Limits and Rates of Change

Instructions These exercises are meant to reinforce what you have learned as well as challenge you on concepts that you have a basic understanding of. You will almost certainly not finish these all during one recitation session. Try picking some of the more straightforward problems to get warmed up, and then try some others you are not sure how to solve. These are all possible exam problems! Your recitation instructor will send out the key later in the week.

1. If a ball is thrown into the air at a velocity of 40 ft/s, its height in feet t seconds later is given by $h(t) = 40t - 16t^2$.

   (a) Find the average velocity for the time period beginning at $t = 2$ and lasting

      i. 0.5 second

      ii. 0.1 second

      iii. 0.001 second

   (b) What is the instantaneous velocity when $t = 2$ seconds?

2. Find the equation of the tangent line to the parabola $y = x^2$ at the point $(1, 1)$.

3. Estimate the instantaneous rate of change of $f(x) = 3\tan(2x)$ at $x = 0$ (x is in radians). Use this to compute the tangent line at the point $(0, 0)$.

4. Use a table of values to evaluate each limit

   (a) $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

   (b) $\lim_{x\to 0} \frac{\tan 3x}{\tan 5x}$

   (c) $\lim_{x\to 0} \frac{9^x-5^x}{x}$

5. Sketch the graph of the function and use it to determine the values of a for which the limit $\lim_{x\to a} f(x)$ exists.

   (a)
$$f(x) = \begin{cases} 1+x, & x < -1 \\ x^2, & -1 \le x < 1 \\ 2-x & x \ge 1 \end{cases}$$

   (b)
$$f(x) = \begin{cases} 1+\sin x, & x < 0 \\ \cos x, & 0 \le x < \pi \\ \sin x & x \ge \pi \end{cases}$$

6. Evaluate the following limits or explain why they do not exist.

(a)
$$\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$$

(b)
$$\lim_{t \to 2} \frac{4 - t^2}{t - 2}$$

(c)
$$\lim_{x \to 3} \frac{x + 3}{x^2 - 9}$$

(d)
$$\lim_{h \to 0} \frac{\sqrt{25 + h} - 5}{h}$$

7. Determine the infinite limit.

(a) $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$

(b) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)}$

(c) $\lim_{x \to \pi^-} \cot x$

8. In the theory of relativity, the mass m of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens to the mass of the particle as $v \to c^-$?