

## Final Practice

1. Find the equation of the tangent line to the curve  $y = \sqrt{2x^2 + 3}$  at  $x = -1$  using the limit definition of the derivative.
2. Where is the function  $f(x) = |\sin x|$  differentiable on the interval  $0 \leq x \leq 2\pi$ .
3. Find  $f'(x)$  and  $f''(x)$  when  $f(x) = \sqrt[5]{x + x^3}$ .
4. **(Challenge)** Find  $f'(x)$  and  $f''(x)$  when  $f(x) = \frac{\sqrt{625 - \sec(x^2)}}{x^{20}}$ .
5. Use implicit differentiation to find the tangent line to  $x^4 = y^2 + x^2$  at  $(2, \sqrt{12})$ .
6. Use differentials to estimate the amount of paint needed to apply a coat of paint 0.03 cm thick to a sphere with diameter 40 meters.
7. A police helicopter is flying at 200 kilometers per hour at a constant altitude of 1 km above a straight road. The pilot uses radar to determine that an oncoming car is at a distance of exactly 2 kilometers from the helicopter, and that this distance is decreasing at 250 kph. Find the speed of the car.
8. The strength of a rectangular beam is proportional to the product of its width  $w$  times the square of its depth  $d$ . Find the dimensions of the strongest beam that can be cut from a cylindrical log of radius  $r$ .
9. Consider the function

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$$

Given that the first and second derivatives are:

$$f'(x) = \frac{1}{8\sqrt{2\pi}} (x-1) e^{-\frac{(x-1)^2}{8}}$$

$$f''(x) = \frac{1}{32\sqrt{2\pi}} (x^2 - 2x - 3) e^{-\frac{(x-1)^2}{8}}$$

- (a) What are the critical points of  $f(x)$ . Is there a local max or min ? (Make sure to give the actual point(s) not just the  $x$ -values).
  - (b) What are the points of inflection ? (Make sure to give the actual points, not just the  $x$  values). Intervals of concavity?
  - (c) Sketch the graph of  $f(x)$ , making sure to label all the points you found above.
  - (d) Bonus question:  $f(x)$  is actually a special type of function. What special function is it?
10. Find the derivative of

$$f(x) = \int_{-\sqrt{x}}^{e^{x^2}} \sec(\pi(t^2 + 2t - 1)) dt$$

11. Evaluate the following integrals:

(a)

$$\int_0^4 \sqrt{2x+1} dx$$

(b)

$$\int \sin^6 2x dx$$

(c)

$$\int_{-1}^1 \frac{x^2}{\sqrt[3]{1-x^3}} dx$$

12. Find the area between  $y = x^2 - 2x$  and  $y = x - 2$ .
13. Find the average value of  $f(x) = \frac{x^2+2}{(6x+x^3)^2}$  over the interval  $[1, 3]$ .
14. Find the volume of the region bounded by  $y = x - x^2$  and the  $x$ -axis rotated about the  $x$ -axis.
15. Suppose that a water tank is shaped like a right circular cone with the tip at the bottom, and has height 10 meters and radius 2 meters at the top. If the tank is full, how much work is required to pump all the water out over the top?
16. Find the arc length of  $f(x) = x^{3/2}$  on  $[1, 4]$ .