Physical Optics

Interference

The redistribution of light energy due to superposition of waves from two coherent sources is called interference.

Coherent sources :

Two sources are said to be coherent if they emit waves of same wavelength or frequency having same phase or constant phase difference.

Types of interference

(i) Constructive interference :
When crest of one wave superimpose with crest of other wave the amplitude will be maximum and intensity also increases. This is called constructive interference. Constructive interference will occur if

path difference $= n\lambda$ ——— (1)

where, n is an integer and λ is the wavelength of light waves superimposing.

(ii) Distructive interference :
When crest of one wave superimpose with trough of other wave the amplitude of resultant wave is minimum and hence intensity becomes minimum. This is called distructive interference. Distructive

interference will occur if

$$\text{path difference} = (2n+1)\frac{\lambda}{2} \quad \text{——— (2)}$$

where, 'n' is an integer and 'λ' is the wavelength of waves superimposing.

Condition for sustained interference

The interference in which the positions of maxima and minima of intensity of light remain fixed with time, all along on the screen, is called sustained or permanent interference pattern. In order to obtain a sustained interference pattern, the following condition should be satisfied.

(i) The two interfering source must be coherent.

(ii) The interfering waves must have equal amplitudes. Otherwise the ~~fringes~~ the minimum intensity will not be zero and there will be general illumination.

(iii) The light used must be monochromatic. Otherwise the fringes of different colours will overlap.

(iv) The two sources must be very close to each other. Otherwise bright and dark pattern produced some distance away would be too close to each other and no interference would then be seen.

(v) The two sources should be very narrow.

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(vi) The two sources should emit waves contineously.

Optical path

Optical path of a medium is defined as a distance which the light travel in a vacuum during the time which it travel in that medium.

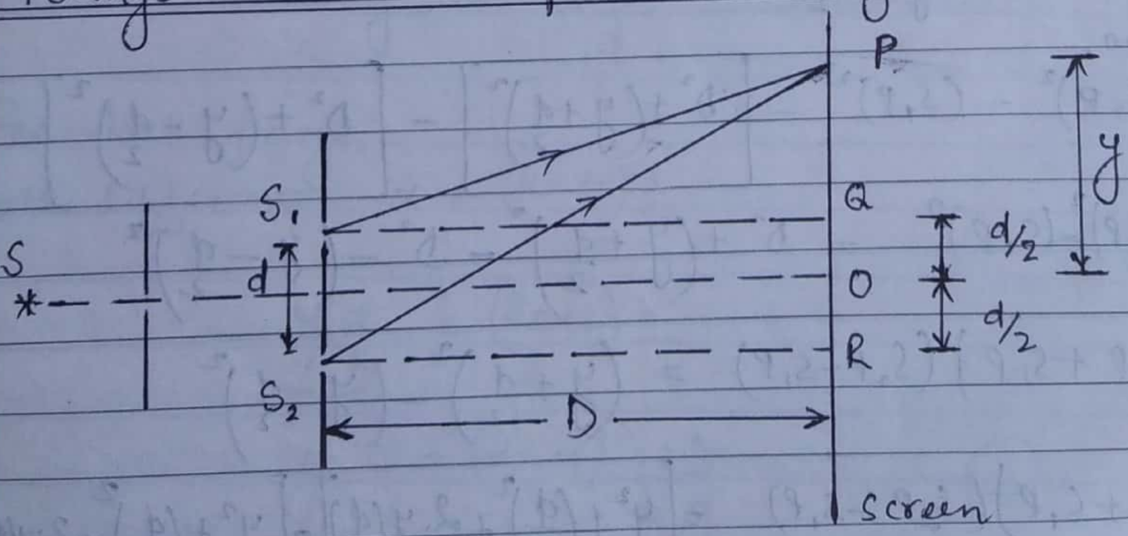
Let 'x' be the distance travelled by the light in a medium of refractive index 'μ' in a given time 't'. We have,

$$\mu = \frac{c}{v}$$

$$\Rightarrow \mu = \frac{\text{optical path}/t}{x/t}$$

$$\Rightarrow \text{optical path} = \mu x \quad \text{——} \quad (1)$$

Young's double slit experiment (Analytical treatement)



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Let S_1 and S_2 be the two coherent sources of light emitting wave of wavelength 'λ' and seperated by a distance 'd' as shown in fig.. Let the screen be placed at a distance D from the coherent sources. Let 'O' be the center of the screen which is at equidistance from source S_1 and S_2. We get central bright fringe at O.

Let us now consider a point 'P' at a distance 'y' from O. Waves from S_1 and S_2 reach at point P after travelling distance S_1P and S_2P respectively and will be in phase or out of phase depending upon the path difference,

$$x = S_2P - S_1P \quad \text{——— (1)}$$

To calculate path difference x, perpendicular S_1Q and S_2Q are drawn from S_1 and S_2 on the screen.

Here,

$$PQ = y - \frac{d}{2} \quad \text{——— (2)}$$

and

$$PR = y + \frac{d}{2} \quad \text{——— (3)}$$

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y+\frac{d}{2}\right)^2\right] - \left[D^2 + \left(y-\frac{d}{2}\right)^2\right]$$

$$\Rightarrow (S_2P)^2 - (S_1P)^2 = D^2 + \left(y+\frac{d}{2}\right)^2 - D^2 - \left(y-\frac{d}{2}\right)^2$$

$$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = \left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2$$

$$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = \left[y^2 + \left(\frac{d}{2}\right)^2 + 2.y\left(\frac{d}{2}\right)\right] - \left[y^2 + \left(\frac{d}{2}\right)^2 - 2.y\left(\frac{d}{2}\right)\right]$$

$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = 4 \cdot y \cdot \left(\frac{d}{2}\right)$

$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = 2yd$

$\Rightarrow (S_2P - S_1P) = \frac{2yd}{(S_2P + S_1P)}$ —— (4)

In practice point P lies very close to O.
So,

$S_2P \approx S_1P \approx D$

$\therefore (S_2P + S_1P) = 2D$ —— (5)

$\therefore$ from equation (4) and (5), we get,

$(S_2P - S_1P) = \frac{2yd}{2D}$

$\Rightarrow x = \frac{yd}{D}$ —— (6)

For bright fringe,
path difference, $x = n\lambda$

$\Rightarrow \frac{yd}{D} = n\lambda$

$\Rightarrow y = n\lambda \frac{D}{d}$ —— (7)

For dark fringe,
path difference, $x = (2n+1)\frac{\lambda}{2}$

$\Rightarrow \frac{yd}{D} = (2n+1)\frac{\lambda}{2}$

$\Rightarrow y = (2n+1)\frac{\lambda}{2}\frac{D}{d}$ —— (8)

The distance between any two consecutive bright fringes or any two dark fringes is called fringe width.

For bright fringe,

$$\text{fringe width} = y_2 - y_1$$

$$= \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$= \frac{\lambda D}{d} \quad \text{———} \quad (9)$$

For dark fringe,

$$\text{fringe width} = y_2 - y_1$$

$$= \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{d} \quad \text{———} \quad (10)$$

From equations (9) and (10) it is seen that the spacing between any two consecutive bright fringes or consecutive dark fringes is same.

