

MATH 1512-Summer 2021-Final Exam

July 30, 2021

NAME (please print) : _____

Instructor's Name: _____

INSTRUCTIONS:

- This is an individual exam based on what you understand.
- Books, notes, calculators, graphing software, etc. are not allowed
- You must leave your answers as exact values, such as $x = \sqrt{5}$, $t = \frac{\ln 3}{2}$, etc.
- To get full credit you must use proper mathematical notation and vocabulary, show all important steps, and present neat and organized work. Use methods covered in this course.
- You will have the entire class period (1 hour and 40 minutes) for the exam.
- May the Force be with you!

1. (10 pts) Compute the derivative of the following functions. You do not need to simplify your answer.

a. $y = \left(x^3 - \frac{7}{x}\right)^{-2}$

b. $y = 10 \sec(e^{-3x^2})$.

2. (15 pts) Find the equation of the tangent line to the curve $x^3 + y^3 = 3xy$ at the point $(\frac{3}{2}, \frac{3}{2})$

3. (15 pts) Find the absolute maximum and minimum values of $f(x) = 4 \sin x + 4 \cos x$ on the interval $[0, 2\pi]$.

4. (20 pts) On a rainy day, a girl broke up with her boyfriend after being together for eight long years. They decided to separate at the place where everything about them began, at the same time. The boy is due north crying and running at a rate of 5 ft/sec and the girl is walking due east at a rate of 1 ft/sec thinking if she made the right decision. How fast are they separating from each other 5 seconds after they started moving to a new life without each other.

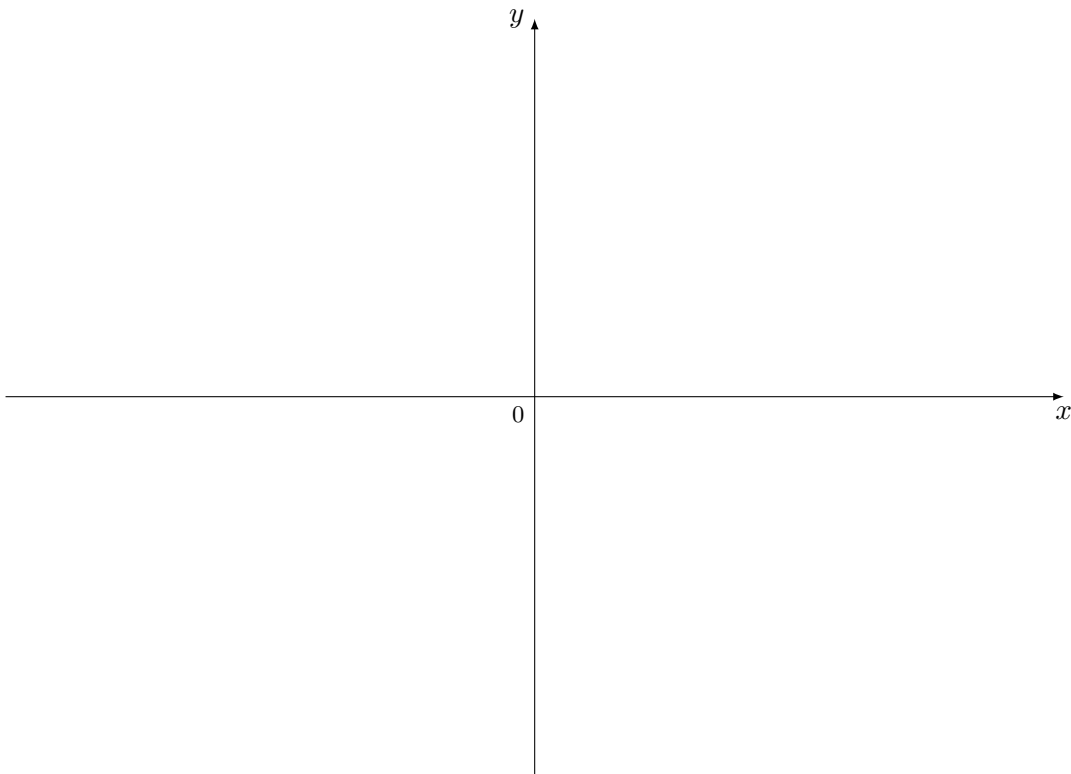
5. (25 pts) Given that

$$f(x) = x^{2/3}(6-x)^{1/3}, f'(x) = \frac{4-x}{x^{1/3}(6-x)^{2/3}}, f''(x) = \frac{-8}{x^{4/3}(6-x)^{5/3}}.$$

a. On what intervals is $f(x)$ increasing or decreasing? Give the coordinates of any local minimums or local maximums.

