Math 1512 Exam 1

NAME:_____

INSTRUCTIONS:

SHOW ALL OF YOUR WORK. Unsupported and illegible answers will not receive credit. Use **proper mathematical notation** to receive full credit. Absolutely NO electronic devices or notes are allowed during this test. May the Force be with you...

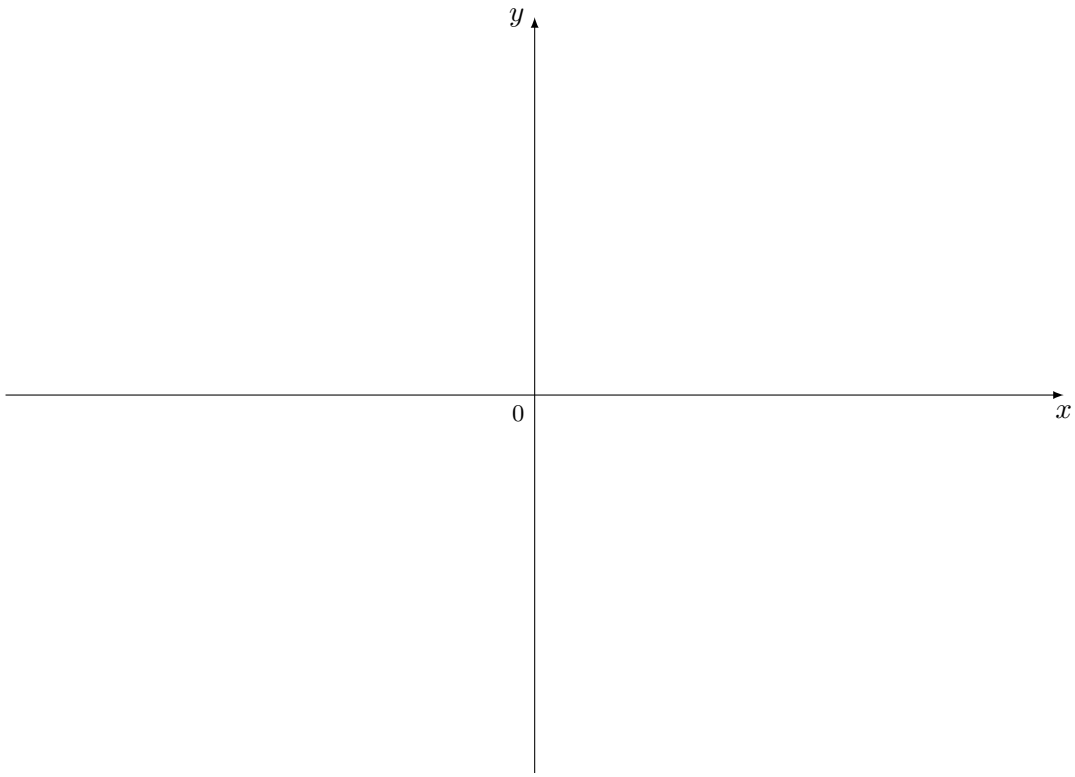
1. (10 pts) Compute the following limits. If a limit does not exist explain why.

   a. $\lim_{x \to 1} \frac{\sqrt{x+8}-3}{x-1}$

   b. $\lim_{x \to 4} \frac{\frac{1}{x}-\frac{1}{4}}{x-4}$

2. (20 pts) For what value of the constant c is the function f continuous on $(-\infty, \infty)$?. Once you have found c, sketch a graph of f. Be sure to label key points.

$$f(x) = \begin{cases} cx^2 + 4, & x < -2 \\ -x - 3c, & x \geq -2 \end{cases}$$



Write a formula for $f'(x)$ as a piece-wise function and determine if f is differentiable on $(-\infty, \infty)$. Justify your answer carefully.

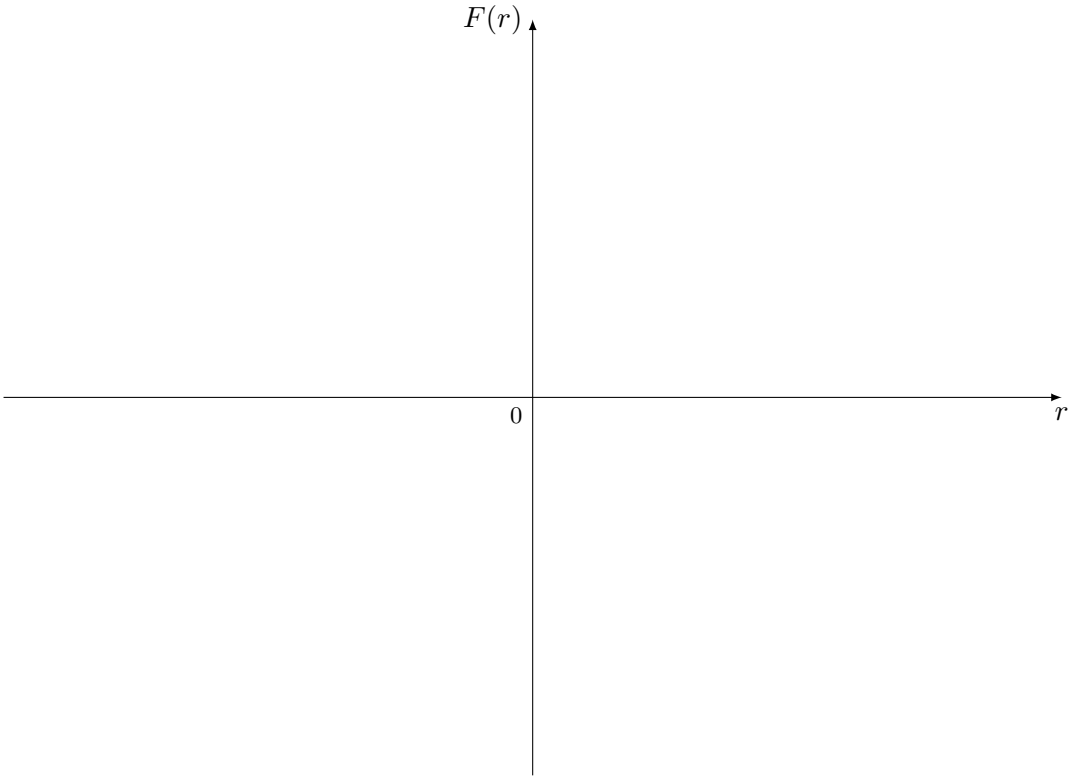
3. (20 pts) The gravitational force exerted by the planet Earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \frac{GMr}{R^3}, & 0 \leq r < R \\ \frac{GM}{r^2}, & r \geq R \end{cases}$$

where M is the mass of the Earth, R is its radius, and G is the gravitational constant.

(i) Is $F(r)$ continuous? Explain.

(ii) Sketch a graph of $F(r)$. Label the key points.



(iii) Compute $F'(2R)$. What does the sign of the derivative indicate?

4. (20 pts)Let $f(x) = -\frac{10}{\sqrt{1-x}}$. Use the limit definition of the derivative to find $f'(x)$.

5. (10 pts) Evaluate and simplify y'

   a. $y = \left(1 + \frac{1}{x^2\sqrt{x}}\right)(x^2 + 1)$

   b. $y = 4t^2 - \frac{2t}{5t+1}$

6. (20 pts) Consider $f(x) = \frac{x^2-9}{x(x-3)}$.

   a. Find the vertical asymptotes of f.

   b. Find any horizontal asymptotes of f

   c. Find the tangent line to f at the point $(2, \frac{5}{2})$