Interference in thin film

An optical medium whose thickness is of the order of wavelength of light in visible region is called thin film. If two bounding surface

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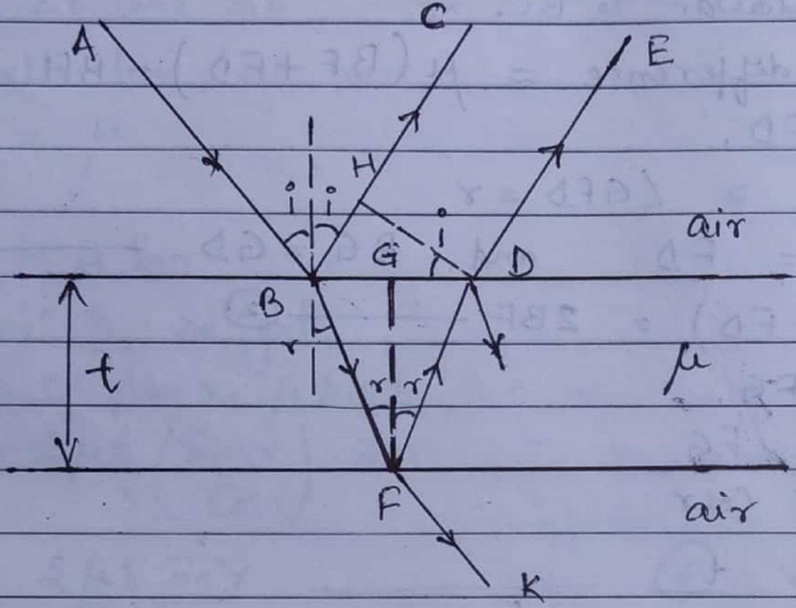
of the thin film is parallel then it is called plane parallel thin film.

(i) Interference due to reflected light



Consider a plane parallel thin film of thickness 't' having refractive index 'μ'. Let ray of light AB incident on the surface of thin film making angle of incidence 'i'. A part of AB is reflected along 'BC' and part of it is transmitted along BF making angle of refraction 'r'. This ray BF is reflected along FD and refracted along FK at the second boundary or surface as shown in fig. A part of reflected ray FD is transmitted at the first surface and travell along DE. Since film surface are parallel so the reflected rays BC and DE are parallel. The reflected rays are from single incident wave, therefore, they are

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coherent. Hence if the reflected rays are made to overlap each other they can form interference pattern.

To measure path difference we draw DH perpendicular to BC.

$\therefore$ path difference $= \mu(BF + FD) - BH$ ——— (1)

In ΔBFD,

$\angle BFG = \angle GFD = r$

$\therefore BF = FD$ and $BG = GD$ ——— (x)

$\therefore (BF + FD) = 2BF$ ——— (2)

In ΔBFG,

$BF = \frac{FG}{\cos r}$

$\Rightarrow BF = \frac{t}{\cos r}$

$\therefore$ from eq. (2),

$(BF + FD) = \frac{2t}{\cos r}$ ——— (3)

In ΔBFG,

$\frac{BG}{FG} = \tan r$

$\Rightarrow BG = FG \tan r$

$\Rightarrow BG = t \tan r$ ——— (4)

Now,

$BD = BG + GD$

$\Rightarrow BD = 2BG$ - - - (from eq (x))

$\Rightarrow BD = 2t \tan r$ - - (5) (from eq. (4))

In ΔBHD,

$$\frac{BH}{BD} = \sin i$$

$$\Rightarrow BH = BD \sin i$$

$$\Rightarrow BH = 2t \tan r \sin i \quad \text{———} \quad (6)$$

from Snell's law, we have,

$$\frac{\sin i}{\sin r} = \mu$$

$$\Rightarrow \sin i = \mu \sin r$$

$\therefore$ from eqⁿ (6), we get,

$$BH = 2t \tan r \cdot \mu \sin r$$

$$\Rightarrow BH = 2\mu t \left(\frac{\sin r}{\cos r}\right) \cdot \sin r$$

$$\Rightarrow BH = 2\mu t \frac{\sin^2 r}{\cos r} \quad \text{———} \quad (7)$$

$\therefore$ from equations (1), (3) and (7), we get,

$$\text{path difference} = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$= \frac{2\mu t \cos^2 r}{\cos r}$$

$$\Rightarrow \text{path difference} = 2\mu t \cos r \quad \text{———} \quad (8)$$

The ray BC suffers path change of $\frac{\lambda}{2}$ at B but ray FD suffers no path change. Therefore the actual (or optical) path difference is given by

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optical path difference $= 2\mu t \cos r - \frac{\lambda}{2}$ ——— (9)

For maxima,

optical path difference $= n\lambda$

$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$\Rightarrow 2\mu t \cos r = n\lambda + \frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ ——— (10)

where, n is an integer.

For minima,

optical path difference $= (2n+1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$\Rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$

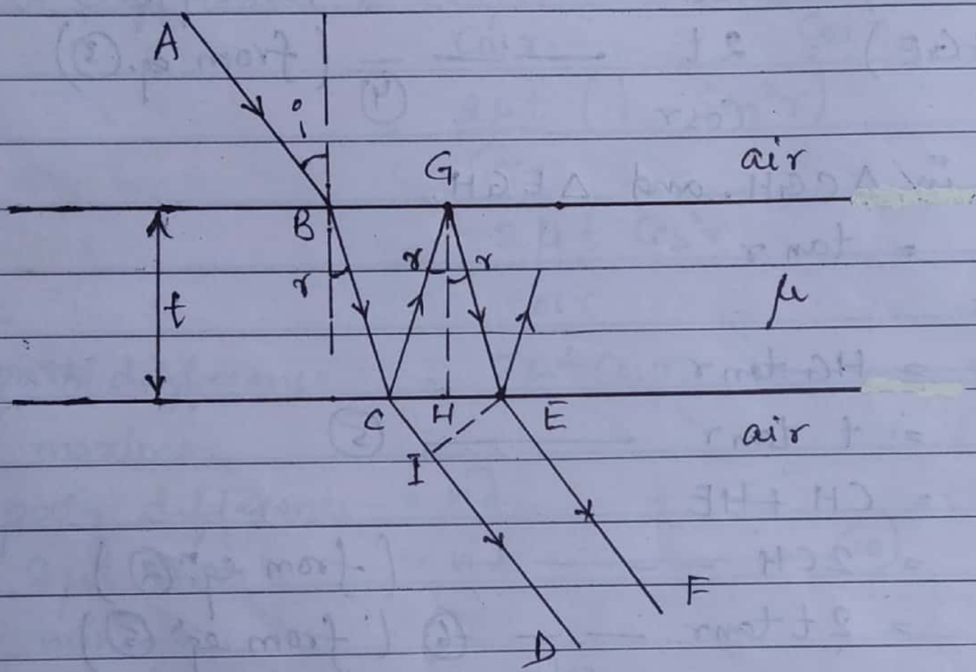
$\Rightarrow 2\mu t \cos r = (n+1)\lambda$

$\Rightarrow 2\mu t \cos r = m\lambda$ ——— (11)

where, m is an integer.

When monochromatic light falls on the thin film the whole film will appear uniformly bright or dark depending on wheather the condition for constructive or distructive interference is satisfied. If a parallel beam of white light falls on parallel thin film, those wavelength for which the path difference is $n\lambda$ will be absent from reflected light. Therefore, the film will appear uniformly coulored with one colour being absent.

(ii) Interference due to transmitted light



Consider a plane parallel thin film of thickness 't' having refractive index 'μ'. Let AB be the incident ray and CD and EF be two transmitted rays as shown in fig. To measure path difference we draw EI parallel to CD.

