

Economics 448: Development and Growth Accounting

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Growth & Development Accounting

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2. How much of the variation in the income per capita among countries is explained by productivity differences?
3. How much does productivity **growth** differ among countries?
4. How much variation in growth rates among countries is explained by variation in productivity growth, and how much by variation in factor accumulation?

Define Terms:

Same basic idea used in **growth** and **development** accounting.

Growth Accounting used with time series data (e.g., annual information on a single country).

Development Accounting used to compare two countries at the same point in time. Typically use cross-sectional data (on countries, geographical regions).

Basic Idea

Production is composed of two parts:

$$\text{Output} = \text{Productivity} \times \text{Factors of Production}$$

Does the USA produces more than UK because of (a) greater productivity; (b) accumulated more factors (physical & human capital) or (c) **both**?

In comparing two countries will want to decompose differences in output into differences in productivity and differences in **accumulation**; factors of production.

Make comparison for any set of countries.

Growth Acct, with Cobb Douglas PF

Use Cobb Douglas (*per capita*) production function :

$$y(t) = A(t)k(t)^\alpha h(t)^{1-\alpha}$$

where $A(t)$ is a general productivity term

$k(t)^\alpha h^{1-\alpha}$ composite term of two factors (physical & human capital)

Development Accounting

Start with basic idea:

$$\text{Output} = \text{Productivity} \times \text{Factors of production}$$

Assume each country $i = 1, 2$ has Cobb–Douglas production function

drop time subscript as doing calculation at the same t

$$Y_i = A_i K_i^\alpha N_i^{(1-\alpha)}$$

where N_i is working population or human capital in country i

A_i measure of productivity

$K_i^\alpha N_i^{1-\alpha}$ composition factor of production

Development Accounting (cont)

Divide p.f of country 1 by p.f. country 2:

$$\frac{y_1}{y_2} = \frac{A_1 K_1^\alpha N_1^{1-\alpha}}{A_2 K_2^\alpha N_2^{1-\alpha}}$$

$$\frac{y_1}{y_2} = \left[\frac{A_1}{A_2} \right] \left(\frac{K_1^\alpha N_1^{1-\alpha}}{K_2^\alpha N_2^{1-\alpha}} \right)$$

$$Q = P \times F$$

or

$$P = \frac{Q}{F} = \frac{y_1 / y_2}{K_1^{\alpha_1} N_1^{1-\alpha} / K_2^{\alpha_2} N_2^{1-\alpha_2}}$$

Example:

Table : Data to Compare Productivity

Country	y	k	h
1	24	27	8
2	1	1	1

Example - Calculation

Assume that countries have same technology with income share of capital $\alpha = 1/3$ and $1 - \alpha = 2/3$ the income share of human capital.

$$\begin{aligned}\frac{A_1}{A_2} &= \frac{\frac{24}{1}}{\frac{27^{1/3} \times 8^{2/3}}{1^{1/3} \times 1^{2/3}}} \\ &= \frac{24}{\frac{3 \times 4}{1}} = 2.\end{aligned}$$

Hence, Country 1 has twice the productivity of Country 2.

Developmental Growth Accounting (2005)

Country	Output Y/P	Phys K/P	Human h/P	Factors $k^{1/3}h^{2/3}$	Productivity A
USA	1.00	1.00	1.00	1.00	1.00
Norway	0.92	1.08	0.97	1.01	0.92
UK	0.76	0.69	0.97	0.87	0.87
Canada	0.75	0.86	1.01	0.96	0.79
Japan	0.69	1.10	0.99	1.02	0.67
S.Korea	0.54	0.73	0.93	0.86	0.63
Mexico	0.29	0.27	0.79	0.56	0.52
Peru	0.14	0.12	0.82	0.44	0.32
India	0.13	0.10	0.74	0.38	0.35
Cameroon	0.13	0.036	0.58	0.23	0.44
Zambia	0.034	0.032	0.65	0.24	0.14

Source: Weil (2009) *Economic Growth*, 2nd Ed. p.193

Developmental Growth Accounting (2009)

