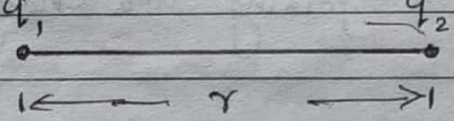
Electrostatics

Coulomb's Law

Statement :

"The force of attraction or repulsion between two stationary point charges is directly proportional to the product of magnitude of charges and inversly proportional to the square of distance between them."

Let q_1 and q_2 be two charges seperated by distance 'r'. According to Coulomb's law the force F is



$$F \propto q_1 q_2 \quad \text{——— ①}$$

$$F \propto \frac{1}{r^2} \quad \text{——— ②}$$

Combining equations ① and ②, we get,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = k \frac{q_1 q_2}{r^2} \quad \text{——— ③}$$

where, k is a constant of proportionality.

If charges are placed in vacuum then

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2 C^{-2}$$

Here, ϵ_0 is absolute permittivity of free space or vacuum and $\epsilon_0 = 8.85 \times 10^{-12} \ C^2 N^{-1} m^{-2}$.

∴ from equation (iii),

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \text{———} \quad (4)$$

$q_1 q_2 > 0$ $\qquad\qquad$ $q_1 q_2 < 0$

The Coulomb's force always acts along the line joining two charges.

Electric field and Electric field Intensity

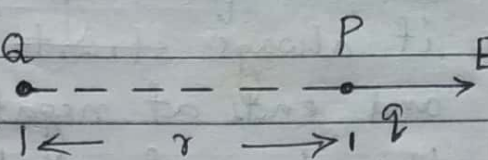
An electric field can be defined as a region where electric force is experienced.

The electric field intensity of an electric field at any point is defined as the force per unit charge at that point. If 'F' is the force on a charge 'q' at any point in the field then the electric field intensity at that point is given by

$$E = \frac{F}{q} \quad \text{———} \quad (1)$$

It is a vector quantity and its SI unit is NC^{-1}.

Electric field due to a point charge



Consider a charge 'q' in the electric field of another charge 'Q'. If 'r' is the seperation between the charges then the electric force between them is given by

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{r^2} \quad \text{——— (1)}$$

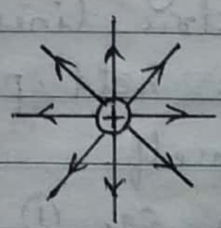
∴ the electric field at point P is

$$E = \frac{F}{q}$$

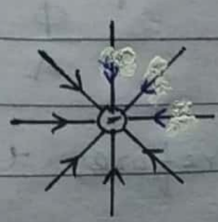
$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{r^2} \Big/ q$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}} \quad \text{——— (2)}$$

Electric Lines of force :



The electric lines of force is a path along which a unit positive charge will move if it is free to do so. It is a way of pictorially representing the



electric field around a charge or group of charges.
It has following properties:

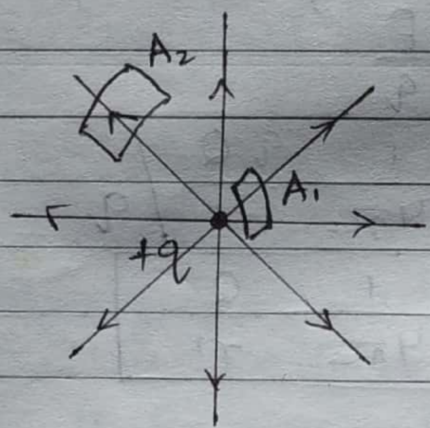
(i) it always ~~starts~~ starts from positive charge and ends at negative charge.

(ii) the tangent on electric line of force at any point gives the direction of resultant electric field at that point.

(iii) the relative closeness of lines of force gives idea about relative intensity of electric field.

(iv) two lines of force never intersect each other.

Electric flux



The quantity

$$\phi = EA \quad \text{——— (1)}$$

is called flux of the electric field through A.

It may also be defined as the total number of electric lines of force passing through surface area held perpendicular to the lines of force themself.

From eq. (1),

$$E = \frac{\phi}{A} \quad \text{——— (2)}$$

Therefore, the electric field intensity at a point can be defined as electric flux per unit area held perpendicular to the direction of lines of force.

Gauss Law

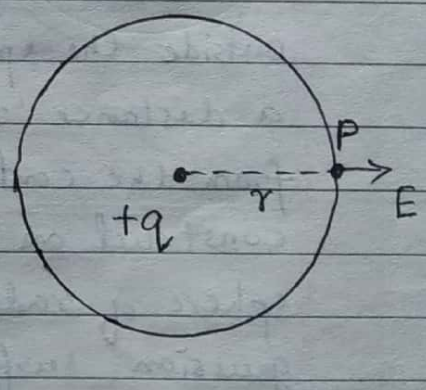
Statement:

"The total electric flux passing through a closed surface is equal to $\left(\frac{1}{\epsilon_0}\right)$ times the net charge enclosed by the surface."

i.e, total electric flux $= \frac{1}{\epsilon_0} \times \left(\text{net charge enclosed by the surface}\right)$

Proof:

Consider a charge $+q$ enclosed in a surface of radius 'r'. Let 'E' be the electric field at point P then



$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad \text{--- (1)}$$

∴ electric flux (total) passing through closed surface is

$$\phi = E \times (\text{surface area})$$

$$\Rightarrow \phi = E \cdot 4\pi r^2$$

$$\Rightarrow \phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2$$

$$\Rightarrow \phi = \frac{1}{\epsilon_0} . q$$

$$\Rightarrow \phi = \frac{q}{\epsilon_0} \quad \text{——— (2)}$$

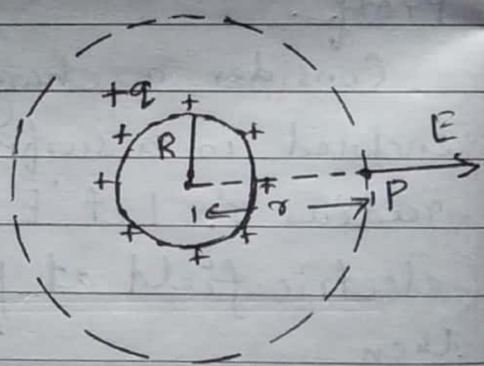
Gaussian Surface

The imaginary close surface that we choose for the application of Gauss law is called gaussian surface.

Electric field due to charged solid sphere

(i) Outside the sphere:

Consider a point P outside the sphere at a distance 'r' away from the center. We construct an imaginery sphere of radius 'r' as gaussian surface. Let 'E' be the electric field at point P on the gaussian surface, then flux through gaussian surface is



$$\phi = E.A$$

$$\Rightarrow \phi = E.4\pi r^2 \quad \text{——— (1)}$$

According to Gauss law,

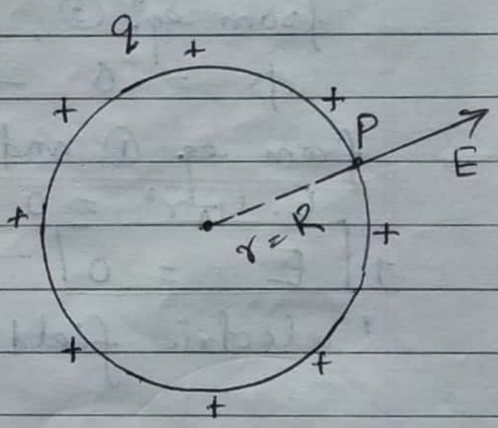
$$\phi = \frac{q}{\epsilon_0} \quad \text{——— (2)}$$

