1) solve the following recurrence relations

a) $x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

$$\boxed{x(n) = x(n-1) + 5} \rightarrow (1)$$

Let $(n = n-1)$ in (1)

$$\boxed{x(n-1) = x(n-1-1) + 5 = x(n-2) + 5} \rightarrow (2)$$

Let $(n = n-2)$ in (1)

$$\boxed{x(n-2) = x(n-3) + 5} \rightarrow (3)$$

Now,

$$x(n) = x((n-2)+5) + 5$$

$$\boxed{x(n) = x(n-2) + 10} \rightarrow (4)$$

Put the value of $(n-2)$ in (3), (4).

$$\boxed{x(n) = x(n-3) + 5 + 5 + 5 = x(n-3) + 15} \rightarrow (5)$$

from eq no (1), (4) & (5)

$$x(n) = x(n-i) + 5i \quad \text{for } i < n$$

Put $i = n-1$,

$$x(n) = x(n-(n-1)) + 5(n-1)x(n)$$

$$= x(1) + 5(n-1)$$

$x(n) = x(1) + 5(n-1)$

$(x(1) = 0)$ then;

$$\boxed{x(n) = 5(n-1)}$$

b) $x(n) = 3x(n-1) \rightarrow (1)$ $\quad (x(1) = 4)$

$n \geq 1$

Let $(n = n-1)$;

$x(n-1) = 3x(n-1-1)$

$x(n-1) = 3x(n-2) \rightarrow (2)$

Let $(n = n-2)$; in (1)

$x(n-2) = 3x(n-2-1)$

$x(n-2) = 3x(n-3) \rightarrow (3)$

Now $x(n-1)$ value in (1),

$x(n) = 3.3x(n-2) = 3^2 x(n-2) \rightarrow (4)$

Put $x(n-2)$ in (4),

$x(n) = 3^3 x(n-3) \rightarrow (5)$

from (1), (4) & (5)

$x(n) = 3^i x(n-i)$ for $i < n$

now, $i = n-1$;

$x(n) = 3^{n-1} x(n-(n-1))$ $\boxed{x(n) = 3^{n-1} x(1)}$

$x(n) = 3^{n-1} x(1)$

but $x(1) = 4$. then, from Question.

$$\boxed{x(n) = 3^{n-1} 4}$$

c) $x(n) = x(n/2) + n$ for $n \geq 0$ $x(1) = 1$ (for $n = 2k$)

$x(n) = x(n/2) + n \rightarrow (1)$

But, $x(1) = 1$;

Sub $(n = 2^k)$ then;

$x(2^k) = x(2^{k-1}) + 2^k \rightarrow (2)$

sub $(2^k = 2^{k-1})$

$x(2^{k-1}) = x(2^{k-2}) + 2^{k-1} \rightarrow (3)$

Sub $(2^k = 2^{k-1})$

$x(2^{k-2}) = x(2^{k-3}) + 2^{k-2} \rightarrow (4)$

$$x(2^k) = x(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k \rightarrow (5)$$

when $(k = 0)$

$$x(2^0) = x(1) = 1$$

from (5)

The sum of the first;

$$2^0 + 2^1 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$$

So we have,

$$x(2^k) = 2^{k+1} - 1$$

Sub $(2^k = n/3)$

$$\boxed{x(n) = 2^{n/3} - 1} = \boxed{2^{n/k} - 1}$$

d) $x(n) = x(n/3) + 1$ for $n \geq 1$ $x(1) = 1$ (solve for $n = 3k$)

$$x(n) = x(n/3) + 1 \rightarrow (1)$$

But $(x(1) = 1)$;

Sub $(n = 3^k)$ in (1)

$$x(3^k) = x\left(\frac{3^k}{3}\right) + 1$$

$$x(3^k) = x(3^{k-1}) + 1 \rightarrow (2)$$

2) Evaluate the following.

i) $T(n) = T(n/2) + 1$ where $n = 2^k$ for all $k \geq 0$

assume $n = 2^k$; $k = \log n$.

$$T(2^k) = T\left[\frac{2^k}{2}\right] + 1$$

$$T(2^k) = T(2^{k-1}) + 1$$

$$= \left(T(2^{k-2}) + 1\right) + 1$$

$$= \left[T(2^{k-3}) + 1\right] + 2$$

$$= T(2^{k-3}) + 3$$

$$T(2^k) = T(2^{k-k}) + k$$

$$= T(2^0) + k$$

$$= T(1) + k$$

$T(1) = 1$ then,

$$T(2^k) = 1 + k$$

$$T(n) = \log n + 1$$

$\therefore$ we got $T(n) = \Theta(\log n)$

2) $T(n) = T(n/3) + T(2n/3) + cn,$

$T(n) \leq T(n/3) + T(2n/3) + cn$

$\leq d(n/3)\log(n/3) + d(2n/3)\log(2n/3) + cn$

$= (d(n/3)\log n - d(n/3)\log 3)$

$+ [d(2n/3)\log n - d(2n/3)\log(3/2)] + cn$

$= dn\log n - d((n/3)\log 3 + (2n/3)\log(3/2)) + cn$

$= dn\log n - dn(\log 3 - 2/3) + cn$

$\leq dn\log n$

$[\because d \geq c/(\log(3) - 2/3)]$

$\therefore$ order $= O(n\log n)$

3) Consider the following recursion algorithm.

```
min1(A[0... n-1])
if n=1 return A[0]
else temp = Min1(A[0.... n-2])
        if temp <= A[n-1] return temp
else
        Return A[n-1]
```

a) The recursive algorithm computes the min value in array A of size n. it does this by comparing the last element of the array A[n-1] with the minimum value of the rest of the array A[0.... n-2] & returning smaller value.

b) the algorithm makes call to min(A[0..n-1]) which is n-1, then,

$$T(n) = T(n-1) + C.$$

C is the constant representing the time taken for the operations outside the call.

```
def min1 (A,n):
    if n==1:
        return A[0]
    else:
        temp = min1 (A, n-1)
        if temp<=A[n-1]:
            return temp
        else:
            return A[n-1]
```