ndx$$

$$\Rightarrow dB = \frac{\mu_0 n I R}{2r^2} \cdot \sin\theta \, dx \quad \text{--- (1)} \quad \left(\text{In } \Delta APN, \frac{R}{r} = \sin\theta\right)$$

In ΔABC

$$\sin\theta = \frac{BC}{AB}$$

$$\Rightarrow BC = AB \sin\theta$$

$\Rightarrow BC = dx \sin\theta$ ——— (2)

In ΔBCP,

$$\sin(d\theta) = \frac{BC}{BP}$$

$$\Rightarrow d\theta = \frac{BC}{r} \text{ ——— } \left(\because d\theta \text{ is small} \quad \therefore \sin(d\theta) \approx d\theta \right)$$

$\Rightarrow BC = r\,d\theta$ ——— (3)

from eq. (2) and (3), we get

$dx \sin\theta = r\,d\theta$ ——— (4)

$\therefore$ from eq. (1) and eq. (4),

$$dB = \frac{\mu_0 n I R}{2r^2} . r\,d\theta$$

$$\Rightarrow dB = \frac{\mu_0 n I R}{2r} d\theta$$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \frac{R}{r} d\theta$$

$$\Rightarrow dB = \frac{\mu_0 n I}{2} \sin\theta\, d\theta$$

$\therefore$ magnetic field due to solenoid at point P is

$$B = \int dB$$

$$\Rightarrow B = \int_{\theta_1}^{\theta_2} \frac{\mu_0 n I}{2} \sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow B = \frac{\mu_0 n I}{2} \left[\cos\theta_1 - \cos\theta_2\right]$$

If the solenoid is long, than
$\theta_1 = 0°$ and $\theta_2 = 180°$

$\therefore B = \frac{\mu_0 n I}{2} (\cos 0° - \cos 180°)$

$\Rightarrow B = \frac{\mu_0 n I}{2} \cdot 2$

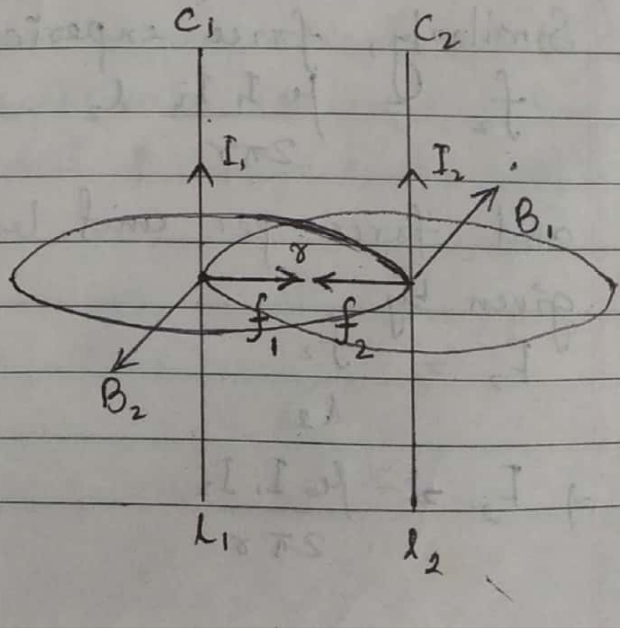
$\Rightarrow \boxed{B = \mu_0 n I}$ ———— (5)

Force on two parallel current carrying conductors

When two current carrying conductors are brought near to each other due to the interaction of their magnetic field force is set up between them. The force may be attractive or repulsive depending on the direction of the current.

(i) When current flows in same direction

Consider two parallel conductors C_1 and C_2 of length l_1 and l_2 carrying current I_1 and I_2 respectively, seperated by distance 'r'. Applying right



hand grip rule, magnetic field due to C_1 acting at C_2 is B_1 which is perpendicul to C_2. The magnetic field due to C_2 acting on C_1 is B_2 and is perpendicular to C_1. let force on C_1 is f_1 and force on C_2 is f_2. Applying Fleming left hand rule it can be seen that force is attractive.