$\therefore$ from eq. (1) and (2),

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}} \quad \text{——— (3)}$$

(ii) <u>On the surface of the sphere:</u>



Consider a point P on the surface of the sphere of radius 'R' then

$$\phi = E \cdot 4\pi R^2 \quad \text{——— (1)}$$

By Gauss Law,

$$\phi = \frac{q}{\epsilon_0} \quad \text{——— (2)}$$

from eqs. (1) and (2), we get,

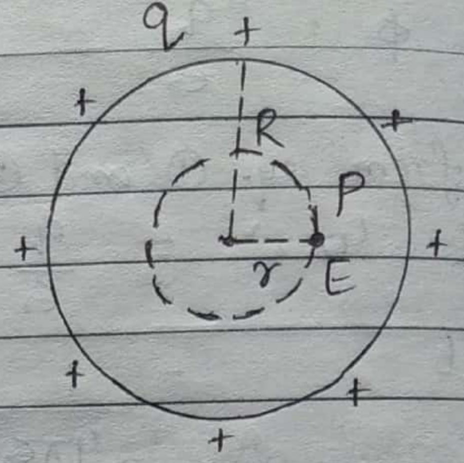
$$E \cdot 4\pi R^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}} \quad \text{——— (3)}$$

(iii) <u>Inside the sphere:</u>

Consider a point 'P' inside the sphere at a distance 'r' away from the center of sphere. We construct a gaussian surface as shown in fig. If 'E' is the electric field at P then flux through gaussian surface is

$$\phi = E\, 4\pi r^2 \quad \text{——— (1)}$$



By Gauss Law,

$$\phi = \frac{q}{\epsilon_0} \quad \text{——— (2)}$$

∵ no charge is enclosed by gaussian surface

∴ $q = 0$

∴ from eqⁿ (2),

$$\phi = 0 \quad \text{——— (3)}$$

from eq. (1) and (3),

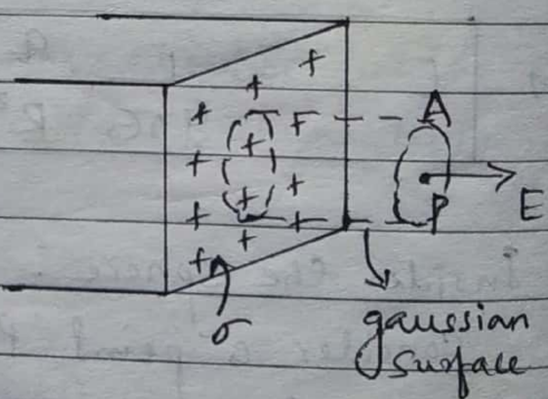
$$E \cdot 4\pi r^2 = 0$$

$$\Rightarrow \boxed{E = 0} \quad \text{——— (4)}$$

∴ electric field inside a charged sphere is zero.

Electric field due to plane charged conductor

Let 'σ' be the surface charge density of a plane charged conductor. We draw a cylindrical gaussian surface with cross-sectional area A



such that one of the base area is inside the surface of the conductor and other base area contains point 'P' where electric field is to be determined. The charge enclosed by gaussian surface is

$$q = \sigma A \quad \text{——} \quad (1)$$

Let 'E' be the electric field at point P then the flux passing through the gaussian surface is given by

$$\phi = EA \quad \text{——} \quad (2)$$

By Gauss Law,

$$\phi = \frac{q}{\epsilon_0} \quad (3)$$

$\therefore$ from eq. (2) and (3), we get,

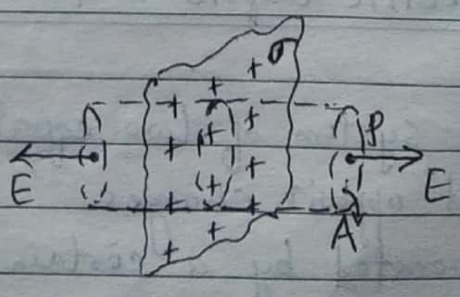
$$EA = \frac{q}{\epsilon_0}$$

$$\Rightarrow EA = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}} \quad \text{——} \quad (4)$$

Electric field due to infinite plane charged sheet

Consider an infinitely thin plane sheet of positive charge having charge density 'σ'. Let 'E' be the electric field intensity at point P.