$\therefore$ path difference $= \mu(CG+GE) - CI$ —— (1)

In Δ CGH and ΔEGH,

$$\left.\begin{array}{l} CG = GE \\ \text{and, } CH = HE \end{array}\right\} \text{—— (2)}$$

In ΔCGH,

$$\frac{GH}{CG} = \cos r$$

$\Rightarrow CG = \frac{GH}{\cos r}$

$\Rightarrow CG = \frac{t}{\cos r}$ ——— (3)

Now,

$(CG + GE) = 2CG$ ——— (from eq. (2))

$\Rightarrow (CG + GE) = \frac{2t}{\cos r}$ ——— (4) (from eq. (3))

Again, in ΔCGH,

$\frac{CH}{HG} = \tan r$

$\Rightarrow CH = HG \tan r$

$\Rightarrow CH = t \tan r$ ——— (5)

$\therefore CE = CH + HE$

$\Rightarrow CE = 2CH$ ——— (from eq^n (2))

$\Rightarrow CE = 2t \tan r$ ——— (6) (from eq^n (5))

In ΔCEI,

$\frac{CI}{CE} = \sin i$

$\Rightarrow CI = CE \sin i$

$\Rightarrow CI = 2t \tan r \sin i$ ——— (7)

$\therefore$ from Snell's law,

$\mu = \frac{\sin i}{\sin r}$

$\Rightarrow \sin i = \mu \sin r$

$\therefore$ from eq^n (7), we get,

$CI = 2t \tan r . \mu \sin r$

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$\Rightarrow$ $CI = 2t \frac{\sin r}{\cos r} \mu \sin r$

$\Rightarrow$ $CI = 2\mu t \frac{\sin^2 r}{\cos r}$ —— (8)

$\therefore$ from equations (1), (4) and (8), we get,

path difference $= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$

$= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$

$= \frac{2\mu t \cos^2 r}{\cos r}$

$\Rightarrow$ path difference $= 2\mu t \cos r$ —— (9)

For maxima,

path difference $= n\lambda$

$\Rightarrow$ $2\mu t \cos r = n\lambda$ —— (10)

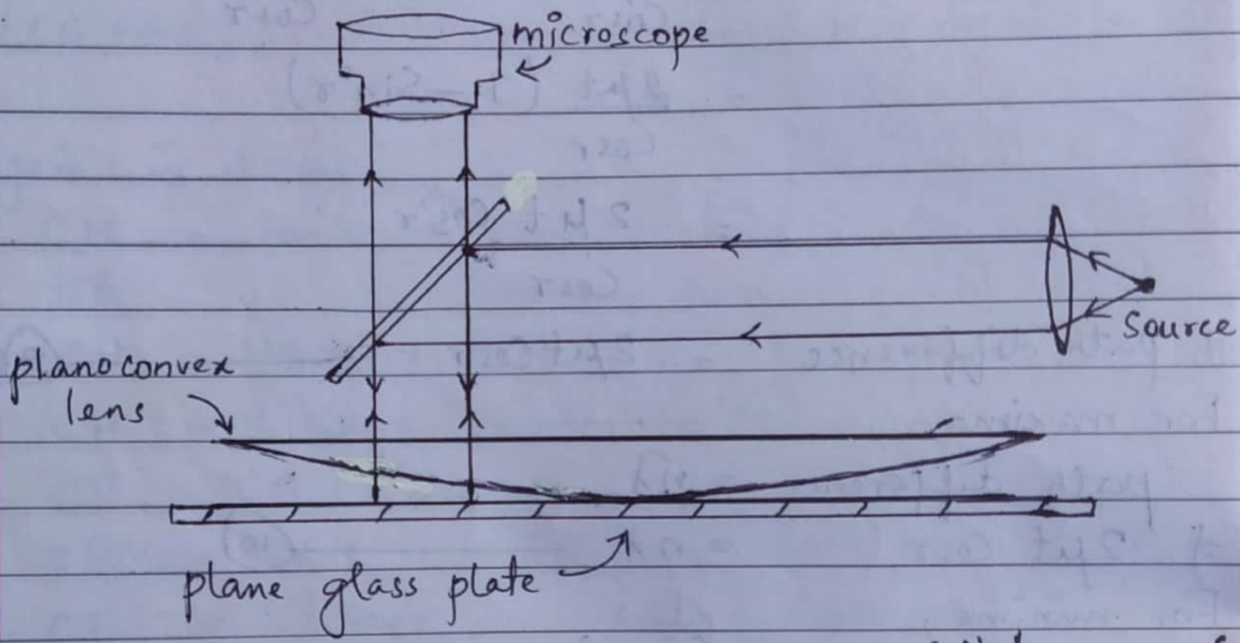
For minima,

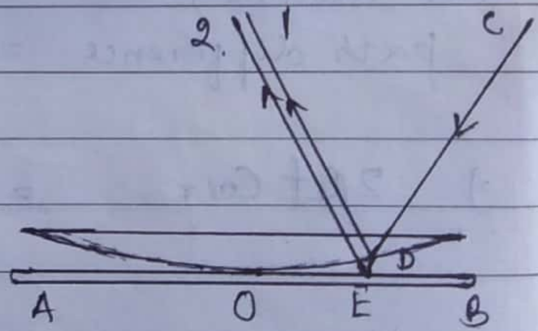
path difference $= (2n+1)\frac{\lambda}{2}$

$\Rightarrow$ $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ —— (11)

Newton's Ring

When plano-convex lens of large focal length is placed on a plane glass plate a thin film of air is enclosed between lower surface of lens and upper surface of plane glass plate. The thickness of air film is very small at the point of contact and gradually increases from center to outwards. The fringe produced by monochromatic light are circular and concentric. The form of bright and dark circular fringes are called Newton's rings.





Theory :
A ray of light incident on the convex lens is partially reflected along direction 1 and partially transmitted along DE, at point D. The ray DE gets reflected at E along direction 2. The two rays are part of single ray so they are coherent and hence they can produce interference

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(i) Condition for bright and dark rings :

The ray reflected at point E suffers a phase change of π (or path difference of $\lambda/2$) whereas ray reflected at D do not suffer any phase change. So, if 't' is the thickness of air film at the point of reflection then the optical path difference between the rays is

$$\text{path difference} = 2\mu t \cos r - \frac{\lambda}{2}$$

But,

$\mu = 1$, for air

and $\cos r = 1$, for normal incidence.

$$\therefore \text{path difference} = 2t - \frac{\lambda}{2} \quad \text{——— (1)}$$

For maxima,

$$\text{path difference} = n\lambda$$

$$\Rightarrow 2t - \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow 2t = (2n+1)\frac{\lambda}{2} \quad \text{——— (2)}$$

where, $n = 0, 1, 2, \ldots$

For minima,

$$\text{path difference} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2t - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow 2t = m\lambda \quad \text{——— (3)}$$

where, $m = 0, 1, 2, \ldots$

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In the arrangement of this experiment the air flim is formed between convex surface of plano-convex lens and the glass plate such that the thickness of thin film is zero at the point of contact and gradually increases as we move outward. The points having same thickness in this thinfilm forms a circle having center at point of contact. This means if any one point satisfy the condition for maxima or minima then other points on the circle will also satisfy the same condition and thus the corrosponding bright or dark circular fringe will appear. Therefore, fringes are circular which are concentric.

At the point of contact or center,

$t = 0$

$\Rightarrow 2t = 0$

this the condition for minima. Therefore the central fringe is always dark in Newton's Rings.

(ii) Radii of dark fringes:

Let 'R' be the radius of curvature of a plano-convex lens. We consider a dark fringe at point P, having thickness 't', of radius 'r_n'.

From ΔPMN,

$$R^2 = r_n^2 + (R-t)^2$$

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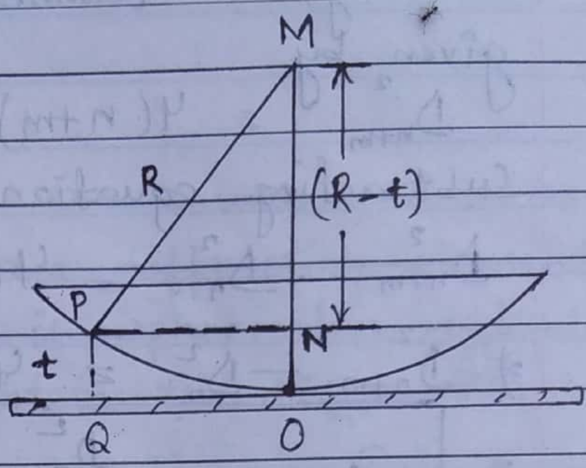
$\Rightarrow R^2 = r_n^2 + R^2 + t^2 - 2Rt$

$\Rightarrow r_n^2 = 2Rt - t^2$

$\because R >> t$

$\therefore 2Rt >> t^2$

and so t^2 can be neglected.



