

MATH 1512-Summer 2021-Final Exam

July 30, 2021

NAME (please print) : _____

Instructor's Name: _____

INSTRUCTIONS:

- This is an individual exam based on what you understand.
- Books, notes, calculators, graphing software, etc. are not allowed
- You must leave your answers as exact values, such as $x = \sqrt{5}$, $t = \frac{\ln 3}{2}$, etc.
- To get full credit you must use proper mathematical notation and vocabulary, show all important steps, and present neat and organized work. Use methods covered in this course.
- You will have the entire class period (1 hour and 40 minutes) for the exam.
- May the Force be with you!

1. (i) State the mathematical definition of the derivative $f'(x)$ of a function $f(x)$ as a limit.

(ii) Use the limit definition to find the derivative of $f(x) = \frac{2}{\sqrt{2x+3}}$. You must use the limit definition to receive any credit.

2. Find the derivatives of the following functions

(i) $f(x) = \frac{\sin^3 x}{e^{x^2}}$

(ii) $g(x) = \sqrt[4]{x^3 - 4x^2 + 2}$

3. Evaluate the following integrals

(i) $\int \frac{2}{x\sqrt{x}} dx$

(ii) $\int_0^{\sqrt{\pi}/2} x \sec^2(x^2) \tan(x^2) dx$

4. Find the area between $y = \cos\left(\frac{\pi x}{2}\right)$ and $y = 1 - x^2$ in the first quadrant.

5. Let S be the region of the xy -plane bounded above by the curve $x^3y = 64$, below by the line $y = 1$, on the left by the line $x = 2$ and on the right by the line $x = 4$.

(i) Find the volume of the solid obtained by rotating S around the x -axis.

(ii) Find the volume of the solid obtained by rotating S around the line $x = 2$.

6. Find the arc length of $y = e^{x/2} + e^{-x/2}$ on the interval $[0, 1]$.

7. Evaluate the following expressions

(i)

$$\int_0^{\pi/2} \frac{d}{dx} [\sin(x/2) \cos(x/2)] dx =$$

(ii)

$$\frac{d}{dx} \int_{x^2}^{\pi/2} \sin(t/2) \cos(t/2) dt =$$

(iii)

$$\frac{d}{dx} \int_0^{\pi/2} \sin(x/2) \cos(x/2) dx =$$

8. A light shines from the top of a pole 20m high. A ball is falling 10 meters from the pole, casting a shadow on a building 30 meters away. When the ball is 25 meters from the ground it is falling at 6 meters per second. How fast is its shadow moving?