We draw a cylinderical gaussian surface of base area A as shown in fig. The charge enclosed by gaussian surface is

$$q = \sigma A \quad \text{——— (1)}$$

The total electric flux passing through the gaussian surface is

$$\phi = E \times (\text{area of two bases})$$

$$\Rightarrow \phi = E \cdot 2A \quad \text{——— (2)}$$

By gauss law, the total flux passing through same gaussian surface is

$$\phi = \frac{q}{\epsilon_0}$$

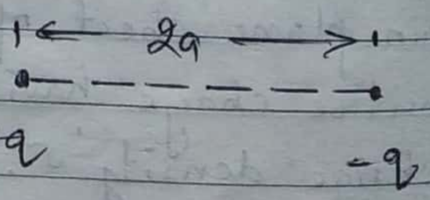
$$\Rightarrow \phi = \frac{\sigma A}{\epsilon_0} \quad \text{——— (3)}$$

From eq. (2) and (3), we get,

$$E \cdot 2A = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}} \quad \text{——— (4)}$$

Electric dipole and electric dipole moment

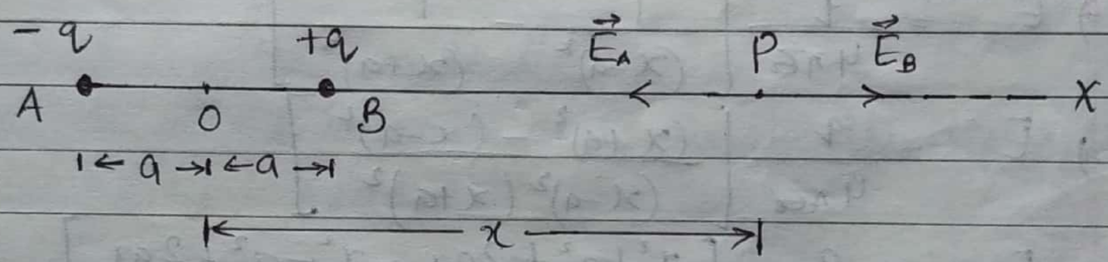


A system of two equal and opposit charges seperated by a certain distance is called electric dipole.

Electric dipole moment of an electric dipole is defined as the product of magnitude of either charge and distance of seperation between them. It is denoted by p.

$$\therefore \quad p = q(2a) \quad \text{——— (1)}$$

Electric field on axial line of an electric dipole



Let P be the point at a distance 'x' away from center 'O' of dipole along the axial line. The electric field at P will be due to ~~E~~ resultant of $\vec{E}_A$ due to $-q$ and $\vec{E}_B$ due to $+q$.

Let $\vec{E}$ be the resultant electric field at P, then

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

Now,

$$|\vec{E}_A| = \frac{1}{4\pi\epsilon_0} \frac{q}{(AP)^2}$$

$$\Rightarrow |\vec{E}_A| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x+a)^2} \text{ along PA}$$

and

$$|\vec{E}_B| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(BP)^2}$$

$\Rightarrow |\vec{E}_B| = \frac{1}{4\pi\epsilon_0} \frac{q}{(x-a)^2}$ along PX

$\because |\vec{E}_B|$ is greater than $|\vec{E}_A|$ but acts in opposit direction so the resultant electric field acts along PX and given by

$$E = |\vec{E}| = |\vec{E}_B| - |\vec{E}_A|$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{q}{(x-a)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(x+a)^2}$$

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right]$$

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \left[\frac{(x+a)^2 - (x-a)^2}{(x-a)^2 (x+a)^2} \right]$$