$\therefore r_n^2 = 2Rt$ ——— (1)

We have, the condition for dark fringe as

$2t = n\lambda$

$\therefore$ from eqn (1), we get

$r_n^2 = n\lambda R$

$\Rightarrow r_n = \sqrt{n\lambda R}$ ——— (2)

i.e., $r_n \propto \sqrt{R}$ and $r_n \propto \sqrt{\lambda}$

The diameter of n^{th} dark fringe is

$D_n = 2r_n$

$\Rightarrow D_n = 2\sqrt{n\lambda R}$ ——— (3)

$\Rightarrow D_n^2 = 4n\lambda R$ ——— (4)

(iii) Wavelength of monochromatic light :

The diameter of n^{th} dark fringe is given by

$D_n^2 = 4n\lambda R$ ——— (1)

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Similarly, the diameter of $(n+m)^{th}$ dark ring is given by

$$D_{n+m}^2 = 4(n+m)\lambda R \quad \text{——— (2)}$$

Subtracting equations (1) and (2), we get,

$$D_{n+m}^2 - D_n^2 = 4(n+m)\lambda R - 4n\lambda R$$

$$\Rightarrow D_{n+m}^2 - D_n^2 = 4m\lambda R$$

$$\Rightarrow \boxed{\lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}} \quad \text{——— (3)}$$

Knowing the values of D_n, D_{n+m}, m, and R value of λ can be calculated.

Diffraction

The bending of light about the corners of an obstacle and spreading into the region of geometrical shadow is called diffraction of light.

The wave are diffracted only when the size of obstacle is comparable to the wavelength of the wave. The diffraction pattern obtained is only due to the interference of the secondary wavelets arising from the same wavefronts.

Types of diffraction

(i) Fresnel Diffraction:

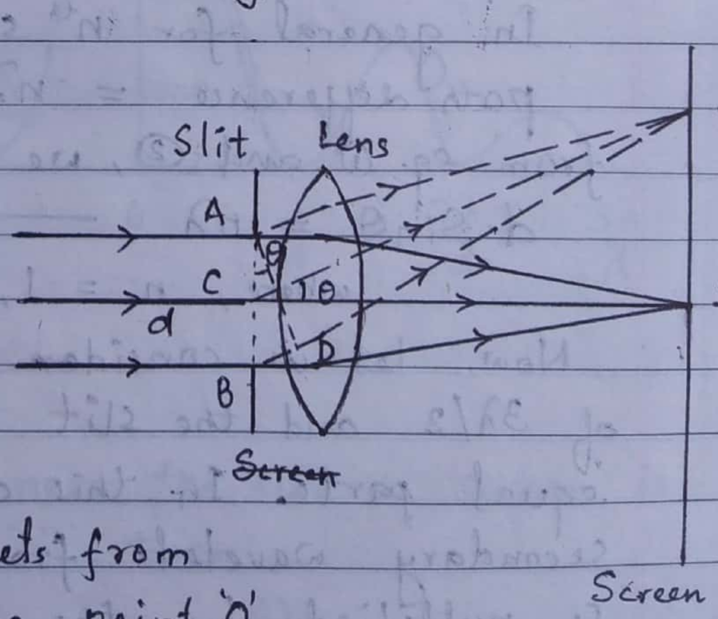
In Fresnel diffraction the source and screen are at finite distance from diffracting element. So, the incident wavefront is spherical or cylinderical. No lens is required to obtain the diffraction pattern.

(ii) Fraunhoffer diffraction:

In Fraunhoffer's diffraction the source and screen are effectively at infinite distance. So, the incident wavefront is plane. The converging lens is required to obtain diffraction pattern.

Fraunhoffer Diffraction at Single Slit:

Consider a parallel beam of monochromatic light incident on a narrow slit AB of width 'd', where 'C' be the center of the slit.



The secondary wavelets from slit AB reach to the point 'O', the center of the screen, in the same time and hence in the same phase. Therefore, 'O' becomes bright and becomes central maxima.

Consider an arbitrary point 'P' which makes an angle 'θ' with CO. Now, the path difference between secondary wavelets from A and B to the point 'P' is given by

path difference = BD

$$= d \sin\theta \quad \text{——} \quad (1)$$

Consider that the path difference is equal to λ and the slit is divided into two equal parts. Then, the secondary wavelets from points A and C or from C and B have path difference $\lambda/2$ to the point P. This means that the secondary wavelets from upper half and lower half of the slit will be out of phase, causing the point P as the secondary minimum.

In general for n^{th} secondary minima,

$$\text{path difference} = n\lambda \quad \text{——} \quad (2)$$

from eq. (1) and (2), we get,

$$d \sin\theta = n\lambda \quad \text{——} \quad (3)$$

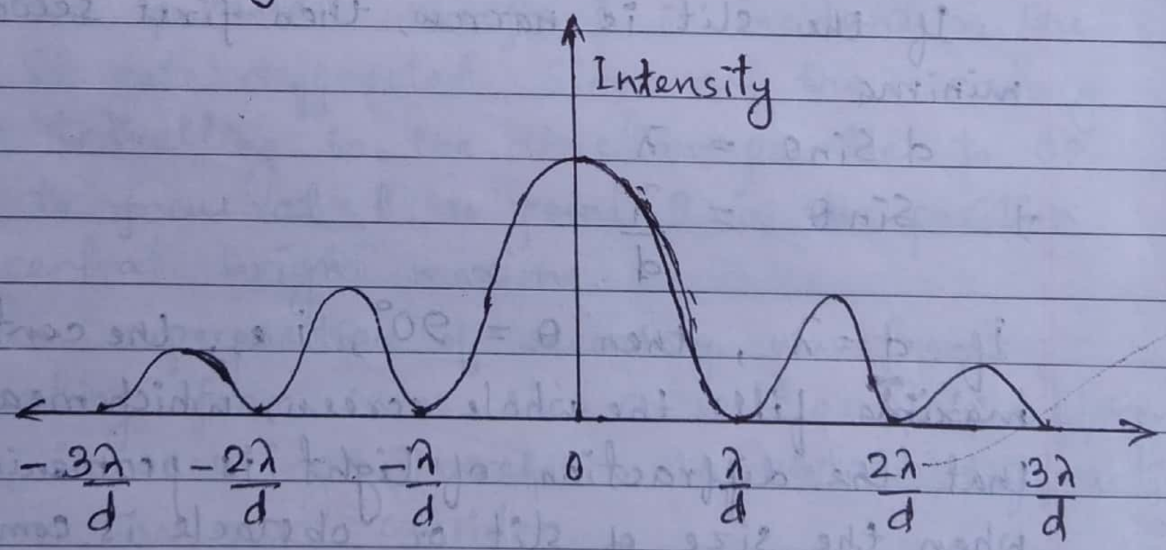
where, $n = 1, 2, 3, \ldots$

Now, let us consider the path difference of $3\lambda/2$ and the slit is divided into three equal parts. In this case, the effect of secondary wavelets from one part of slit is nullified by the secondary wavelets from other part but the secondary wavelet from remaining part of the slit makes point 'P' as secondary maxima.

