EEE102
Sample Problems for Fall 2009
15-10-2009
KARNAUGH MAP MINIMIZATION

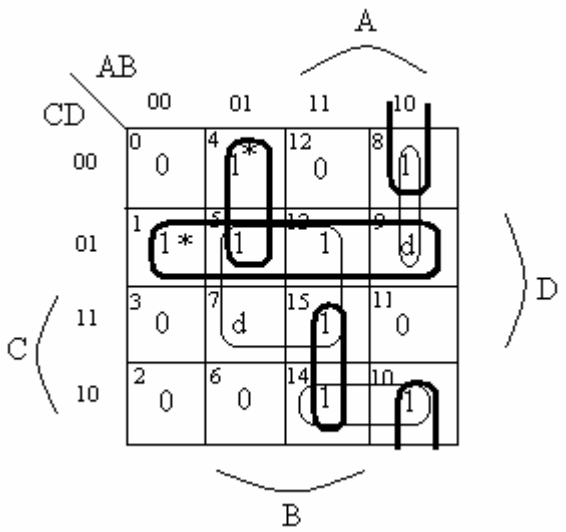
Q1.

Given the function $F(A,B,C,D) = \Sigma_{A,B,C,D}\ (1,4,5,8,10,13,14,15) + d(7,9)$.

a) Find all minimal sums for F.
b) Find all minimal products for F
c) Which of the above minimal sums are equivalent to which of the above minimal products? Equivalent means having the same Truth Table.

Solution:

a)



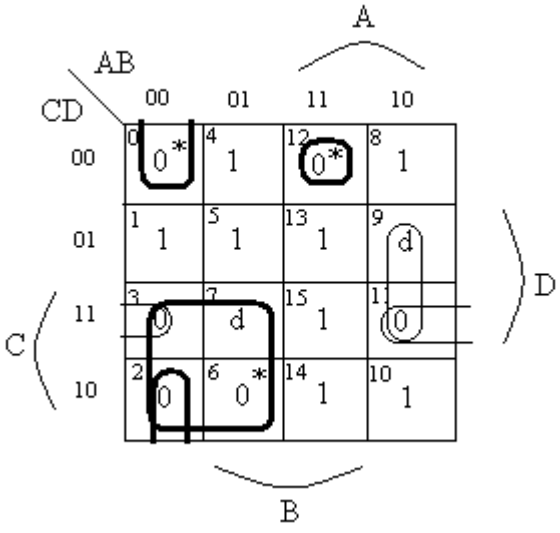
SOP1:

F = C'D+A'BC'+ABC+AB'D'

7 is zero

9 is one

b)



POS1:

F =

(A+B+D)(A+C')(A'+B'+C+D)(B+C'+D')

7 is zero

9 is one

POS2:

F = (A+B+D)(A+C')(A'+B'+C+D)(A'+B+D')

7 is zero

9 is zero

c) SOP1 is equivalent to POS1 because in each case 7 is zero, 9 is one.

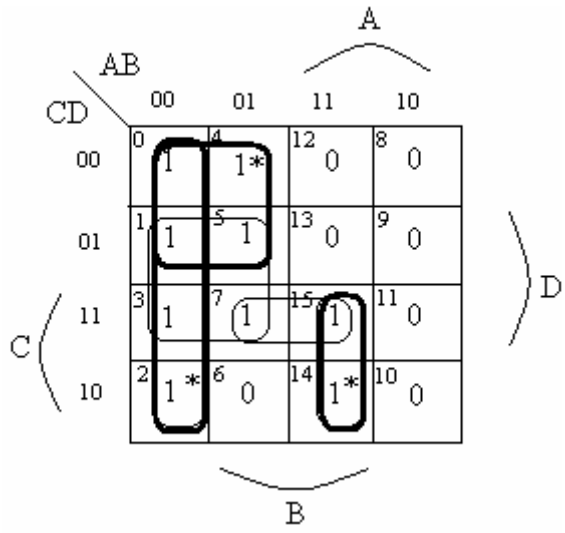
Q2.

Find a minimal SOP expression for

$$F = \Sigma_{A,B,C,D}\,(0,1,2,3,4,5,7,14,15)$$

using a Karnaugh map. In so doing, clearly show the distinguished 1-cells and also draw the reduced map.

