EEE102
Sample Problems for Fall 2009
6-10-2008
BOOLEAN ALGEBRA

Q1.
Prove that

a) X(X+1)=X
b) (X+X)(X+X')=X
c) XY+XYZ'=XY
d) XY+XY'+X'Y=X+Y
e) ab+a'c+bc+abd=a'c+ab

Do not use perfect induction (i.e. truth table method)
At each step of your proof indicate the name of the axiom or theorem you have used.

Solution:

a) X(X+1)=X
   X(1)=X null element
   X=X identity
b) (X+X)(X+X')=X
   (X)(X+X')=X idempotency
   (X)(1)=X complement
   X=X identity
c) XY+XYZ'=XY
   XY =XY covering
d) XY+XY'+X'Y=X+Y
   X+X'Y=X+Y distributivity and complement
   X+X'Y + Y =X+Y consensus
   X+Y =X+Y covering
e) ab+a'c+bc+abd=a'c+ab
   ab+a'c+bc=a'c+ab covering
   ab+a'c=a'c+ab consensus

Q2.

a) Complement F1=A'B(CD'E+(AD')'BC)+DE' algebraically
b) Complement F2=((A+B)')+D+((A+B)CD) algebraically

Solution:

a) F1={A'B[CD'E+(AD')'BC]}+(DE')
   F1'={A'B[CD'E+(AD')'BC]}'(D'+E)
   F1'={A+B'+[CD'E+(AD')'BC]'}(D'+E)
   F1'={A+B'+[(C'+D+E')(AD'+B'+C')]}(D'+E)
b) F2=((A+B)')+D+((A+B)CD)
   F2'=(A+B)D'((A+B)CD)'
   F2'=(A+B)D'((A'B'+C'+D')

Q3.
Prove each of the following using axioms and theorems of Boolean algebra:

a) X'Y'+X'Y+XY = X'+Y
b) A'B+B'C'+AB+B'C = 1

c) $Y+X'Z+XY' = X+Y+Z$

d) $X'Y'+Y'Z+XZ+XY+YZ' = X'Y'+XZ+YZ'$

Solution:

a) $X'Y'+X'Y+XY$

$= X'(Y'+Y)+XY$

$= X'+XY$

$= (X'+X)(X'+Y)$

$= X'+Y$

b) $A'B+B'C'+AB+B'C$

$= B(A'+A)+B'(C'+C)$

$= B+B'$

$= 1$

c) $Y+X'Z+XY'$

$= (Y+X)(Y+Y')+X'Z$

$= Y+X+X'Z$

$= Y+(X+X')(X+Z)$

$= X + Y + Z$

d) $X'Y'+Y'Z+XZ+XY+YZ'$

$= X'Y'+Y'Z(X+X')+XZ+XY(Z+Z')+YZ'$

$= X'Y'+\cancel{Y'ZX}+\cancel{Y'ZX'}+XZ+\cancel{XYZ}+\cancel{XYZ'}+YZ'$

$= X'Y'+XZ+YZ'$

Q4.

Given that $AB = 0$ and $A+B=1$, prove using algebraic manipulation that $AC+A'B+BC = B+C$.

Solution:

$AC+A'B+BC$

$= C(A+B)+A'B$

$= C + A'B$ (since $A+B = 1$)

$= C+A'B+AB$ (Since $AB = 0$ it can be added)

$=C + B(A' + A)$

$= C + B = B + C$ (OK)

Q5.

a) Prove $F(A,B) = B\cdot F(A,1) + B'\cdot F(A,0)$ (This is Shannon's expansion theorem used to expand $F(A,B)$ with respect to B)

b) Add out $V\cdot W + Y\cdot Z$

c) State duality principle

Solution:

a) $F(A,B) = B.F(A,1) + B'.F(A,0)$

Proof:

For B = 0

F (A,0) = 0.F(A,1) + 1.F(A,0)

= F(A,0) (OK)

For B = 1

F (A,1) = 1.F(A,1) + 0.F(A,0)

= F(A,1) (OK)

Since it is true for all values of B irrespective of the value of A, the equality is proven.

b) VW + YZ

= (V+Y)(V+Z)(W+Y)(W+Z)

c) *Principle of Duality:* Any theorem or identity in switching algebra remains true if 0 and 1 are swapped and . and + are swapped throughout.

