Recitation 2

EE102, Spring 2024-25

Q4.

For the TT given below find

a) two-level canonical SOP expression for F

b) two-level canonical POS expression for F

X	Y	Z	F	
0	0	0	1	SOP
0	0	1	1	SOP
0	1	0	1	SOP
0	1	1	0	POS
1	0	0	0	POS
1	0	1	1	SOP
1	1	0	0	POS
1	1	1	1	SOP

F is the output and X,Y,Z are inputs.

Solution:

a) F = X'Y'Z' + X'Y'Z + X'YZ' + XY'Z + XYZ

b) F = (X+Y'+Z')(X'+Y+Z)(X'+Y'+Z)

Q5.

. For the TT given below find

a) two-level canonical SOP expression for F

b) two-level canonical POS expression for F

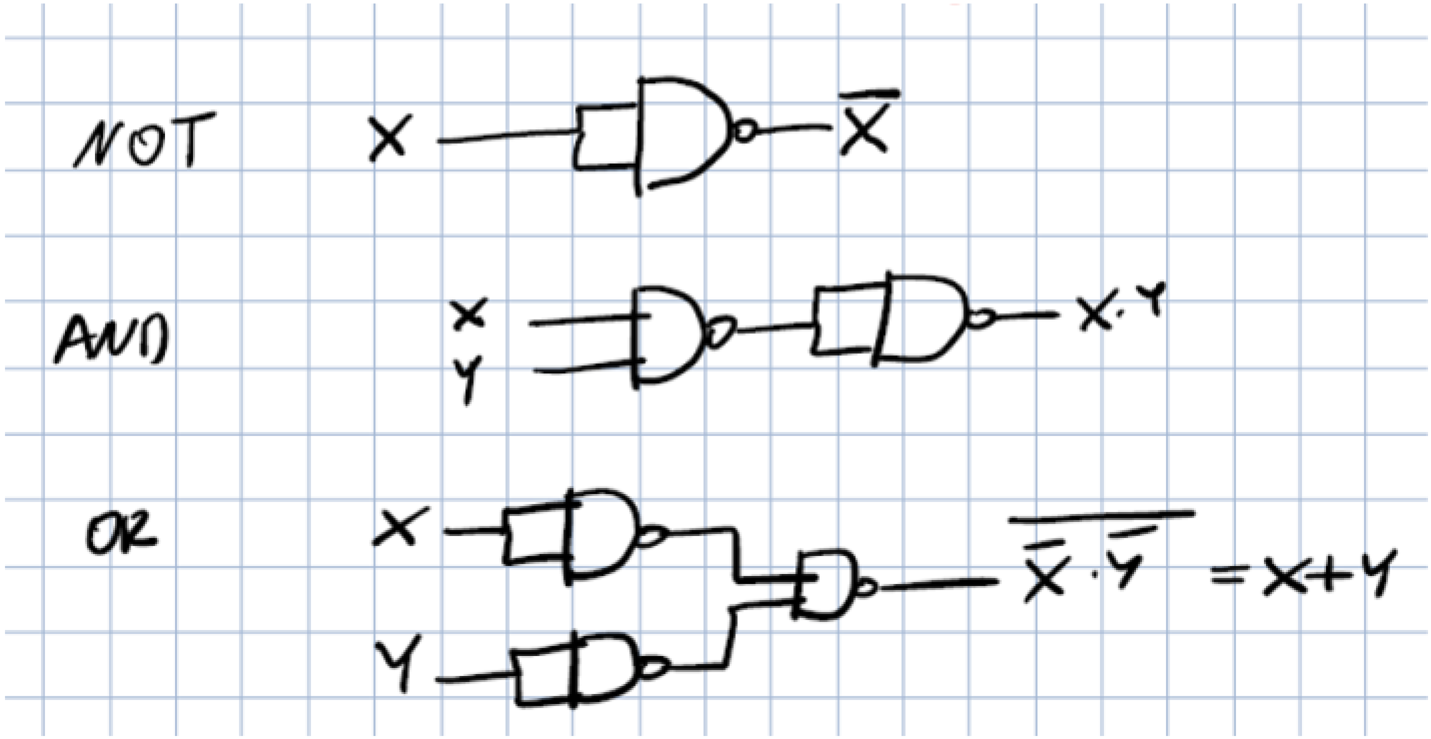
X	Y	Z	F	
0	0	0	1	SOP
0	0	1	1	SOP
0	1	0	0	POS
0	1	1	1	SOP
1	0	0	0	POS
1	0	1	1	SOP
1	1	0	0	POS
1	1	1	1	SOP

F is the output and X,Y,Z are inputs

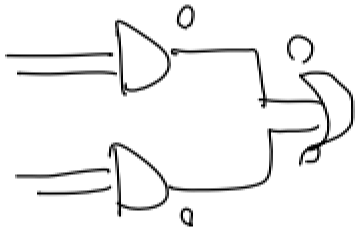
Solution:

a) $F = X'Y'Z' + X'Y'Z + X'YZ + XY'Z + XYZ$

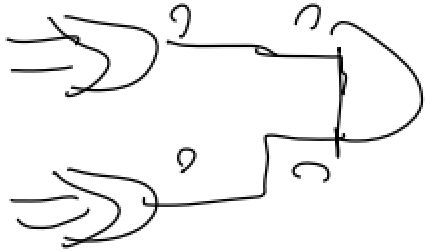
b) $F = (X+Y'+Z)(X'+Y+Z)(X'+Y'+Z)$

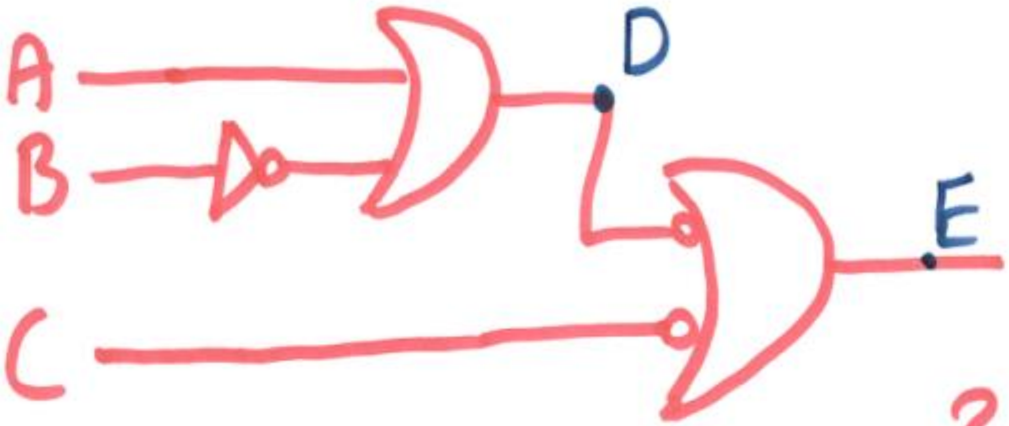


SOP using NAND



POS using NOR

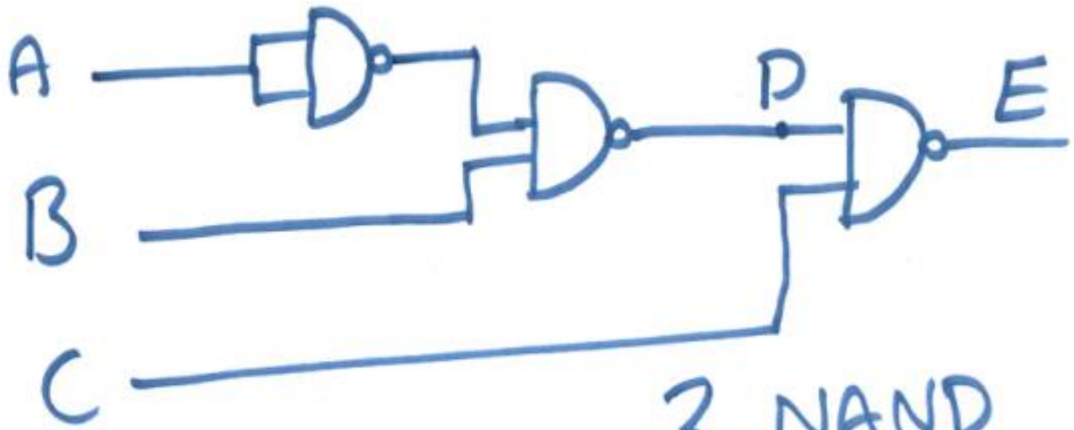




use NAND gates only?

$$D = A + B' \underset{\text{rule}}{\overset{\text{demorgan's}}{=}} (A'B)'$$

$$E = (CD)'$$



3 NAND gates

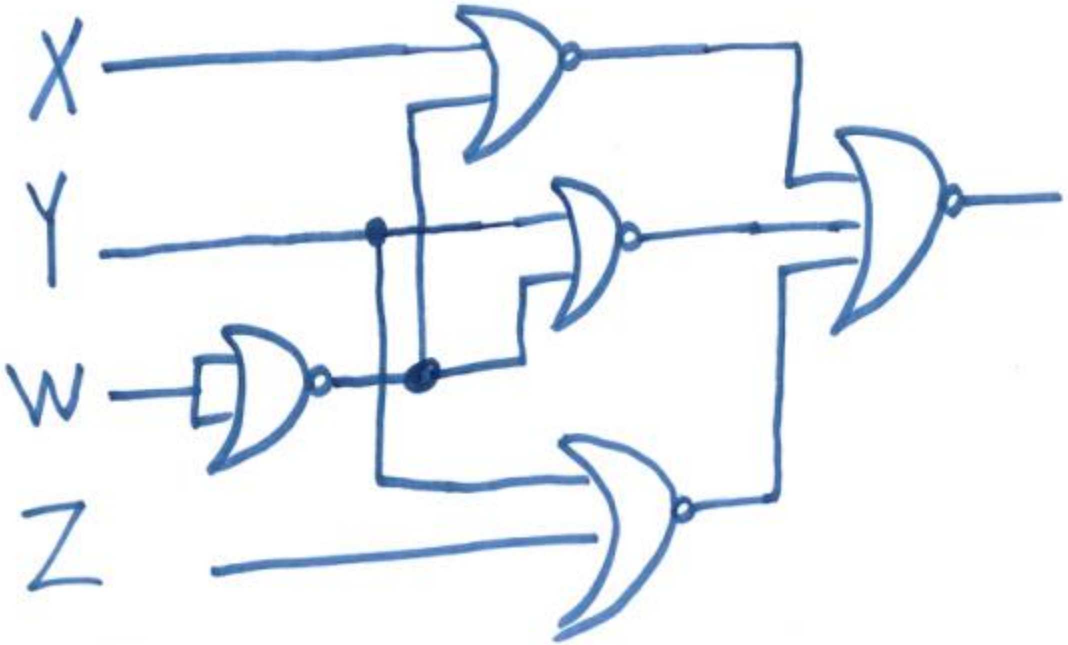
$$F = \underbrace{(W' + X)}_{A}\underbrace{(W' + Y)}_{B}\underbrace{(Y + Z)}_{C}$$

draw the schematic's of F using NOR

$$F = ABC \xrightarrow[\text{rule}]{\text{dem.}} (A' + B' + C')'$$

$$A' = (W' + X)' \; ; \; B' = (W' + Y)'$$

$$C' = (Y + Z)'$$



KARNAUGH MAPS

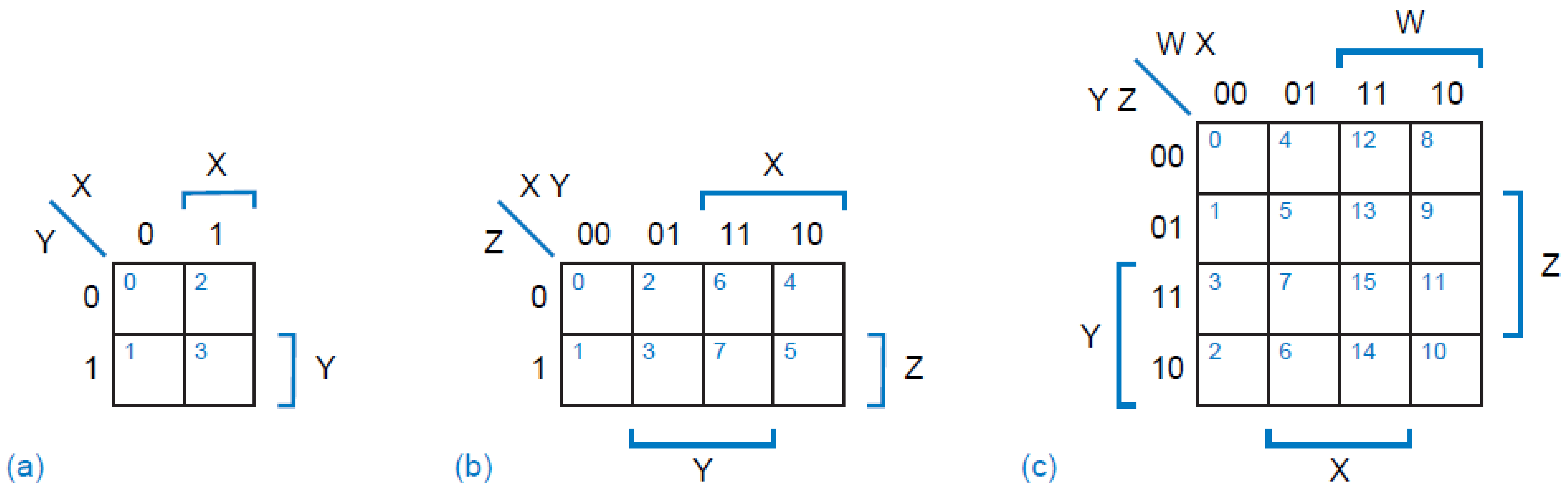


Figure 4-26 Karnaugh maps: (a) 2-variable; (b) 3-variable; (c) 4-variable.

Q1.

