30-10-2008

BILKENT UNIVERSITY

Department of Electrical and Electronics Engineering

EEE102 Introduction to Digital Circuit Design

Midterm Exam I SOLUTION

Surname: ________________________________

Name: ________________________________

ID-Number: ________________________________

Signature: ________________________________

Duration is 120 minutes. Solve all 6 questions. Show all your work.

Q1 (12 points	
Q2 (18 points)	
Q3 (15 points)	
Q4 (15 points)	
Q5 (10 points)	
Q6 (30 points)	
Total	

Q1.

a) **Convert the decimal numbers 75 and 80 to 8-bit two's complement representation. Subtract 80 from 75 using binary arithmetic, to obtain an 8-bit two's complement binary number. Show all carry/borrow bits used in finding the result. Indicate whether your result is valid and explain why.**

b) **Convert the decimal numbers 75 and 80 to 8-bit two's complement representation. Subtract 80 from -75 using binary arithmetic, to obtain an 8-bit two's complement binary number. Show all carry/borrow bits used in finding the result. Indicate whether your result is valid and explain why.**

c) **What is the decimal equivalent of the two's complement binary number 1111111110101 ?**

d) **Write down the BCD code for the decimal number 1756.**

Solution:

a) 75 = 01001011
80 = 01010000
-80 = 10110000

75-80:

```
 000000000  (carries)
  01001011
  10110000
+-------------
  11111011
```

The result is valid because there is no overflow. This is so because one of the numbers that we add is positive and the other is negative (or equivalently the last two carries are the same).

b) 75 = 01001011
-75 = 10110101
80 = 01010000
-80 = 10110000

-75 -80 :

```
  101100000  (carries)
   10110101
   10110000
+---------------
  101100101
```

We throw away the 9th bit and the answer is 01100101. This answer is not valid because there is overflow. This is so because the two numbers we add are negative but the result is positive (or equivalently the last two carries are different).

c) 1111111110101 is a negative number and it is equivalent to 10101 which is decimal -11.

d) The BCD code for the decimal number 1756 is 0001011101010110

Q2. a) Represent each of the following functions by a minterm list using the Σ notation:
i) $x_1 \oplus x_2 \oplus x_3$ **; ii)** $x_1 + x_2' + x_3'$ **; iii)** $x_1 x_2 + x_3 x_4$ **.**

b) Represent each of the following functions by a maxterm list using the Π notation:
i) $a \,.\, (b \oplus c \oplus d)'$ **; ii)** $ab + c(a' + d')$ **; iii)** $a + (b + (c + d + e))$**.**

Solution:

a) i) $x_1 \oplus x_2 \oplus x_3 = \sum_{x_1,x_2,x_3} (1,2,4,7)$

ii) $x_1 + x_2' + x_3' = \sum_{x_1,x_2,x_3} (0,1,2,4,5,6,7)$

iii) $x_1 x_2 + x_3 x_4 = \sum_{x_1,x_2,x_3,x_4} (3,7,11,12,13,14,15)$

b) i) $a(b \oplus c \oplus d)' = \prod_{a,b,c,d} (0,1,2,3,4,5,6,7,9,10,12,15)$

ii) $ab + c(a' + d') = \prod_{a,b,c,d} (0,1,4,5,8,9,11)$

iii) $a + (b + (c + d + e)) = \prod_{a,b,c,d,e} (0)$

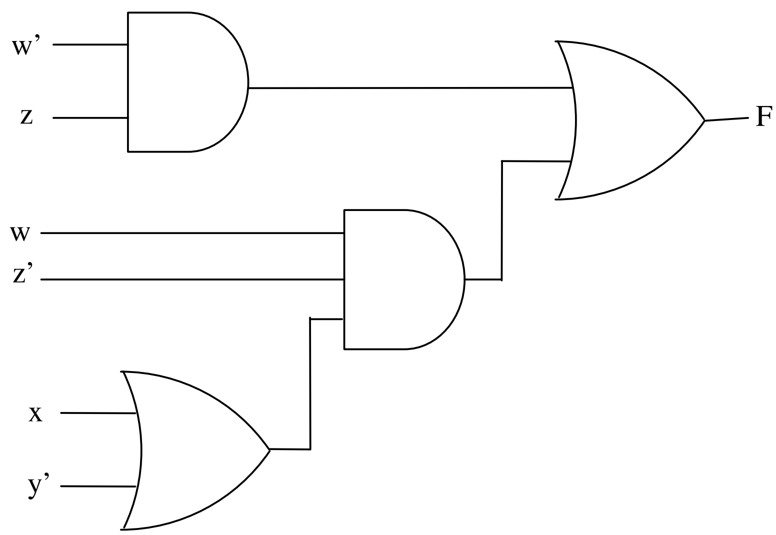
Q3. a) Is the set {AND, XOR} functionally complete? Prove your answer. (In this context “functionally complete” means “being able to implement any combinational circuit”.)

