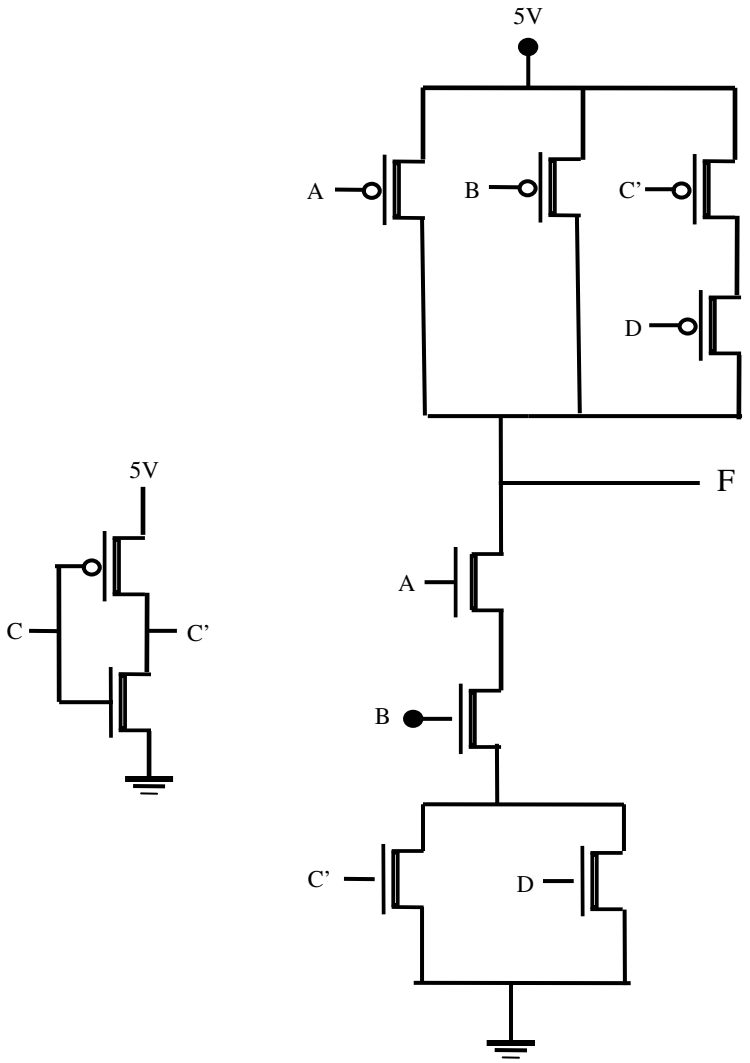
BILKENT UNIVERSITY
Department of Electrical and Electronics Engineering
EEE102 Introduction to Digital Circuit Design
MidTerm Exam I SOLUTION
8-03-2006
Duration 110 minutes

Q1. (20 points)
Draw a circuit, using NMOS and PMOS transistors, which implements the function $F = A' + B' + CD'$. Complements of the input variables are not available.

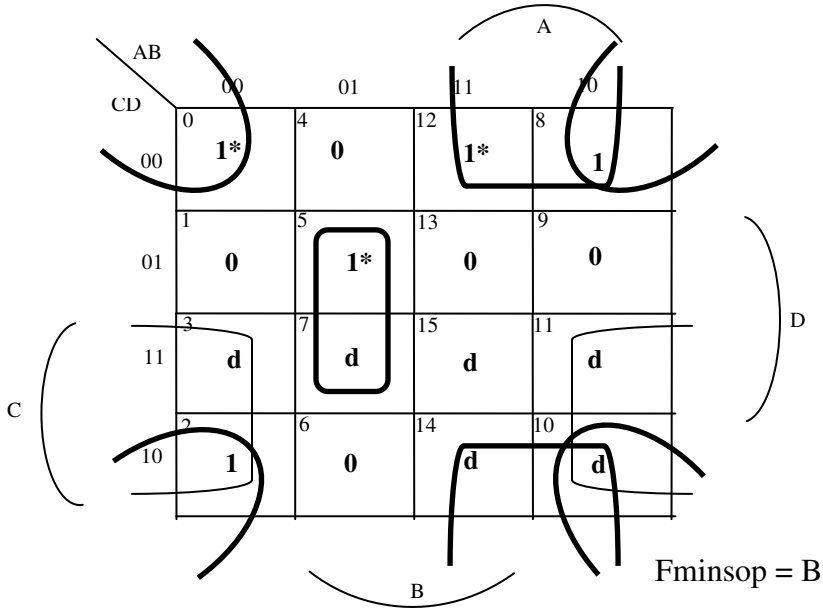
Solution: $F = A' + B' + CD' = \left[A \cdot B \cdot (C' + D)\right]'$



