# **Theorem Prover**

ATLS 5241: Gregory Greenstreet, Brian Newsom Sushma Akoju

#### **Project Details**

Project name: Theorem Prover

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A link to or PDF of your presentation slides:

https://docs.google.com/presentation/d/1G3IEAArSBkxVCxcbiNA-CP7ZTPym5P4DYHeqA or-DA/edit?usp=sharing

A link to your project: <a href="https://theorem-prover-4182022.uc.r.appspot.com/">https://theorem-prover-4182022.uc.r.appspot.com/</a>

Video:

https://drive.google.com/drive/folders/11CYQdIHaEHFdWmHj36qu4WJO3829uT 0?usp=s haring

Repository: <a href="https://github.com/sushmaakoju/demo-ATLS5214">https://github.com/sushmaakoju/demo-ATLS5214</a>

### **About the conceptual part of the Project**

- Theorem Provers have been there all the time: Compilers and Interpreters.
- It was Theorem Proving concept that inspired Compilers.
- We extend Theorem Proving concept to solve real world problems Natural Language Problems.

## Example description for this project

- Smoking causes cancer
- We need to stop people from smoking
- It's hard to do that since people are influenced by friends
- If friends keep smoking, they are likely to continue smoking

#### Peer influence doubles smoking risk for adolescents

Teens from collectivistic cultures also more swayed by peers than those in individualistic cultures

Date: August 21, 2017

Source: University of Pennsylvania

Summary: Having friends who smoke doubles the risk that youth ages 10 to 19 will pick up the

habit, finds new meta-analysis of 75 longitudinal teen smoking studies. This influence is

more powerful in collectivistic cultures than in individualistic ones.

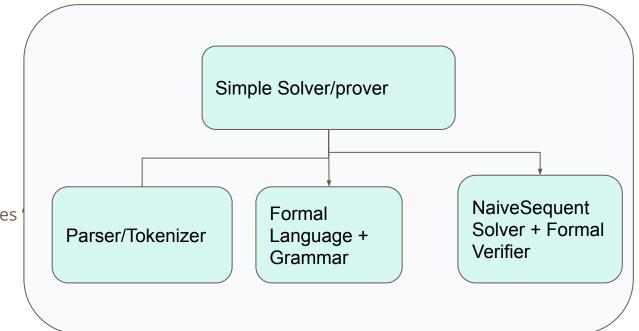
https://www.sciencedaily.com/releases/2017/08/170821102718.htm

#### Natural Language Beliefs and Logic Statements

- Let us say, we have following beliefs from previous slide:
  - Smoking causes cancer
  - Friends have similar habits
  - Let Alice and Bob be two friends
  - Alice Smokes and has Cancer
  - Alice has Smoking habit
  - Bob has Smoking habit
  - Can Bob get Cancer?

Simple Theorem Prover (Entscheidungsproblem)

- General Substitution
- General Unification
- Entscheidungsproblem -
- Universally validity and defines
- Using Hilbert's version



Reference: https://www2.karlin.mff.cuni.cz/~stovicek/math/decidability.pdf

#### **About Universal Validity**

**Hilbert:** Is there an algorithm which, given an effectively described theory, such as Peano Arithmetics, and a sentence  $\xi$  in the theory decides, whether  $\xi$  is or is not provable from the axioms?

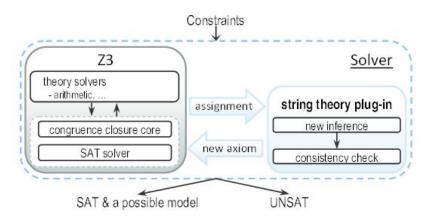
**Halting problem:** Is there an algorithm (program) Halt(P, F) which, given a source code P of another program and its input file F, decides whether P halts on the input F ?

**Turing:** There is no such algorithm. Therefore, the halting problem is undecidable.

-> There is no such Universally valid algorithm

#### **Z3 Solver and Prover (Microsoft Research)**

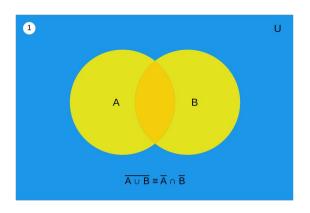
- Built from advanced concepts of Satisfiability Theories (Modulo)
- Proof by Refutation
- Z3 first tries to prove theorem is wrong from set of axioms
- If it fails to prove theorem is wrong from set of facts, then theorem is true.

