BILKENT UNIVERSITY
Department of Electrical and Electronics Engineering
EEE102 Introduction to Digital Circuit Design
MidTerm Exam I Solution
14-03-2005
Duration 110 minutes

1. (16 points)
   a) Show that A·B = A·C does not imply that B = C.
   b) Show that NAND and NOR operators are not associative.
   c) Given that X·Y’ + X’·Y = Z, show that X·Z’ + X’·Z = Y.
   d) Convert the expression {[(A+B+A’·C’)·C+D]’+A·B’} into sum of minterms.

Solution:
1a)

	A	B	C	A·B	A·C
0	0	0	0	0	0
1	**0**	**0**	**1**	**0**	**0**
2	**0**	**1**	**0**	**0**	**0**
3	0	1	1	0	0
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	1	0
7	1	1	1	1	1

Note that in rows 0,1,2,3,4, and 7, A·B and A·C have the same values, but in rows 1 and 2, B and C are not equal.

1b) If nand operator is associative then [(A nand B) nand C] must be equal to [A nand (C nand B)].
(A nand B) nand C = [(A·B)’·C]’ = A·B+C’ (1)
A nand (C nand B) = [A·(B·C)’]’ = A’+B·C (2)
Expressions (1) and (2) are not equal because forexample if A = 0 and C = 1, then expression (1) is 0 but expression (2) is 1.

If nor operator is associative then [(A nor B) nor C] must be equal to [A nor (C nor B)].
(A nor B) nor C = [(A+B)’+C]’ = (A+B)·C’ (1)
A nor (C nor B) = [A+(B+C)’]’ = A’·(B+C) (2)
Expressions (1) and (2) are not equal because forexample if A = 0 and C = 1, then expression (1) is 0 but expression (2) is 1.

1c) XY’+X’Y=Z is given.

$XZ'+X'Z = X(XY'+X'Y)' + X'(XY'+X'Y)$ substituting Z

$= X(X'+Y)(X+Y') + X'XY' + X'X'Y$

$= X(X'X+X'Y'+YX+YY') + X'Y$

$= XY + X'Y$

$= (X+X')Y$

$= Y$

$XY'+X'Y=Z$ means $Z=X \oplus Y$

Shorter solution: $XZ'+X'Z = X \oplus Z = X \oplus (X \oplus Y) = (X \oplus X) \oplus Y$

$= 0 \oplus Y = Y$

1d) $[(A+B+A'\cdot C')\cdot C+D]'+A\cdot B'$

$= [AC+BC+D]'+AB'$

$= (A'+C')(B'+C')D' + AB'$

$= (A'B'+A'C'+B'C'+C')D'+AB'$

$= (A'B'+C')D'+AB'$

$= A'B'D'+ C'D'+AB'$

$= A'B'(C'+C)D'+(A'B'+A'B+AB'+AB)C'D'+AB'(C'D'+C'D+CD'+CD)$

$= A'B'C'D'+A'B'CD'+A'B'C'D'+A'BC'D'+AB'C'D'+ABC'D'$
$+AB'C'D'+AB'C'D+AB'CD'+AB'CD$

$= A'B'C'D'+A'B'CD'+A'BC'D'+AB'C'D'+ABC'D'$
$+AB'C'D+AB'CD'+AB'CD$

Another method of solution: F = $[(A+B+A'\cdot C')\cdot C+D]'+A\cdot B'$ has the TT

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

We can then write the sum of minterms

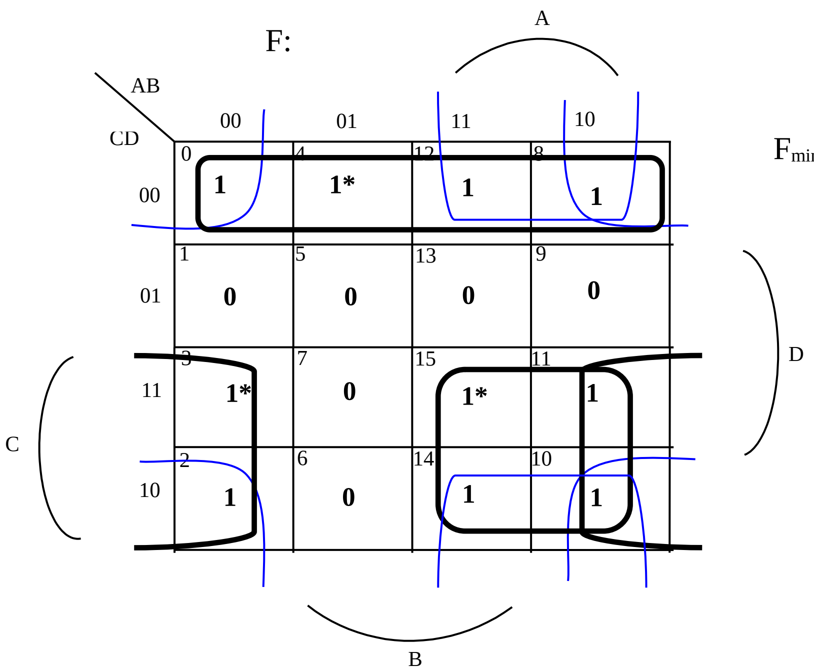
$F = A'B'C'D'+A'B'CD'+A'BC'D'+AB'C'D'+AB'C'D+AB'CD'+AB'CD+ABC'D'$

2. (12 points) Find all minimal sum expressions and all minimal product expressions for

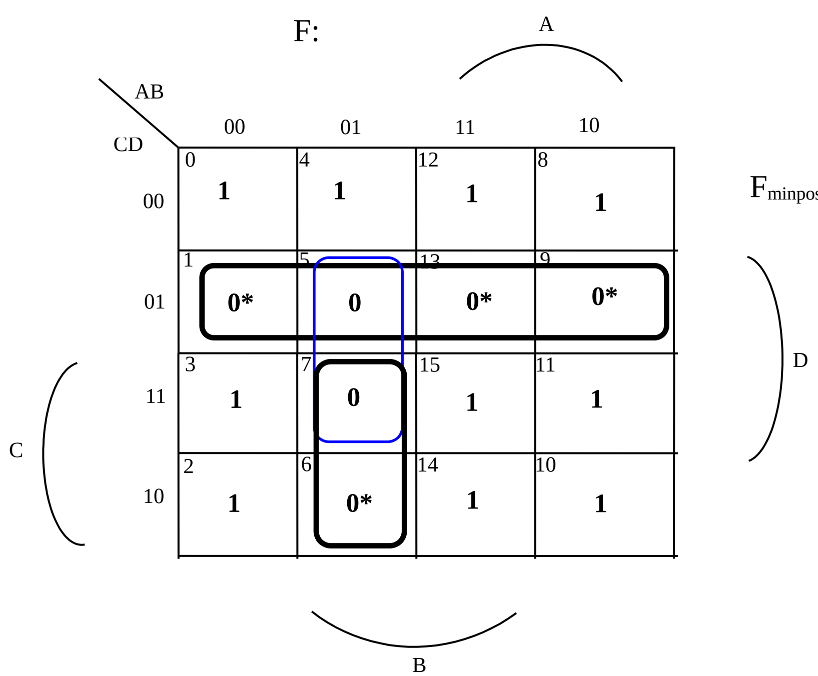
$$F = \sum_{A,B,C,D}(0,2,3,4,8,10,11,12,14,15).$$

(Draw Karnaugh Maps, indicate prime implicants and distinguished 1s and 0s).

Solution:



F_{minsop} = C’D’+B’C+AC

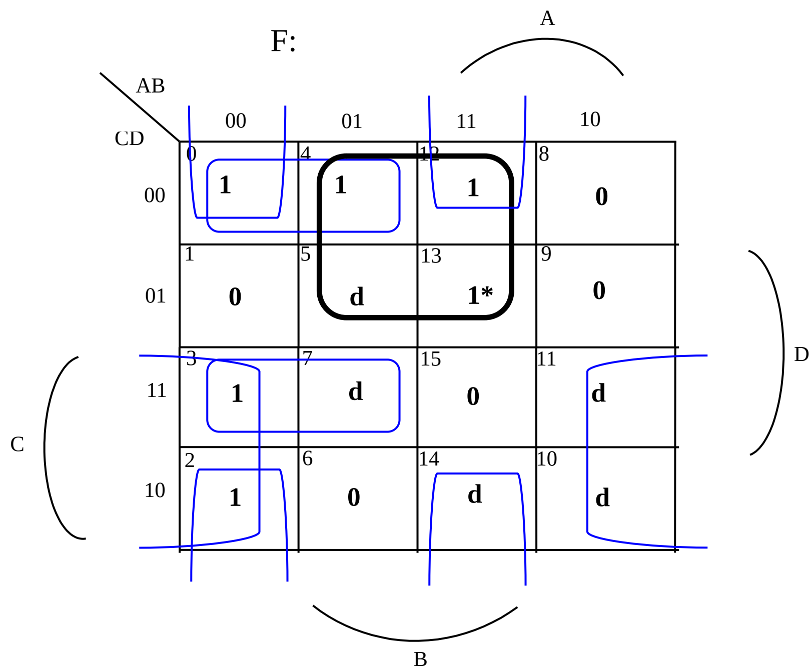


F_{minpos} = (C+D’)(A+B’+C’)

3. (16 points) Find all minimal sum expressions and all minimal product expressions for

$$F = \sum_{A,B,C,D}(0,2,3,4,12,13) + d(5,7,10,11,14).$$

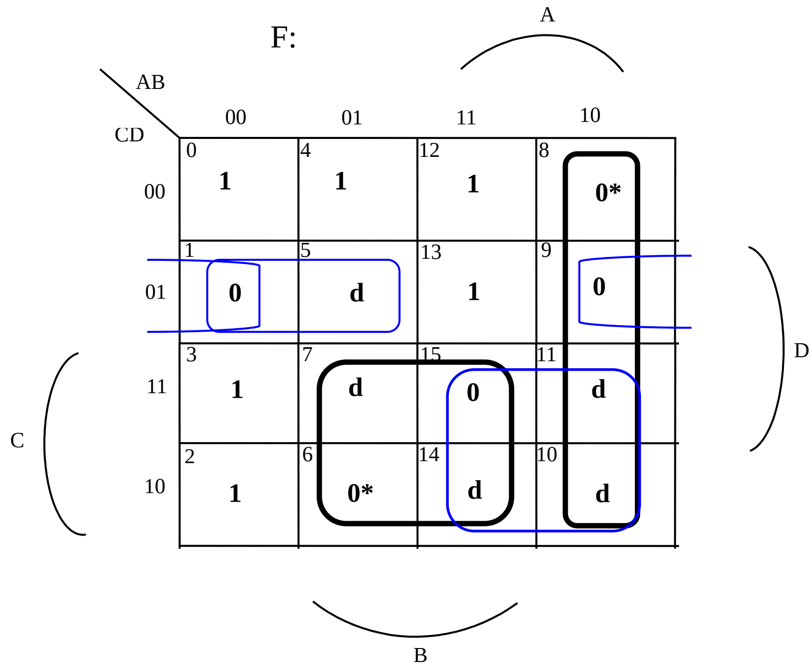
Determine which of these minimal expressions are equivalent. Explain why. (Draw Karnaugh Maps, indicate prime implicants and distinguished 1s and 0s).



$F_{minsop1}$ = BC'+B'C+A'B'D'   5710111410110

5710111410110

$F_{minsop2}$ = BC'+B'C+A'C'D'



$F_{minpos1} = (B'+C')(A'+B)(A+C+D')$ 5710111400000

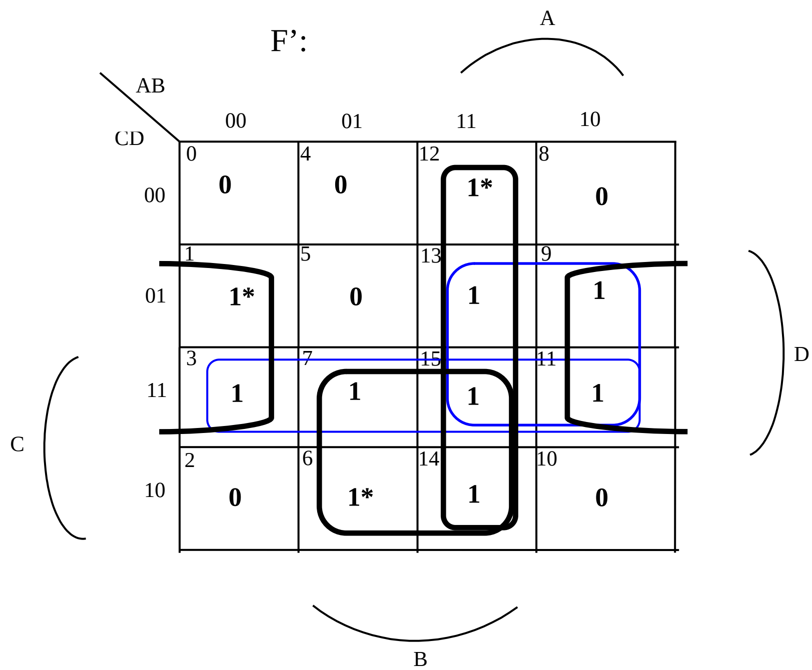
$F_{minpos2} = (B'+C')(A'+B)(B+C+D')$ 5710111410000

$F_{minsop1}$ and $F_{minsop2}$ are equivalent because don't cares have the same values . All the other solutions have different don't care values.

4. (12 points) Find all minimal sum expressions for

$$F = \prod_{A,B,C,D}(1,3,6,7,9,11,12,13,14,15).$$

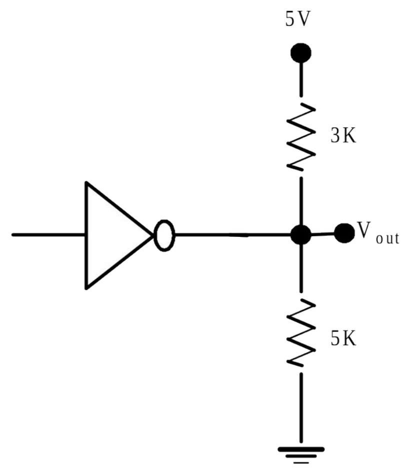
Also find all minimal product expressions for F by using the methodology of finding minimal sums.
(Draw Karnaugh Maps, indicate prime implicants and distinguished 1s and 0s).



F'_{minsop} = B'D + AB + BC

F'_{minpos} = (B+D')(A'+B')(B'+C')

5. (16 points) A CMOS inverter working from 5V supply has a resistive load as shown below.



Suppose this CMOS inverter has the following specifications:

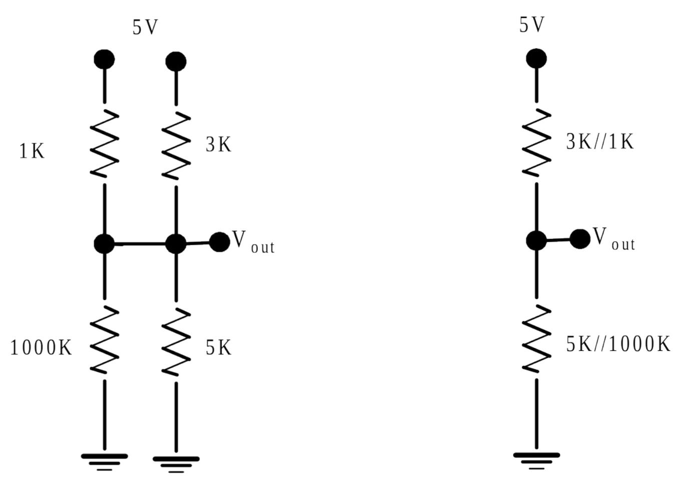
V_{IHmin} = 3.5 V, V_{ILmax} = 1.5 V , I_{ILmax} = -10 μA, I_{IHmax} = 15 μA.

The ON and OFF resistances of the NMOS and PMOS transistors are 1KΩ and 1000 KΩ respectively.

a) Find V_{out} for when it is HIGH and also for when it is LOW.
b) How many additional CMOS inverters of the same type can be connected to V_{out} ?

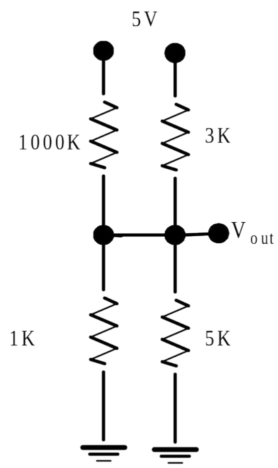
Solution:

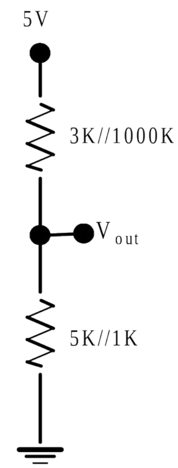
a) Output HIGH:



$$3K\,//\,1K = 0.75K$$
$$1000K\,//\,5K \simeq 5K$$
$$V_{out} = 5V \times \frac{5}{5+.75} = 4.35V$$

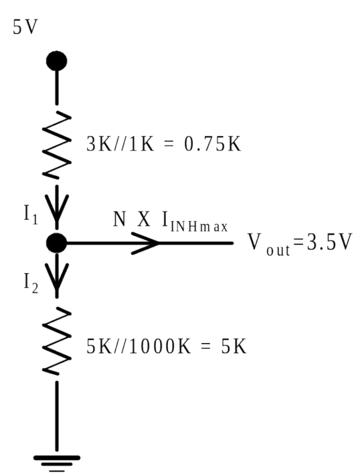
Output LOW:





$$3K\,//\,1000K \simeq 3K$$
$$1K\,//\,5K \simeq 0.833K$$
$$V_{out} = 5V \times \frac{0.833}{3+.833} = 1.09V$$