Country	Output Y/P	Phys K/P	Human h/P	Factors $k^{1/3}h^{2/3}$	Productivity A
USA	1.00	1.00	1.00	1.00	1.00
Norway	0.92	1.32	0.98	1.08	1.04
UK	0.82	0.68	0.87	0.80	1.03
Canada	0.80	0.81	0.96	0.91	0.88
Japan	0.73	1.16	0.98	1.04	0.70
S.Korea	0.62	0.92	0.98	0.96	0.64
Turkey	0.37	0.28	0.78	0.55	0.68
Mexico	0.35	0.33	0.84	0.61	0.56
Brazil	0.20	0.19	0.78	0.48	0.42
India	0.10	0.089	0.66	0.34	0.31
Kenya	0.032	0.022	0.73	0.23	0.14
Malawi	0.018	0.029	0.57	0.21	0.087

Source: Weil (2012) *Economic Growth*, 3rd Ed. p.186

Growth Accounting

Now want to make comparison over time to compare **rates** of change of output, factor accumulation and productivity.
Once again start with the per capita Cobb–Douglas production function

$$y(t) = A(t)k^{\alpha}(t)h(t)^{1-\alpha}$$

Take logs to yield:

$$\ln y(t) = \ln A(t) + \alpha \ln k(t) + (1 - \alpha) \ln h(t)$$

Time Derivatives of $\ln(z(t))$

Recall that the time derivative of $\ln(z(t))$ is:

$$\frac{d \ln(z(t))}{dt} = \frac{1}{z} \frac{dz}{dt}$$

Growth Accounting

Take derivative w.r.t. time t

$$\frac{1}{y} \frac{dy}{dt} = \frac{1}{A} \frac{dA}{dt} + \alpha \frac{1}{k} \frac{dk}{dt} + (1 - \alpha) \frac{1}{h} \frac{dh}{dt}$$

Represent time derivative by a dot above the variable, $\dot{z} = \frac{dz}{dt}$.
Use “carrot” to denote a percent change $\frac{\dot{z}}{z} = \hat{z}$.

$$\hat{y} = \hat{A} + \alpha \hat{k} + (1 - \alpha) \hat{h}$$

Recall that α is the income share of capital while $1 - \alpha$ is the income share of human capital.

Growth Accounting

Thus the rate of growth of output is the sum of productivity growth and the share weight sum the growth of factors of production.

We observe: y , k , h . Requires effort and much attention to detail. Calculation where the devil is in the details.

Direct measurement of the rate of growth of productivity is not credible. (You could try, but no matter the estimate, no one would believe it.)

Hence, “measure” growth rate of productivity as **residual**

$$\hat{A} = \hat{y} - \alpha \hat{k} - (1 - \alpha) \hat{h}$$

Growth Accounting

The above formulation assumes data on education (to measure HC) is available.

Show for yourself that if the production function is:

$$Y(t) = A(t)K(t)^\alpha P(t)^{1-\alpha}$$

then the growth accounting equation is:

$$\hat{y} = \alpha \hat{k} + (1 - \alpha) \hat{P} + \hat{A}$$

Comparison with Textbook

$$\hat{y} = \alpha \hat{k} + (1 - \alpha) \hat{P} + \hat{A}$$

Textbook:

$$\frac{\Delta Y(t)}{Y(t)} = \sigma_k(t) \frac{\Delta K(t)}{K(t)} + \sigma_P(t) \frac{\Delta P(t)}{P(t)} + \text{TFPG}(t)$$

$$\text{TFPG} = \hat{A}$$

Ray's formulation allows income shares of capital and labor to vary over time.

Comments on TFP

Important $P(t)$ should be the **working** population. Sometimes well approximated by total population, sometimes times not.

Total population not accurate for labor force if major changes in labor force composition (entry by women, or longer schooling period or declining retirement age).

TFP Growth

- ▶ Units of A are arbitrary so **level** of A is meaningless. What's important is the **rate of change** of TFP.
- ▶ Assumed production function exhibits constant returns to scale. **where assumed?**
- ▶ If production function exhibits increasing return to scale the observed factor shares **underestimate** the true productivity of factors. Which implies we **overestimate** the rate of technical progress.