Force on C_1 is

$$f_1 = B_2 I_1 l_1 \quad ---(\theta = 90^\circ \quad \therefore \sin\theta = 1)$$

$$\because B_2 = \frac{\mu_0 I_2}{2\pi r}$$

$$\therefore f_1 = \frac{\mu_0 I_2}{2\pi r} I_1 l_1$$

$$\Rightarrow f_1 = \frac{\mu_0 I_1 I_2 l_1}{2\pi r} \quad \text{———} \quad (1)$$

$\therefore$ force per unit length of conductor C_1 is given by

$$F_1 = \frac{f_1}{l_1} = \frac{\mu_0 I_1 I_2 l_1}{2\pi r} / l_1$$

$$\Rightarrow F_1 = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{———} \quad (2)$$

Similarly, force experienced by C_2 is given by

$$f_2 = \frac{\mu_0 I_1 I_2}{2\pi r} l_2 \quad \text{———} \quad (3)$$

and force per unit length of conductor C_2 is given by

$$F_2 = \frac{f_2}{l_2}$$

$$\Rightarrow F_2 = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{———} \quad (4)$$

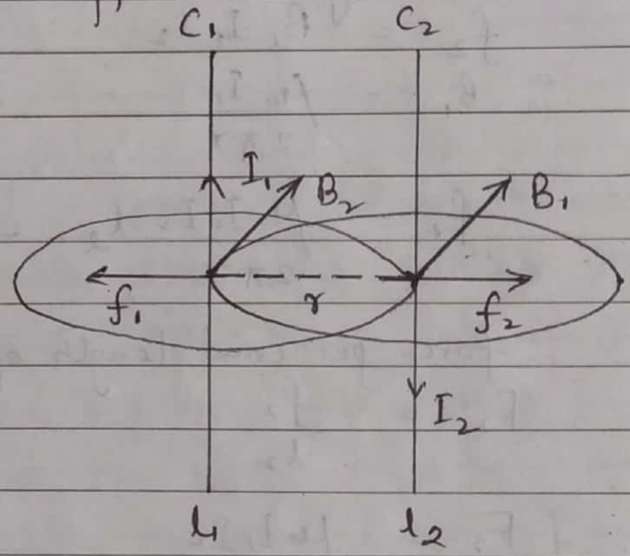
Scanned with CamScanner

$\therefore$ from eq. ② and ④,

$$F_1 = F_2 = \frac{\mu_0 I_1 I_2}{2\pi r} \quad \text{⑤}$$

$\therefore$ when current flows in same direction in two parallel conductors the conductors experience attractive force of magnitude $\frac{\mu_0 I_1 I_2}{2\pi r}$ per unit length.

(ii) <u>When current flows in opposit direction</u>



Consider two parallel conductors C_1 and C_2 of length l_1 and l_2. Carrying current I_1 and I_2 respectively and seperated by distance 'r'. Applying right hand grip rule the magnetic field due to C_1 is B_1 and is perpendicular to C_2 and the magnetic field due to C_2 is B_2 and is perpendicular to C_1. Applying Fleming ~~left~~ hand rule the force acting on C_1 and C_2 are f_1 and f_2 respectively and are attractive in nature.

The force on C_1 is

$$f_1 = B_2 I_1 l_1$$

$$\because B_2 = \frac{\mu_0 I_2}{2\pi r}$$

$\therefore f_1 = \frac{\mu_0 I_1 I_2}{2\pi r} l_1$ ——— (1)

force per unit length of conductor C_1 is

$F_1 = \frac{f_1}{l_1}$

$\Rightarrow F_1 = \frac{\mu_0 I_1 I_2}{2\pi r}$ ——— (2)

Similarly, the force on C_2 is

$f_2 = B_1 I_2 l_2$

$\because B_1 = \frac{\mu_0 I_1}{2\pi r}$

$\therefore f_2 = \frac{\mu_0 I_1 I_2}{2\pi r} l_2$ ——— (3)

$\therefore$ force per unit length of conductor C_2 is

$F_2 = \frac{f_2}{l_2}$

$\Rightarrow F_2 = \frac{\mu_0 I_1 I_2}{2\pi r}$ ——— (4)

from eq. (2) and (4),

$F_1 = F_2 = \frac{\mu_0 I_1 I_2}{2\pi r}$ ——— (5)

$\therefore$ When current flows in opposit direction of the two conductors (parallel) repulsive force of magnitude $\frac{\mu_0 I_1 I_2}{2\pi r}$ is established per unit length.