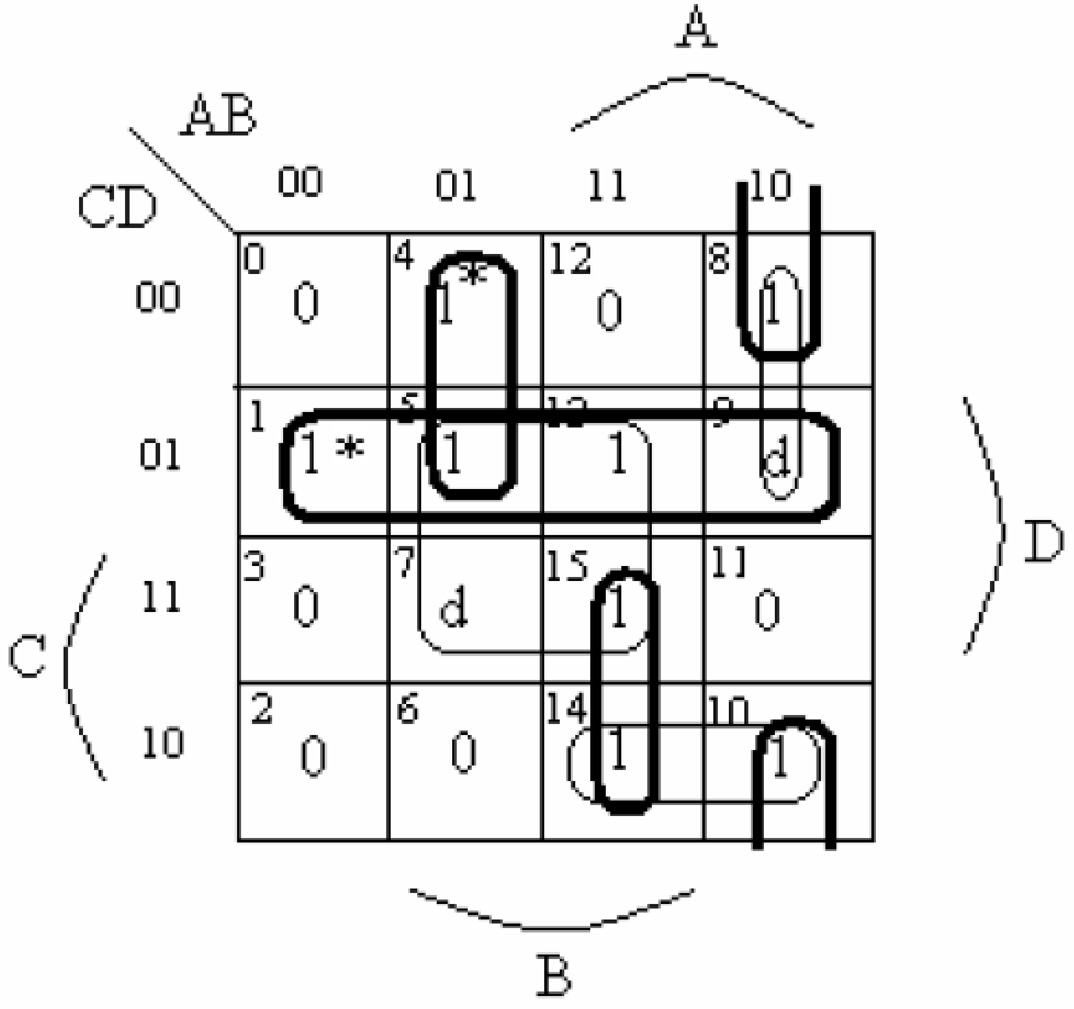
Given the function $F(A,B,C,D) = \Sigma_{A,B,C,D}(1,4,5,8,10,13,14,15) + d(7,9)$.

a) Find all minimal sums for F.

b) Find all minimal products for F

c) Which of the above minimal sums are equivalent to which of the above minimal products? Equivalent means having the same Truth Table.

a)



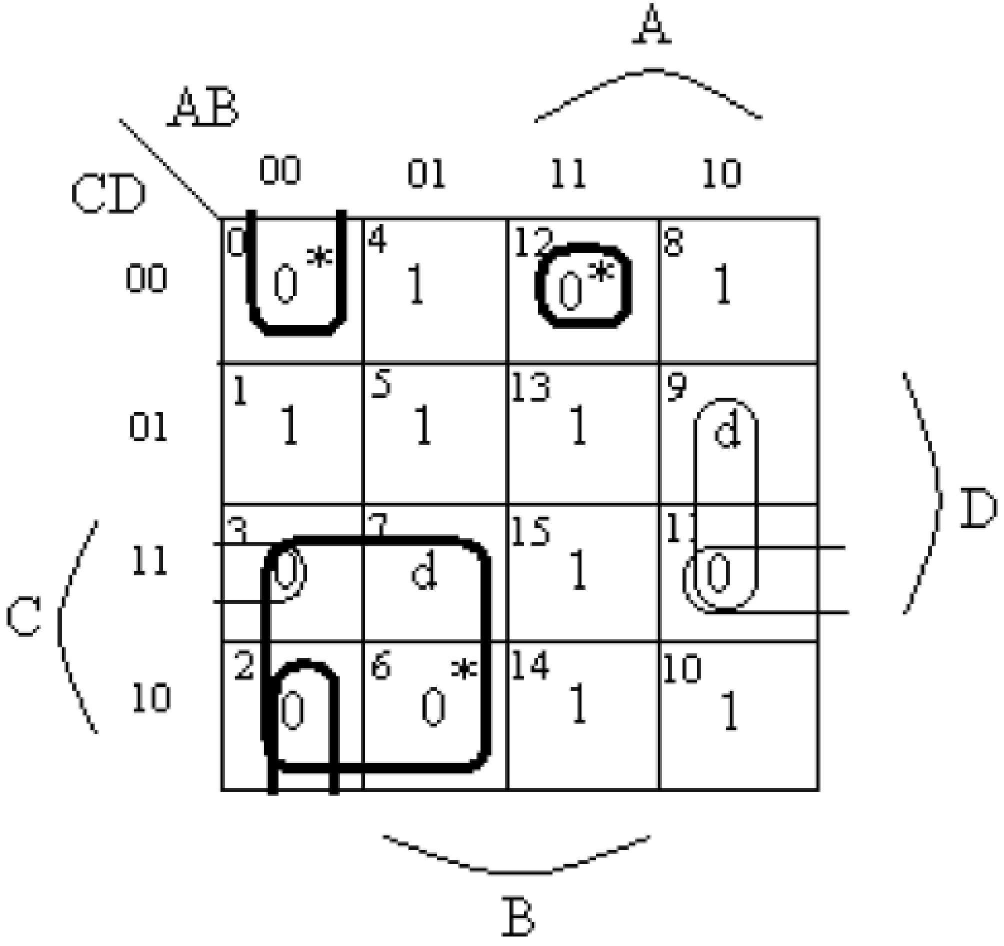
<u>SOP1:</u>

F = C’D+A’BC’+ABC+AB’D’

7 is zero

9 is one

b)



POS1:

F =

(A+B+D)(A+C')(A'+B'+C+D)(B+C'+D')

7 is zero

9 is one

POS2:

F = (A+B+D)(A+C')(A'+B'+C+D)(A'+B+D')