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \left[\frac{x^2+a^2+2ax-x^2-a^2+2ax}{(x^2-a^2)^2} \right]$$

$$\Rightarrow E = \frac{q}{4\pi\epsilon_0} \frac{4ax}{(x^2-a^2)^2}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2 \cdot q(2a)x}{(x^2-a^2)^2}$$

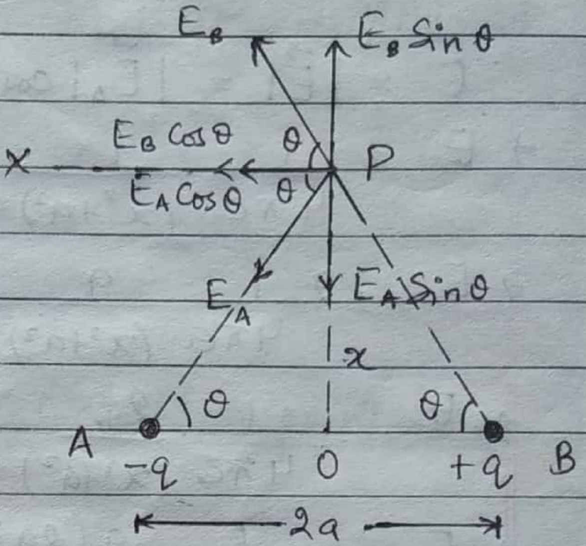
$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{2px}{(x^2-a^2)^2} \text{ along PX} \quad (1)$$

if $x >> a$, then

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2px}{x^4}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{x^3} \quad (2)$$

Electric field on equitorial line of an electric dipole



Let 'P' be a point at a distance 'x' away from 'O'. The electric field intensity at 'P' will be due to resultant of $\vec{E}_A$ due to $-q$ and $\vec{E}_B$ due to $+q$.

Let $\vec{E}$ be the resultant electric field intensity at P due to the dipole, then

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

Now,

$$|\vec{E}_A| = \frac{1}{4\pi\epsilon_0} \frac{q}{(AP)^2}$$

$$\Rightarrow |\vec{E}_A| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x^2+a^2)} \text{ along } PA$$

and

$$|\vec{E}_B| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(BP)^2}$$

$$\Rightarrow |\vec{E}_B| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x^2+a^2)} \text{ along } BP.$$

Let 'θ' be the angle made by electric dipole with point P. At point P, the sine component of $|\vec{E}_A|$ and $|\vec{E}_B|$ being equal and opposit cancel each other whereas the cos component being in same direction gives resultant electric field at P, given by

$$E = |\vec{E}| = |\vec{E}_A| \cos\theta + |\vec{E}_B| \cos\theta$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x^2+a^2)} \cos\theta + \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x^2+a^2)} \cos\theta$$

$$\Rightarrow E = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q}{(x^2+a^2)} \cos\theta$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{(x^2+a^2)} \cdot \frac{a}{(x^2+a^2)^{1/2}}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(2a)}{(x^2+a^2)^{3/2}}$$

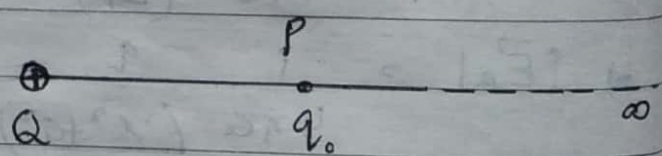
$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{p}{(x^2+a^2)^{3/2}} \text{ along } PX$$

If $x >> a$, then

$$\vec{E} = \frac{p}{4\pi\epsilon_0 x^3}$$

Electric Potential

The electric potential at any point in the field of a charge is defined

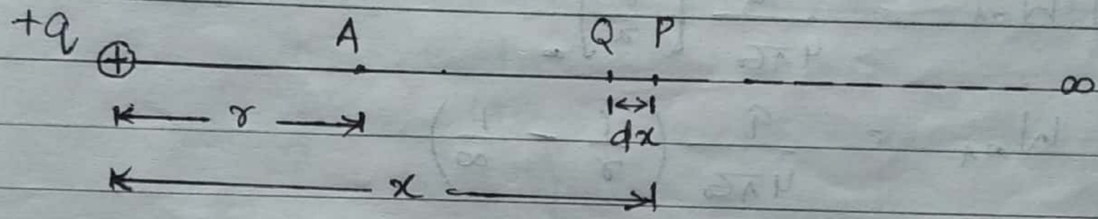


as the work done in bringing unit positive charge from infinity to that point against the repulsive force of the field.