Date ____
Page ____

Defination of Ampere

The force acting per unit length in two parallel current carrying conductors is

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

If $I_1 = I_2 = 1\,A$ and $r = 1\,m$

then

$$F = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1}$$

$$\Rightarrow F = 2 \times 10^{-7}\ N m^{-1}$$

Therefore, 1 Ampere current is that current which when flows through the two parallel conductors placed 1m apart produces force of $2 \times 10^{-7}\ N m^{-1}$.

Ampere's law

Ampere's law states that line integral of magnetic field around a close loop (path) is equal to μ_0 times the current enclosed by that loop.

Mathematically,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

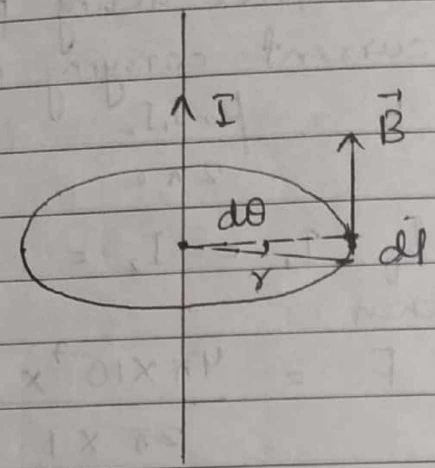
$$\Rightarrow \oint B\, dl \cos\theta = \mu_0 I$$

where, θ is the angle between magnetic field and small length element.

I is the current enclosed by close loop.

Proof:

Consider a conductor carrying current I. Let us consider closed circular path, around the conductor, of radius 'r' as shown in fig. The magnetic field at any point on this circular path is given by



$$B = \frac{\mu_0 I}{2\pi r} \quad \text{--- (1)}$$

Let 'dl' be the small elementary length of the closed path making an angle $d\theta$ with the center.

$$\therefore \; dl = r\,d\theta \quad \text{--- (2)}$$

Now,

$$\oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_0 I}{2\pi r}\cdot r\,d\theta$$

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \oint \frac{\mu_0 I}{2\pi}\,d\theta$$

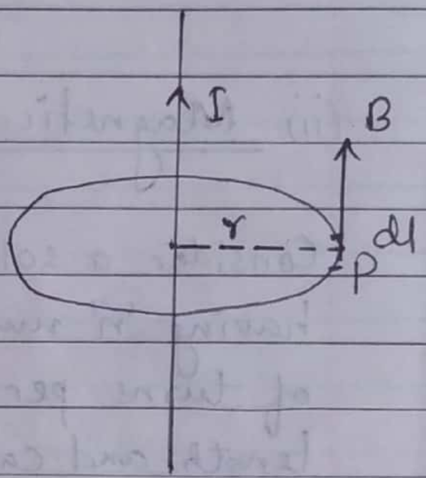
$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\theta$$

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \frac{\mu_0 I}{2\pi}\cdot(2\pi)$$

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{--- (3)}$$

Application of Ampere's law

i) Magnetic field due to straight conductor



Consider a straight conductor carrying current I. Let P be any point at a perpendicular distance 'R' from the conductor. To find magnetic field B at point P we draw a circle of radius 'R' such that the conductor passes through its center. By symmetry the magnitude of the magnetic field at all points on the circle are equal but the direction is given by tangent at its point.

Let 'dl' be the small elementary length on the path. Applying Ampere's law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\Rightarrow \oint B\, dl \cos\theta = \mu_0 I$$

$\because \theta = 0°$

$\therefore \cos\theta = 1$

$$\therefore \oint B\, dl = \mu_0 I$$

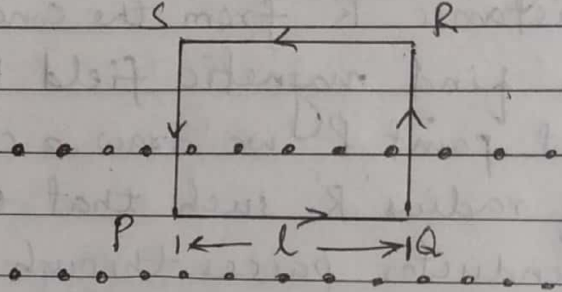
$$\Rightarrow \int_0^{2\pi R} B\, dl = \mu_0 I$$

$$\Rightarrow B \int_0^{2\pi R} dl = \mu_0 I$$

$$\Rightarrow B(2\pi R) = \mu_0 I$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi R} \quad \text{——— (1)}$$

(ii) <u>Magnetic field due to solenoid</u>



Consider a solenoid having 'n' number of turns per unit length and carrying I. Applying right hand grip rule, the magnetic field due to solenoid is parallel to the axis. Let P be any point on its axis where magnetic field is to be found.