b) Output HIGH



$V_{out} = 3.5V$

$$I_1 = \frac{5 - 3.5}{0.75K} = 2mA$$

$$I_2 = \frac{3.5}{5} = 0.7mA$$

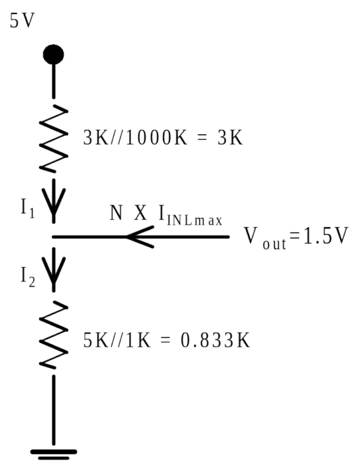
$N \times 15\mu A = I_1 - I_2 = 2 - 0.7 = 1.3mA$

$$N = \frac{1.3mA}{15\mu A} = 86.7$$

N must be an integer less than or equal to 86.7

Therefore take $N = 86$

Output LOW



$V_{out} = 1.5V$

$$I_1 = \frac{5 - 1.5}{3K} = 1.167mA$$

$$I_2 = \frac{1.5}{0.833} = 1.8mA$$

$N \times 10\mu A = I_2 - I_1 = 1.8 - 1.167 = 0.633mA$

$$N = \frac{0.633mA}{10\mu A} = 63.3$$

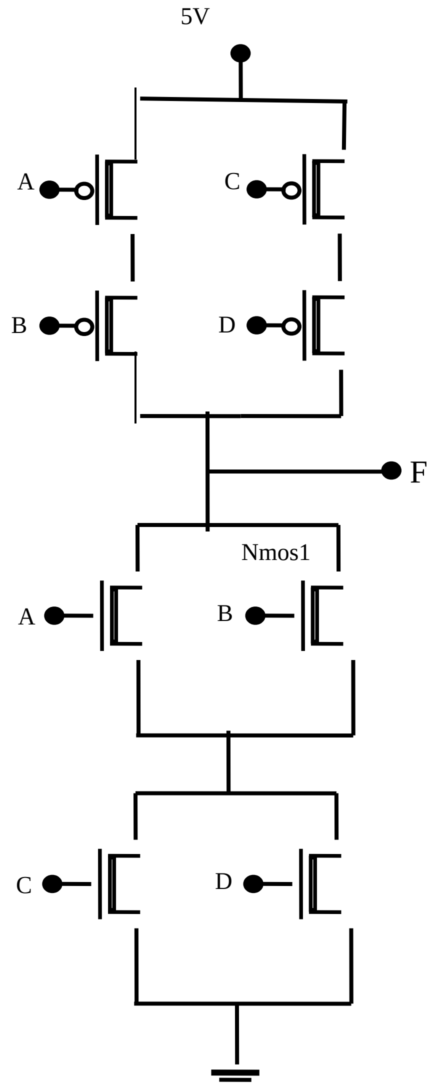
N must be an integer less than or equal to 63.3

Therefore take $N = 63$

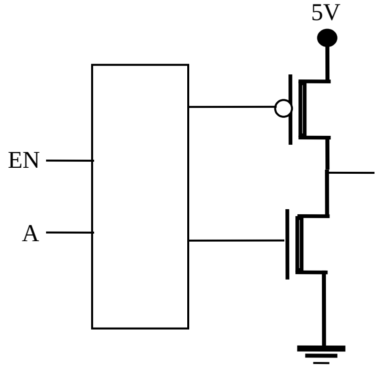
$Fanout = \min\{86, 63\} = 63$

6. (12 points) Draw the internal circuit of a CMOS circuit (using NMOS and PMOS transistors) which has the logic function F = A’·B’ + C’·D’.

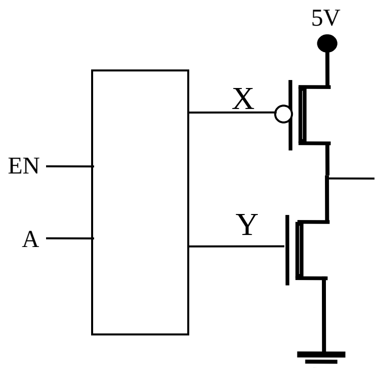
Solution: F = A’·B’ + C’·D’= [(A+B).(C+D)]’



7. (16 points) The rectangular block in the below drawing represents a logic circuit (logic block). Design this logic circuit so that if EN is 0 then F is in Hi-Z state and if EN is 1 then F = A'. In order words, you are to design the inside of the logic block so that the whole circuit becomes a three-state inverter. Make your design using gates (not transistors). Draw the whole circuit.



Solution:



EN	A	X	Y
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

X=EN'+A
Y=EN·A