Solution:

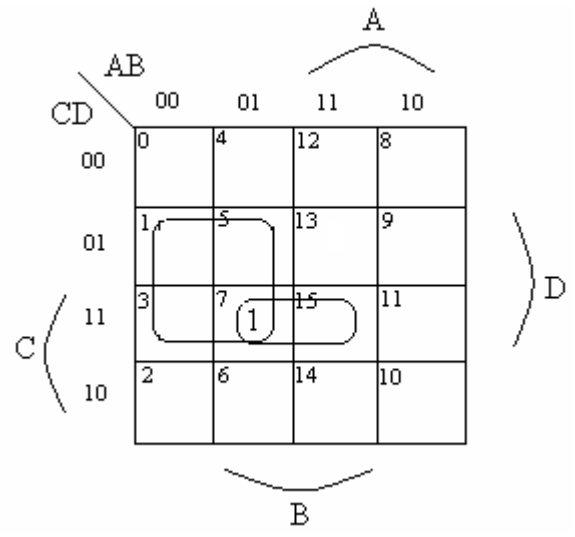


F:

Expression including essential prime implicants

A'C'+A'B'+ABC

Reduced Map:



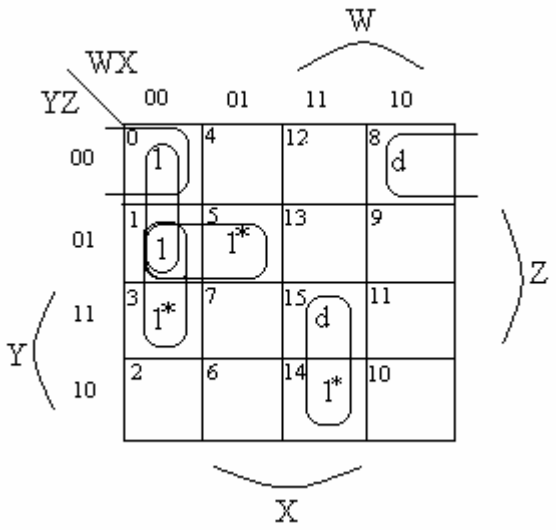
minimal SOP:

F = A'C'+A'B'+ABC+A'D

Q3.
Find all minimal SOPs for each of the following logic functions:

a) $F = \Sigma_{W,X,Y,Z} (0,1,3,5,14) + d (8,15)$
b) $F = \Sigma_{W,X,Y,Z} (2,5,13,15,10) + d (8,0,12)$
c) $F = \Pi_{W,X,Y,Z} (4,5,6,7,10,12,13,14) + d (3,9,15)$
d) $F = \Sigma_{W,X,Y,Z} (0,1,3,5,14) + d (8,12,15)$
e) $F= \Sigma_{W,X,Y,Z} (0,2,4,6,8,10,12,14) + d (9,13)$
f) $F= \Sigma_{W,X,Y,Z} (0,2,8,10) + d (5,13)$

Solution:
a) $F = \Sigma_{W,X,Y,Z} (0,1,3,5,14) + d (8,15)$



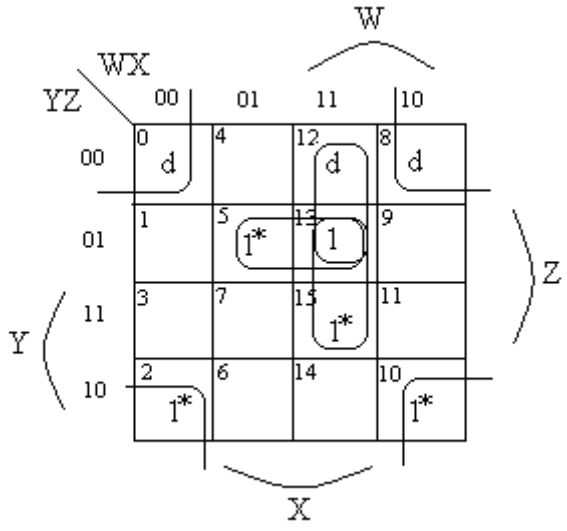
F = W'Y'Z+ W'X'Z+WXY+ X'Y'Z'

or

F = W'Y'Z +W'X'Z+WXY+ W'X'Y'

