15-03-2008

BILKENT UNIVERSITY

Department of Electrical and Electronics Engineering

EEE102 Introduction to Digital Circuit Design

Midterm Exam I SOLUTION

Surname: ______________________________

Name: ______________________________

ID-Number: ______________________________

Signature: ______________________________

Duration is 120 minutes. Solve all 7 questions. Show all your work.

Q1 (14 points	
Q2 (14 points)	
Q3 (16 points)	
Q4 (14 points)	
Q5 (14 points)	
Q6 (14 points)	
Q7 (14 points)	
Total	

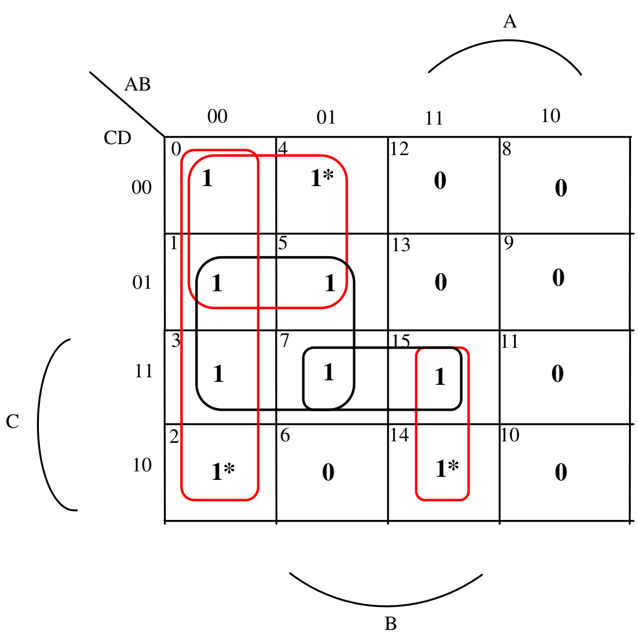
Q1. For the function $F=\Sigma_{A,B,C,D}(0,1,2,3,4,5,7,14,15)$ use the Karnaugh Map method

a) **to find a minimal SOP form, and**

b) **to find a minimal POS form.**

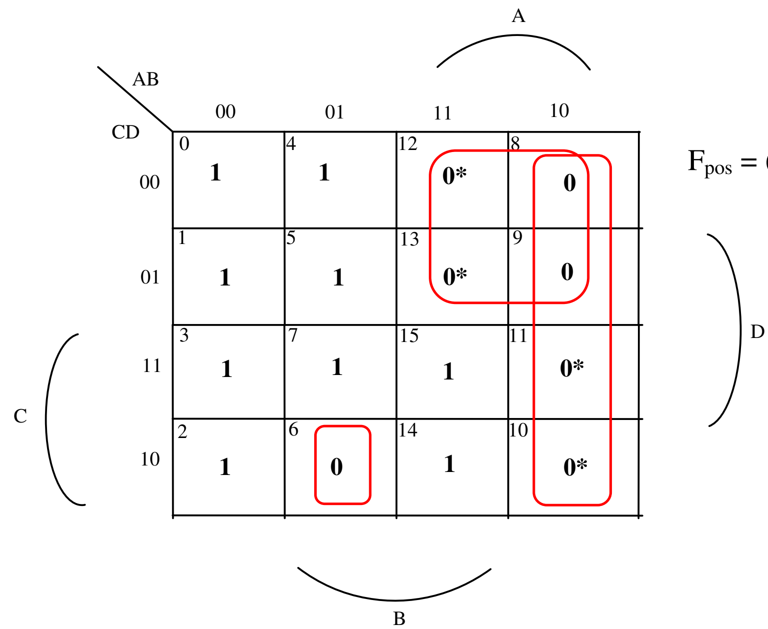
c) **Which minimal solution(s) would you choose for implementation? Why?**

Solution: a)



$F_{sop} = A'B'+A'C'+ABC+A'D$

b)

