MATH 1512-Summer 2021-Final Exam

July 30, 2021

NAME (please print) : ____________________

Instructor's Name: ____________________

INSTRUCTIONS:

- This is an individual exam based on what you understand.
- Books, notes, calculators, graphing software, etc. are not allowed
- You must leave your answers as exact values, such as $x = \sqrt{5}$, $t = \dfrac{\ln 3}{2}$, etc.
- To get full credit you must use proper mathematical notation and vocabulary, show all important steps, and present neat and organized work. Use methods covered in this course.
- You will have the entire class period (1 hour and 40 minutes) for the exam.
- May the Force be with you!

1. (i) State the mathematical definition of the derivative $f'(x)$ of a function $f(x)$ as a limit.

(ii) Use the limit definition to find the derivative of $f(x) = \frac{2}{\sqrt{2x+3}}$. You must use the limit definition to receive any credit.

2. Find the derivatives of the following functions

(i) $y = [(2x+1)^{-1} + 3]^{-1}$

(ii) $f(x) = \frac{\sin^3 x}{e^{x^2}}$

(iii) $g(x) = \sqrt[4]{x^3 - 4x^2 + 2}$

3. Evaluate the following integrals

(i) $\int \frac{2}{x\sqrt{x}}\,dx$

(ii) $\int \frac{x^2}{\sqrt[3]{1-x^3}}\,dx$

(iii) $\int_0^{\sqrt{\pi}/2} x\sec^2(x^2)\tan(x^2)\,dx$

4. The error function (denoted as erf) is defined as

$$\text{erf } x = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

(i) Find the first and second derivatives of the error function.

(ii) The error function has two horizontal asymptotes - $\lim_{x\to\infty} \text{erf } x = 1$ and $\lim_{x\to-\infty} \text{erf } x = -1$. Using this and the first and second derivatives from the previous part, sketch a graph of the error function labeling the points of inflection and local extrema if any exist.

5. Find the area between $y = \cos\left(\frac{\pi x}{2}\right)$ and $y = 1 - x^2$ in the first quadrant.

6. Let S be the region of the xy-plane bounded above by the curve $x^3y = 64$, below by the line $y = 1$, on the left by the line $x = 2$ and on the right by the line $x = 4$.

   (i) Find the volume of the solid obtained by rotating S around the x-axis.

   (ii) Find the volume of the solid obtained by rotating S around the line $x = 2$.

7. Find the arc length of $y = e^{x/2} + e^{-x/2}$ on the interval $[0, 1]$.

DIRECTIONS: Pick **ONE** problem from Problems 8-10 to complete. Make sure it is clear which problem you have chosen and which ones you have not chosen.

8. Evaluate the following expressions

(i)

$$\int_0^{\pi/2} \frac{d}{dx}[\sin(x/2)\cos(x/2)]\,dx =$$

(ii)

$$\frac{d}{dx}\int_{x^2}^{\pi/2} \sin(t/2)\cos(t/2)\,dt =$$

(iii)

$$\frac{d}{dx}\int_0^{\pi/2} \sin(x/2)\cos(x/2)\,dx =$$

9. A light shines from the top of a pole 20m high. A ball is falling 10 meters from the pole, casting a show on a building 30 meters away. When the ball is 25 meters from the ground it is falling at 6 meters per second. How fast is its shadow moving?

10. You want to see a certain number n of items in order to maximize your profit. Market research tells you that if you set the price at $\$1.50$, you will be able to sell 5000 items and for every 10 cents you lower the price below $\$1.50$ you will be able to sell another 1000 items. Suppose that your fixed costs total \$2000, and the per item cost of production is \$0.50. Find the price to set per item and number of items sold in order to maximize profit. Recall that profit = revenue - cost and revenue = number of items sold times price per item.