1. **[25 pts]** Implement the following four functions (at the same time) using only three half adders. Complements of the variables and logic levels 0 and 1 are NOT available.
$A = X \oplus Y \oplus Z$
$B = X'YZ + XY'Z$
$C = X'Z + XYZ' + Y'Z$
$D = XYZ$

2. **[25 pts]** Design a circuit that takes 2's complement of any four-bit number by using only 3 $XOR$ and minimum number of $OR$ gates.

3. **[25 pts]** Design an even parity check circuit with 3 binary inputs, which counts the number of 1's in its input. If the number of 1's are even, then the output of the circuit is 1. If the number of 1s are odd, then the output is 0.

   - **[12 pts]** Implement in minimal Sum-of-Product form.
   - **[13 pts]** Implement in minimal Product-of-Sum form.

4. [**25 pts**] Implement the following Boolean function using a single 4-to-1 multiplexer and a **minimum** number of combinational gates:

$$F(X, Y, Z, W) = \bar{X}Z + \bar{Z}W + XW + \bar{X}YZ + XYZW.$$

Clearly present your truth table, multiplexer decomposition and schematic circuit.

5. (25 pts) Let $F(X_1, X_2, X_3, X_4) = (X_1 + X_2)\ \overline{X_3} + X_1\ X_2\ X_4 + \overline{X_1}\ \overline{X_3}$

(i) Fill the Karnaugh map for $F$ given below. (7pts)

KMAP for F

| $X_3X_4 \backslash X_1X_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

(ii) Find the Boolean expression for the minimum sum of products form of $F$ using the method of Karnaugh map (**your answer to part i must be correct to get credit from this part**). (10pts)

(iii) Implement $F$ using minimum number of NAND gates (any input size is allowed). Complements of the variables and logic levels 0 and 1 are **not** available (you need to produce them). (8pts)

6. (25 pts) Design a circuit that converts any 3-bit number to its negative in two's complement system using only minimum number of full adders. Use of any other gates is not allowed. The complements of the variables are not available. You can use logic levels 0 and 1

7. (15 pts)

(i) Convert $16.15_8$ to binary. (5pts)
(ii) Convert $10111.001101_2$ to hexadecimal. (5pts)
(iii) Convert $112.5_{10}$ to binary (5pts)