Quiz Problems

1. Consider the function

$$f(x) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(x-1)^2}{8}}$$

Given that the first and second derivatives are:

$$f'(x) = \frac{1}{8\sqrt{2\pi}}(x-1)e^{-\frac{(x-1)^2}{8}}$$
$$f''(x) = \frac{1}{32\sqrt{2\pi}}(x^2-2x-3)e^{-\frac{(x-1)^2}{8}}$$

(i) What are the critical points of $f(x)$. Is there a local max or min ? (Make sure to give the actual point(s) not just the x-values).

(ii) What are the points of inflection ? (Make sure to give the actual points, not just the x values). Intervals of concavity?

(iii) Sketch the graph of $f(x)$, making sure to label all the points you found above.

(iv) Bonus question: $f(x)$ is actually a special type of function. What special function is it?

2. Find the arc length of $y = \frac{1}{3}(x^2+2)^{3/2}$ on $[0, 1]$

3. Find the volume of the solid of revolution created by rotating the region bounded by $y = 1, y = \sqrt{\sin x}$, when $0 \le x \le \frac{\pi}{2}$ about the x-axis.

4. Let $f(x) = x^{3/2}$ and $g(x) = x^{2/3}$. Find the area between these two curves.

5. Find the area between $f(x) = x^2$ and $g(x) = x^4 - x^2$ when x is negative

6. Evaluate the following integrals using substitution -

a.
$$\int \frac{x^2}{\sqrt{x+2}}\, dx$$

b.
$$\int_2^3 x\sqrt{x^2-4}\, dx$$

7. Evaluate the following definite integrals by using geometry.

a.
$$\int_{-4}^{2} (2x+4)\, dx$$

b.
$$\int_{-1}^{3} \sqrt{4-(x-1)^2}\, dx$$

8. Evaluate the following integrals

a.
$$\int \frac{12t^8 - t}{t^{3/2}}\, dt$$

b.
$$\int \sec\theta(\tan\theta + \sec\theta + \sin 2\theta)\, d\theta$$

(Hint: $\sin 2\theta$ can be written as what using double angle formulas?)

9. Compute the linearization $L(x)$ of $f(x) = \sqrt{1+x}$ at $x = 8$ and use this to estimate the value of $\sqrt{9.2}$.

10. Consider the function
$$f(x) = \frac{x^2}{x^2+9}.$$
Find the intervals of increase/decrease as well as the intervals on which f is concave up and concave down. Then using the test of your choice find where f has local extreme values.

11. Find the absolute extreme values, so both absolute maximum and absolute minimum, of $f(x) = x + 2\sin x \cos x$ on the interval $[0, \frac{\pi}{2}]$.

12. Find the equation of the tangent line to the curve $(x^2+y^2)^2 = \frac{25}{4}xy^2$ at the point (1,2)

13. Sand is poured onto a surface at 15 cm^3/sec, forming a conical pile whose base diameter is always equal to its altitude ($h = d$). How fast is the altitude of the pile increasing when the pile is 3 cm high? (Hint: the volume of a cone was mentioned in class. You will need to use the fact that $h = d$ and the relationship between diameter and radius of a circle).

14. Differentiate the following functions. Simplify your answers.

    a. $y = \frac{4u^2+u}{8u+1}$

    b. $y = 5x^3 - 3x + \sqrt{x} - \frac{4}{x^2\sqrt{x}}$

15. Let
$$y = \frac{f(x)}{g(x)}$$
. Show that
$$y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$
. **Do not use the quotient rule!** *Hint:* Rewrite y as a product and then differentiate.

16. Find the derivative function using the definition of $f(x) = \frac{1}{x}$ and find the equation of the tangent line at $a = -5$

17. Find the point at which the tangent line to the curve given by $y = x^3 - 2x^2 + x + 8$ has the smallest slope. In other words, find the point at which the slope of the tangent line is at a minimum. What is the slope at that point?