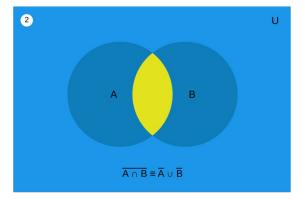


## **De Morgan's Law**

The negation of a disjunction is the conjunction of the negations

The negation of a conjunction is the disjunction of the negations







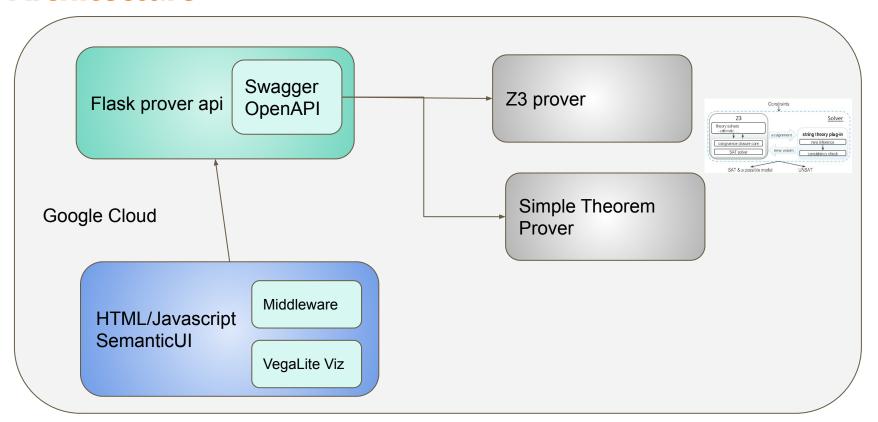








#### **Architecture**



### **Challenges & Solutions**

- Included complex methods for api methods
- Used Textbox instead of dropdown they are not universal provers!
- Vega is not converted to HTML using D3
- Dataset NOT received as it is pending approval from researcher

#### Solutions:

- Use simple abstract api methods
- Use dropdown
- Use VegaEmbed to simply populate Vega visualizations directly into HTML

### Google Cloud deployment & documentation

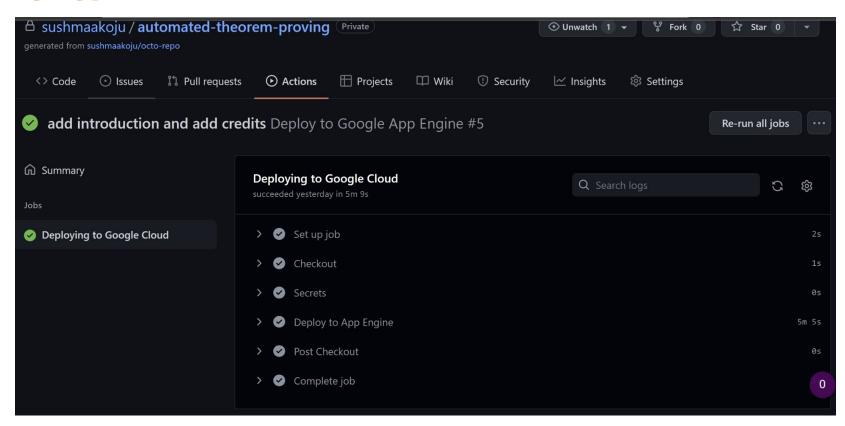
Deployed with help and guidance from Sagar - due to similarity of architecture.

I wrote this document to write down details of what I learnt from Google Cloud:

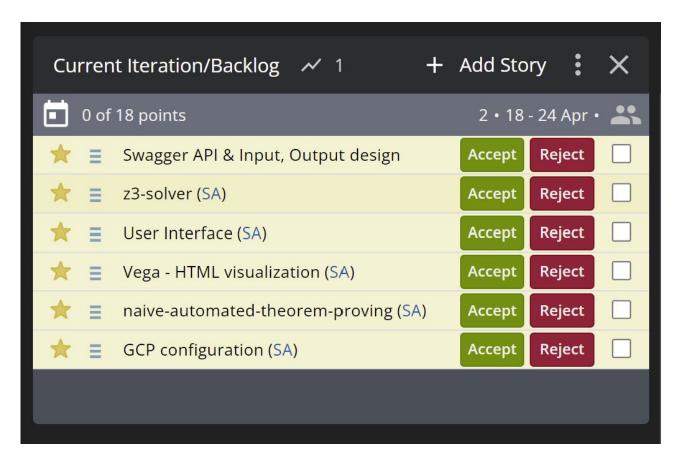
https://docs.google.com/document/d/1WvGXkunjrA8 EkvV5E-KLgv-t5DTDvTVog|ZCgk5mSA/edit?usp=sharing

Reviewed by Sagar.