If 'W' is the work done in bringing 'q_0' from infinity to point P then the electric potential at point P is given by

$$V = \frac{W}{q_0} \quad \text{—— (1)}$$

Expression of electric potential at a point



Consider a point 'A' at a distance 'r' away from charge 'q'. If $W_{\infty A}$ is the amount of work done in bringing unit positive charge from infinity to A then the potential at A is given by

$$V = W_{\infty A} \quad \text{—— (1)}$$

Suppose a unit positive charge is at P at a distance x away from 'O' at any instant of time. The work done to move it from P to Q is given by

$$dW = -F\,dx$$

$$\Rightarrow dW = -E\,dx \quad \text{- - - - -} \quad \left(\because F = q_0E, \because q_0 = 1 \quad \therefore F = E \right)$$

$$\Rightarrow dW = -\frac{1}{4\pi\epsilon_0}\frac{q}{x^2}\,dx \quad \text{—— (2)}$$

$\therefore$ the work done in moving unit positive charge from infinity to point A is given by

$$W_{\infty A} = \int_{\infty}^{A} dW$$

$$\Rightarrow W_{\infty A} = \int_{\infty}^{r} -\frac{q}{4\pi\epsilon_0 x^2}\, dx$$

$$\Rightarrow W_{\infty A} = -\frac{q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{x^2}\, dx$$

$$\Rightarrow W_{\infty A} = -\frac{q}{4\pi\epsilon_0} \left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow W_{\infty A} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow W_{\infty A} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$\Rightarrow W_{\infty A} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad \text{——— (3)}$$

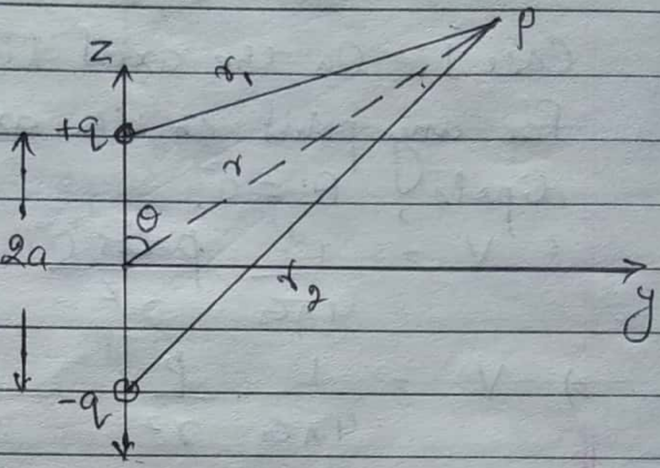
from eq. (1) and (3), we get.

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad \text{——— (4)}$$

Equation (4) is the equation of electric potential due to charge +q at point A.

Potential due to electric dipole :

Consider a dipole as shown in fig.. Let P be a point at a distance 'r' away from the center of dipole O making an angle θ with the axis joining the charge +q and −q. If the distance of point P from +q and −q are r_1 and r_2 resp. then the potential at point P due to dipole is given by



V = (potential due to +q) + (potential due to −q)

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{-q}{r_2}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{r_2 - r_1}{r_1 r_2}\right) \quad \text{———} \quad (1)$$

In practical, generally, point P is very far from the dipole, i.e., $r >> a$.

In this case,

$$r_2 - r_1 \approx 2a \cos\theta$$

and

$$r_1 . r_2 \approx r^2$$

Substituting values in eq. (1), we get,

$$V = \frac{q}{4\pi\epsilon_0}\cdot\frac{2a\cos\theta}{r^2}$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q.(2a)\cos\theta}{r^2}$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos\theta}{r^2} \quad \text{——— (2)}$$

Case I: On the axial line;
For any point on the axial line of electric dipole, $\theta = 0°$.

$$\therefore V = \frac{1}{4\pi\epsilon_0} \frac{p \cos 0°}{r^2}$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2} \quad \text{——— (3)}$$

Case II: On the equitorial line;
For any point on the equitorial line of electric dipole, $\theta = 90°$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos(90°)}{r^2}$$

$$\Rightarrow V = 0 \quad \text{——— (4)}$$

Electrical potential due to quadrupole

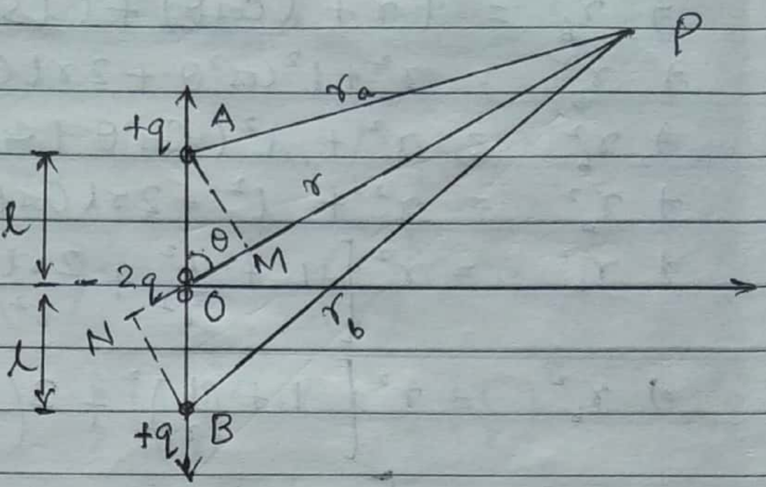
Formula:

$$(a+x)^n = a^n + n a^{n-1} x + \frac{n(n-1)}{2!} a^{n-2} x^2 + \dots$$

$$(1+x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right) a^{-\frac{1}{2}-1} x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2} a^{-\frac{1}{2}-2} x^2 + \dots$$

$$\Rightarrow (1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \dots$$

Consider an electric quadrupole formed of two electric dipoles, each having charge $(-q, +q)$ at seperation 'L'. Consider a point P at a distance 'r' from the center of quadrupole making an angle 'θ' with axis of quadrupole. Clearly, electric quadrupole may be regarded as a system of three point charges $+q$, $-2q$ and $+q$ at position A, O and B resp. Let r_a, r_b and r be the distance of charge $+q$, $+q$ and $-2q$ resp.



The potential at P due to quadrupole is given by

$$V = \begin{pmatrix}\text{potential due to} \\ +q \text{ at } A\end{pmatrix} + \begin{pmatrix}\text{potential due to} \\ +q \text{ at } B\end{pmatrix} + \begin{pmatrix}\text{potential due to} \\ -2q \text{ at } O\end{pmatrix}$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\frac{q}{r_a} + \frac{1}{4\pi\epsilon_0}\frac{q}{r_b} + \frac{1}{4\pi\epsilon_0}\frac{-2q}{r}$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_a} + \frac{1}{r_b} - \frac{2}{r}\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0 r}\left[\left(\frac{r}{r_a}\right) + \left(\frac{r}{r_b}\right) - 2\right] \quad \text{——— (1)}$$