In general, for n^{th} secondary maxima,
path difference $= (2n+1)\frac{\lambda}{2}$ —— (4)

from equations (1) and (4), we get,
$d \sin\theta = (2n+1)\frac{\lambda}{2}$ —— (5)

Thus, diffraction pattern due to single slit consists of central bright maximum at 'O' followed by secondary minima and maxima on both sides. The intensity distribution is shown below.



Case I:

If lens 'L' is very near the slit or screen is very far away from 'L', then
$\sin\theta = \frac{x}{f}$ —— (6)

where, f = focal length of L.

$\therefore \sin\theta = \frac{\lambda}{d}$, for $n = 1$. ——— (7)

$\therefore$ from eq^n (6) and (7),

$$\frac{x}{f} = \frac{\lambda}{d}$$

$$\Rightarrow x = \frac{\lambda f}{d} \quad \text{——— (8)}$$

where, 'x' is the distance of secondary minima from center 'O'.

$\therefore$ width of central maxima $= 2x$

$$= \frac{2\lambda f}{d} \quad \text{——— (10)}$$

Case II :

If the slit is narrow, then first secondary minima

$$d \sin\theta = \lambda$$

$$\Rightarrow \sin\theta = \frac{\lambda}{d}$$

If $d = \lambda$, then $\theta = 90^\circ$, i.e., the central maxima fills the whole screen, which means that the diffraction of light is promanient when the size of slit or obstacle is comparable to the wavelength of the light itself.

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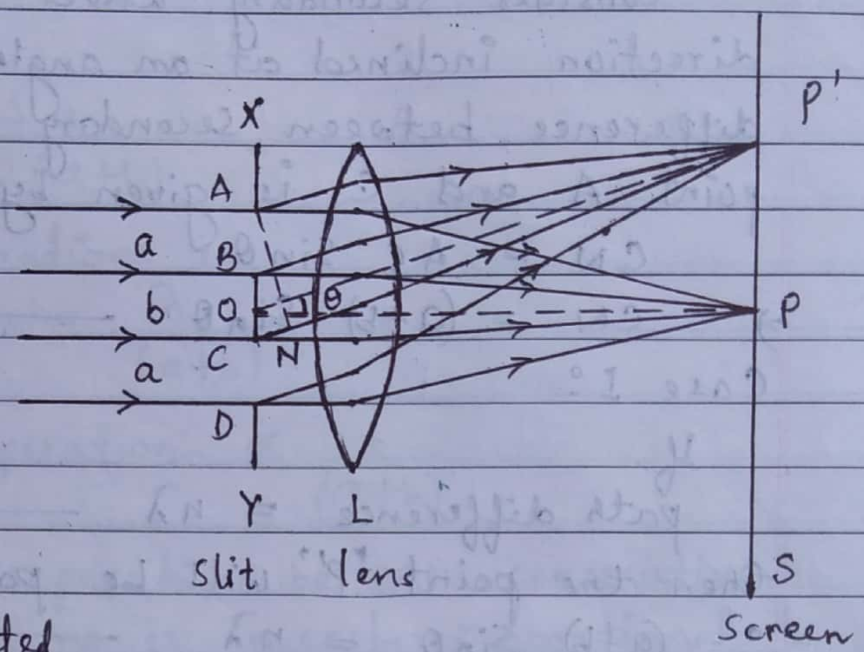
Fraunhoffer Diffraction at Double Slits:



Let XY be the double slit, L be the lens and S be the screen. Let AB and CD be the two parallel slits having width 'a' and seperated by opaque portion of width 'b'.

When parallel wavefront is incident on the slits it gets diffracted. Since all the secondary waves travelling in the direction parallel to OP come to focus at P so point P is the position of central bright maxima.

Here, superposition of secondary wavefronts emanating from same slit as well as two different slits is occuring. Therefore, to obtain diffraction pattern we must consider.

(i) interference phenomena due to secondary waves of two slits, and

(ii) diffraction phenomenon due to secondary waves from two slits individually.

(i) Interference maxima and minima :
Consider secondary waves travelling in the direction inclined at an angle 'θ'. The path difference between secondary waves from point A and C is given by

$$CN = AC \sin\theta$$

$$\Rightarrow CN = (a+b)\sin\theta \quad \text{——— (1)}$$

Case I :

If

$$\text{path difference} = n\lambda \quad \text{——— (2)}$$

then the point 'P'' will be position of maxima.

$$\therefore (a+b)\sin\theta = n\lambda \quad \text{——— (3)}$$

$$\Rightarrow \sin\theta = \frac{n\lambda}{(a+b)} \quad \text{where, } n = 1, 2, 3, \ldots$$

Case II :

If

$$\text{path difference} = (2n+1)\frac{\lambda}{2} \quad \text{——— (4)}$$

then the point P' will be the position of minima.

$$\therefore (a+b)\sin\theta = (2n+1)\frac{\lambda}{2} \quad \text{——— (5)}$$

$$\Rightarrow \sin\theta = \frac{(2n+1)\lambda}{(a+b)\,2} \quad \text{where, } n = 1, 2, 3, \ldots$$

If θ_1 and θ_2 are the angular directions of minima corresponding to $n=1$ and $n=2$ then

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and

$$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$\therefore$ angular seperations is

$$\sin\theta_2 - \sin\theta_1 = \frac{\lambda}{(a+b)}$$

i.e., angular seperation $\propto \frac{1}{(a+b)}$

$\therefore$ the angular seperation between consecutive minima or maxima is inversly proportional to $(a+b)$.

(ii) Diffraction maxima and minima:

Consider a secondary waves travelling in a direction inclined at an angle ϕ. If the path difference between secondary waves from points A and B is λ then the direction ϕ will be the direction of diffraction minima. If the path difference between the secondary waves from points A and B is odd multiple of $\lambda/2$ then the corrosponding direction will give the direction of diffraction maxima.

In general,

$a \sin\phi = n\lambda$, for minima

where, $n = 1, 2, 3, \ldots$

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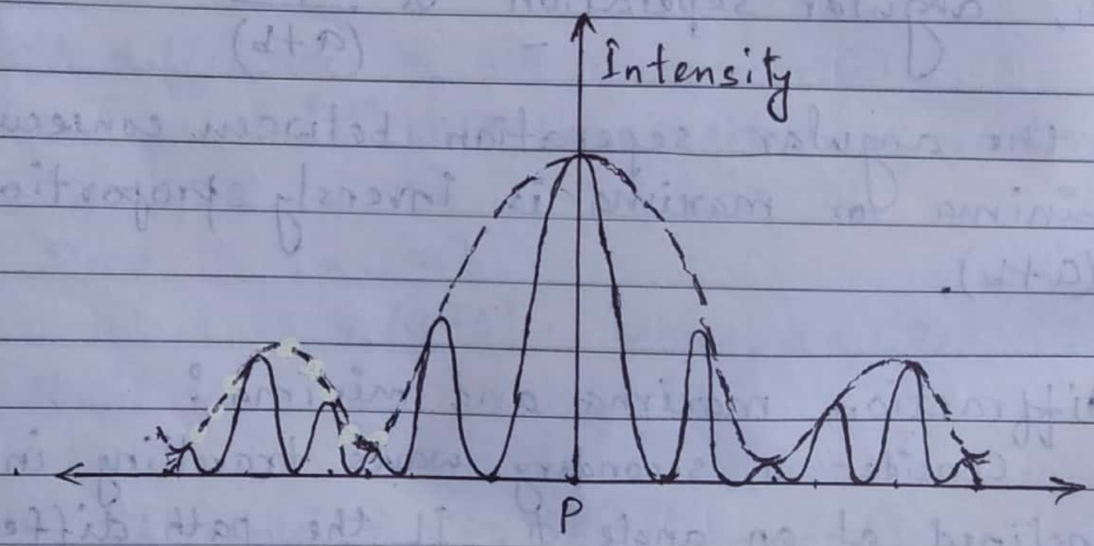
and

$$a \sin\phi = (2n+1)\frac{\lambda}{2}, \text{ for maxima}$$

where, $n = 1, 2, 3, \ldots$

The intensity distribution is shown in fig below. The dark line represents interference maxima and minima and dotted line represents the diffraction maxima and minima.