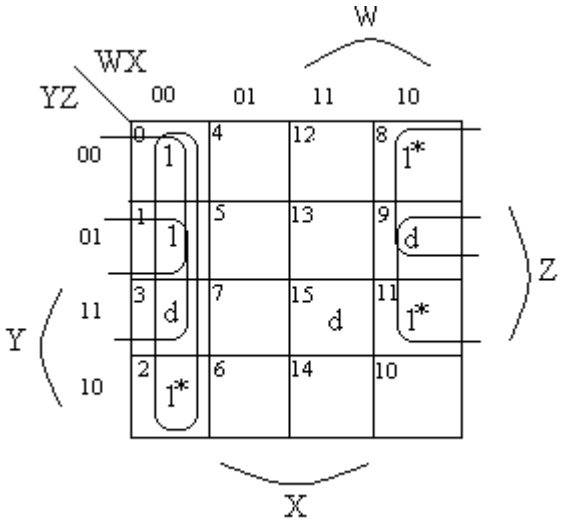
There are two minimal SOPs

b) $F = \Sigma_{W,X,Y,Z} (2,5,13,15,10) + d (8,0,12)$



F = X'Z' + ZWX + XY'Z
There is only one minimal SOP

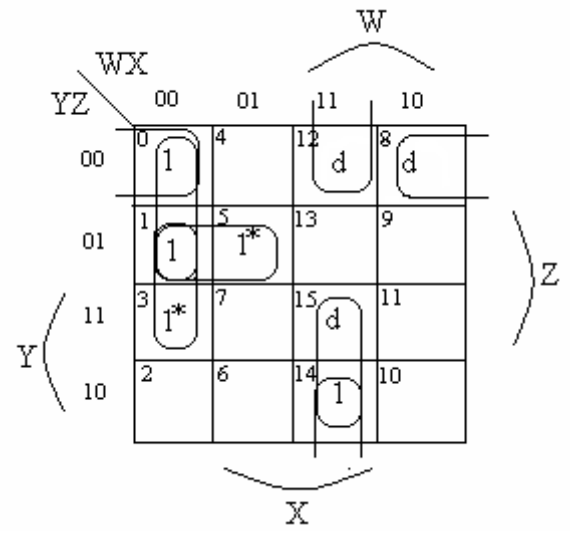
c) $F = \Pi_{W,X,Y,Z}(4,5,6,7,10,12,13,14) + d(3,9,15)$



