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Q.3 MLE for kumarswamy Distribution.

$$x_i = \text{kumarswamy}(a, b)$$

$$f(x|a,b) = a \cdot b \cdot x^{a-1} (1-x^a)^{(b-1)}$$

Assume a is known.

Question: Derive a formula for estimating b.

Solution: Derive the likelihood.

$$P(x_1, x_2, \dots x_n | a, b) = \prod_{k=1}^{n} P(x_k | a, b)$$

$$\& \quad P(x_k | a, b) = a \cdot b \cdot x^{a-1} (1-x^a)^{(b-1)}$$

Step 1: Applying log i.e. ln

$$\ln [P(x_k | a, b)] = \sum_{k=1}^{n} \ln [a \cdot b \cdot x_k^{a-1} (1-x_k^a)^{(b-1)}]$$

for simplicity, assume $\ln [P(x_k | a, b)] = f(x)$

$$f(x) = \ln [a \cdot b \cdot x_1^{a-1} (1-x_1^a)^{b-1}] + \dots + \ln [a \cdot b \cdot x_n^{a-1} (1-x_n^a)^{b-1}]$$

$$f(x) = [\ln (a \cdot b) + \ln (x_1)^{a-1} + \ln (1-x_1^a)^{b-1}] + \dots + [\ln (a \cdot b) + \ln (x_n)^{a-1} + \ln (1-x_n^a)^{b-1}]$$

$$f(x) = [\ln (a \cdot b) + (a-1) \ln (x_1) + (b-1) \ln (1-x_1^a)] + \dots + [\ln (a \cdot b) + (a-1) \ln (x_n) + (b-1) \ln (1-x_n^a)]$$

$$f(x) = n \cdot \ln (a \cdot b) + (a-1) [\ln (x_1) + \dots + \ln (x_n)] + (b-1) [\ln (1-x_1^a) + \dots + \ln (1-x_n^a)]$$

Step 2: Differentiating w.r.t. b

$$\frac{\partial f(x)}{\partial b} = n \cdot \frac{1}{a \cdot b} \cdot a + 0 + [\ln (1-x_1^a) + \dots + \ln (1-x_n^a)](1)$$

$$= \frac{n}{b} + [\ln (1-x_1^a) + \dots + \ln (1-x_n^a)]$$

Step 3: Equating to 0

$$\therefore \frac{n}{b} + [\ln (1-x_1^a) + \dots + \ln (1-x_n^a)] = 0$$

$$[\ln (1-x_1^a) + \dots + \ln (1-x_n^a)] = \frac{-n}{b}$$

$$\therefore b = \frac{-n}{[\ln (1-x_1^a) + \dots + \ln (1-x_n^a)]}$$

$$\boxed{\therefore b = \frac{-n}{\sum_{k=1}^{n} [\ln (1-x_k^a)]}}$$