(iii) Missing orders in double slits diffraction pattern :

In the intensity pattern, the envelop of the intensity variation of interference fringes follows the diffraction pattern variation due to single slit. In general, if I_i is the intensity at a point due to interference between slits and I_d due to diffraction of the slit, then the resultant intensity I is given by

$$I = I_i \cdot I_d \quad \text{——— (1)}$$

Hence, if $I_d = 0$ at any point, then $I = 0$ at this point irrespective of the value of I_i.

The direction of interference maxima is given by

$$(a+b) \sin\theta = n\lambda \quad \text{——— (2)}$$

The direction of diffraction minima is given by

$$a \sin\theta = p\lambda \quad \text{——— (3)}$$

where, n and p are integers.

If values of a and b are such that both the equations are satisfied simultaneously for the same value of θ, then position of certain interference maxima corresponds to diffraction minima on the same position on the screen.

(a) If $a = b$,
then

$$2a \sin\theta = n\lambda$$

$$a \sin\theta = p\lambda$$

∴ Dividing, we get

$$2 = \frac{n}{p}$$

$$\Rightarrow n = 2p \quad \text{——— (4)}$$

if $p = 1, 2, 3, \ldots$

then, $n = 2, 4, 6, \ldots$

∴ the orders 2, 4, 6, ... etc. of the interference maxima will be absent in the diffraction pattern.

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(b) If $2a = b$

then

$$3a \sin\theta = n\lambda$$

$$a \sin\theta = p\lambda$$

Dividing, we get,

$$3 = \frac{n}{p}$$

$$\Rightarrow n = 3p \quad \text{——— (5)}$$

∴ the orders 3, 6, 9, ... etc. of the interference maxima will be absent in the diffraction pattern.

Plane Diffraction Grating

A diffraction grating consists of a large number of equidistance parallel slits of same width ruled on glass or metal surface and is made by drawing lines with fine diamond point.

Generally, there are two types of grating :

(i) plane transmission grating

(ii) reflection grating

In a grating, line acts as obstacle and spacing between them acts as aperture. If 'a' and 'b' are size of obstacle and aperture then the value (a+b) is called grating element. If 'N' is the number of lines drawn per inch then

$$N(a+b) = 1 \text{ inch}$$

$$\Rightarrow N = \frac{1}{(a+b)} \text{ inch}^{-1} \quad \text{——— (1)}$$

$$\Rightarrow N = \frac{2.54}{(a+b)} \text{ cm}^{-1}$$

If 'N' is the number of lines drawn per cm then

$$N(a+b) = 1 \text{ cm}$$

$$\Rightarrow N = \frac{1}{(a+b)} \text{ cm}^{-1} \quad \text{——— (2)}$$

When parallel beam of monochromatic light incident normally on a slit it is diffracted and diffraction pattern is obtained. At the center of screen all the secondary waves superimpose in phase and so it is point of central maxima.

For secondary maxima,

$$(a+b)\sin\theta = n\lambda \quad \text{——— (3)}$$

where, $n = 1, 2, 3, \ldots$

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Polarization

The phenomenon in which the vibration of light are restricted to a perticular plane is called polarization. It is characteristics of transverse wave.

Plane polarized light :

A light in which the vibration of light is confined to a perticular plane perpendicular to the direction of propogation of light is called plane polarized light. It is also called linearly polarized light.

Unpolarized light

A light in which the vibration of light take place in all possible plane perpendicular to direction of propogation of light is called unpolarized light.

Plane of vibration

A plane in which the vibration of light take place is called plane of vibration.

Plane of polarization

A plane perpendicular to plane of vibration is called plane of polarization.

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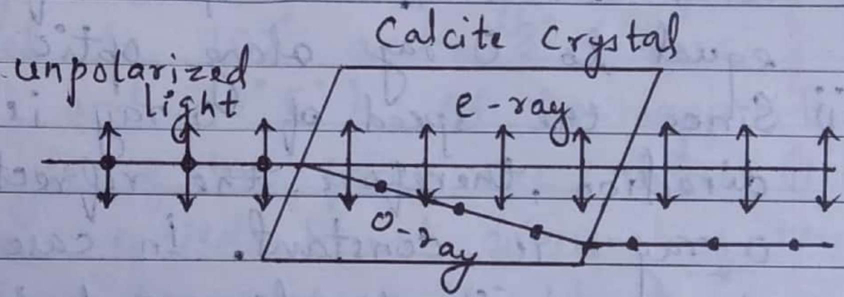
Double Refraction

When light is incident on the surface of calcite crystal the refracted rays splits into two rays with different properties. The phenomenon of splitting of incident ray into two refracted rays by certain crystal is called double refraction. It is also called birefringence. The crystal showing birefringence is called birefringent.

The two rays produced in double refraction are alway plane polarized in mutually perpendicular direction. One



of the rays obey's Snell's law and hence is called Ordinary ray or O-ray and other does not obeys Snell's law and hence is called extraordinary ray or e-ray.

If, by any method, one ray is eliminated the other transmitted ray is plane polarized. Therefore, by eleminating one ray the plane polarized light can be obtained.

O-ray and e-ray

(i) Both these rays have equal inensity and move in crystal with different velocities in slightly different directions. There is a special direction in the crystal along which double refraction does not take place called optic axis.

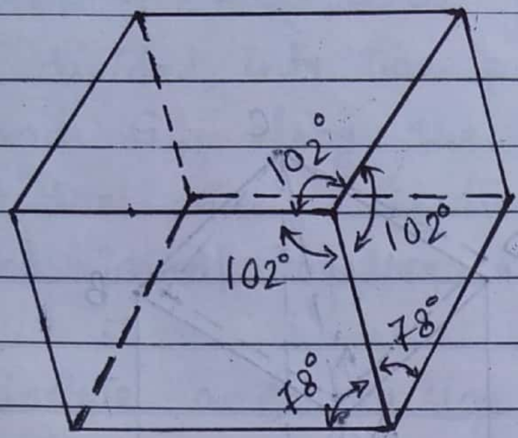
(ii) The speed of o-ray is same in all directions whereas speed of e-ray changes with direction. The speed of e-ray will be equal to O-ray along optic axis direction.

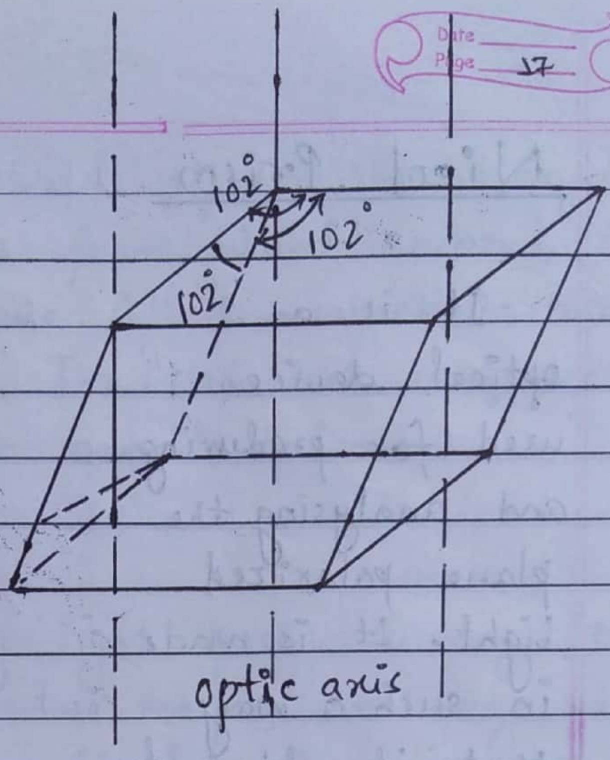
(iii) Since the speed of O-ray is same in all direction, therefore, the refrective index of O-ray is constant. In case of e-ray speed changes with direction and hence refractive index for e-ray changes with direction.

(iv) The distinction of o-ray and e-ray exist within the crystal. Outside the crystal they only differ in their plane of polarization.

Calcite Crystal

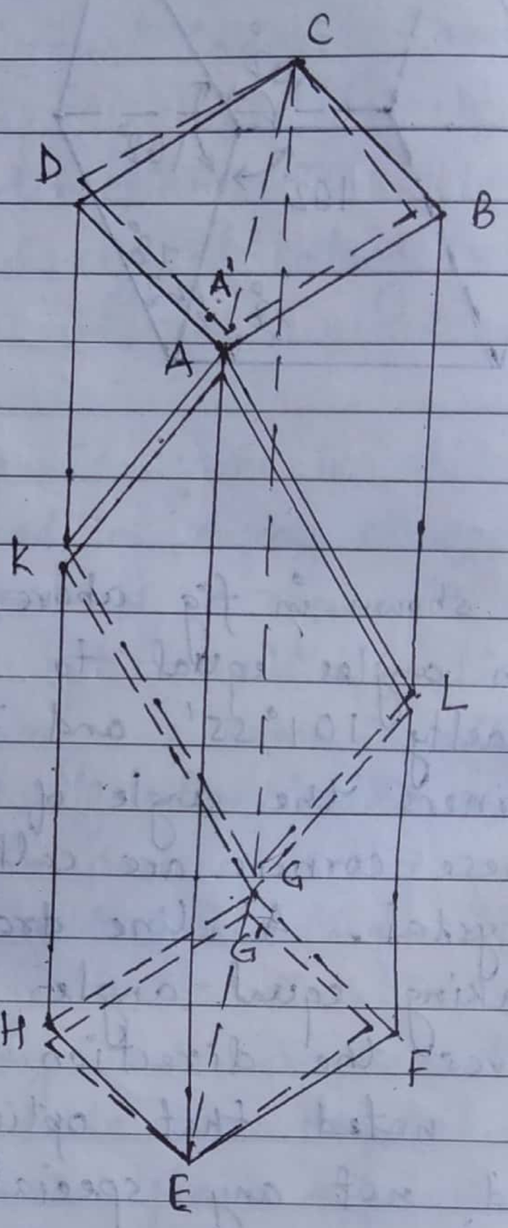
Calcite crystal, also called island spar, is a transparent crystal of calcium carbonate ($CaCO_3$). It is rhombohedron in shape





as shown in fig above, bounded by six parallelograms with angles equal to 102° and 78° (more exaclty 101°55' and 78°5'). At two opposit corners the angle of the faces are obtuse (102°). These corners are called blunt corners of the crystal. A line drawn through a blunt corner making equal angle with each of three edges gives the direction of optic axis. It is to be noted that optic axis is a direction and not any special line. Therefore, any line parallel to the line through blunt corner obtained above is also an optic axis. It is also noted that optic axis is not defined by joining two blunt corners. It is so only when three edges of the crystal are equal.

Nicol Prism



It is an optical device used for producing and analysing the plane polarized light. It is made in such a way that it eliminates one of the two rays (O-ray and e-ray) by total internal reflection.

Construction:

Nicole prism is made up of calcite crystal whose length is three times its breadth as shown in fig. In fig A'BCDEFG'H is a calcite crystal with A and G as its blunt corners and A'CG'E as one of its principle section in which ∠A'CG' = 71°.

To make Nicole prism face A'BCD and EFG'H are grounded in such a way that

$\angle ACG = \angle AEG = 68^\circ$ instead of 71°. Now, crystal is divided into two parts from blunt corners A and G, along the plane AKGL, which is normal to the principle section. The two cut surfaces are joined together with canada balsam.

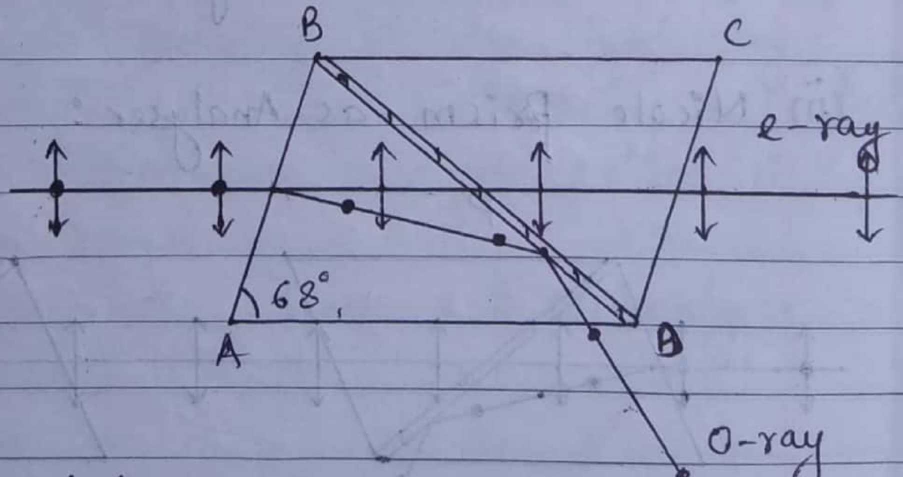
Principle and Working:

When ray of ordinary light passes through the crystal it is broken into two rays, ordinary rays whose vibration are perpendicular to principle section and extraordinary rays whose vibration are in principle plane.

The nicole prism eliminates the O-ray by total internal reflection where as passes only e-rays through it. It is also used to detect or analyse the polarized light.

(i) Nicole Prism as Polariser:

Consider a principle section ABCD of a nicole prism. The refractive index ($\mu_e = 1.55$) of canada balsam lies between refractive index of O-ray ($\mu_o = 1.658$)



and refractive index of e-ray ($\mu_o = 1.486$), for the principle plane direction.

When a beam of light incident on a nicole prism parallel to the principle plane it is divided into o-rays and e-rays that travels through the crystal and strike at canada balsam layer.

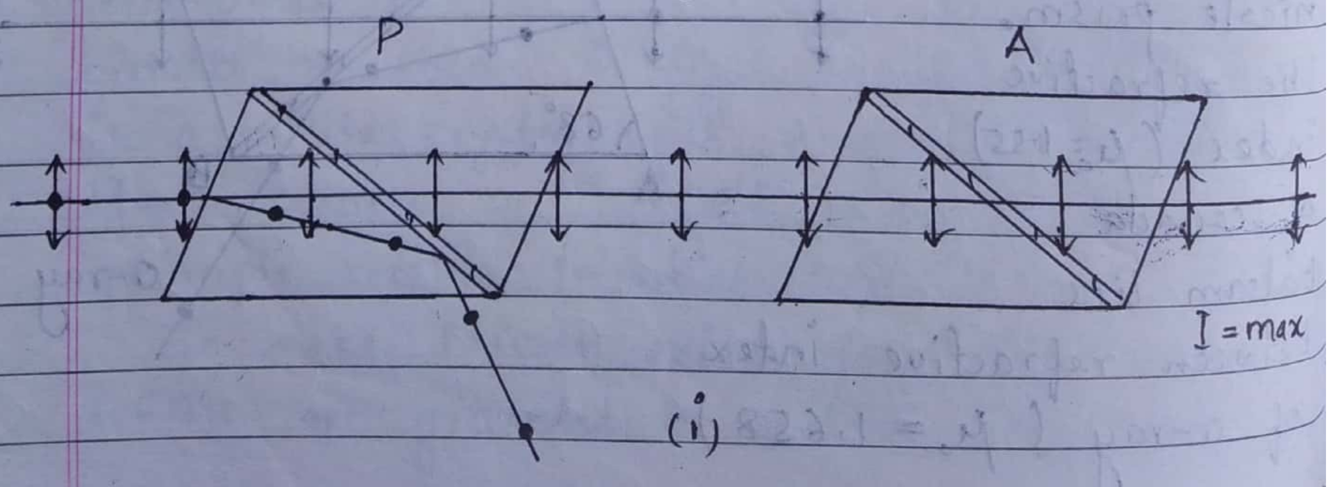
For o-rays,

$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1.55}{1.658}\right) = 69^\circ$$