Draw a close loop PQRS whose sides RS is very far from the axis. Let this loop enclose 'N' number of turns. Applying Ampere's law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 (NI) \quad \text{——— (1)}$$

Here,

$$\oint \vec{B}\cdot\vec{dl} = \int_P^Q \vec{B}\cdot\vec{dl} + \int_Q^R \vec{B}\cdot\vec{dl} + \int_R^S \vec{B}\cdot\vec{dl} + \int_S^P \vec{B}\cdot\vec{dl}$$

$$\text{——— (2)}$$

Now,

$$\int_P^Q \vec{B}\cdot\vec{dl} = \int_P^Q B\,dl\,\cos(0^\circ)$$

$$= \int_P^Q B\,dl$$

$$= B\int_P^Q dl$$

$$= B\,l \quad \text{——— (3)}$$

$\because$ QR and SP are perpendicular to B.

$$\therefore \int_Q^R \vec{B}\cdot\vec{dl} = \int_S^P \vec{B}\cdot\vec{dl} = 0 \quad \text{——— (4)}$$

Also,

$\because$ RS is very far from solenoid

$\therefore$ $B = 0$

$$\therefore \int_R^S \vec{B}\cdot\vec{dl} = 0 \quad \text{——— (5)}$$

$\therefore$ from eq^n (2), (3), (4) and (5),

$$\oint \vec{B}\cdot\vec{dl} = B\,l \quad \text{——— (6)}$$

from eq. (1) and (6), we get,

$$B\,l = \mu_0 N I$$

$$\Rightarrow B = \frac{\mu_0 N I}{l}$$

$$\Rightarrow \boxed{B = \mu_0 n I} \quad \text{——— (7)}$$

where, $\frac{N}{l} = n$, is number of turns per unit length.

Date ____
Page ____

Faraday's discovery

In 1831, Faraday performed an experiment on the coil connected with a galvanometer as a circuit. When he moved the magnet toward or away from the coil he got deflection on the galvanometer. Similar result was obtained when the magnet was kept stationary and coil was moved. He then concluded that when there was relative motion between coil and magnet the deflection on galvanometer was noticed due to induced current in the coil. The emf due to which induced current flows is known as induced emf. The phenomenon of producing induced emf on conductor due to relative motion between the conductor and magnet is known as electromagnetic induction

Magnetic flux and flux linked

The product

$$\phi = BA \quad \text{——— (1)}$$

is called flux of magnetic field through A. It may also be defined as total number of magnetic lines of force passing through the area held perpendicular to line of force themselves.

Scanned with CamScanner

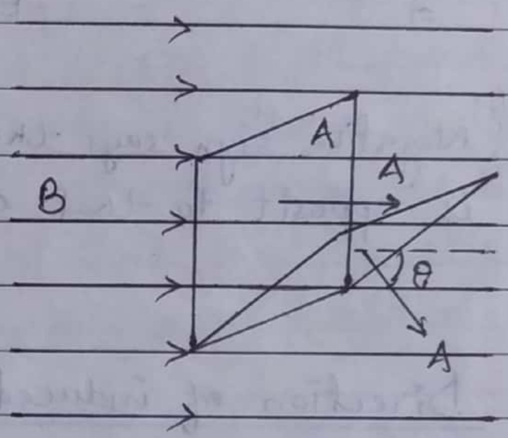
from eq^n ①,

$$B = \frac{\Phi}{A} \quad \text{——— ②}$$

∴ magnetic field intensity at a point is defined as the magnetic lines of force (flux ϕ) passing through unit area held perpendicular to lines of force.

We have flux through area A as

$$\Phi = BA$$



If B is not perpendicular to surface then we take component of area (A) along B. Therefore, the flux is given by

$$\phi = BA\cos\theta \quad \text{——— ③}$$

If there are N surface then the flux through them or flux linked to N surface is given by

$$\phi = NBA\cos\theta \quad \text{——— ④}$$

Faraday's law of electromagnetic induction

I^{st} law :

It states that "whenever there is change in magnetic flux linked with a coil, an emf is induced in it which lasts as long as the change in magnetic flux exist."

IInd law :

"The induced emf is directly proportional to the rate of change of magnetic flux linked with the coil."

i.e., induced emf, $E \propto -\frac{d\phi}{dt}$

$$\Rightarrow \quad E = -\frac{d\phi}{dt} \quad \text{——— (1)}$$

Negative sign says that the direction of induced emf is opposit to that cause it.

Direction of induced emf and current

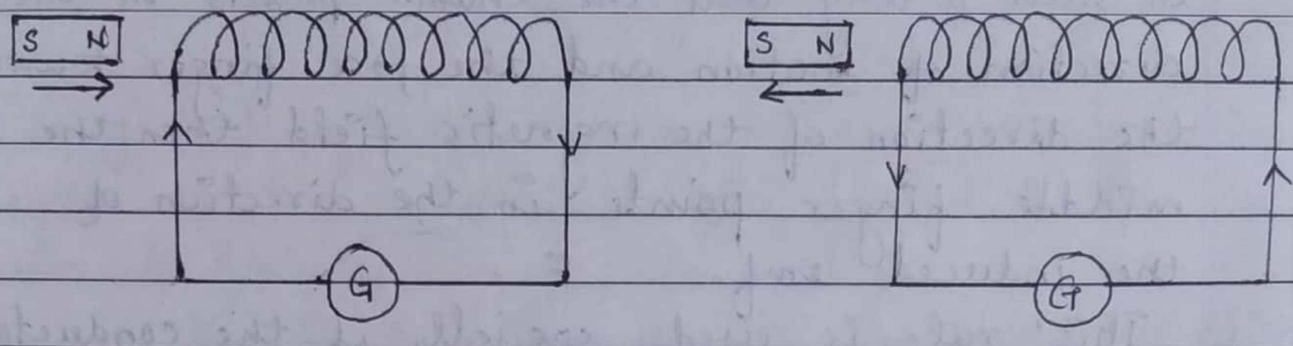
The direction can be obtained in two different ways :

(i) Lenz law :

This law states that the direction of induced emf in a coil is such that it always opposes the cause or change that produce it.

Explaination :

Consider a coil of wire (solenoid) connected with galvanometer. Let a magnet is moved toward one end of coil as shown in fig(i). Due to motion of magnet (cause or change) current is induced in the coil. This induced current produces its own magnetic field and this field opposes



(i) (ii)

the cause (motion of magnet). Applying right hand grip rule the direction of induced current is shown in fig (i).

Similarly, if the magnet is moved away the direction of induced current changes so that the magnetic field due to this current opposes the cause (motion of magnet).

Lenz law obeys the law of conservation of energy since the induced current always opposes the cause or change that produce it. To continue the induction of current on coil external mechanical work has to be done against the opposition. This mechanical work done converted to electrical energy. The electrical energy is not created on the coil by itself. It is only the conversion of mechanical energy to electrical energy.

(ii) Fleming Right hand rule

According to this rule, if the fore finger, the middle finger and the thumb of our right hand are stretched mutually perpendicular to each other

in such a way that the thumb points in the direction of motion and the fore finger towards the direction of the magnetic field then the middle finger points in the direction of the induced emf.
This rule is used specially if the conductor is moving in a magnetic field.

Self - induction

Self-induction is the phenomenon of inducing emf in a coil or conductor by changing the current flowing through it. The direction of induced emf always opposes the cause or change that produce it.
Consider a coil in which current 'I' is flowing and the flux linked due to this current with coil is ϕ. If we change the current the magnetic flux or field produce by it also changes which changes the flux linked with the coil. Due to this the emf is induced in the coil.
It is found that

$$\phi \propto I$$

$$\Rightarrow \phi = LI \quad \text{--- (1)}$$

where, L is a constant of proportionality called coefficient of self induction or self-inductance.