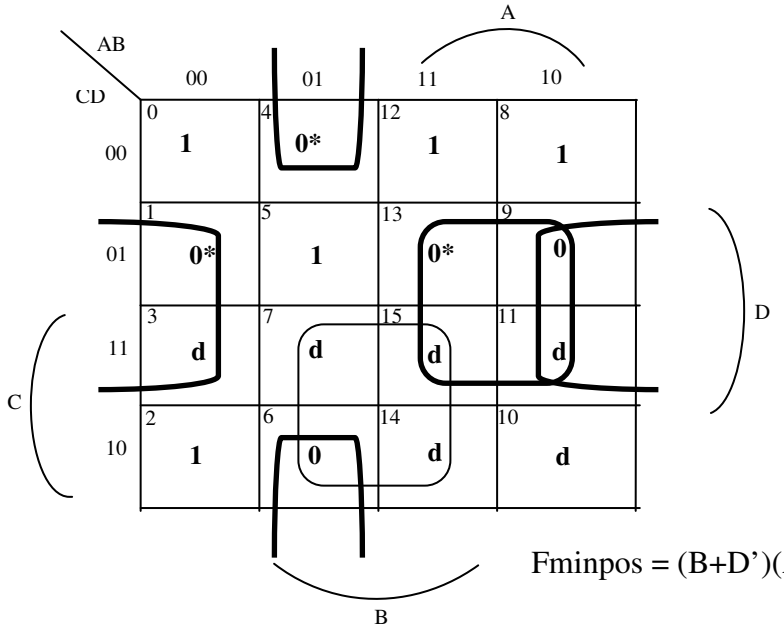
Q2. (20 points)
Find all minimal sums and all minimal products for

$$F = \sum_{A,B,C,D} (0,2,5,8,12) + d(3,7,10,11,14,15) .$$

Which of the solutions you have found are equivalent, why? Equivalent means having the same truth table.
Solution:



Fminsop = B'D'+A'BD+AD'



Fminpos = (B+D')(A'+D')(A+B'+D)

In finding Fminsop we have made the following assignments to the don't cares:

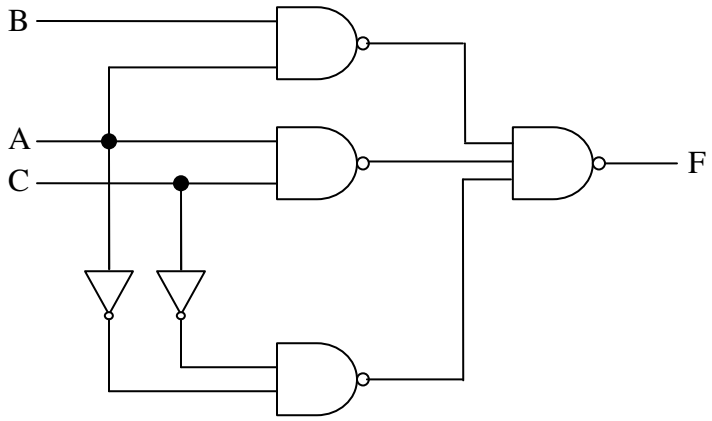
3	7	10	11	14	15
0	1	1	0	1	0

In finding Fminpos we have made the following assignments to the don't cares:

3	7	10	11	14	15
0	1	1	0	1	0

Thus in both cases the don't cares take the same values, and therefore the two solutions are equivalent.

Q3. (20 points)
Given the circuit below



a) Find an expression for F.
b) Write down the truth table of F given that the input combinations ABC = 000, ABC = 011, and ABC = 111 never occur.
c) Find and draw a minimal 2-level NAND-NAND circuit which does the same job as the above drawn circuit for the input combinations which occur.

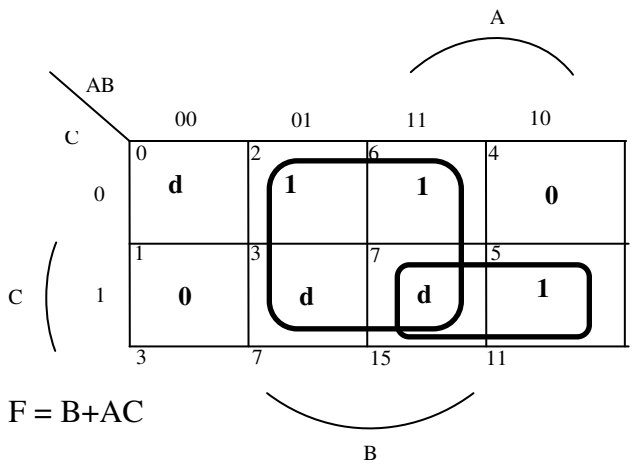
Solution:

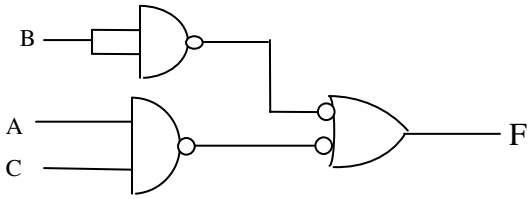
a) $F = [(AB)'(AC)'(A'C')']' = AB + AC + A'C'$

b)

A	**B**	**C**	**F**
0	0	0	d
0	0	1	0
0	1	0	1
0	1	1	d
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	d

c)





Q4. a) (10 points) The operator with symbol ** is defined as A**B=A'B'. Is this operator commutative, why? Is this operator associative, why?

b) (10 points) The statement $A \Rightarrow B$ (which reads as A implies B) is a function of A and B, with the truth table

A	B	$A \Rightarrow B$
0	0	1
0	1	1
1	0	0
1	1	1

Consider the statement G: $A \Rightarrow B + C$, and also the statement H: $AB' \Rightarrow C$. Show that G=H.

Solution:

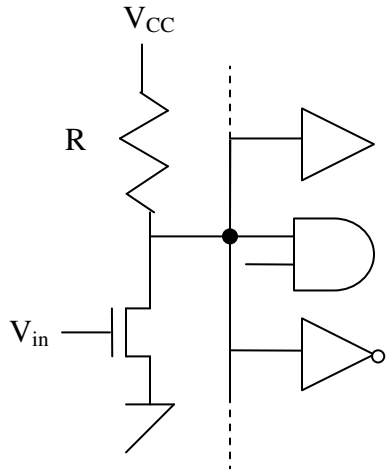
a) A**B=A'B' = B'A' = B**A Therefore ** is commutative

A**(B**C) = A**(B'C') = A'(B'C')' = A'(B+C) = A'B+A'C

(A**B)**C = (A'B')**C = (A'B')'C' = (A+B)C' = AC'+BC'
Thus A**(B**C) ≠ (A**B)**C and ** is not associative.

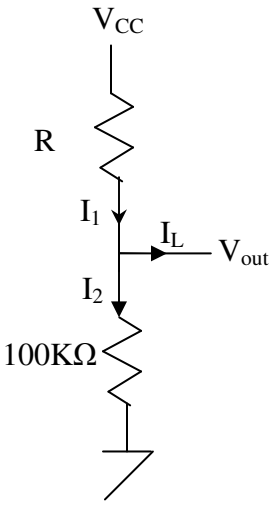
b) $A \Rightarrow B \equiv A' + B$ from the truth table.
Therefore $G : A \Rightarrow B + C \equiv A' + B + C$
and $H : AB' \Rightarrow C \equiv (AB')' + C = A' + B + C$.
Thus $G = H$.

Q5. (20 points)
An open drain NMOS transistor with pull up resistor R drives 10 various CMOS gates as shown below. The whole circuit operates from a supply of V_{CC} = 5V.



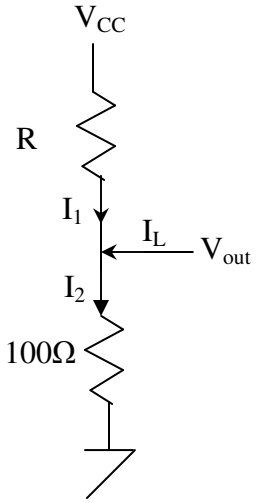
The CMOS family has the following specifications: For a supply of 5V
V_{IHmin} = 3.5 V (Minimum voltage accepted as HIGH)
V_{ILmax} = 1.5 V (Maximum voltage accepted as LOW)
I_{ILmax} = -1 μA (Maximum input current for LOW input)
I_{IHmax} = 1.5 μA (Maximum input current for HIGH input)
(Currents are defined as into the gate)
The ON and OFF resistances of the NMOS transistor are 100Ω and 100KΩ respectively.
a) Find the maximum value of R for a High Noise Margin of 0.5 volts.
b) Find the minimum value of R for a Low Noise Margin of 0.5 volts.

Solution:
a)Output High case:



$$I_L = 15\mu A$$
$$V_{out} = 4V$$
$$I_2 = \frac{4}{100K\Omega} = 40\mu A$$
$$I_1 = 15\mu A + 40\mu A = 55\mu A$$
$$R = \frac{5-1}{55\mu A} = 18.18K\Omega$$

b) Output Low case:



$$I_L = 10\mu A$$
$$V_{out} = 1V$$
$$I_2 = \frac{1}{100\Omega} = 10mA$$
$$I_1 = 10mA - 10\mu A \approx 10mA$$
$$R = \frac{5-1}{10mA} = 400\Omega$$