$F = W'X' + X'Y' + ZX'$

There is only one minimal SOP

d) $F = \Sigma_{W,X,Y,Z}(0,1,3,5,14) + d(8,12,15)$



$F_1 = W'Y'Z+W'X'Z+W'X'Y'+YWX$

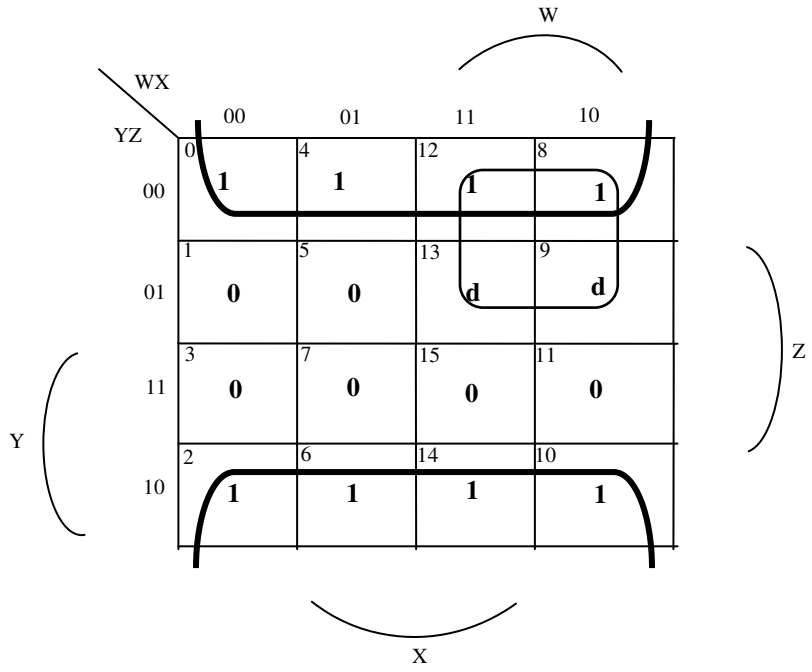
$F_2 = W'Y'Z+W'X'Z+W'X'Y'+WXZ'$

$F_3 = W'Y'Z+W'X'Z+X'Y'Z'+YWX$

$F_4 = W'Y'Z+W'X'Z+X'Y'Z'+WXZ'$

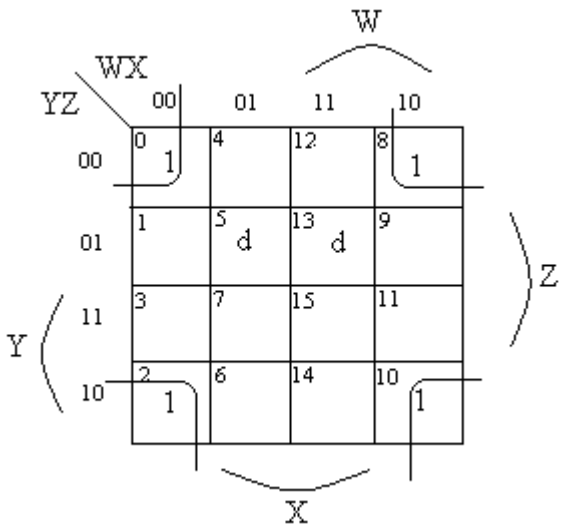
There are 4 minimal SOPs

e) $F = \Sigma_{W,X,Y,Z}(0,2,4,6,8,10,12,14) + d(9,13)$



F = Z’ There is only one minimal SOP

f) F= $\Sigma_{W,X,Y,Z}$ (0,2,8,10) + d (5,13)



F = X’Z’

There is only one minimal SOP

Q4.

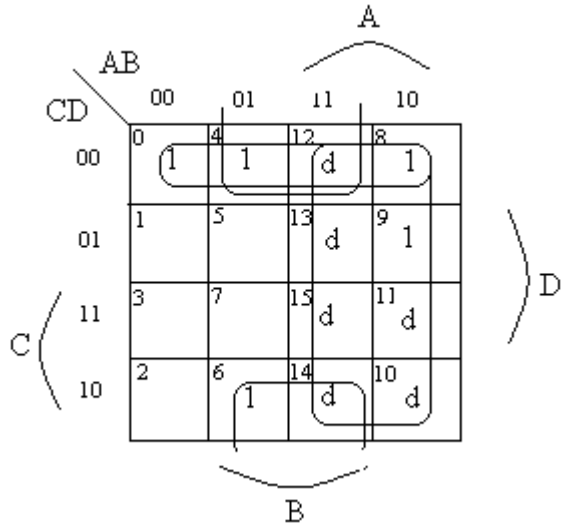
Find the lowest cost two-level realization for

$$F = \Sigma_{A,B,C,D}\,(0,4,6,8,9) + d\,(10,11,12,13,14,15).$$

Solution:

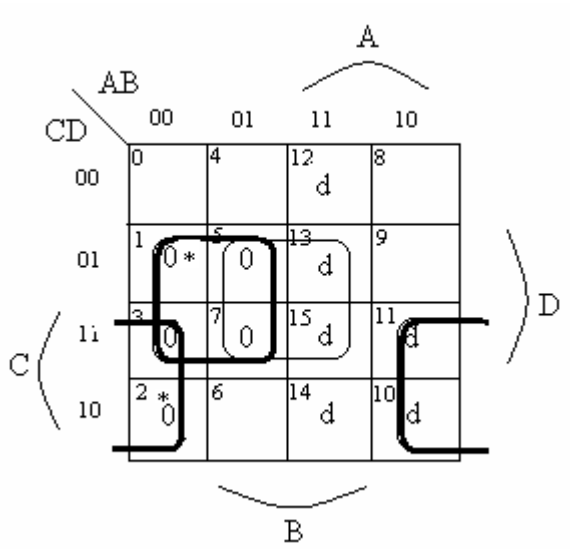
First let us find a minimal SOP expression