Scanned with CamScanner

$\therefore L = \frac{\Phi}{I}$ ——— (2)

$\therefore$ coefficient of self-induction or self-inductance is equal to the ratio of the flux linked to the current flowing through the coil.

If 'E' is the induced emf due to self-induction then by Faraday's law

$$E = -\frac{d\Phi}{dt}$$

$$\Rightarrow E = -\frac{d}{dt}(LI)$$

$$\Rightarrow E = -L\frac{dI}{dt} \quad \text{——— (3)}$$

$$\Rightarrow L = -\frac{E}{(dI/dt)}$$

If $\frac{dI}{dt} = 1\,A\,s^{-1}$

then,

$$L = -E$$

$\therefore$ coefficient of self-induction is also defined as induced emf in the coil when rate of change of current is 1 ampere per second.

Now,

$$L = -\frac{E}{(dI/dt)} = V s A^{-1}$$

Also,

$$L = \frac{\Phi}{I} = Weber\ A^{-1}$$

$\therefore$ S.I. unit of L is VsA^{-1} or WbA^{-1}. It is also called Henry (H).

Again,

$$1\,H = \frac{1\,V}{1\,As^{-1}}$$

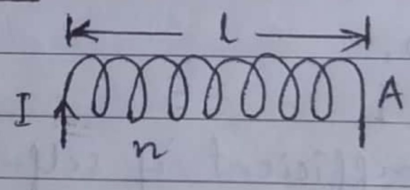
One Henry is the self-inductance of that coil in which 1 volt emf is induced when the rate of change of current is 1 ampere per second.

Also,

$$1\,H = \frac{1\text{ weber}}{1\text{ ampere}}$$

One Henry is self inductance of that coil in which one weber flux is linked when one ampere current flows through it.

Self-inductance of solenoid



Consider a long solenoid of length 'l', area of cross-section 'A' and number of turns per unit length 'n'. Suppose that a current 'I' is passed through the solenoid. The magnetic field inside the solenoid is given by

$$B = \mu_0 n I \quad \text{——— (1)}$$

In case of long solenoid, the magnetic field produced within the solenoid is practically constant.

$\therefore$ total magnetic flux linked with the solenoid is,

ϕ = (magnetic flux linked with one coil) $\times$ (total no. of turns)

$\Rightarrow \phi = B \cdot A (n l)$

$\Rightarrow \phi = \mu_0 n I A n l$

$\Rightarrow \phi = \mu_0 n^2 l A I$. ——— (2)

If 'L' is the self-inductance of the coil then

$L = \frac{\phi}{I}$

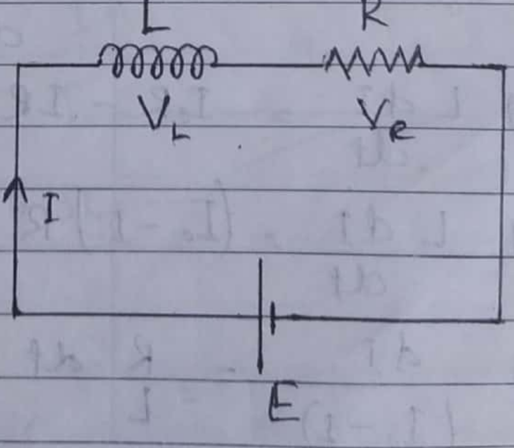
$\Rightarrow L = \frac{\mu_0 n^2 l A I}{I}$

$\Rightarrow \boxed{L = \mu_0 n^2 l A}$ ——— (3)

LR circuit

(i) Growth of current:

Consider a circuit containing an inductor (L) and resistance (R) in series connected to a battery (E). Initially, the current in the circuit



is zero and with time it increases. When current is increasing, due to self induction, an induced emf is setup in the inductor which opposes the growth in it. If 'I' is the current and $\left(\frac{dI}{dt}\right)$ is the rate of change of current at any instant in the circuit, then

p.d. across R, $V_R = IR$

and p.d. across L, $V_L = L\frac{dI}{dt}$

Applying Kirchhoff's voltage Law (KVL), we get

$$E = V_R + V_L$$

$$\Rightarrow E = IR + L\frac{dI}{dt} \quad \text{——— (1)}$$

When current in the circuit attains maximum value, i.e., $I = I_0$, then $\frac{dI}{dt} = 0$