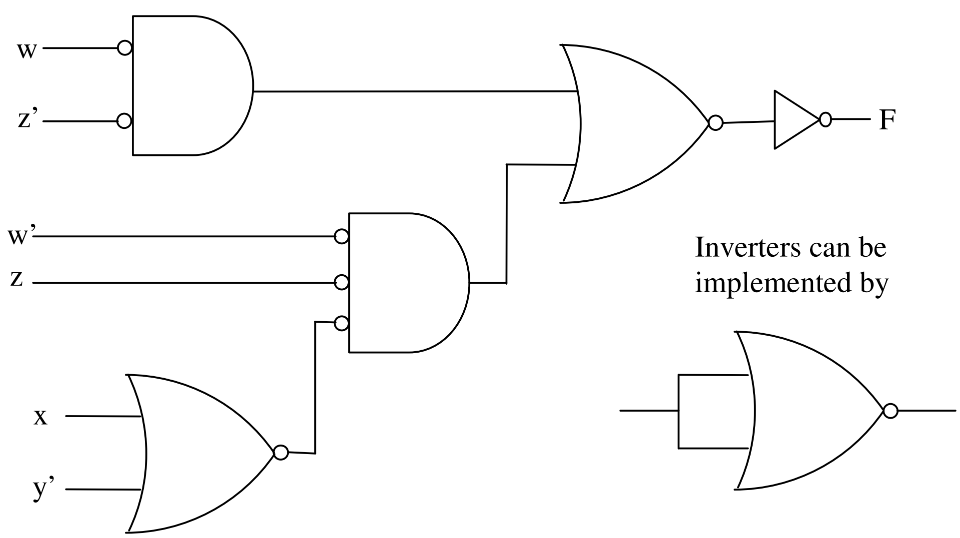
b) **First, draw the logic circuit for the function** $F(w,x,y,z) = w'z + wz'(x+y')$. **Second, draw the same circuit using NOR gates only.**

Solution:

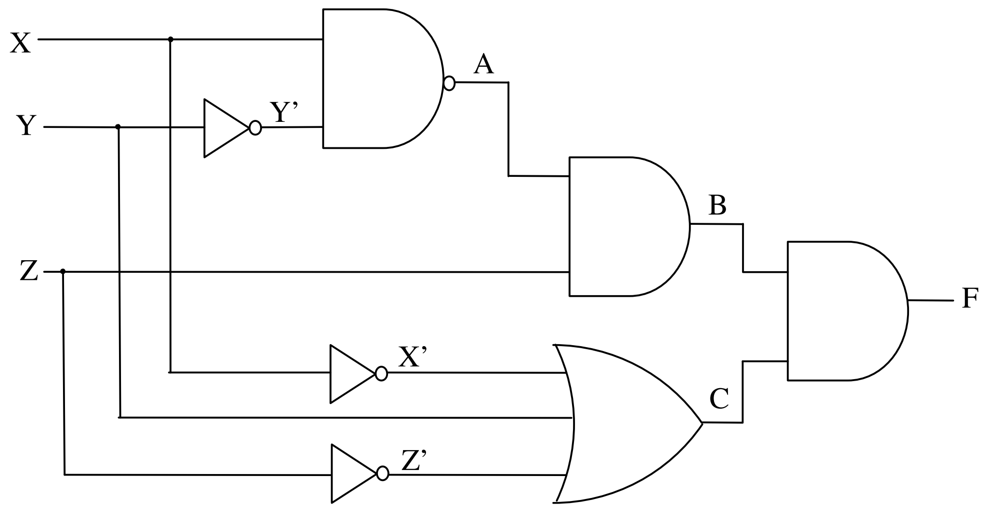
a) **Yes because given AND and XOR gates we can implement all gates.**
Proof: $XOR(x,y) = x \oplus y = xy' + x'y \Rightarrow XOR(x,1) = x' = NOT(x)$
Thus we have AND and NOT gates from which we can obtain NAND gates. We know that NAND gates are functionally complete.

b)

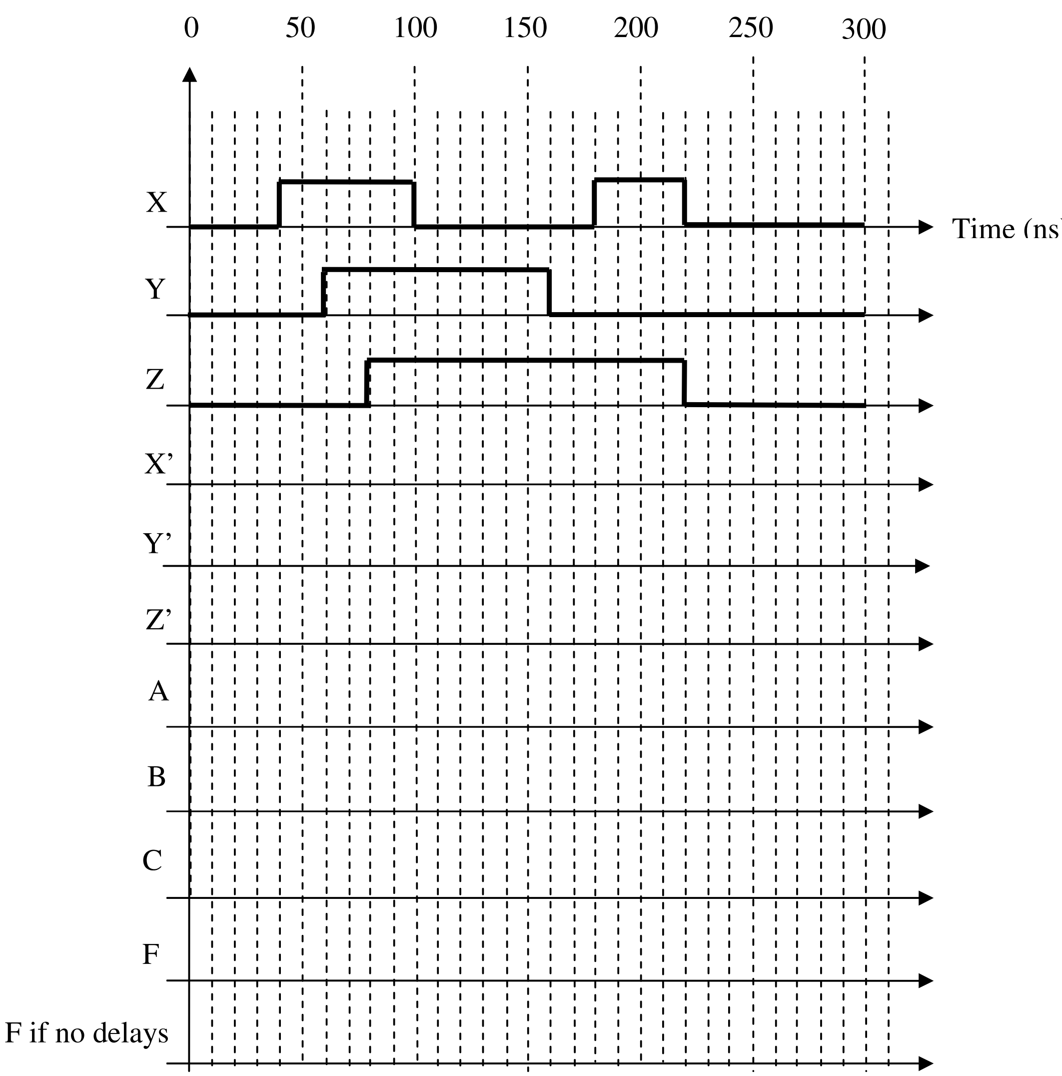




Q4. For the circuit given below draw the waveforms of X’ ,Y’, Z’, A, B, C, and F for the given waveforms of X, Y, and Z if each gate has 10 ns delay. Also draw F if the gates do not have any delays.

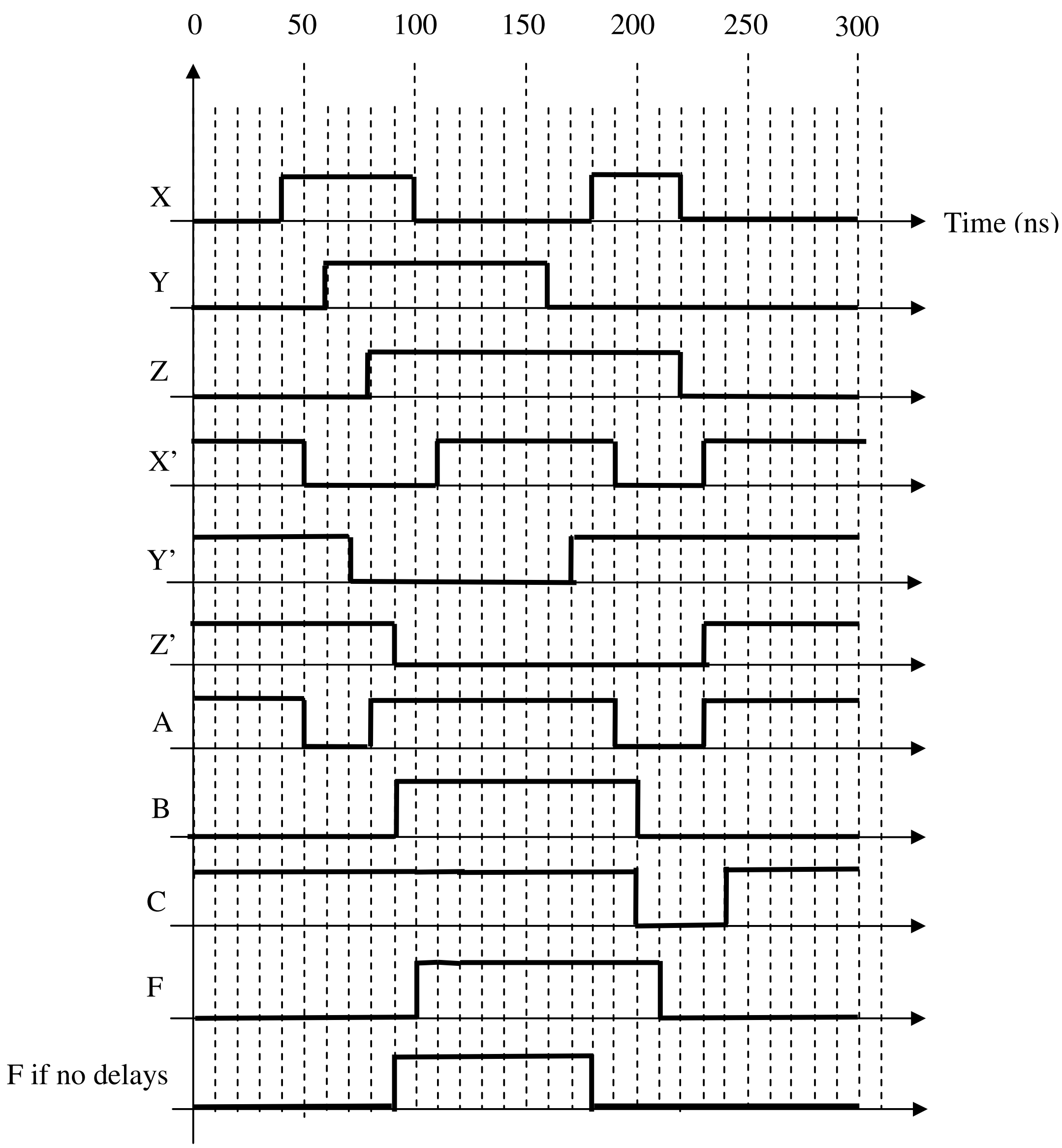


X
Y
Z
Y'
A
B
F
X'
C
Z'



0
50
100
150
200
250
300
X
Time (ns)
Y
Z
X'
Y'
Z'
A
B
C
F
F if no delays

Solution: If the gates do not have any delays then F = (XY')'Z(X'+Y+Z') = (X'+Z)(X'Z+YZ) = X'Z+X'YZ+X'Z+YZ = X'Z+YZ = (X'+Y)Z



Q5)

- **a) State the concensus theorem and prove it.**
- **b) Expand the function F = AB' + C' + BC with respect to C using Shannon's theorem.**
- **c) Find the complement of the function F = A + BC'**
- **d) Find the dual of the function F = A + BC' + 1**

Solution:

a) **XY+X'Z+YZ=XY+X'Z is the consensus theorem.**

Proof: XY+X'Z+YZ = XY+X'Z+(X+X')YZ

= XY+X'Z+XYZ+X'YZ

= (XY+XYZ)+(X’Z+X’YZ)
= XY+X’Z

b) F = AB’ + C’ + BC =C(AB’ + B)+C’(AB’ + 1) = C(AB’ + B)+C’

c) F = A + BC’ = A + (BC’)   F’= A’(B’+C)

d) F = A + (BC’) + 1 F^D = A(B+C’) 0 = 0

Q6)

a) **For the function F given below find all minimal sum of products and all minimal products of sums. Show all prime implicants, all distinguished 1s and 0s, all essential prime implicants, and draw the reduced maps if there are any.**

F:

CD \ AB	00	01	11	10
00	0: **1**	4: **1**	12: **1**	8: **0**
01	1: **1**	5: **1**	13: **1**	9: **0**
11	3: **1**	7: **1**	15: **1**	11: **1**
10	2: **1**	6: **0**	14: **0**	10: **0**

A B C D

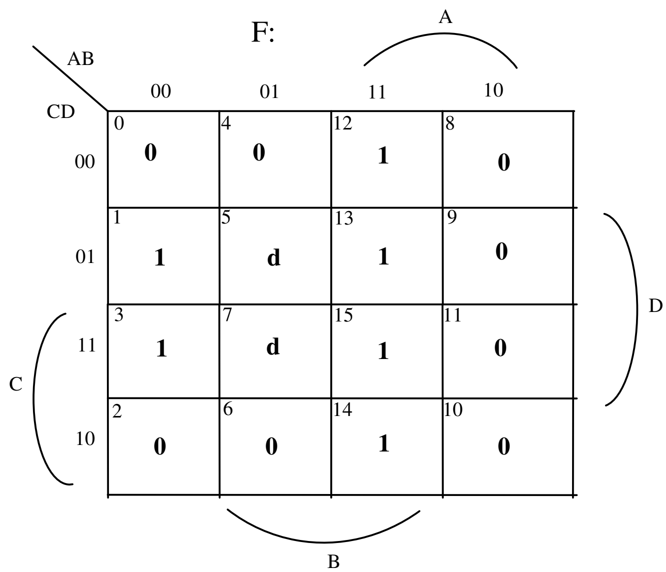
b) **For the function given below**

i) Find all minimal sums of products. Show all prime implicants, all distinguished 1s, all essential prime implicants, and draw the reduced map if there is one.

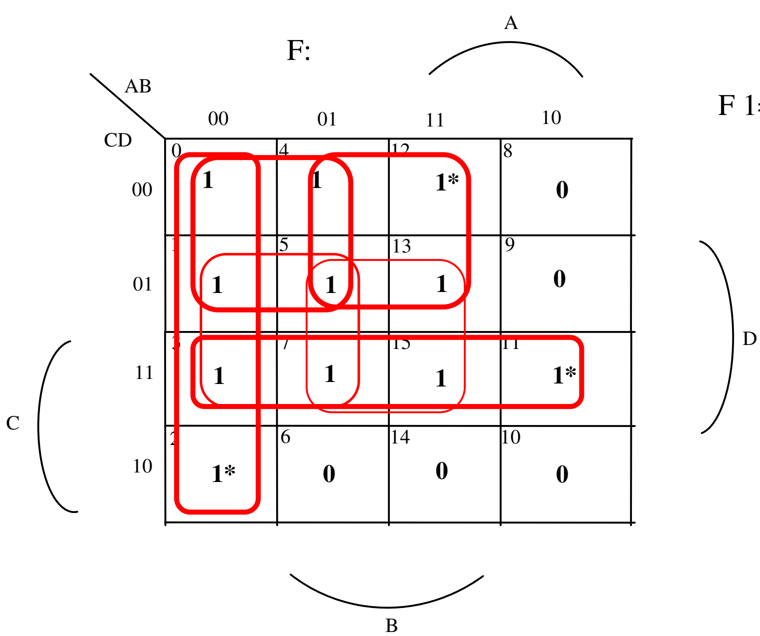
ii) Find all minimal products of sums. Show all prime implicants, all distinguished 0s, all essential prime implicants, and draw the reduced map if there is one.

iii) Which of the solutions in i) and ii) are equivalent?