In ΔPNB,

$$(PB)^2 = (PN)^2 + (BN)^2$$

$$\Rightarrow (PB)^2 = (PO + ON)^2 + (BN)^2$$

$$\Rightarrow r_b^2 = (r + l\cos\theta)^2 + (l\sin\theta)^2$$

$$\Rightarrow r_b^2 = r^2 + l^2\cos^2\theta + 2rl\cos\theta + l^2\sin^2\theta$$

$$\Rightarrow r_b^2 = r^2 + l^2(\cos^2\theta + \sin^2\theta) + 2rl\cos\theta$$

$$\Rightarrow r_b^2 = r^2 + l^2 + 2rl\cos\theta$$

$$\Rightarrow r_b^2 = r^2\left[1 + \frac{l^2}{r^2} + \frac{2rl\cos\theta}{r^2}\right]$$

$$\Rightarrow r_b^2 = r^2\left[1 + \left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right]$$

$$\Rightarrow \frac{r_b^2}{r^2} = \left[1 + \left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\}\right]$$

$$\Rightarrow \left(\frac{r_b}{r}\right)^2 = \left[1 + \left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\}\right]$$

$$\Rightarrow \left(\frac{r_b}{r}\right) = \left[1 + \left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\}\right]^{1/2}$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = \left[1 + \left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\}\right]^{-\frac{1}{2}}$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \frac{1}{2}\left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\} + \frac{3}{8}\left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\}^2 - \ldots$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \frac{1}{2}\left\{\left(\frac{l}{r}\right)^2 + 2\left(\frac{l}{r}\right)\cos\theta\right\} + \frac{3}{8}\left\{\left(\frac{l}{r}\right)^4 + 4\left(\frac{l}{r}\right)^2\cos^2\theta + 4\left(\frac{l}{r}\right)^3\cos\theta\right\} - \ldots$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \frac{1}{2}\left(\frac{l}{r}\right)^2 - \frac{2}{2}\left(\frac{l}{r}\right)\cos\theta + \frac{3}{8}\left(\frac{l}{r}\right)^4 + \frac{3}{8}\times 4\left(\frac{l}{r}\right)^2\cos^2\theta + \frac{3}{8}\times 4\left(\frac{l}{r}\right)^3\cos\theta - \ldots$$

Neglecting higher order terms, we get,

$$\left(\frac{r}{r_b}\right) = 1 - \frac{1}{2}\left(\frac{l}{r}\right)^2 - \frac{2}{2}\left(\frac{l}{r}\right)\cos\theta + \frac{3 \times 4}{8}\left(\frac{l}{r}\right)^2 \cos^2\theta$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \frac{1}{2}\left(\frac{l}{r}\right)^2 - \left(\frac{l}{r}\right)\cos\theta + \frac{3}{2}\left(\frac{l}{r}\right)^2 \cos^2\theta$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \left(\frac{l}{r}\right)\cos\theta + \frac{3}{2}\left(\frac{l}{r}\right)^2 \cos^2\theta - \frac{1}{2}\left(\frac{l}{r}\right)^2$$

$$\Rightarrow \left(\frac{r}{r_b}\right) = 1 - \left(\frac{l}{r}\right)\cos\theta + \frac{1}{2}\left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1) \quad \text{——— (2)}$$

Similarly,

$$\left(\frac{r}{r_a}\right) = 1 + \left(\frac{l}{r}\right)\cos\theta + \frac{1}{2}\left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1) \quad \text{——— (3)}$$

From eq. (1), (2) & (3), we get,

$$V = \frac{q}{4\pi\epsilon_0 r}\left[1 + \left(\frac{l}{r}\right)\cos\theta + \frac{1}{2}\left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1) + 1 - \left(\frac{l}{r}\right)\cos\theta + \frac{1}{2}\left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1) - 2\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0 r}\left[2 + \left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1) - 2\right]$$

$$\Rightarrow V = \frac{q}{4\pi\epsilon_0 r}\left(\frac{l}{r}\right)^2 (3\cos^2\theta - 1)$$

$$\Rightarrow \boxed{V = \frac{ql^2 (3\cos^2\theta - 1)}{4\pi\epsilon_0 r}} \quad \text{——— (4)}$$