Infinite Limits and Limits at Infinity

1. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following functions and state whether they have horizontal asymptotes or not.

   (i)
   $$f(x) = \frac{4x}{20x+1}$$

   Answer: $y = \frac{1}{5}$

   (ii)
   $$f(x) = \frac{12x^8 - 3}{3x^8 - 2x^7}$$

   Answer: $y = 4$

   (iii)
   $$f(x) = \frac{6x^2+1}{\sqrt{4x^4+3x+1}}$$

   Answer: $y = 3$

2. Determine the following infinite limits.

   (i)
   $$\lim_{x\to\infty}(5 + \frac{1}{x} + \frac{10}{x^2})$$

   Answer: 5

   (ii)
   $$\lim_{\theta\to\infty} \frac{\cos\theta}{\theta^2}$$

   (Hint: similar to an example from the lecture) *Answer*: 0