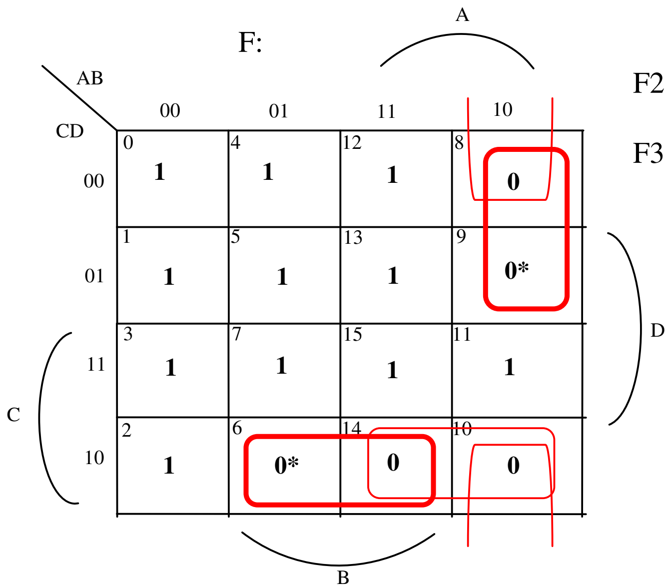
iv) Which of the solutions in i) and ii) would you actually implement? Why?



Solution:
a)



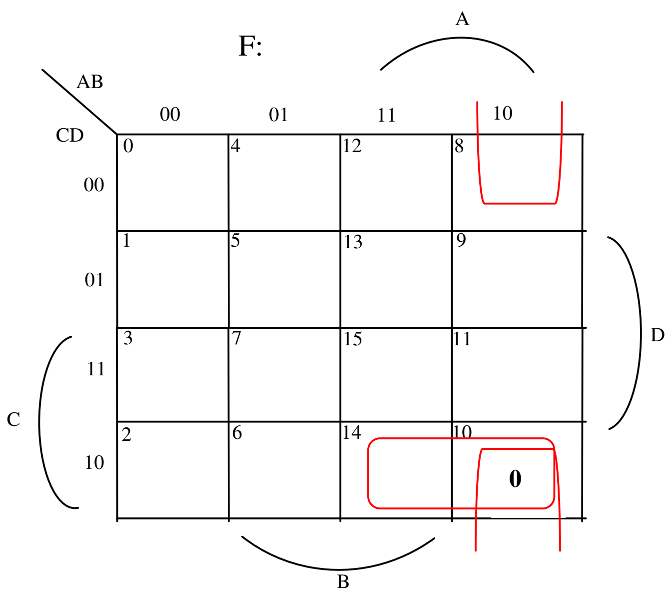
F 1= A'B'+BC'+CD



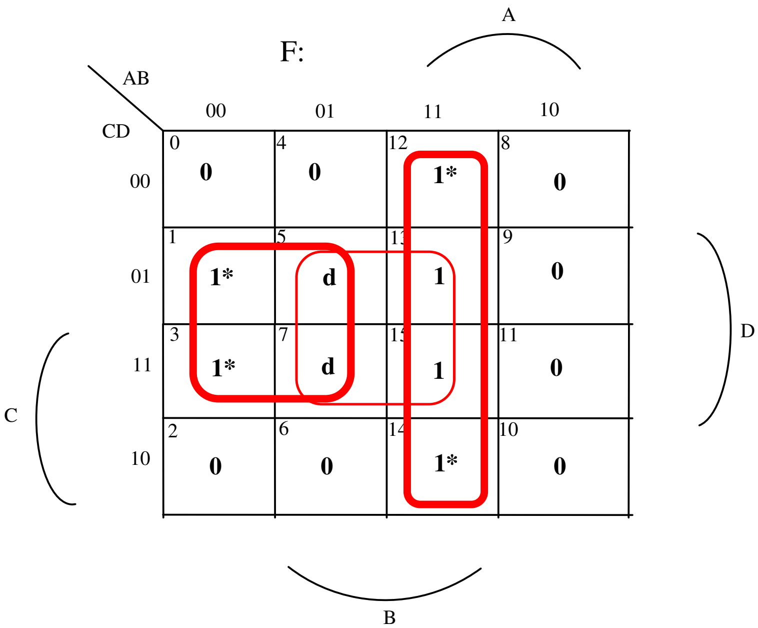
F2 = (B’+C’+D)(A’+B+C)(A’+B+D)

