# Homework 1

Due: Feb 15th before class

1. We discussed two variations of the TTF simulation in Lecture 1. Provide an event graph model of the variation 2 of the TTF simulation that has three components and two repair persons. Clearly define the events, state variables, and inputs of the model. Assume that the simulation continues even when system failure happens.

2. Given a triangle distribution, Triangle(a, b, c), derive an inversion algorithm that generates the triangle random variable from U(0,1).

3. #3.2 & #3.3 and describe & compare your observations from the plots.

4. #3.5 (no credit if you do not use induction to prove; here's an example of proof by induction)

5. In addition to the AR(1), another surrogate model that is used to represent steady-state simulation output, but shows slightly different behavior is the MA(1):

$$Y_i = \mu + \theta X_{i-1} + X_i, i = 1,2, \dots, m,$$

where $X_1, X_2, \dots$ are i.i.d. $(0, \sigma^2)$ random variables and $|\theta| < 1$.

(1) Derive expressions for $V(Y_i)$ and $C(Y_i, Y_j), i \neq j$.

(2) Suppose you simulate the sequence of $Y_i$'s with $X_0 = 1$. You designed the following two experiments:

   a. Run 1 replication and generate $Y_1, Y_2, \dots, Y_{10}$.

   b. Run 2 replications and in each generate $Y_1, Y_2, \dots, Y_5$.

   For each case, compute the mean and the variance of the sample average of all $10$ $Y_i$'s you simulated and compare them for when $\theta > 0$ and $\theta < 0$, respectively.

(3) In class, for M/G/1 simulation we discussed the tradeoff between simulating one long sequence of waiting time from a single replication and simulating multiple shorter replications given the same simulation budget when we want to estimate the expected waiting at the steady-state with the sample average of the generated sequences. From this exercise, what can you deduce about the tradeoff between two approaches in terms of a) bias and b) variance?

6. Use the M/M/∞ simulator you created in the lab to answer the following questions.

   a. We discussed in Lecture 2 that the steady-state number of cars in the system for M(t)/M//∞ queue has Poisson distribution. Derive what $m(t)$ is for the case of homogeneous Poisson arrivals we implemented in the lab. According to this distribution, what is the steady-state expected number of cars in the parking lot at any time point?

b. Construct a 99% confidence interval for the expected number of cars in the garage from 10000 replications of your model. Does the confidence interval cover the steady-state expected number of cars you found in (2)? If not, what are the reasons you see discrepancy between your simulation model and the steady-state analytical model?
c. Suppose you changed the simulation logic so that the parking lot is open 24 hours a day. You would like to initialize the simulator so that it starts with 10 cars in the parking lot. How do you make sure these 10 cars leave the parking lot after a statistically correct amount of time? Clearly mention which parts of your Python code must be changed. Submit your modified Python code on Canvas.