$F_{SOP} = A+C'D'+BD'$



Second let us find a minimal POS expression

$F_{POS} = (A+D').(B+C')$



F_{SOP} has 3 terms

F_{POS} has 2 terms

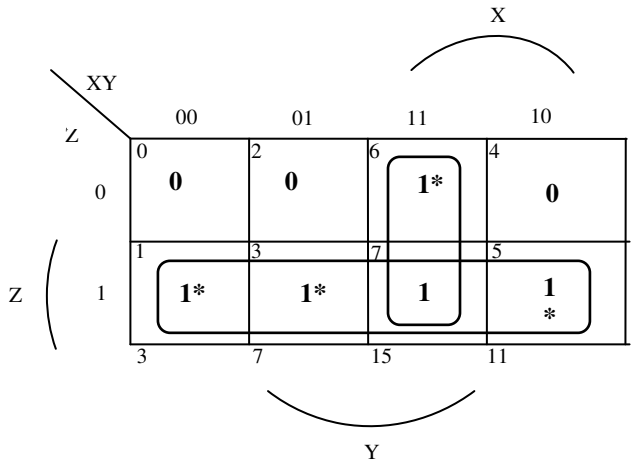
Therefore F_{POS} is of lowest cost.

Q5.

Solve exercise 4.58 from Wakerly 4th Edition

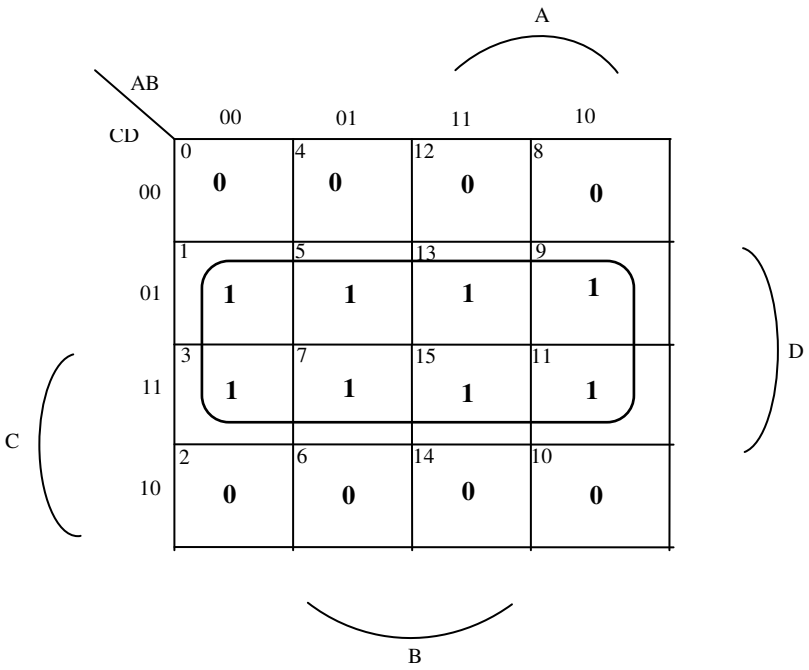
Solution:

a) F = X'Z+XY+XY'Z



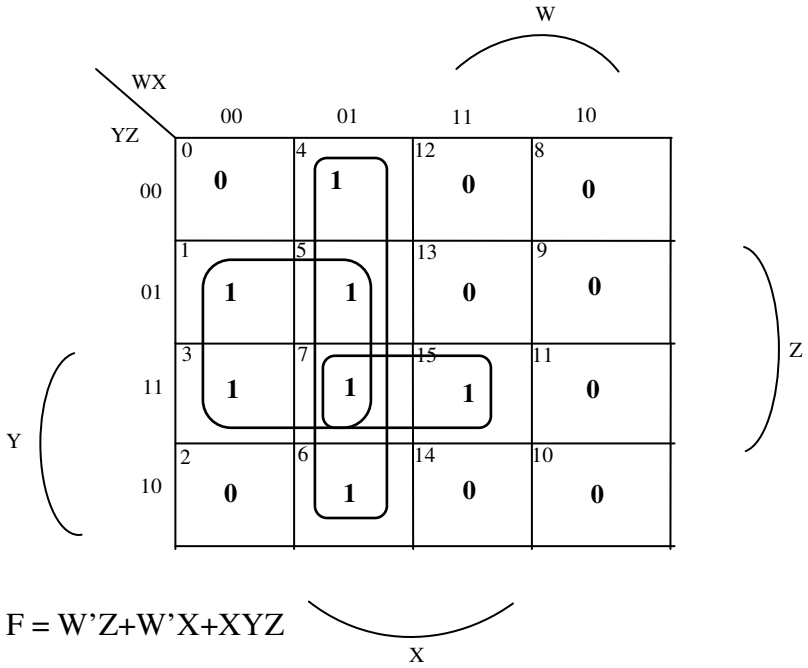
F = Z+XY

b) F = A'C'D+B'CD+AC'D+BCD



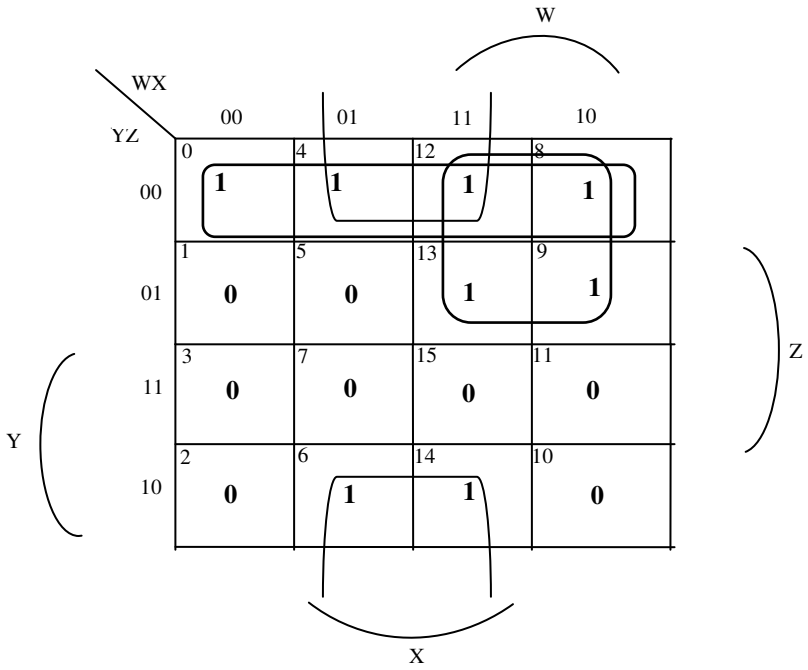
F = D

c) F = W'XZ'+WXYZ+W'Z



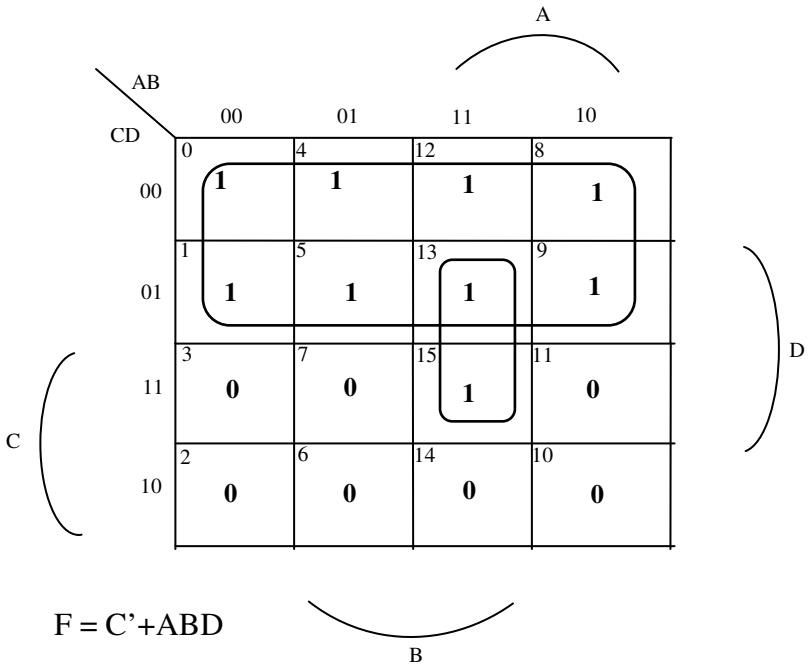
F = W'Z+W'X+XYZ

d) F = (W+Z')(W'+Y'+Z')(X+Y'+Z)



F = Y'Z'+WY'+XZ'

e) F = A'B'C'D'+A'C'D+BC'D'+ABD+AB'C'



Q6.