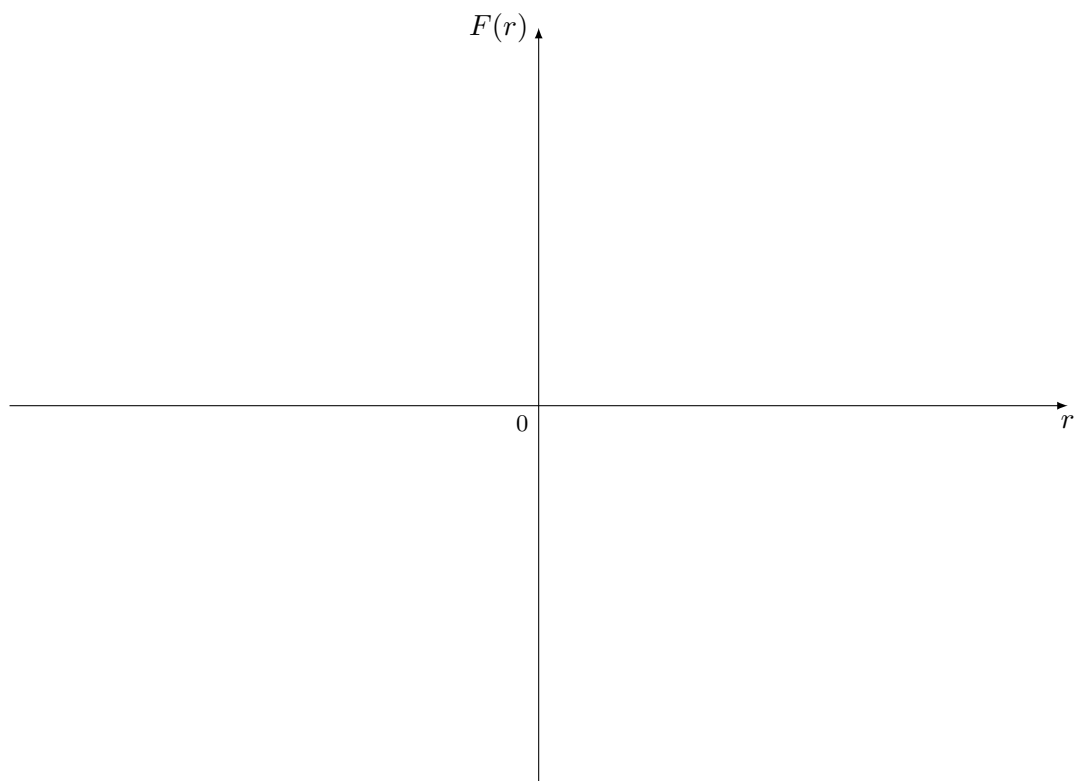
b. On what intervals is the graph of $f(x)$ concave up or down? Find any points of inflection.

c. Neatly sketch a plot of the graph of $f(x)$. Be sure to label CLEARLY all of the points of interest.



DIRECTIONS: Pick **ONE** problem from Problems 6-8 to complete. Make sure it is clear which problem you have chosen and which ones you have not chosen.

6. (15 pts) Compute the linearization of $f(x) = \sqrt{8-x}$ at $a = 4$. Use this to approximate $\sqrt{4.1}$. Sketch a graph of $f(x)$ and $L(x)$ on the same axes. Label all key points **CLEARLY**.



7. (15 pts) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$. Sketch a graph that illustrates the situation.

8. (15 pts) Evaluate the following limits.

a.

$$\lim_{x \rightarrow 1} \frac{4 \sin \pi x}{x + \cos \pi x}$$

b.

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1}$$

c.

$$\lim_{v \rightarrow 3} \frac{v - 1 - \sqrt{v^2 - 5}}{v - 3}$$