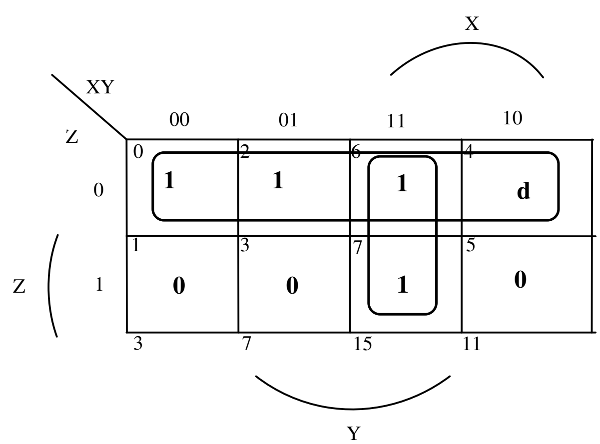


$F_{pos} = (A'+C)(A'+B)\ (A+B'+C'+D)$

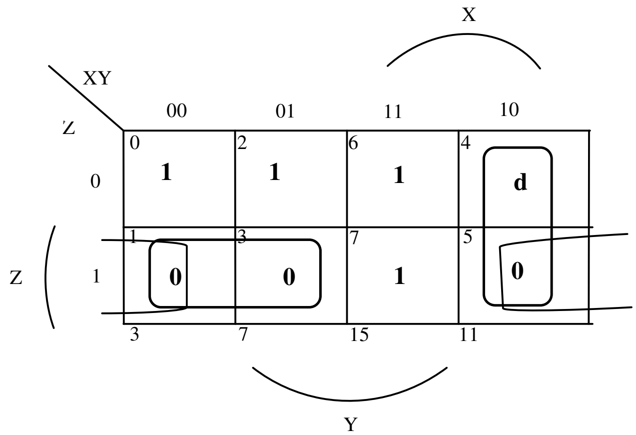
c) **Minimal POS is preferred because it has fewer terms.**

Q2. Suppose you have a computer program which can find minimal SOP expressions for logic functions. How would you use this program to find minimal POS expressions for logic functions?
Solution:
For a function F, first we obtain the TT of the complement of the function, F', by switching 1s and 0s. Then using the program we find the minimal SOP for F'. Then we complement this minimal SOP to obtain the minimal POS of F.

Q3. Give a simple Karnaugh Map example for which the Truth Table of a minimal SOP expression is different from the Truth Table of a minimal POS expression.
Solution:



F1 = Z'+XY   cell 4 is 1



F2 = (X+Z')(Y+Z')   cell 4 is 1
F3= (X+Z')(X'+Y)   cell 4 is 0

F1 and F3 have different truth tables.

Q4. You are given below the Truth Table of a 3-input logic function, F. Find and draw the Truth Table of the dual of F. Explain and justify your answer.

A	**B**	**C**	**F**
0	**0**	**0**	$\mathbf{F_0}$
0	**0**	**1**	$\mathbf{F_1}$
0	**1**	**0**	$\mathbf{F_2}$
0	**1**	**1**	$\mathbf{F_3}$
1	**0**	**0**	$\mathbf{F_4}$
1	**0**	**1**	$\mathbf{F_5}$
1	**1**	**0**	$\mathbf{F_6}$
1	**1**	**1**	$\mathbf{F_7}$

Solution:

F^D(A,B,C) = [F(A, B, C, +, ., 1, 0)]^D = F(A, B, C, ., +, 0, 1)

F'(A,B,C) = [F(A, B, C, +, ., 1, 0)]' = F(A', B', C', ., +, 0, 1)
or equivalently F'(A',B',C') = F(A'', B'', C'', ., +, 0, 1) = F(A, B, C, ., +, 0, 1)

Thus F^D(A,B,C) = F'(A',B',C')

Thus the TT of F^D is the same as the TT of F' except that inputs are complemented. This means that the TT of F^D from top to bottom is the same as the TT of F' from bottom to up.

A	B	C	**F'(A, B, C)**
0	0	0	F0'

0	0	1	F1’
0	1	0	F2’
0	1	1	F3’
1	0	0	F4’
1	0	1	F5’
1	1	0	F6’
1	1	1	F7’

A	B	C	**F’(A’, B’, C’)**
0	0	0	F7’
0	0	1	F6’
0	1	0	F5’
0	1	1	F4’
1	0	0	F3’
1	0	1	F2’
1	1	0	F1’
1	1	1	F0’

Q5. Prove that the exclusive-NOR (XNOR) operation is both commutative and associative.
You can use $X \oplus Y = X'Y + XY'$ and $(X \oplus Y)' = XY + X'Y'$
Solution:
XNOR is commutative

$$(X \oplus Y)' = XY+X'Y' = YX + Y'X' = (Y \oplus X)'$$

XNOR is associative

$$((X \oplus Y)' \oplus Z)' = ((XY+X'Y') \oplus Z)'$$
$$= (XY+X'Y')Z +(XY+X'Y')'Z'$$
$$= (XY+X'Y')Z +(X'+Y')(X+Y)Z'$$
$$= (XY+X'Y')Z +(X'X+X'Y+Y'X+Y'Y)Z'$$
$$= (XY+X'Y')Z +(X'Y+Y'X)Z'$$
$$= XYZ + X'Y'Z + X'YZ' + XY'Z'$$
$$= X(YZ+Y'Z') + X'(Y'Z+YZ')$$
$$= X(Y \oplus Z)'+X'(Y \oplus Z)$$
$$= X(Y \oplus Z)'+X'(Y \oplus Z)''$$
$$= (X \oplus (Y \oplus Z)')'$$

Q6. Two 2’s complement binary numbers are A=1111101010 and B=110001.

a) **Find a 2’s complement 6-bit binary number C which is equal to A minus B.**

b) Find a 2's complement 6-bit binary number C which is equal to A plus B.

Solution:

A = 1111101010 = 101010 = -22

B = 110001 = -15 Two's complement of B is 001110+1 = 001111 = 15

a) C = A minus B

```
  0011100   carries
   101010
   001111
 +-----------
   111001   = -7
```

b) C= A plus B

```
 1000000   carries
  101010
  110001
+-----------
 1011011   = 27
```

There is overflow. We cannot find a 6-bit two's complement number.