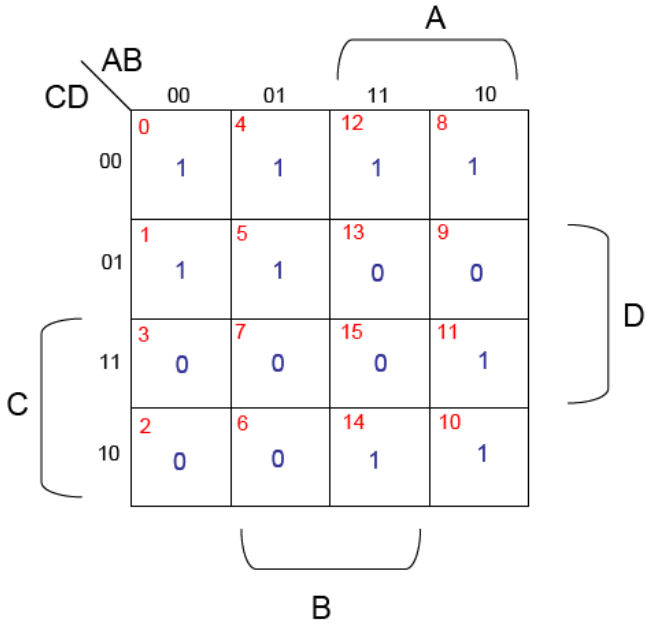
Use the Karnaugh Map method to answer the following questions for the function

F = A'B'C'D' + A'BC'D' + ABC'D' + AB'C'D' + A'B'C'D + A'BC'D +AB'CD+ ABCD' + AB'CD'

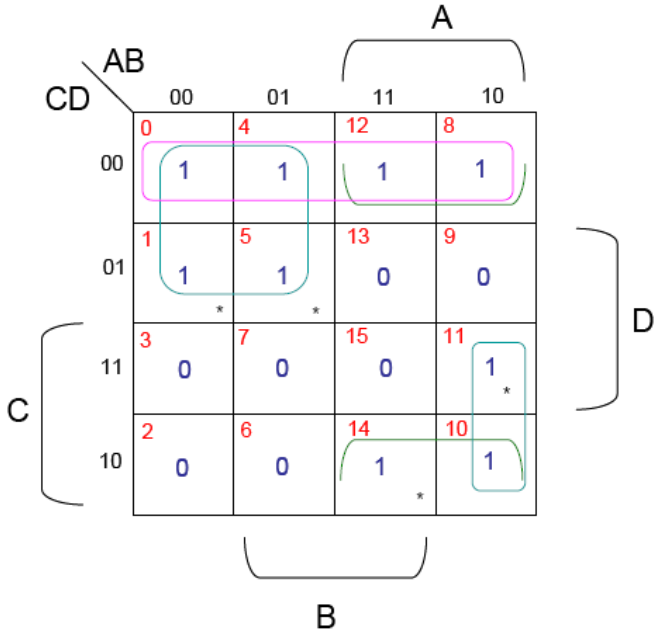
a) **Draw the Karnaugh map for F**
b) **Find a minimal SOP expressions for F.**
c) **Find a minimal POS expressions for F.**
d) **Which of the above found expressions would you use for implementation. Why?**

Solution:

a)



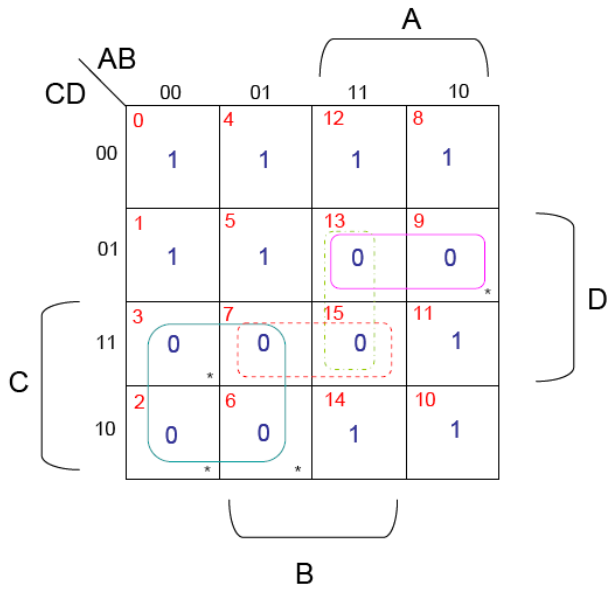
b)



In this case, essential prime implicants cover all ones.

F = A′ C′ + A D′ + A B′ C

c)



In this case, after essential prime implicants are chosen, only one 0-cell is left uncovered (cell 15). This cell can be covered by the maxterm (A′ + B′ + D′) which is depicted with dash-dot in the graph, or by the maxterm (B′ + C′ + D′) which is depicted with dash in the graph. Both choices are equally good.

F = (A + C′) (A′ + C + D′) (A′ + B′ + D′)

or

F = (A + C′) (A′ + C + D′) (B′ + C′ + D′)

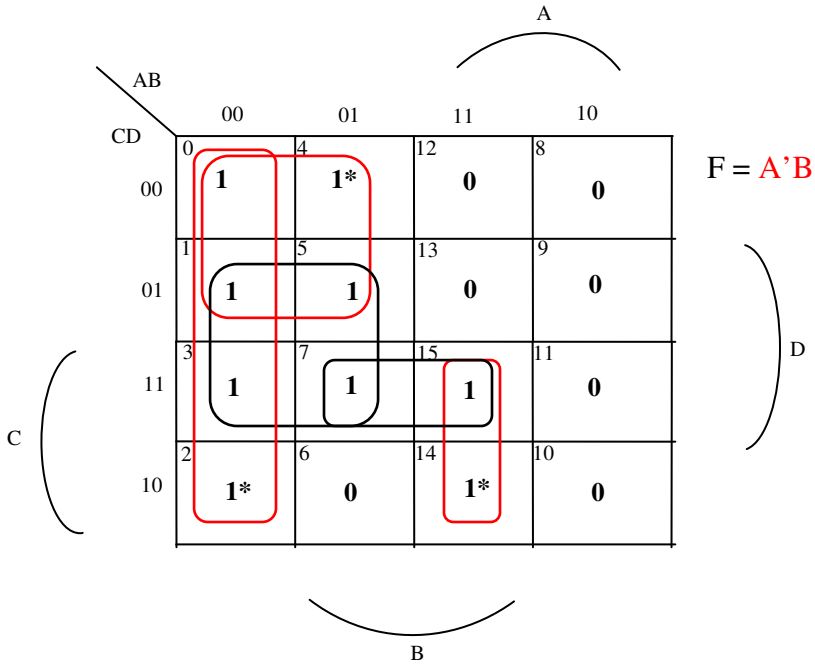
d)

SOP form has 3 terms and 7 literals. POS form has 3 terms and 8 literals. We would use SOP form, since it has no more terms and fewer literals, thus would result in fewer or smaller gates.

Q7.

Use the Karnaugh Map method to find a minimal SOP form for the function $F=\Sigma_{A,B,C,D}(0,1,2,3,4,5,7,14,15)$

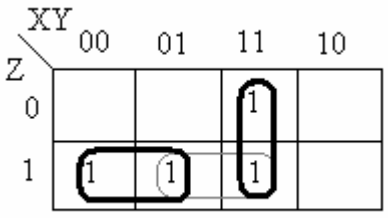
Solution:



F = A'B' + A'C' + ABC + A'D

Q8.
Write down the concensus theorem and show using a Karnaugh map what it means in terms minimization.

Solution:
XY + X'Z = XY + X'Z + YZ



The light (not-dark) gray prime implicant is not necessary for a minimum function as YZ is not necessary in the expression