Dual of a theorem is also true.

Q6.
a)Explain what we mean by "multiplying out an expression to obtain a SOP form", by giving a simple example.
b) Explain what we mean by "adding out an expression to obtain a POS form", by giving a simple example.

Solution:
a) (A+B)(C+D) = AC+AD+BC+BD

b) AB+CD = (A+C)(A+D)(B+C)(B+D)

Q7.
Prove that the exclusive-OR (XOR) operation is both commutative and associative.

Solution:
XOR is commutative

$X \oplus Y = X'Y+XY' = Y'X+YX' = Y \oplus X$

XOR is associative

$(X \oplus Y) \oplus Z = (X'Y+XY') \oplus Z$

$= (X'Y+XY')'Z + (X'Y+XY')Z'$

$= (X+Y')(X'+Y)Z + X'YZ' + XY'Z'$

$= XYZ + X'Y'Z + X'YZ' + XY'Z'$

$= X'(Y'Z+YZ') + X(YZ+Y'Z')$

$= X'(Y \oplus Z)+X(Y \oplus Z)'$

$= X \oplus (Y \oplus Z)$

Q8.
Expand $F = (X+Y').Z + (X'.Y.Z')$ with respect to Y using Shannon's expansion theorem.

Solution:
Using $F(X1, X2, \ldots. Xn) = X1.F(1,X2,,\ldots..Xn)+ X1'.F(0,X2,,\ldots..Xn)$

$F = Y(XZ+X'Z') + Y'(Z)$

Using $F(X1, X2, \ldots. Xn) = [X1+F(0,X2,,\ldots..Xn)]+ [X1'+F(1,X2,,\ldots..Xn)]$

$F = [Y+(Z)][Y'+(XZ+X'Z')] = [Y+Z][Y'+ XZ+X'Z']$

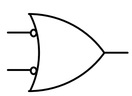
Q9.
Prove using algebraic manipulation that

$(X+Y').Z + (X'.Y.Z') = (X+Y'+Z').(X'+Z).(Y+Z)$

Solution:
Duality: $(XY'+Z)(X'+Y+Z')=(XY'Z')+(X'Z)+(YZ)$
Multiply Out Left Side: $XY'Z'+X'Z+YZ$ (Note that $XX'=YY'=ZZ'=0$)

Q10.

a) Show using DeMorgan's Theorem that



is a symbol for a NAND gate.

b) Find the complement of the function $F = A + BC'(D + E+ 1)$. Do not simplify the expression.
c) Find the dual of the function $F = A + BC'(D + E+ 1)$. Do not simplify the expression.
d) Expand the function $F = A B + CD$ with respect to A using Shannon's theorem.
e) Add out the function $F = A + BC + D$ to obtain a POS form.
f) The Boolean variables A, B, and C have values that we don't know, but we know that $AB = AC$. If $C = 0$, what can we say about B.

Solution:
a)

De Morgan's Theorem states that

$(A\cdot B)' = A' + B'$.

The left-hand side of this equation is the standard NAND definition, and the right-hand side is the alternative representation depicted in the above symbol.

b)

We apply De Morgan's Rule repeatedly.

$F' = (A + BC'(D+E+1))' = A'\cdot(BC'(D+E+1))' = A'\cdot(B' + C + (D+E+1)')$
$= A'\cdot(B' + C + (D'\cdot E'\cdot 0))$

c)

$F^D = (A + (BC'(D+E+1)))^D = (A\cdot(B + C' + (D\cdot E\cdot 0)))$

d)

$F(A,B,C) = A' \cdot F(0,B,C) + A \cdot F(1,B,C) = A' \cdot (C \cdot D) + A \cdot (B + C \cdot D)$

e)

$F = A + BC + D = (A + B)(A+C) + D = (A+B+D)(A+C+D)$

f)

If A=1, then B=C, which means B=0.

If A=0, then both sides of AB = AC will be 0, regardless of the values of B and C. In this case, we cannot say anything about the value of B. (It can be 0 or 1)

In short, without knowing the value of A, we cannot say anything about the value of B.