7 is zero

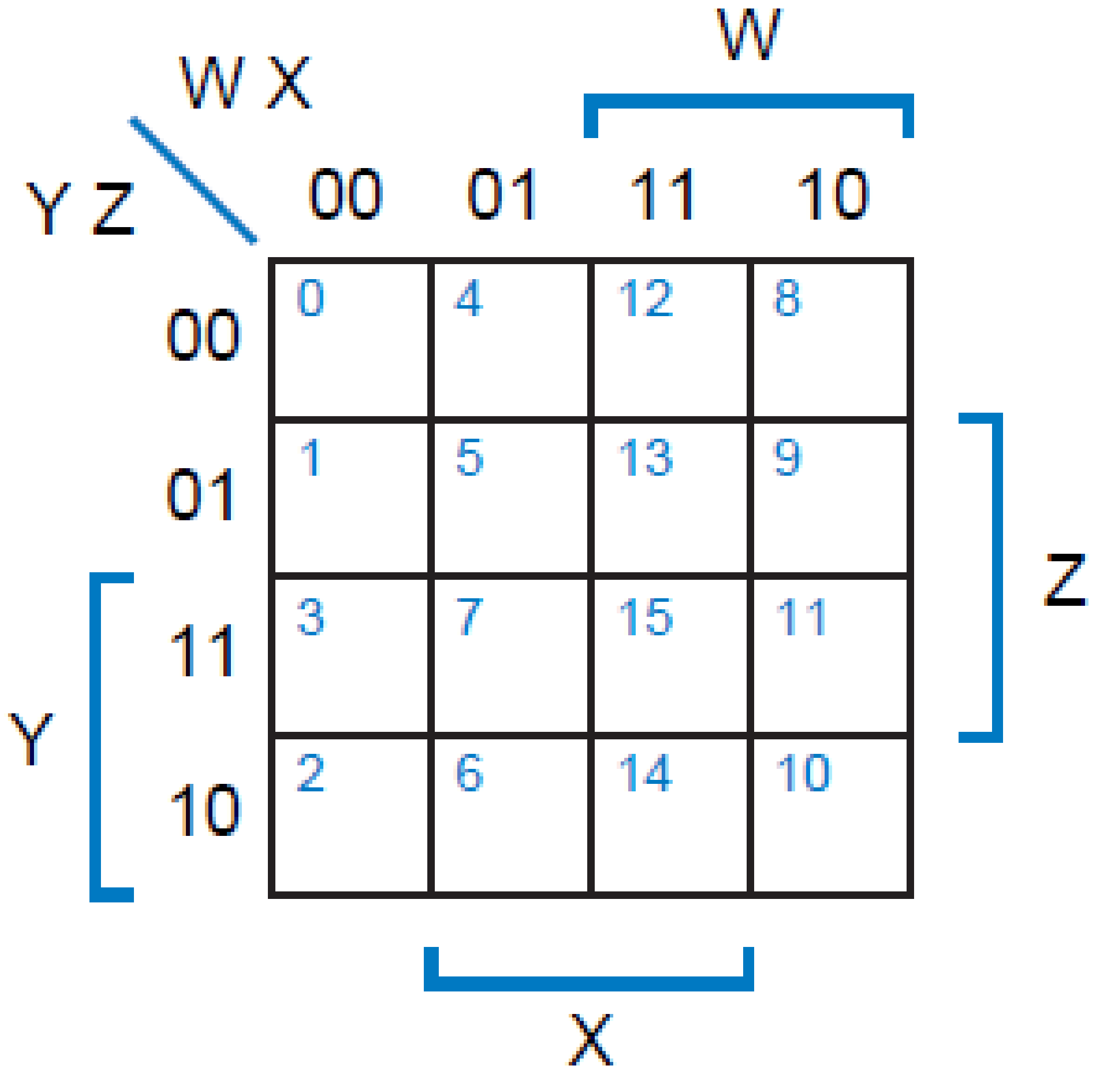
9 is zero

c) SOP1 is equivalent to POS1 because in each case 7 is zero, 9 is one.

Q3.

Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\,(0,1,3,5,14) + d\,(8,15)$
b) $F = \Sigma_{W,X,Y,Z}\,(2,5,13,15,10) + d\,(8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\,(4,5,6,7,10,12,13,14) + d\,(3,9,15)$
d) $F = \Sigma_{W,X,Y,Z}\,(0,1,3,5,14) + d\,(8,12,15)$
e) $F= \Sigma_{W,X,Y,Z}\,(0,2,4,6,8,10,12,14) + d\,(9,13)$
f) $F= \Sigma_{W,X,Y,Z}\,(0,2,8,10) + d\,(5,13)$



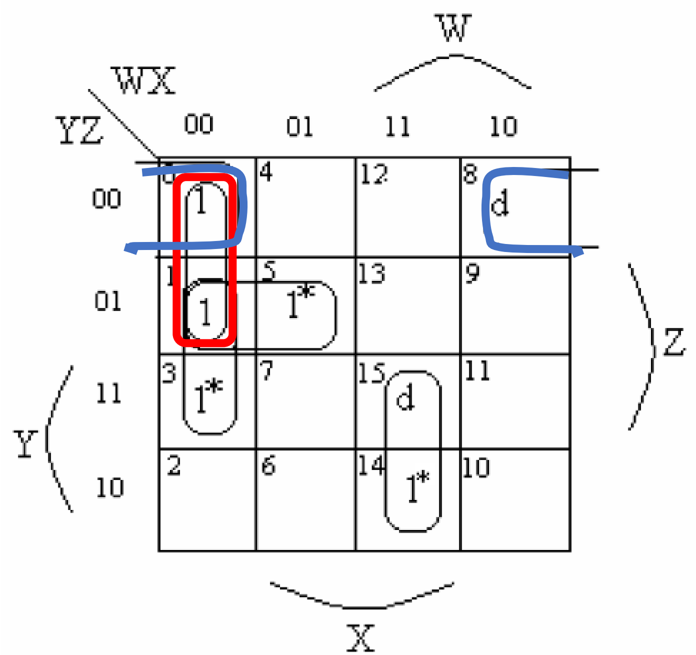
Q3.

Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,15)$
b) $F = \Sigma_{W,X,Y,Z}\ (2,5,13,15,10) + d\ (8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\ (4,5,6,7,10,12,13,14) + d\ (3,9,15)$
d) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,12,15)$
e) $F= \Sigma_{W,X,Y,Z}\ (0,2,4,6,8,10,12,14) + d\ (9,13)$
f) $F= \Sigma_{W,X,Y,Z}\ (0,2,8,10) + d\ (5,13)$

Solution:

a) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,15)$



$F = W'Y'Z + W'X'Z + WXY + \underline{X'Y'Z'}$

or

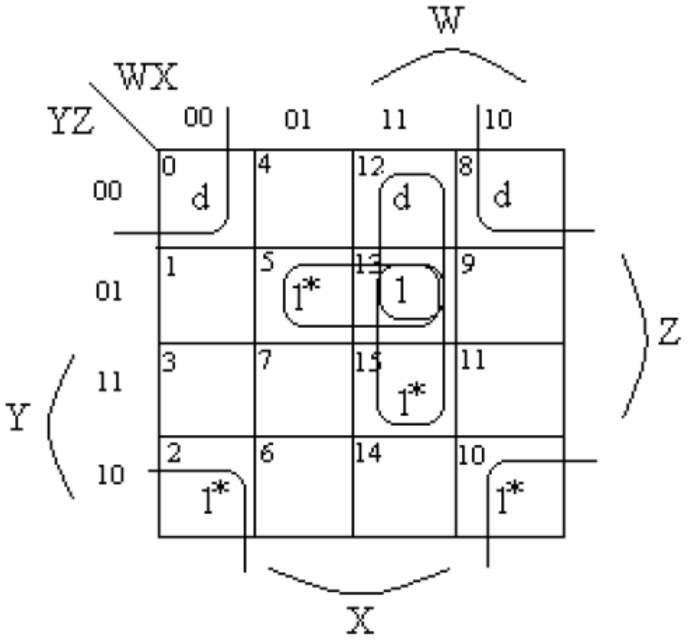
$F = W'Y'Z + W'X'Z + WXY + \underline{W'X'Y'}$

