Chapter 1: Digital Information, Number Systems

A. Positional Number Systems

Unsigned Numbers (non-negative)

Decimal System

$D = \underbrace{342}_{\text{whole}}.\underbrace{12}_{\text{fractional part}} = 3\times100 + 4\times10 + 2\times1 + 1\times10^{-1} + 2\times10^{-2}$

whole | fractional part

radix point

$d_i \in \{0,1,2,3,4,5,6,7,8,9\}$

↑

ith digit — Arabic numerals

base = 10

342.12_{10}

$D = d_2 d_1 d_0 . d_{-1} d_{-2}$

$V(D) = d_2\times10^2 + d_1\times10^1 + d_0\times10^0 + d_{-1}\times10^{-1} + d_{-2}\times10^{-2}$

octal : base 8 $\quad d_i \in \{0,1,2,3,4,5,6,7\}$

$342.12_8 = 3\times8^2 + 4\times8^1 + 2\times8^0 + 1\times8^{-1} + 2\times8^{-2}$

Binary System

base 2 $\quad d_i \in \{0,1\}$ (bit) $\quad$ 0: transistor off , voltage = low

8 bits = Byte $\quad$ 1: on , " = high

$D = \overset{4}{1}\overset{3}{0}\overset{2}{0}\overset{1}{1}\overset{0}{1}_2$

$V(D) = 1\times2^4 + 1\times2^1 + 1\times2^0 = 16+2+1 = 19_{10}$

$D = 10011.\overset{-1}{0}\overset{-2}{1}\overset{-3}{1}_2$

$V(D) = 19_{10} + 1\times2^{-2} + 1\times2^{-3} = 19 + 0.25 + 0.125 = 19.375_{10}$

Counting up

decimal : 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12 - ...

binary : 1 - 10 - 11 - 100 - ...

$100111_2 = 39_{10}$

Octal System

base 8 $\quad d_i \in \{0, 1, 2, 3, 4, 5, 6, 7\}$

octal digit	3-bit binary string
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

$$123_8 = 001\,010\,011_2 = 1010011_2$$

$$111.011_2 = 7.3_8$$

$$\underbrace{001}_{1}.\underbrace{010}_{2}\,\underbrace{100_2}_{4} = 1.24_8$$

101111000 1000 1000 100111 —

less digits with octal representation

Binary to Octal Conversion

Octal to Binary Conversion

Hexadecimal System

base 16

$d_i \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \underset{10}{A}, \underset{11}{B}, \underset{12}{C}, \underset{13}{D}, \underset{14}{E}, \underset{15}{F}\}$

$EE.102_{16} = 11101110.000100000010_2$

hex. dig.	binary strings (4 bit)
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Binary to Hexadecimal Conversion

$\underbrace{0101}_{5}.\underbrace{01\,00}_{4}{}_2 = 5.4_{16}$

General Base-r System

$D = (d_{p-1} d_{p-2} \cdots d_1 d_0 . d_{-1} d_{-2} \cdots d_{-n})_r$ ← base (integer)

Most significant digit (MSD)

radix point

least significant digit (LSD)

$d_i \in \{0, 1, \ldots, r-1\}$

Base-r to Decimal Conversion

$$V(D) = \sum_{i=-n}^{p-1} d_i r^i$$

$$436.5_8 = 4 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 + 5 \times 8^{-1} = 4 \times 64 + 3 \times 8 + 6 + 5 \times 0.125$$
$$= 256 + 24 + 6 + 0.625 = 286.625_{10}$$

Decimal to Base-r Conversion

- Take decimal representation, keep dividing by r until quotient is zero
- Record the remainder at each step
- Last remainder is the MSD. First remainder is the LSD

$179_{10} = (\cdot)_2$?

	quotient	remainder	
179/2	89	1	→ LSD
89/2	44	1	
44/2	22	0	
22/2	11	0	
11/2	5	1	
5/2	2	1	
2/2	1	0	

$1/2$ $\quad 0 \quad 1 \rightarrow MSD$

$= 10110011_2$

most significant bit $\quad$ least significant bit

Decimal Fraction to Base-2

$0.6875_{10} = (.)_2$ $\qquad$ $0.1_{10} = (.)_2$

d_{-1}

$0.6875 \times 2 = 1.3750$

d_{-2}

$0.3750 \times 2 = 0.750$

d_{-3}

$0.75 \times 2 = 1.5$

d_{-4}

$0.5 \times 2 = 1.0$

$= 0.1011_2$

Base r to Base k conversion

- **Convert base r to decimal**
- **Convert decimal to base k**

231_4 to base 5 $\qquad 140_5$

$231_4 = 2 \times 4^2 + 3 \times 4^1 + 1 \times 4^0 = 32 + 12 + 1 = 45_{10}$

	quotient	remainder
$45/5$	9	$0 \rightarrow LSD$
$9/5$	1	4
$1/5$	0	$1 \rightarrow MSD$

$= 140_5$

B. Non-positional Number System

Celek: //// = 4

/// = 3

Roman numerals: VI : 6_{10}

V : 5_{10}

VII : 7_{10}