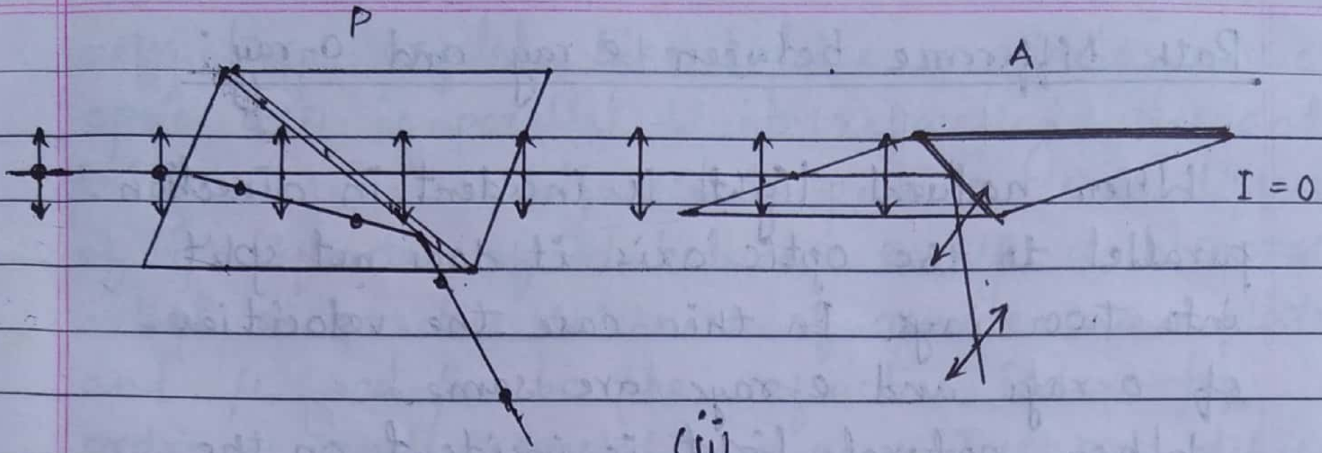
Therefore, if o-ray incident at an angle greater than critical angle (69°) it if totally reflected.

But for e-rays canada balsam is denser medium as compared to calcite. Therefore, e-rays passes through the opposit face since total internal reflection does not occur at the canada balsam layer.

(ii) Nicole Prism as Analyser:



(i)



(ii)

When ordinary light passes through nicole prism P the transmitted light is plane polarized light having vibration on principle section of prism P. In order to analyse, it is passed through other nicole prism A. If the vibration of light is parallel to the principle section of prism A then the light will act as e-rays for prism A and are transmitted.

Now prism A is rotated keeping prism P as it is. Due to rotation if the principle section of prism A becomes perpendicular to the plane of vibration of light entering it then light ray will act as O-ray and total internally reflected. As a result no light pass through prism A and hence zero intensity.

In this way nicole prism acts as analyser.

Path Difference between e-ray and o-ray:

When natural light is incident in direction parallel to the optic axis it does not split into two rays. In this case the velocities of o-rays and e-rays are same.

When natural light is incident on the crystal in the direction perpendicular to the principle axis it does not split into two rays, but the two rays travels with different velocity. Therefore, in this case, when the rays have travelled through the thickness 'd' in the crystal, a phase difference and hence path difference is introduced between them.

Let 'd' be the thickness of the crystal and μ_o and μ_e be the refractive index of o-rays and e-rays then
the optical path for o-ray $= \mu_o d$
the optical path for e-ray $= \mu_e d$

$\therefore$ optical path difference, $\Delta = (\mu_o - \mu_e) d$.

Quarter Wave Plate:

Quarter wave plate is a thin plate of double refracting crystal, like calcite or quartz, whose optic axis is parallel to its refracting faces and thickness is so adjusted to provide path difference of $\frac{\lambda}{4}$ (quarter of λ) between e-rays and o-rays.

Let 'd' be the thickness of quarter wave plate and μ_o and μ_e be the refractive index for ordinary and extraordinary rays. The path difference introduced by this plate is given by

$$\text{path difference} = (\mu_o - \mu_e)\, d \quad \text{———} \quad (1)$$

For quarter wave plate,

$$\text{path difference} = \frac{\lambda}{4} \quad \text{———} \quad (2)$$

from eq. (1) and (2), we get,

$$(\mu_o - \mu_e)\, d = \frac{\lambda}{4}$$

$$\Rightarrow \boxed{d = \frac{\lambda}{4(\mu_o - \mu_e)}} \quad \text{———} \quad (3)$$

It is used to produce elliptically or circularly polarized light.

Half Wave Plate :

Half wave plate is a thin plate of double refracting crystal, like calcite or quartz, whose optic axis is parallel to its refracting faces and thickness is so adjusted to provide path difference of $\frac{\lambda}{2}$ (half of λ) between e-rays and o-rays.

Let 'd' be the thickness of half wave plate and μ_o and μ_e be the refractive index for o-rays and e-rays. The path difference introduced by this plate is given by

path difference, $= (\mu_o - \mu_e) d$ ——— (1)

For half wave plate,

path difference $= \frac{\lambda}{2}$ ——— (2)

From eq. (1) and (2),

$$(\mu_o - \mu_e) d = \frac{\lambda}{2}$$

$$\Rightarrow \boxed{d = \frac{\lambda}{2(\mu_o - \mu_e)}} \quad \text{——— (3)}$$

Optical Activity :

Some crystals rotate the plane of polarization of light. This phenomenon of rotating the plane of polarization by certain crystal or substance is known as optical activity. The crystal or substance showing optical activity is called optically active substance. Examples of active substances are quartz, sugar solution, turpentine, etc..

Some ~~of~~ substance rotates the plane of vibration to the right and they are called dextro-rotatory or right-handed. For eg., quartz.

Some substance rotates the plane of vibration to the left and they are called leavo-rotatory or left-handed. For eg, sugar solution.

Specific Rotation :

The optically active substance rotates the plane of vibration of linearly polarized light. The angle of rotation of plane of vibration by optically active substance depends on

(i) thickness of medium,

i.e., $\theta \propto t$ ——— (1)

(ii) concentration of solution or density of the material

i.e., $\theta \propto c$ ——— (2)

(iii) wavelength of light,

i.e., $\theta \propto \lambda$ ——— (3)

(iv) temperature,

i.e., $\theta \propto T$ ——— (4)

For a perticular light and at a fixed temperature,

$$\theta \propto tc$$

$$\Rightarrow \theta = Stc$$

$$\Rightarrow S = \frac{\theta}{tc} \quad \text{——— (5)} \quad (\text{in dc})$$

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where, s is a constant of proportionality called specific rotation.

The specific rotation is defined as rotation produced by a decimeter long column of liquid containing one gram of active substance in one cubic cm of solution.

From eq. (5),

$$s = \frac{10\,\theta}{t\,(\text{in cm})\,c} \qquad (6)$$

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