There are two minimal SOPs

Q3.
Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,15)$
b) $F = \Sigma_{W,X,Y,Z}\ (2,5,13,15,10) + d\ (8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\ (4,5,6,7,10,12,13,14) + d\ (3,9,15)$
d) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,12,15)$
e) $F= \Sigma_{W,X,Y,Z}\ (0,2,4,6,8,10,12,14) + d\ (9,13)$
f) $F= \Sigma_{W,X,Y,Z}\ (0,2,8,10) + d\ (5,13)$

b) $F = \Sigma_{W,X,Y,Z}\ (2,5,13,15,10) + d\ (8,0,12)$



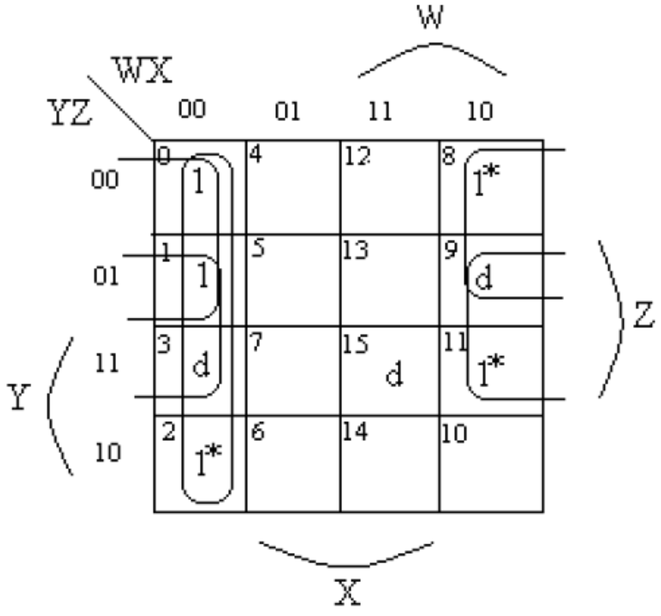
$F = X'Z' + ZWX + XY'Z$
There is only one minimal SOP

Q3.

Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\,(0,1,3,5,14) + d\,(8,15)$
b) $F = \Sigma_{W,X,Y,Z}\,(2,5,13,15,10) + d\,(8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\,(4,5,6,7,10,12,13,14) + d\,(3,9,15)$
d) $F = \Sigma_{W,X,Y,Z}\,(0,1,3,5,14) + d\,(8,12,15)$
e) $F = \Sigma_{W,X,Y,Z}\,(0,2,4,6,8,10,12,14) + d\,(9,13)$
f) $F = \Sigma_{W,X,Y,Z}\,(0,2,8,10) + d\,(5,13)$

c) $F = \Pi_{W,X,Y,Z}\,(4,5,6,7,10,12,13,14) + d\,(3,9,15)$



$F = W'X' + X'Y' + ZX'$

There is only one minimal SOP

Q3.

Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,15)$

b) $F = \Sigma_{W,X,Y,Z}\ (2,5,13,15,10) + d\ (8,0,12)$

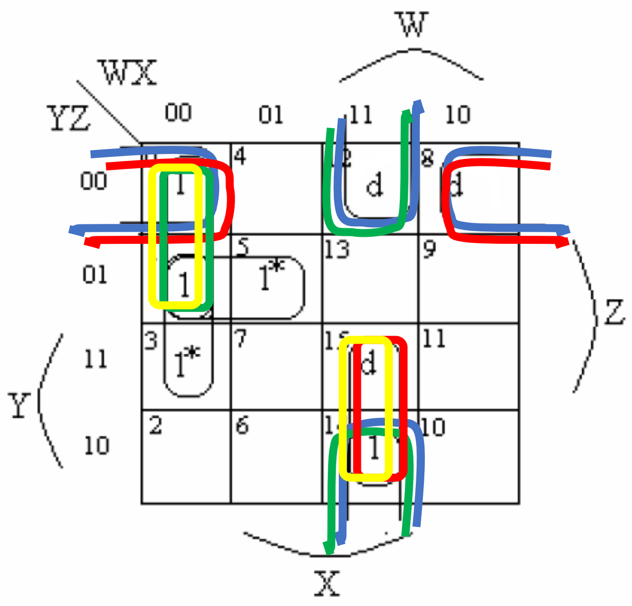
c) $F = \Pi_{W,X,Y,Z}\ (4,5,6,7,10,12,13,14) + d\ (3,9,15)$

d) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,12,15)$

e) $F= \Sigma_{W,X,Y,Z}\ (0,2,4,6,8,10,12,14) + d\ (9,13)$

f) $F= \Sigma_{W,X,Y,Z}\ (0,2,8,10) + d\ (5,13)$

d) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,12,15)$

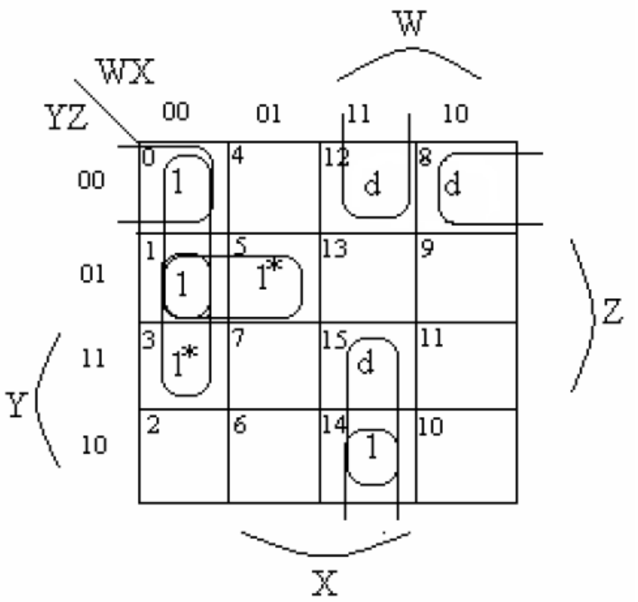


$F_1 = W'Y'Z+W'X'Z+W'X'Y'+YWX$

$F_2 = W'Y'Z+W'X'Z+W'X'Y'+WXZ'$

$F_3 = W'Y'Z+W'X'Z+X'Y'Z'+YWX$

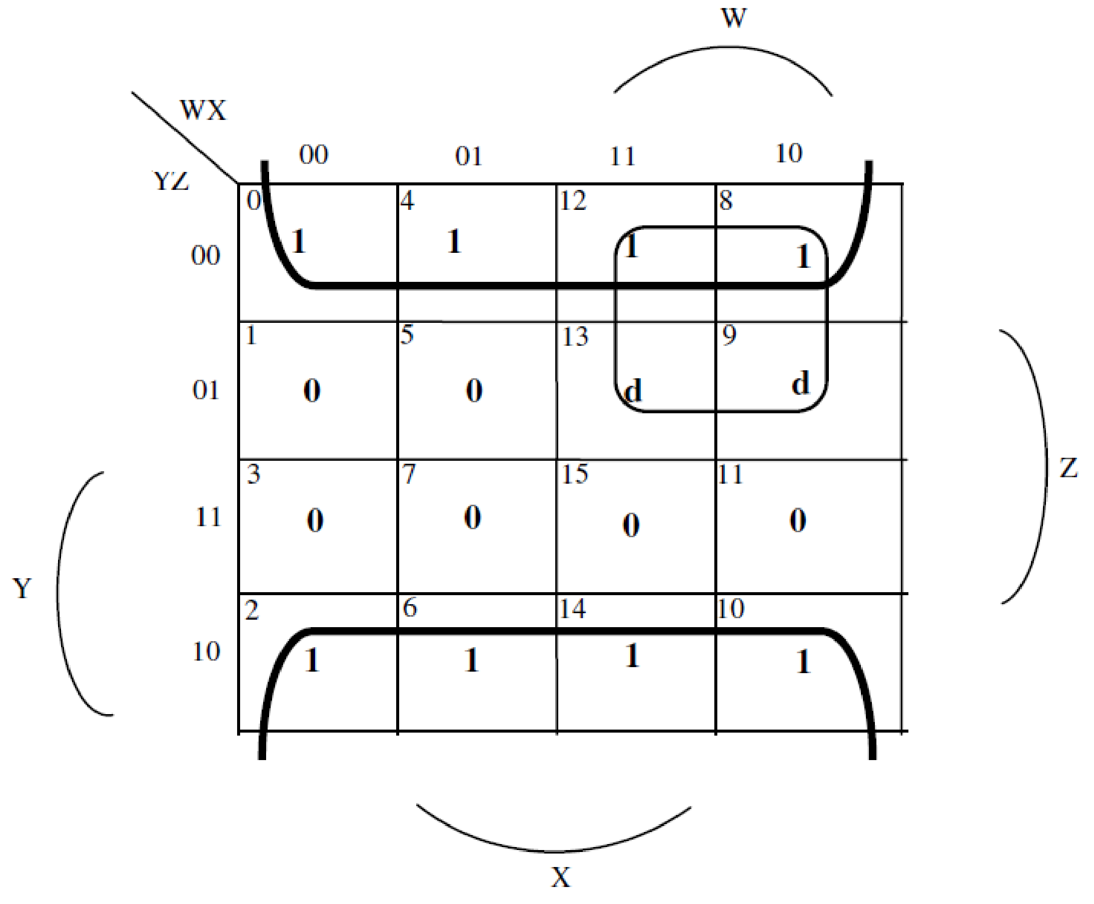
$F_4 = W'Y'Z+W'X'Z+X'Y'Z'+WXZ'$



There are 4 minimal SOPs

Q3.
Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,15)$
b) $F = \Sigma_{W,X,Y,Z}\ (2,5,13,15,10) + d\ (8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\ (4,5,6,7,10,12,13,14) + d\ (3,9,15)$
d) $F = \Sigma_{W,X,Y,Z}\ (0,1,3,5,14) + d\ (8,12,15)$
e) $F= \Sigma_{W,X,Y,Z}\ (0,2,4,6,8,10,12,14) + d\ (9,13)$
f) $F= \Sigma_{W,X,Y,Z}\ (0,2,8,10) + d\ (5,13)$



F = Z'   There is only one minimal SOP

Q3.

Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z}\,(0,1,3,5,14) + d\,(8,15)$
b) $F = \Sigma_{W,X,Y,Z}\,(2,5,13,15,10) + d\,(8,0,12)$
c) $F = \Pi_{W,X,Y,Z}\,(4,5,6,7,10,12,13,14) + d\,(3,9,15)$
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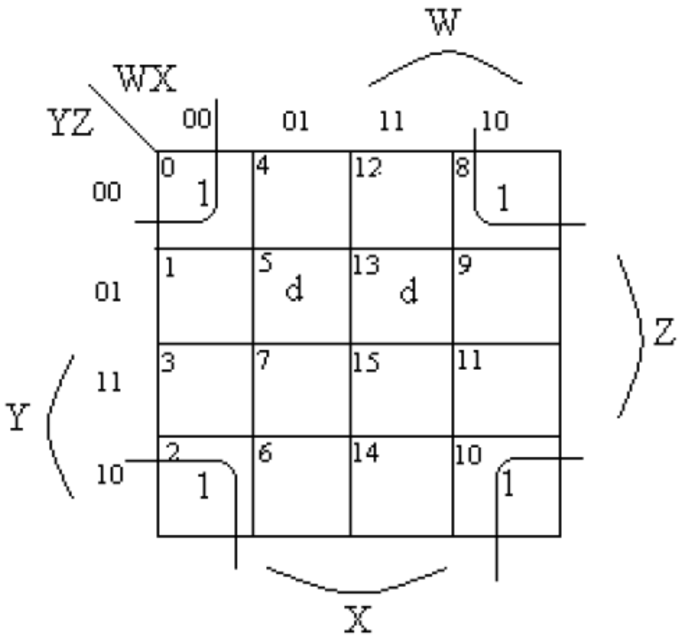


$F = X'Z'$

There is only one minimal SOP

(2’s-Complement)

Most significant bit is the sign bit:

MSB = 0: positive integer

MSB = 1: negative integer

Range: -2^{n-1} to $+2^{n-1} - 1$

0 1 1 0 1 1 1 0 ← Original binary value

1 0 0 1 0 0 0 1 ← 1’s complement

$$\begin{array}{lr} & 1\,0\,0\,1\;\;0\,0\,0\,1 \\ + & 1 \\ \hline & 1\,0\,0\,1\;\;0\,0\,1\,0 \end{array}$$

← 2’s complement

Two’s complement binary	Decimal
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	0
1111	-1
1110	-2
1101	-3
1100	-4
1011	-5
1010	-6
1001	-7
1000	-8

-12

8-bit two's complement?

$$\begin{array}{r|l} 12 & 2 \\ \hline 12 & 6 \\ \hline 0 & \end{array} \qquad \begin{array}{r|l} 6 & 2 \\ \hline 6 & 3 \\ \hline 0 & \end{array} \qquad \begin{array}{r|l} 3 & 2 \\ \hline 2 & 1 \\ \hline 1 & \end{array}$$

$$\underbrace{00001100}_{+12}$$

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 11110011 \\ +1 \\ \hline 11110100 \end{array}$$

Decimal	8-bit two's complement
25	
120	
82	
-42	
-6	
-111	

25	00011001
120	01111000
82	01010010
-42	11010110
-6	11111010
-111	10010001

- Add the following pairs of binary numbers, showing all carries:

		1	1	0	0	1	1
			1	1	0	1	0
sum	1	0	0	1	1	0	1

		1	1	0	0	1	1	0
		1	1	1	1	0	0	1
sum	1	1	0	1	1	1	1	1

a)

carries	1	1	0	0	1	0	0
		1	1	0	0	1	1
			1	1	0	1	0
sum	1	0	0	1	1	0	1

d)

carries	1	1	0	0	0	0	0	0
		1	1	0	0	1	1	0
		1	1	1	1	0	0	1
sum	1	1	0	1	1	1	1	1

- Indicate whether or not overflow occurs when adding the following 8-bit two’s complement numbers:

(a) $\begin{array}{r} 11010100 \\ +\ 10101011 \\ \hline \end{array}$ (b) $\begin{array}{r} 10111001 \\ +\ 11010110 \\ \hline \end{array}$ (c) $\begin{array}{r} 01011101 \\ +\ 00100001 \\ \hline \end{array}$ (d) $\begin{array}{r} 00100110 \\ +\ 01011010 \\ \hline \end{array}$

Q16.
Add the following 5-bit binary numbers in two's complement representation to obtain 5-bit binary results in two's complement representation. For each case, <u>indicate whether overflow has occurred or not</u>.

```
     100100
a)    11010
      10011
   +________
      01101        overflow has occured
```

```
     111100
b)    11110
      11111
   +________
      11101        overflow has not occured
```

```
      111100
c)     01010
       11111
    +________
       01001        overflow has not occured

      011100
d)     01110
       01111
    +________
       11101        overflow has occured
```

4-Bit Unsigned Comparator

$AeqB = i_3 i_2 i_1 i_0$, $AgtB = a_3 b_3' + i_3 a_2 b_2' + i_3 i_2 a_1 b_1' + i_3 i_2 i_1 a_0 b_0'$

$AltB = (AeqB + AgtB)' = AeqB'AgtB'$

A	B	
6	15	Decimal Number
0110	1111	Binary Number
$a_3\ a_2\ a_1\ a_0$	$b_3\ b_2\ b_1\ b_0$	

$i_0 = \overline{a_0 \oplus b_0} = 0$

$i_1 = \overline{a_1 \oplus b_1} = 1$

$i_2 = \overline{a_2 \oplus b_2} = 1$

$i_3 = \overline{a_3 \oplus b_3} = 0$

$AeqB = 0.1.1.0 = 0$ ✗

$AgtB = a_3 b_3' + i_3 a_2 b_2' + i_3 i_2 a_1 b_1' + i_3 i_2 i_1 a_0 b_0'$

$= 0 + 0 + 0$ ✗

$AltB = (AeqB + AgtB)' = (0+0)' = 1$ ✓

$A < B$

$6 \quad 15$

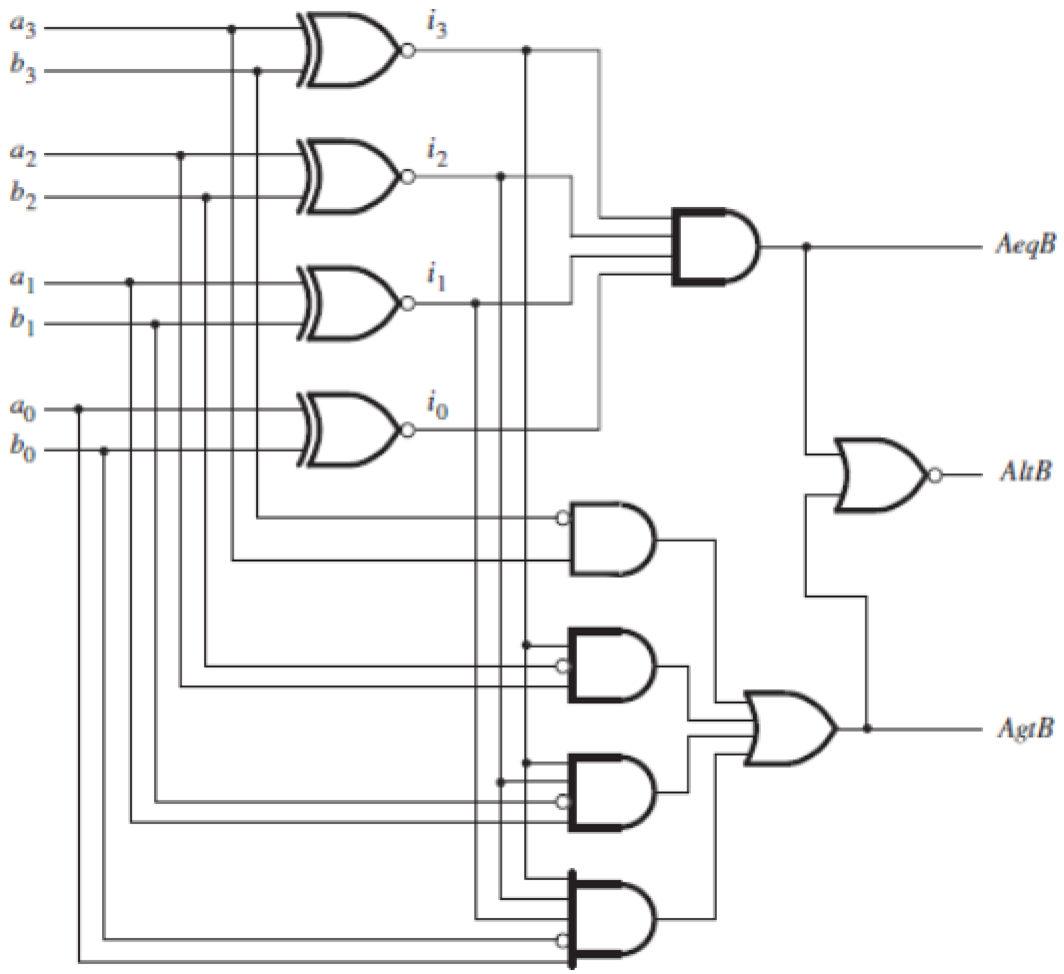
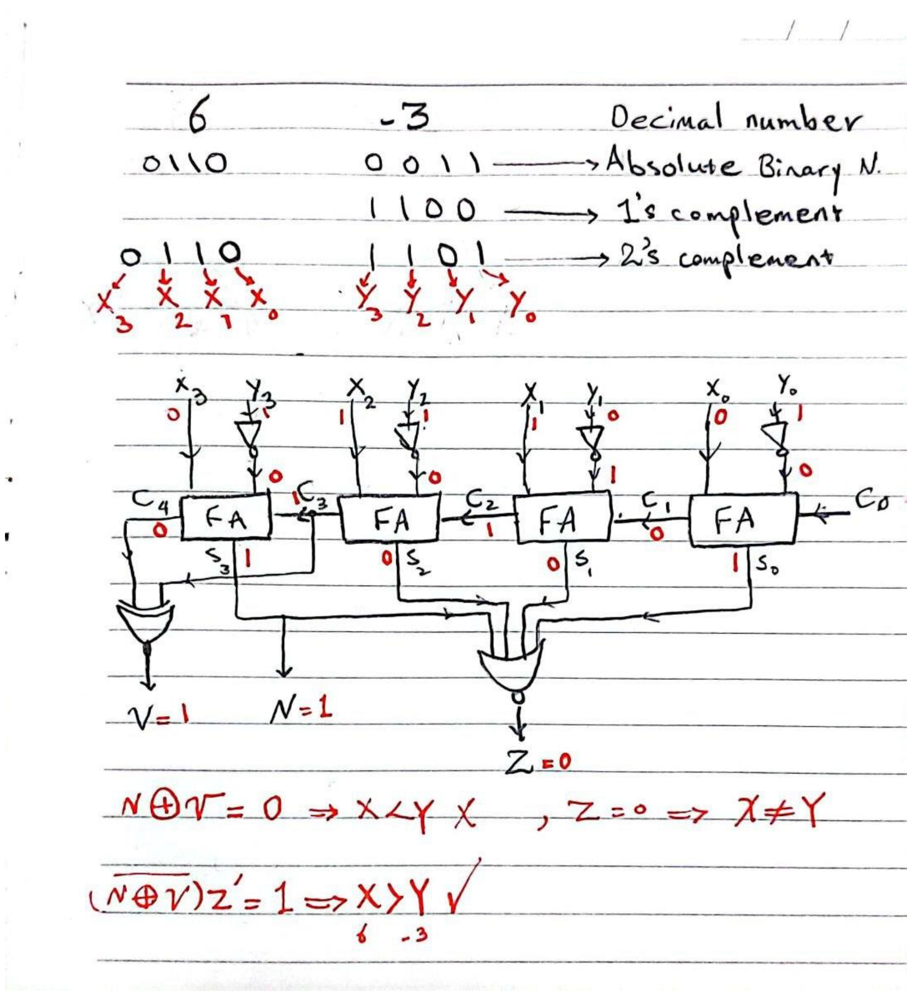
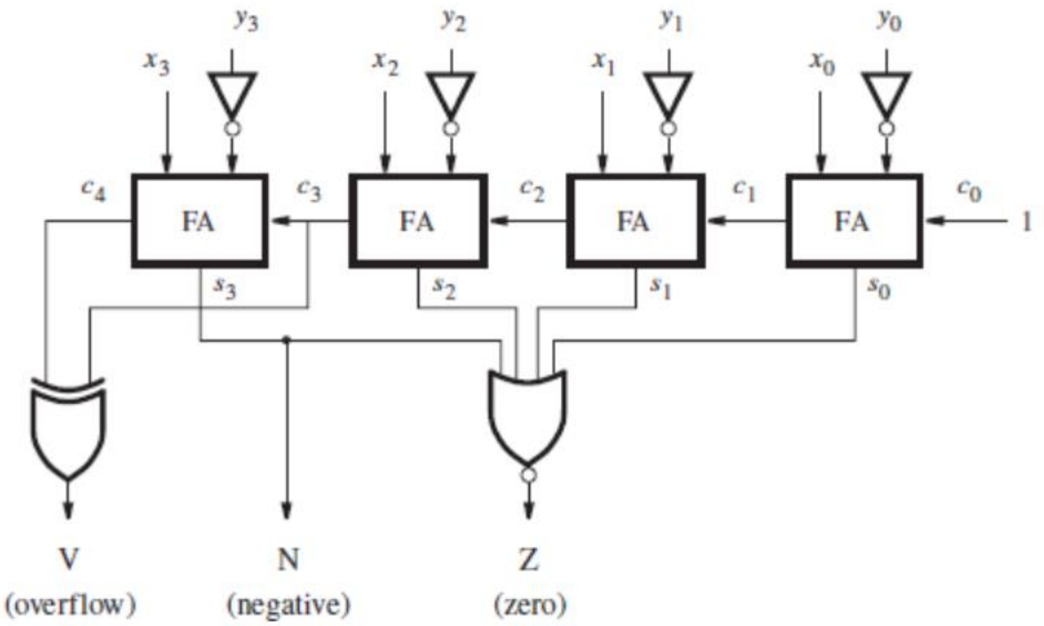


