Final Practice

1. Find the equation of the tangent line to the curve $y = \sqrt{2x^2 + 3}$ at $x = -1$ using the limit definition of the derivative.
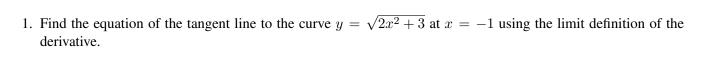
2. Where is the function $f(x) = |\sin x|$ differentiable on the interval $0 \le x \le 2\pi$.

3. Find $f'(x)$ and $f''(x)$ when $f(x) = \sqrt[5]{x} + x^3$.

4. (**Challenge**)Find $f'(x)$ and $f''(x)$ when $f(x) = \frac{\sqrt{625-\sec(x^2)}}{x^{20}}$.

5. Use implicit differentiation to find the tangent line to $x^4 = y^2 + x^2$ at $(2, \sqrt{12})$.

6. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.03 cm thick to a sphere with diameter 40 meters.

7. A police helicopter is flying at 200 kilometers per hour at a constant altitude of 1 km above a straight road. The pilot uses radar to determine that an oncoming car is at a distance of exactly 2 kilometers from the helicopter, and that this distance is decreasing at 250 kph. Find the speed of the car.

8. The strength of a rectangular beam is proportional to the product of its width w times the square of its depth d. Find the dimensions of the strongest beam that can be cut from a cylindrical log of radius r.

9. Consider the function

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$$

Given that the first and second derivatives are:

$$f'(x) = \frac{1}{8\sqrt{2\pi}}(x-1)e^{-\frac{(x-1)^2}{8}}$$

$$f''(x) = \frac{1}{32\sqrt{2\pi}}(x^2 - 2x - 3)e^{-\frac{(x-1)^2}{8}}$$

(a) What are the critical points of $f(x)$. Is there a local max or min ? (Make sure to give the actual point(s) not just the x-values).

(b) What are the points of inflection ? (Make sure to give the actual points, not just the x values). Intervals of concavity?

(c) Sketch the graph of $f(x)$, making sure to label all the points you found above.

(d) Bonus question: $f(x)$ is actually a special type of function. What special function is it?

10. Find the derivative of

$$f(x) = \int_{-\sqrt{x}}^{e^{x^2}} \sec(\pi(t^2 + 2t - 1))\, dt$$

11. Evaluate the following integrals:

(a)
$$\int_0^4 \sqrt{2x+1}\ dx$$

(b)
$$\int \sin^6 2x\ dx$$

(c)
$$\int_{-1}^1 \frac{x^2}{\sqrt[3]{1-x^3}}\ dx$$

.

12. Find the area between $y = x^2 - 2x$ and $y = x - 2$.

13. Find the average value of $f(x) = \frac{x^2+2}{(6x+x^3)^2}$ over the interval $[1, 3]$.

14. Find the volume of the region bounded by $y = x - x^2$ and the x-axis rotated about the x-axis.

15. Suppose that a water tank is shaped like a right circular cone with the tip at the bottom, and has height 10 meters and radius 2 meters at the top. If the tank is full, how much work is required to pump all the water out over the top?

16. Find the arc length of $f(x) = x^{3/2}$ on $[1, 4]$.