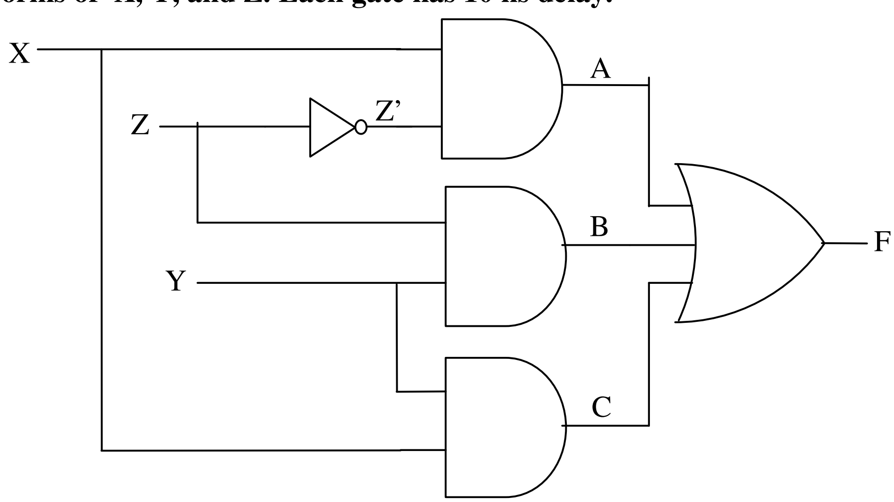
Another solution:

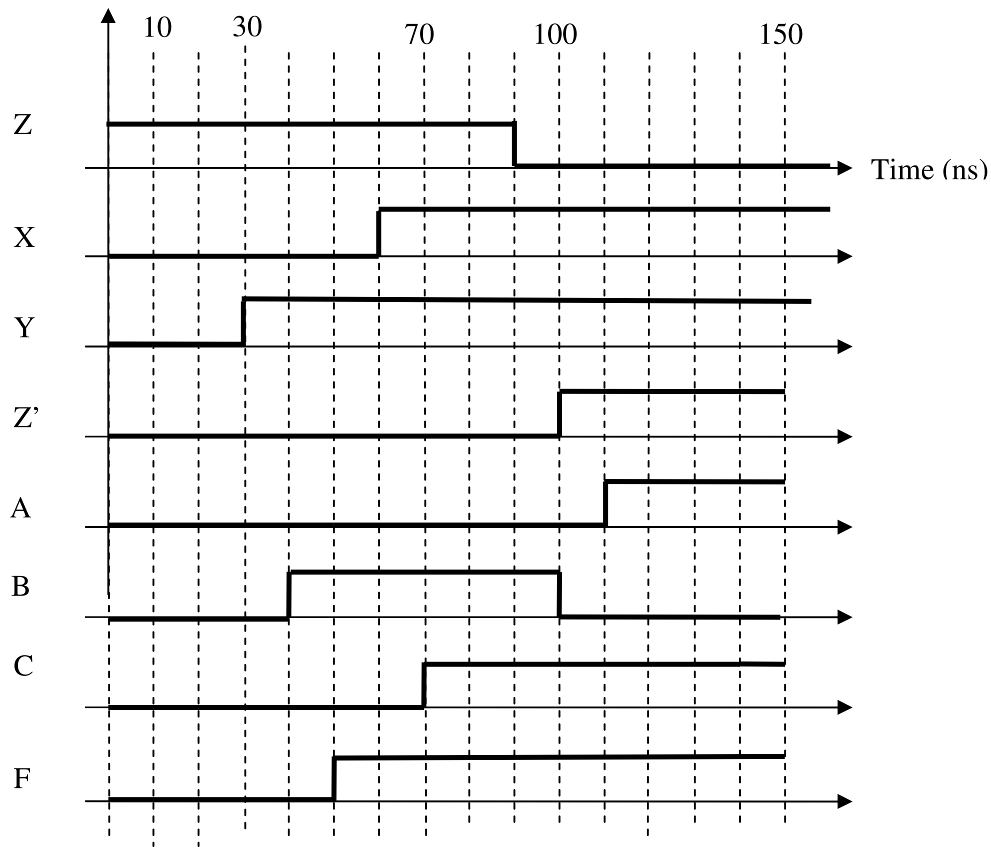
A = -22,   B = -15

A minus B = -22 – (-15) = -7 which is 100111 in the 6-bit 2's complement representation.

A plus B = -22 – 15 = -37 which is out of the range of 6-bit 2's complement representation which is -32 to 31. Therefore A plus B cannot be represented by a 6-bit 2's complement binary number.

Q7. For the circuit given below draw the waveforms of Z', A, B, C, and F for the given waveforms of X, Y, and Z. Each gate has 10 ns delay.




10
30
70
100
150
Z
Time (ns)
X
Y
Z'
A
B
C
F