#### CI-CD

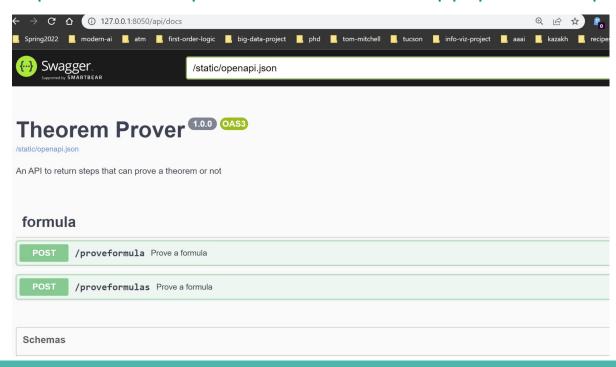


#### **Pivotal Tracker**

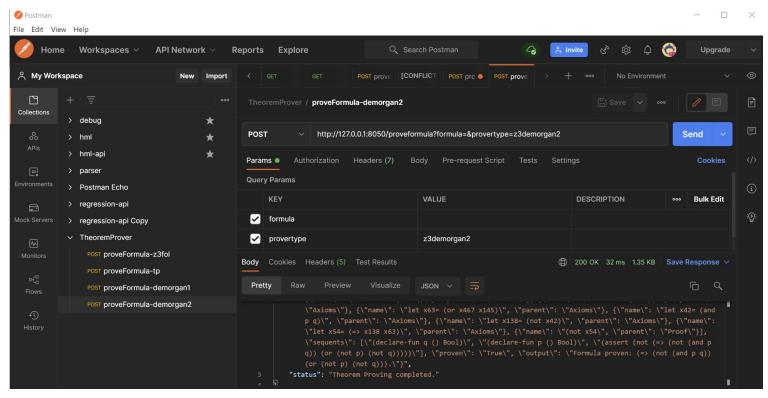


## Swagger (OpenAPI) standard

https://theorem-prover-4182022.uc.r.appspot.com/api/docs



## **Postman API Testing**



## **Simple Theorem Prover: Additional theory**

- 1) Define class for Variables/symbols
- 2) define Functions/methods to Instantiate:
- "Not", "And", "Or", "Implies", "ForAll", "ThereExists"
- 3) define keywords, to separate tokens, identify tokens
- Valid tokens: ['not', 'implies', 'and', 'or', 'forall', 'exists']
- 4) Parse and typecheck (we need to know if types are valid)
- 5) Substitution: to substitute ground terms (such as Alice, Bob)

- 3) First each of function/operation, create a Sequent
- Note: A sequent is a conditional or unconditional assertion.
- 4) Unify each sequent. -> 3 unification strategies:
  - a) Does sequent consist of a list? then unify list
  - b) Find all unifiable pairs

What we mean by unification?

f(g(x, b), f(x, z)) = f(y, f(g(a, b), c)) can be written as

$$x = g(g(a, b), b), y = g(a, b), z = c$$

For more in-depth reading: please do, refer this: <a href="https://www.cs.le.ac.uk/events/mgs2009/courses/struth/slides.pdf">https://www.cs.le.ac.uk/events/mgs2009/courses/struth/slides.pdf</a>.

I would be glad to discuss further.

#### **Unification pseudo code: Naive version**

Goal: Identify two symbolic expressions.

Method: Replace certain subexpressions (variables) by

other expressions

Refer: Syntactic Unification for Inferential Logic.

https://github.com/aimacode/aima-pseudocode/blob/master/md/Unify.md

#### **Credits**

Thank to Sagar for help with Google Cloud.

Thank to Saumya for sharing the guidelines.

Thank you Brian and Greg for allowing me to work on this project.