Figure 4.22 A four-bit comparator circuit.

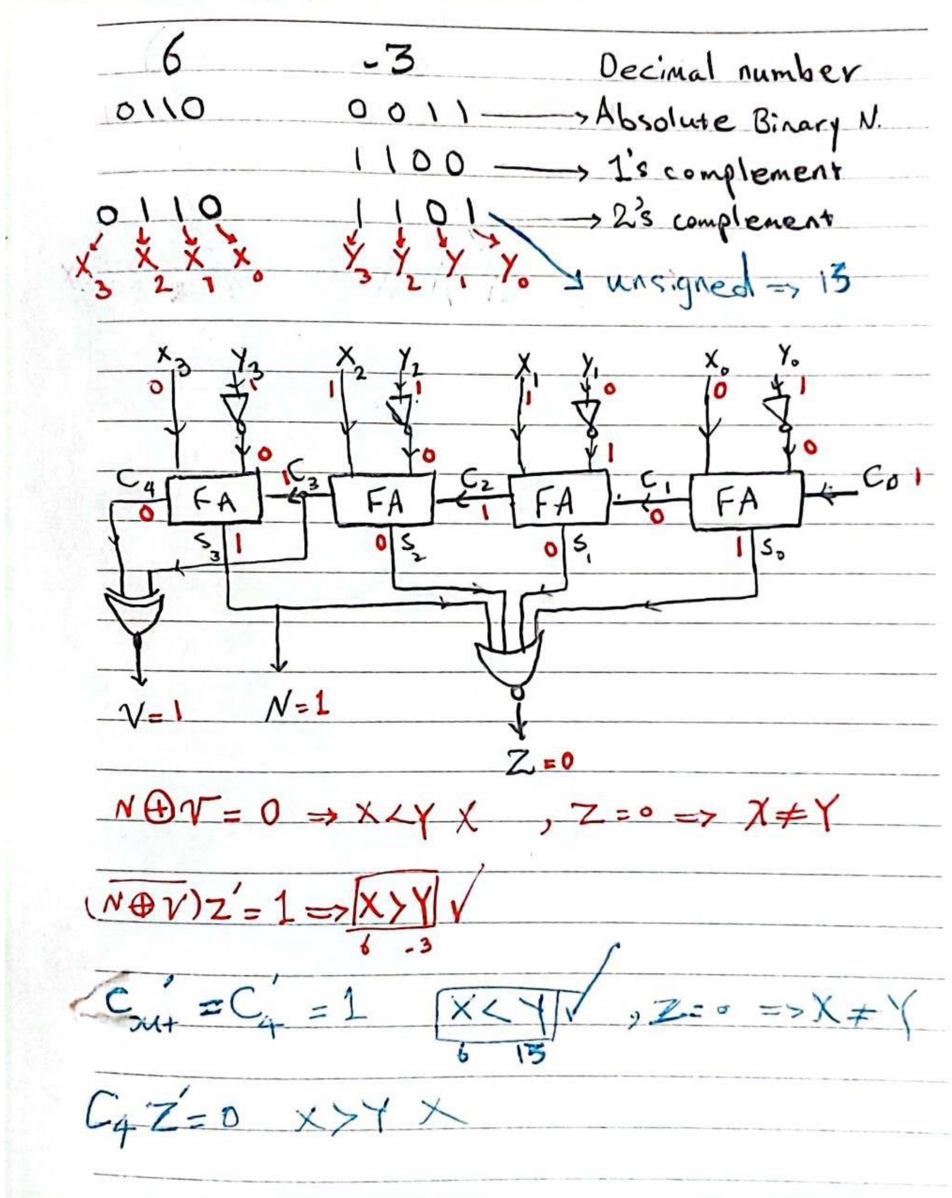
4-Bit Comparator



	Unsigned	Signed
X = Y	Z	Z
X < Y	C_{out}'	$N \oplus V$
X > Y	$C_{out}Z'$	$(\overline{N \oplus V})Z'$



4-Bit Comparator



	Unsigned	Signed
X = Y	Z	Z
X < Y	C_{out}'	$N \oplus V$
X > Y	$C_{out}Z'$	$(\overline{N \oplus V})Z'$