Applying this condition, we get,

$$E = I_0 R \quad \text{——— (2)}$$

∴ from eq. (1) and (2), we get,

$$I_0 R = IR + L\frac{dI}{dt}$$

$$\Rightarrow L\frac{dI}{dt} = I_0 R - IR$$

$$\Rightarrow L\frac{dI}{dt} = (I_0 - I)R$$

$$\Rightarrow \frac{dI}{(I_0 - I)} = \frac{R}{L}\,dt$$

Scanned with CamScanner

Integrating both sides, we get,

$$\int_0^I \frac{dI}{(I_0 - I)} = \int_0^t \frac{R}{L}\,dt$$

$$\Rightarrow \left[-\log_e (I_0 - I)\right]_0^I = \frac{R}{L}[t]_0^t$$

$$\Rightarrow \left[\log_e (I_0 - I)\right]_0^I = -\frac{R}{L}t$$

$$\Rightarrow \log_e (I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

$$\Rightarrow \log_e \left(\frac{I_0 - I}{I_0}\right) = -\frac{R}{L}t$$

$$\Rightarrow \left(1 - \frac{I}{I_0}\right) = e^{-\frac{R}{L}t}$$

$$\Rightarrow \frac{I}{I_0} = 1 - e^{-\frac{R}{L}t}$$

$$\Rightarrow I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{——— (3)}$$

In eqn (3), the term $\left(\frac{L}{R}\right)$ has dimension of time and is called inductive time constant or time constant of the circuit.

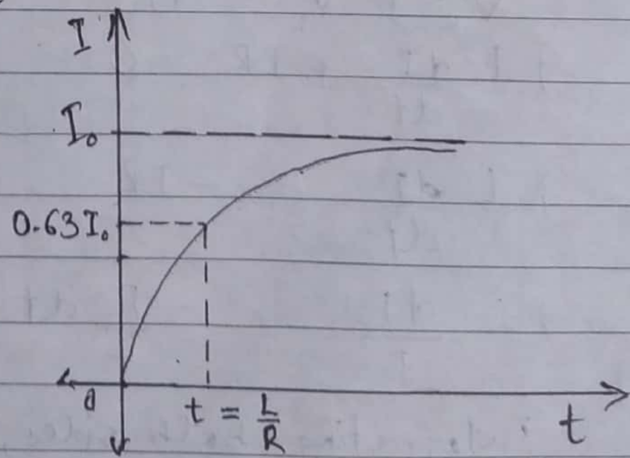
If $t = (L/R)$, then

$$I = I_0(1 - e^{-1})$$

$$\Rightarrow I = I_0(1 - 0.37)$$

$$\Rightarrow I = 0.63\, I_0$$

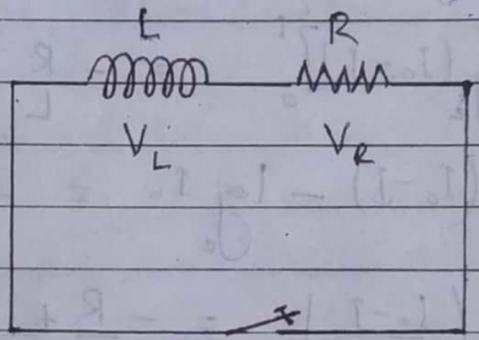
$$\Rightarrow I = 63\% \text{ of } I_0$$



∴ Inductive time constant of LR circuit

may be defined as the time in which the current grows from zero to 63% of its maximum value.

(ii) Decay of current:



Let I_0 be the maximum current initially in the circuit. As there is no battery connected the current decreases with time. The decreasing current produces induced emf in the circuit and it opposes the decay of the current.

Let at any instant of time 't' during decay process 'I' be the current in the circuit and $\frac{dI}{dt}$ be the rate of decay of current.

Applying kirchhoff's voltage law (KVL), we get,

$V_R + V_L = 0$

$\Rightarrow L\frac{dI}{dt} + IR = 0$

$\Rightarrow L\frac{dI}{dt} = -IR$

$\Rightarrow \frac{dI}{I} = -\frac{R}{L}dt$

Integrating both sides, we get,