F3 = (B’+C’+D)(A’+B+C)(A’+C’+D)

The reduced map is

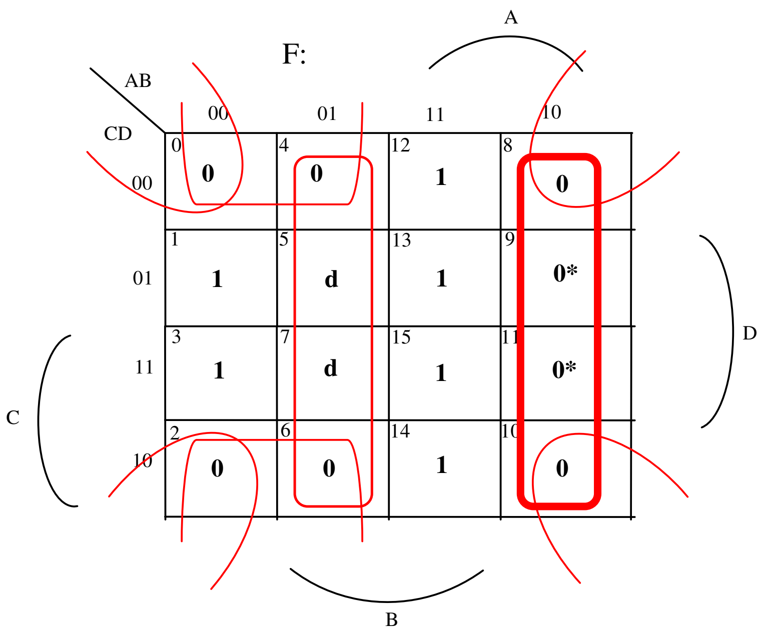


b) i)



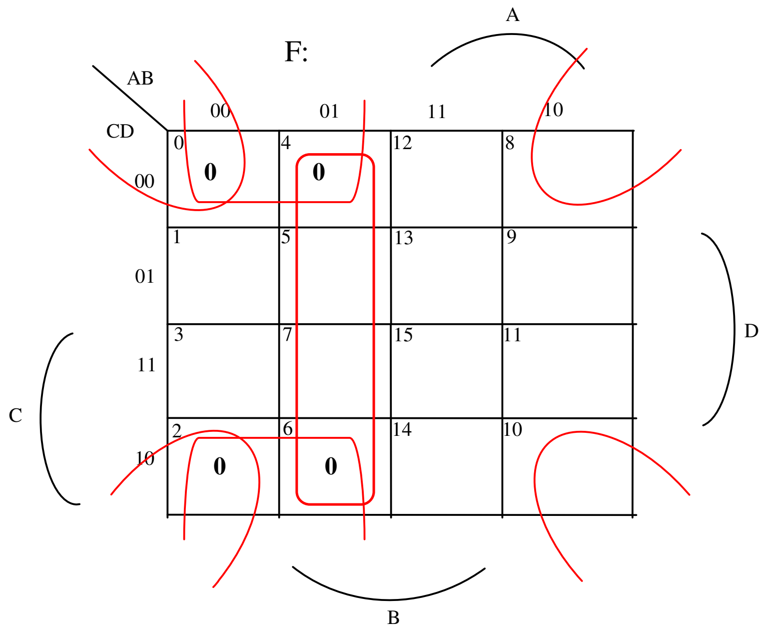
F = AB + A'D

ii)



F = (A'+B)(A+ D)

The reduced map is



iii) The two solutions are equivalent because in both cases the ds are taken as 1s.
iv)Any one of the solutions can be implemented because they both have 2 terms and 4 literals.