

[L]MATHEMATICS [C] [R] [C]2019-20

blue5.1 Introduction

This chapter is essentially a continuation of our study of differentiation of functions in Class XI. We had learnt to differentiate functions, of course, assuming that every function is continuous. The graph of this function is given in Figure 5.1. One

blue5.2 Continuity

We start the section with two informal examples to get a feel of continuity. Consider the function:

r0.4 [width=1]IMG_0250312_01905.jpg5.1 Graph of the function

In particular, the left and right hand limits do not coincide. We also observe that the value of the function at $x = 0$ coincides with the right hand limit. Now, consider the function defined as follows:

r0.4 [width=1]IMG_0250312_0185848.jpg5.1 Graph of the function
This function is also defined at every point. The left and right hand limits at $x = 0$ are both equal to 1. However, we note that we cannot draw the graph of the function without lifting the pen. This is yet another instance of discontinuity. Naively, one might say that a function is continuous at a fixed point if we can draw the graph of the function at that point. Mathematically, the concept of continuity can be phrased more precisely as follows:

blueDefinition 1 Suppose f is a real function on a subset of the real numbers, and let c be a point in the domain of f .

More elaborately, if the left-hand limit, right-hand limit, and the value of the function at $x = c$ exist and are equal to each other, then f is continuous at $x = c$. Hence, we may also rephrase the definition of continuity as follows: a function f is continuous at $x = c$ if: