CH 1. Basics of Engineering Economics

Economics:

:- Economics is the social science that examines how people choose to use limited resources in attempting to satisfy their unlimited wants.

: Economics can be broken down into two parts:

(a) Microeconomics :- Focuses on the actions of individual agents within the economy, like households, workers and businesses.

(b) Macroeconomics :- looks at the economy as a whole.

Engineering Economics:

:- Engineering Economics is the study of how to make economic decisions in engineering projects.

:- Engineering Economics is the application of economic techniques for the evaluating of design and engineering alternative.

Engineering projects :- complete within possible short period
:- economically
:- without compromising on quality

Role of Engineering Economics:

: Access the appropriateness of a given project
: Estimate its value
: Justify it from an engineering point of view

Types of Engineering Economics decisions:

1. Material & process Selection
2. Equipment replacement
3. New product and product expansion

4. cost reduction
5. Service improvement
6. Capital budgets allocation

Role of Engineer's in decision making:
1. Understand the problem and define the objectives
2. collect relevant information
3. Define the feasible alternative solution and make realistic estimates.
4. Identify the criteria for decision making using one or more attribute.
5. Evaluate each alternative, using sensitivity analysis to enhance the evolution.
6. select the best alternatives
7. Implement the solution and monitor the result.

principle of Engineering Economics:

1. Develop the alternatives:
: There must be more than one alternatives, the choice is made among these after subsequent analysis.

2. Focus on the differences:
: only the differences in expected outcomes of the feasible alternatives
: Be very neutral & impartial between the alternatives.

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3. Use a consistant viewpoint: (economic viewpoint)
: what is your position? Be consistent
: The prospective outcomes of the alternatives, selection of the criteria and other, should be consistently developed from a defined viewpoint. (~~per~~

4. Use a common unit of measure
: Uniformity in measurement, analysis and result. Helps interpret easily.

5. Consider all relevant criteria: (Social & Environmental aspects)
: Selection of best alternative required consideration of almost all relevant criteria. This include both the outcomes specified in the monetary unit & those expressed in some other unit of measurement.

6. Make uncertainty explicit:
: Risk and uncertainty are inherent in estimating the future outcomes of the alternatives and should be recognized in the analysis and comparison.
: Don't fire blindly.

7. Revisit your decisions:
: The initial projected outcomes of the selected alternative should be subsequently compared with the actual results achieved.
: Revisiting of decision ensure the good result of the final decision.

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Why is it important to study engineering economics for engineers?

:- The field of engineering economy is concerned with the systematic evaluation of the benefits and costs of the projects involving engineering design and analysis.

:- Engineering economy quantifies the benefits and costs associated with engineering projects to determines if they make (or save) enough money to warrant their capital investment.

:- In manufacturing or construction, engineering is involved in every detail of a products production (about 85%) from conceptual design to distribution.

:- It is mathematical modeling with emphasis on the economic effects in the primary ~~and~~ analytical technique to select between defined feasible alternatives

:- Engineering economy requires the application of technical and economic analysis with the goal of deciding best meets technical performance criteria and uses scare capital in a prudent manner.

:- The study of engineering economy draws upon knowledge of engineering and economics to address problems of allocating scarce resources.

:- The techniques and models of engineering economy assist people in making decisions.

:- There are numerous examples of structures, machines, processes, and systems that exhibit excellent physical design but have little economic merit.

:- In fact, not understanding the economic issues will guarantee that you will not reach your full potential as an engineer.

:- Engineering economics involves the evaluation of costs of benefits of proposed projects.
:- So, studying engineering economics provides foundation for engineers to make good decisions in business environment.

CHAPTER 1
Basics of Engineering Economics

Demand :

- Demand refers to the willingness and ability of consumers to purchase a given quantity of a good or service.
- Demand in economics is the quantity of goods and services bought at various prices during a period of time.
- It's the key driver of economic growth.

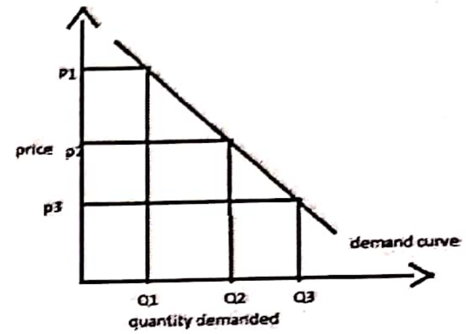
Law of Demand :

- The law of demand states that other factors being constant, price and quantity demand of any good and service are inversely related to each other. When the price of a product increases, the demand for the same product will fall.

- Law of demand explains consumer choice behavior when the price changes.
- In the market, assuming other factors affecting demand being constant, when the price of a good rises, it leads to a fall in the demand of that good.
- This is the natural consumer choice behavior. This happens because a consumer hesitates to spend more for the good with the fear of going out of cash.

Demand Schedule

Price of Apple (Per Kg)	Quantity Demanded
100	200
90	300
80	400
70	500
60	600
50	700



- The above diagram shows the demand curve which is downward sloping. Clearly when the price of the commodity increases from price p3 to p2, then its quantity demand comes down from Q3 to Q2 and then to Q3 and vice versa.

Factors influencing to Demand :

1. Price of the Given Commodity
2. Price of Related Goods
3. Income of the Consumer
4. Tastes and Preferences
5. Expectation of Change in the Price in Future
6. Size of population
7. Weather

Utility:

- ➢Utility is a term in economics that refers to the total satisfaction received from consuming a good or service.
- ➢Utility, in economics, refers to the usefulness or enjoyment a consumer can get from a service or good.
- ➢Economic utility can decline as the supply of a service or good increases.
- ➢**Utility** simply means the ability to satisfy a want. A commodity may have **utility** but it may not be useful to the consumer.
- ➢For instance—A cigarette has **utility** to the smoker but it is injurious to his health.
- ➢Example: Bread satisfies hunger, TV satisfies the want for entertainment.
- ➢However, demand for a commodity depends on its **utility** rather than its usefulness.

Marginal Utility(MU)

- ➢Marginal utility, in economics, the additional satisfaction or benefit (utility) that a consumer derives from buying an additional unit of a commodity or service.
- ➢Marginal utility refers to the extra utility a consumer gets from one additional unit of a specific product.
- ➢It is the utility derived by single unit of consumption.
- ➢MU= $\frac{\textit{Change in total utility}}{\textit{Change in quantity consumption}}$

Law of Diminishing Marginal Utility:

- ➢When a consumer consumes a particular commodity continuously then the utility derives from each successive unit goes on diminishing.
- ➢Assumptions:

1. Units consumed must be homogeneous.
2. Units consumed must be of standard size.
3. Units consumed must be continuous.
4. Only one commodity must be consumed at a time.
5. Income must be constant.
6. No substitute or compliment be introduced.

Diagrammatic Explanation of Law of DMU:

- Let us understand the law with the help of Table and Fig. :
- **Table: Law of Diminishing Marginal Utility**

Units of Ice Cream	Total Utility (in utils)	Marginal Utility (in utils)
1	20	20
2	36	16
3	46	10
4	50	4
5	50	0
6	44	-6

Law of Diminishing Marginal Utility

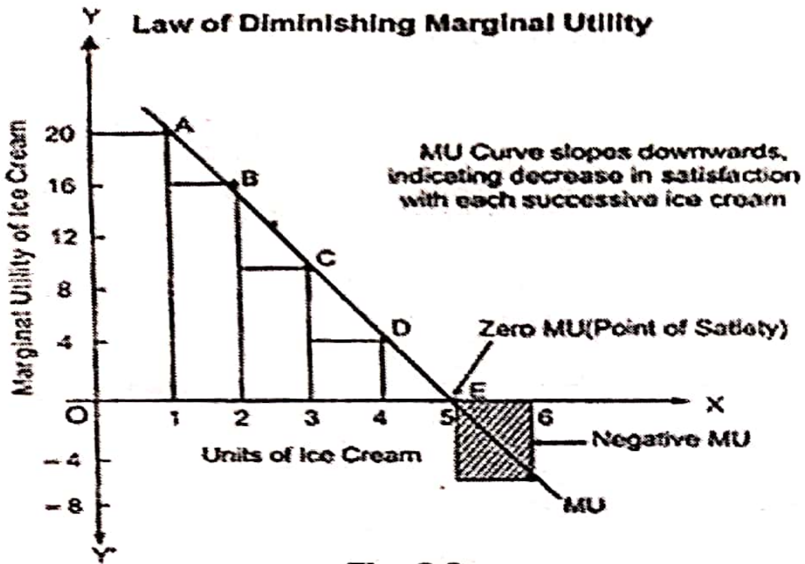
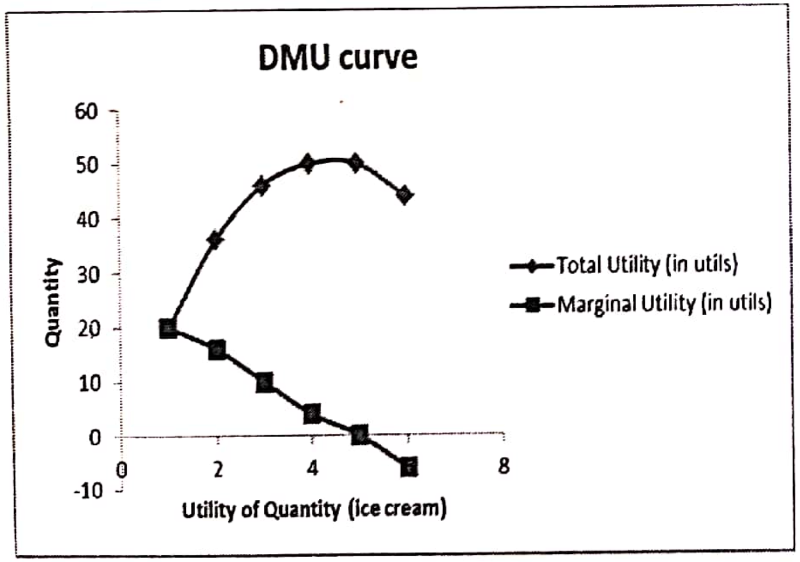


Fig. 2.2



- In the diagram, units of ice-cream are shown along the X-axis and MU along the Y-axis. MU from each successive ice-cream is represented by points A, B, C, D and E. As seen, the rectangles (showing each level of satisfaction) become smaller and smaller with increase in consumption of ice-creams.
- MU falls from 20 to 16 and then to 10 utils, when consumption is increased from 1st to 2nd and then to 3rd ice-cream. 5th ice-cream has no utility (MU= 0) and this is known as the 'Point of satiety'. When 6th ice-cream is consumed, MU becomes negative. MU curve slopes downwards showing that MU of successive units is falling.
- In the graph below, the total utility (TU curve) is increasing at decreasing rate (MU Curve) and becomes maximum when MU is zero. When MU negative TU declines.

Supply:

Supply is a fundamental **economic** concept that describes the total amount of a specific good or service that is available to consumers.

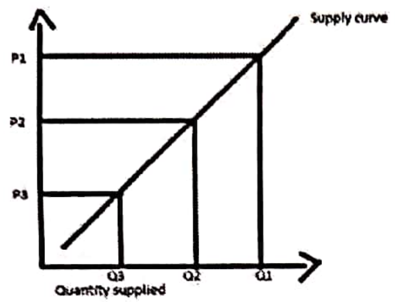
Law of Supply:

Law of supply states that other factors remaining constant, price and quantity supplied of a good are directly related to each other.

In other words, when the price paid by buyers for a good rises, then suppliers increase the supply of that good in the market.

Table 4.1. Supply Schedule

Price (Rs.)	Quantity Supplied
225	100
275	200
325	300
375	400
425	500



- The above diagram shows the supply curve that is upward sloping (positive relation between the price and the quantity supplied). When the price of the good was at P3, suppliers were supplying Q3 quantity. As the price starts rising, the quantity supplied also starts rising.

Factors influencing to supply:

1. Price of commodity
2. Price of factors of production
3. Price of related goods
4. Production technology
5. Change in money income
6. Taxes and subsidies
7. Development of infrastructures
8. State of natural resource etc.

Economics:

➢Economics is a social science that examines how people choose to use limited resources in attempting to satisfy their unlimited wants.

➢Economics is a social science concerned with the production, distribution, and consumption of goods and services.

➢It studies how individuals, businesses, governments, and nations make choices on allocating resources to satisfy their wants and needs, trying to determine how these groups should organize and coordinate efforts to achieve maximum output.

Two major types of economics:

1. *Microeconomics*, which focuses on the behavior of individual consumers and producers.
2. *Macroeconomics*, which examine overall economies on a regional, national, or international scale.

Engineering Economics:

- ➤Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives.
- ➤Engineering economics is the study of how to make economic decisions in engineering projects.
- ➤Engineering Projects:-

-complete within possible short period

-economically

-without compromising on quality

- ➤Engineering economics involves the systematic evaluation of the economic benefits of proposed solutions to engineering problems.
- ➤The engineering economics involves technical analyzing with emphasis on the economic aspects and has the objective of assisting decisions.

Principles of Engineering Economics:

1. Develop the Alternatives
2. Focus on the Differences
3. Use a Consistent Viewpoint
4. Use a Common Unit of Measure
5. Consider All Relevant Criteria
6. Make Uncertainty Explicit
7. Revisit Your Decisions

Elasticity of Demand (Ed) :

- Elasticity of demand is the responsiveness of the quantity demanded of a commodity to change in one of the variables (price, income and so on) on which demand depends.
- The change in quantity demanded due to change in price, income etc. is called elasticity of demand.
- It may also be defined as the ratio of the percentage change in demand to the percentage change in variables (price, income etc.) of particular commodity.
- Mathematically,

$$Ed = \frac{\textit{proportionate change in quantity demanded}}{\textit{proportionate change in any one of variable}}$$

Types of elasticity of demand

1. **Price Elasticity of Demand**
2. **Income Elasticity of Demand**
3. **Cross Elasticity of Demand**

1. Price Elasticity of Demand:

Price Elasticity of demand is the responsiveness of the quantity demanded of a commodity to change in its price.

It may also be defined as the ratio of the percentage change in demand to the percentage change in price of particular commodity.

- Price elasticity of demand is represented by E_P and it is calculated using the following formula:

$$E_P = \frac{\Delta Q/Q}{\Delta P/P}$$

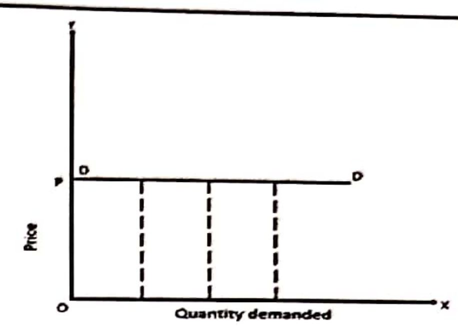
Where, E_P= Price elasticity of demand
Q= Original quantity demanded
ΔQ = Change in quantity demanded
p= Original price
Δp = Change in price

Types or degrees of price elasticity of demand:

There are 5 types of elasticity of demand:

I. Perfectly Elastic Demand ($E_P = \infty$):

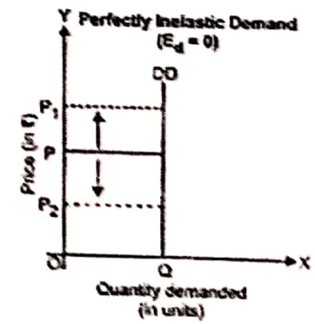
The demand is said to be perfectly elastic if the quantity demanded increases infinitely (or by unlimited quantity) with a small fall in price or quantity demanded falls to zero with a small rise in price. Thus, it is also known as infinite elasticity. It does not have practical importance as it is rarely found in real life.



- In the given figure, price and quantity demanded are measured along the Y-axis and X-axis respectively. The demand curve DD is a horizontal straight line parallel to the X-axis. It shows that negligible change in price causes infinite fall or rise in quantity demanded.

II. Perfectly Inelastic Demand ($E_P = 0$):

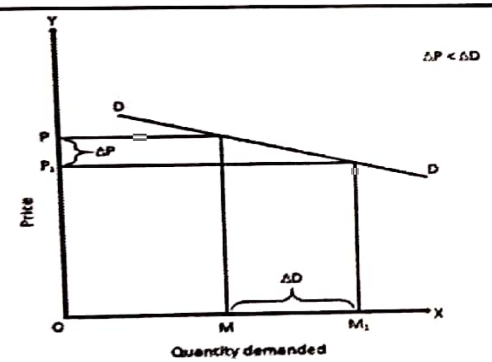
The demand is said to be perfectly inelastic if the demand remains constant whatever may be the price (i.e. price may rise or fall). Thus it is also called zero elasticity. It also does not have practical importance as it is rarely found in real life.



- In the given figure, the demand curve DD is a vertical straight line parallel to the Y-axis. It shows that the demand remains constant whatever may be the change in price.

III. Relatively Elastic Demand (E_p> 1):

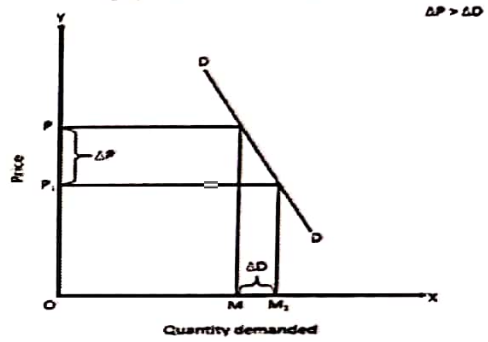
The demand is said to be relatively elastic if the percentage change in demand is greater than the percentage change in price i.e. if there is a greater change in demand there is a small change in price. It is also called highly elastic demand or simply elastic demand.



- In the given figure, the demand curve DD is more flat, which shows that the demand is elastic. The small fall in price from OP to OP_1 has led to greater increase in demand from OM to OM_1. Likewise, demand decrease more with small increase in price.

IV. Relatively Inelastic Demand (E_p< 1):

The demand is said to be relatively inelastic if the percentage change in quantity demanded is less than the percentage change in price i.e. if there is a small change in demand with a greater change in price. It is also called less elastic or simply inelastic demand.

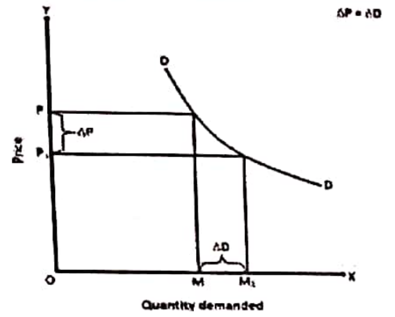


- In the given figure, the demand curve DD is steeper, which shows that the demand is less elastic. The greater fall in price from OP to OP_1 has led to small increase in demand from OM to OM_1. Likewise, greater increase in price leads to small fall in demand.

V. Unitary Elastic Demand (E_p = 1):

The demand is said to be unitary elastic if the percentage change in quantity demanded is equal to the percentage change in price. It is also called unitary elasticity.

This type of demand is an imaginary one as it is rarely applicable in our practical life.



- In the given figure, the demand curve DD is a rectangular hyperbola, which shows that the demand is unitary elastic. The fall in price from OP to OP_1 has caused equal proportionate increase in demand from OM to OM_1. Likewise, when price increases, the demand decreases in the same proportion.

2. Income Elasticity of Demand:

➢Income elasticity of demand measures the responsiveness of demand for a particular good to changes in consumer income.

➢Income elasticity of demand means the ratio of the percentage change in the quantity demanded to the percentage change in income.

$$E_Y = \frac{\Delta Q/Q}{\Delta Y/Y}$$

Where, E_Y = Income Elasticity of demand

Q = Original quantity demanded

ΔQ = Change in quantity demanded

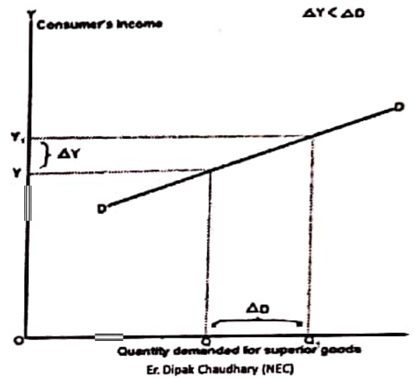
Y = Original consumer's income

ΔY = Change in consumer's income

There are five types of income elasticity of demand:

I. Income elasticity greater than unity (E_Y > 1):

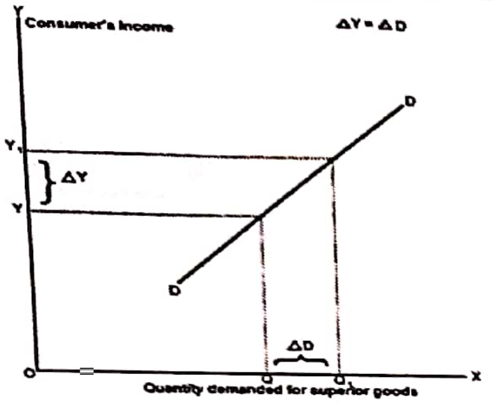
If the percentage change in quantity demanded for a commodity is greater than percentage change in income of the consumer, it is said to be income greater than unity.



- In the given figure, quantity demanded and consumer's income is measured along X-axis and Y-axis respectively. The small rise in income from OY to OY_1 has caused greater rise in the quantity demanded from OQ to OQ_1 and vice versa. Thus, the demand curve DD shows income elasticity greater than unity.

II. Income elasticity equal to unity (E_Y = 1):

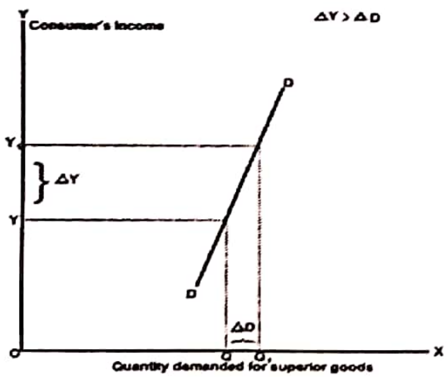
- If the percentage change in quantity demanded for a commodity is equal to percentage change in income of the consumer, it is said to be income elasticity equal to unity.



- In the given figure, the small rise in income from OY to OY_1 has caused equal rise in the quantity demanded from OQ to OQ_1 and vice versa. Thus, the demand curve DD shows income elasticity equal to unity.

III. Income elasticity less than unity ($E_Y < 1$):

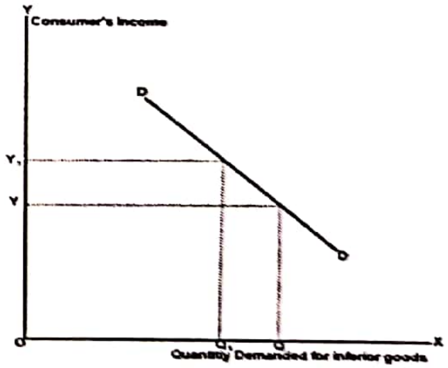
- If the percentage change in quantity demanded for a commodity is less than percentage change in income of the consumer, it is said to be income greater than unity.



- In the given figure, the greater rise in income from OY to OY_1 has caused small rise in the quantity demanded from OQ to OQ_1 and vice versa. Thus, the demand curve DD shows income elasticity less than unity.

IV. Negative income elasticity of demand (E_Y<0):

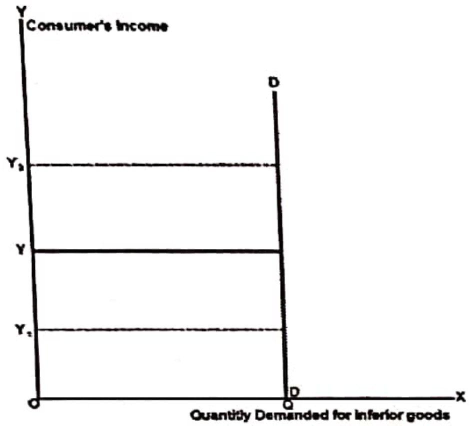
- If there is inverse relationship between income of the consumer and demand for the commodity, then income elasticity will be negative. That is, if the quantity demanded for a commodity decreases with the rise in income of the consumer and vice versa, it is said to be negative income elasticity of demand.



- In the given figure, when the consumer's income rises from OY to OY_1 the quantity demanded of inferior goods falls from OQ to OQ_1 and vice versa. Thus, the demand curve DD shows negative income elasticity of demand.

V. Zero income elasticity of demand (E_Y=0):

- If the quantity demanded for a commodity remains constant with any rise or fall in income of the consumer and, it is said to be zero income elasticity of demand.



- In the given figure, the consumer's income may fall to OY_1 or rise to OY_2 from OY, the quantity demanded remains the same at OQ. Thus, the demand curve DD, which is vertical straight line parallel to Y-axis shows zero income elasticity of demand.

There are five types of income elasticity of demand:(in short)

- High: A rise in income comes with bigger increases in the quantity demanded.
- Unitary: The rise in income is proportionate to the increase in the quantity demanded.
- Low: A jump in income is less than proportionate than the increase in the quantity demanded.
- Zero: The quantity bought/demanded is the same even if income changes
- Negative: An increase in income comes with a decrease in the quantity demanded.

3. Cross Elasticity of Demand:

- The cross-price elasticity of demand is the degree of responsiveness of quantity demanded of a commodity due to the change in price of another commodity.
- Cross elasticity of demand is the percentage change in the quantity demanded of good X due to certain percent change in the price of good Y.

Mathematically, it is expressed as:

$$\text{Cross elasticity of demand} = \frac{\%\ \text{change in quantity demanded for good } x}{\%\ \text{change in price of good } y}$$

Symbolically, it is expressed as:

$$E_C = \frac{\Delta q_x}{\Delta p_y} \times \frac{p_y}{q_x}$$

Where, E_C= Cross elasticity of demand

q_x= initial quantity demanded for good x

Δq_x= change in quantity demanded of good x

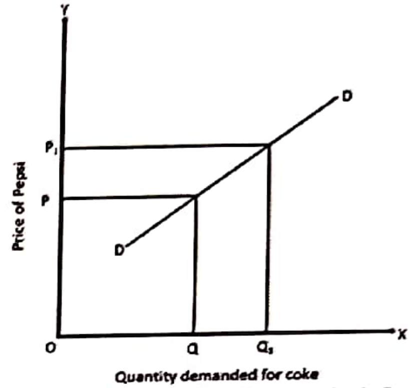
p_y= initial price of good y

Δp_y= change in price of good y

Types of Cross Elasticity of Demand:

I. Positive cross elasticity of demand (E_C>0)

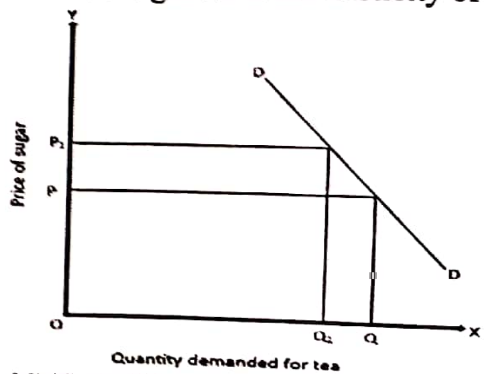
- If rise in price of one good leads to rise in quantity demanded of other good of a similar nature and vice versa, it is known as positive cross elasticity of demand. Positive cross elasticity exists between two goods which are substitutes of each other.



- In the above figure, quantity demanded for Coke and price of Pepsi are measured along X-axis and Y-axis respectively. When the price of Pepsi increases from *OP* to *OP₁*, quantity demanded for coke rises from *OQ* to *OQ₁* and vice versa. Thus, the demand curve *DD* shows positive cross elasticity of demand.

II. Negative cross elasticity of demand (E_C<0):

- Two goods which are complementary have negative cross elasticity of demand. If the rise in price of one good leads to fall in quantity demanded of its complementary good and vice versa, it is known as negative cross elasticity of demand.



- In the above figure, quantity demanded for Tea and price of Sugar are measured along X-axis and Y-axis respectively. When the price of Sugar increases from OP to OP_1, quantity demanded for Tea falls from OQ to OQ_1 and vice versa. Thus, the demand curve DD shows negative cross elasticity of demand.

THE END

Cost:

:- It is the expenses incurred in the production.
:- Cost can be defined as monetary valuation of efforts, materials, resources, time and utilities consumed, risk incurred and opportunity forgone in production of a good or service.

Elements of Cost:

Following are the three broad elements of cost:

1. Material:

The substance from which a product is made is known as material. It may be in a raw or a manufactured state.

2. Labour:

For conversion of materials into finished goods, human effort is needed and such human effort is called labour. cost of unskilled as well as skilled labour employed in construction or production process fall into this category.

3. Expenses:

These include cost of special design, drawing or layout, cost of purchase or hire of tools and plants for a particular Job and maintenance of such tools and equipment, etc.

Classification of cost:

- Cost
  - 1. Manufacturing cost
    - I. Direct cost (Prime cost)
      - Direct material cost
      - Direct labour cost
      - Direct Expenses
    - II. Overhead cost (Indirect cost)
      - Indirect Material cost
      - Indirect Labour cost
      - Indirect expenses
  - 2. Non Manufacturing cost

1. Manufacturing cost:

: Manufacturing cost include all costs incurred by a manufacturer in operating a factory while converting raw materials into complited products.

: Manufacturing cost can further be devided into

I. Direct cost or prime cost:

: Those costs related to a given cost object and that can be traced into ~~in~~ it in economically feasible way. prime costs is the aggregate of following three elements:

(a) Direct material cost:

:- It is the cost associated with those materials and components that can be directly and conveniently traced to a unit of product.

Eg: cost of paper in book, wood in furniture, ~~stell~~ steel in bridge, gearbox in a car etc.

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(b) ~~Ind~~Direct labour cost:

: The costs of production labour that can be directly and conveniently traced to a unit of production are called direct labour cost.

Eg: Welders in metal fabrication, carpenters in building works

(c) Direct Expenses:

: The production expenses that can be directly and conveniently traced to a unit of product are called direct expenses.

: Eg: special design & drawings, hire of special tools and equipment for manufacturing Job and their maintenance.

II. Indirect cost Or overhead cost:

: overhead costs are related to the particular cost object that can not be traced to specific unit of product.

: overhead cost is the aggregate of following three elements.

(a) Indirect Material cost:

: The cost of materials not directly traceable, and those ~~extremely~~ extremely small in monetory value are called indirect material cost.

(b) Indirect labour cost:

:- The labour costs that are not directly traceable, or those extremely small in monetory value are called indirect labour cost.

Eg: cost associated with storekeepers, night security guards

(c) Indirect Expenses:

:- The expenses that are not directly traceable, or those extremely small in monetory value are called indirect expenses.

:- Eg: Factory rent, lighting, insurance charges etc.

2. Non manufactoring cost:

:- Non manufactoring cost include all costs associated with the activities carried out in support of any manufacturing operations.

Eg: Selling & Marketing

: Distribution

: Research & development

: General & Administrative

: Finance

Costs for Business Decision:

Differential cost ~~& Revenue~~:

:- Difference in costs between any two alternatives known as differental cost.

:- Differential cost is the difference in total cost that results from selecting one alternative instead of another.

Differential Revenue:

:- Difference in revenue between any two alternatives is known as differential revenue.

Eg: You have a Job paying Rs 30000 per month in your hometown. You have a Job offer in neighbouring city that pays Rs 35000 per month. The commuting costs ~~to the city is Rs 2000~~. are Rs 1000 per month in your hometown and Rs 3000 per month to the city.

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Hence, differential cost = Rs 2000 = (3000 - 1000)

differential revenue = Rs 35000 - Rs 30000
= Rs 5000

Opportunity cost:

:- The cost of sacrificing one opportunity is known as opportunity cost.

:- Opportunity lost is opportunity cost

:- The loss of other alternatives when one alternative is chosen.

:- The profit lost when one alternative is selected over another.

Eg:- Suppose a bank would pay 3.5%/year but you decide to keep $10000 in your mattress. Opportunity cost of keeping in mattress as opposed to the bank is $350/year.

Eg:-

Project A	Project B
5%	8%
Accepted	X

i.e opportunity cost = 3%.

Sunk cost:

: A cost that has already been paid and can not be recovered.

Eg: once rent is paid, that dollar amount is no longer recoverable it is sunk.

:- Sunk costs are not relevant to decision making process because they cannot be changed regardless of what decision is made now or in the future.

Marginal cost:

: Represents the incremental costs incurred when producing additional units of a good or service.

:

$$\text{Marginal cost} = \frac{\text{change in costs}}{\text{change in quantity}}$$

Eg: current unit of production = 1000
current cost of production = \$ 100000
Future unit of production = 2000
Future cost of production = \$125000

$$\text{Marginal cost} = \frac{125000 - 100000}{2000 - 1000}$$

$$= \frac{25000}{1000}$$

$$= 25$$

Fixed cost:

: Fixed costs are those costs which are not output dependent.

: Fixed costs are those costs that the firm has to pay independently of whether it is operating or not.

: Eg. rent of building, ~~cost~~ insurance premium, equipment cost

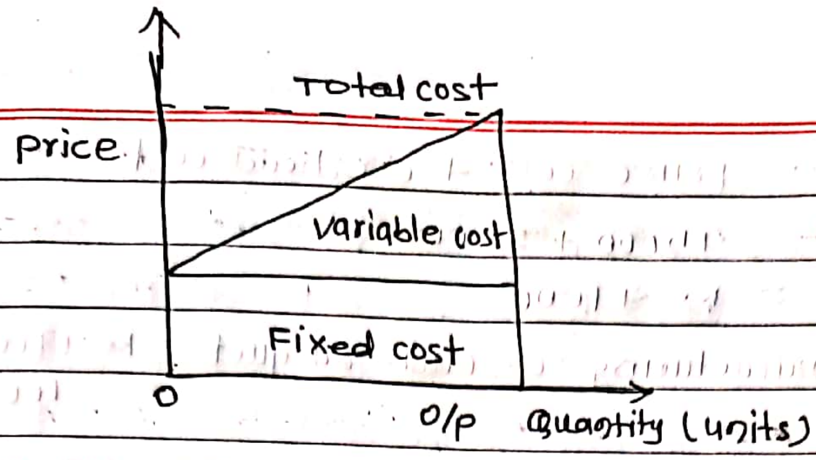
variable cost:

: variable costs are those cost which are output dependent.

:- variable costs that relate directly to the production or sale of a product.

: eg: price of raw materials, wages of labours transport expenditure etc.

Total cost is the summation of fixed cost and variable cost.



Eg: Following are the data for manufacturing of 100 tennis racquets

Timber $1m^3$ @ Rs $20000/m^3$
Gut 50m @ Rs 150/m
paints 25 Kg @ Rs 300/Kg
Direct expenses = Rs 5000
Labour hour needed 200 hrs @ Rs 25/hr
Annual Factory overhead = Rs 2000000
Annual labour hours = 200000 hrs

Breakdown the costs into component of prime cost and overhead cost and find out the manufacturing cost of each racquet.

Soln:

1. prime cost

(a) Direct material cost

Timber = Rs 20000
Gut = Rs 50*150 = Rs 7500
paints = Rs 25*300 = Rs 7500

Direct material cost = 20000 + 7500 + 7500 = Rs 35000

(b) Direct labour cost = 200*25 = Rs 5000

(c) Direct expenses = Rs 5000

prime cost = 35000 + 5000 + 5000 = Rs 45000

2. Overhead cost

Factory overhead cost = $\frac{2000000}{200000} * 200$ = Rs 2000

3. Total cost = prime cost + overhead cost

= 45000 + 2000

= Rs 47000

cost of manufacturing of each racquet = Rs $\frac{47000}{100}$

= Rs 470 Ans.

CH-3 Time value of Money

Time value of Money:

: The time value of money (TVM) is the idea that money available at the present time is worth more than the same amount in the future due to its potential earning capacity.

:- Money has the ability to earn interest, its value increases with time. Hence it is the relationship between interest and time.

:- It has earning and purchasing power over time.

Interest Rate:

:- It is a percentage that is periodically applied and added to an amount of money over a specified length of time.

:- It is expressed at percentage per time period.

Simple Interest :

: In this scheme, interest earned is only on the principal amount during each interest period and does not earn additional interest in the remaining periods, even though you don't withdraw it.

In general, for a deposit of P dollars at a simple interest ~~in the remaining periods~~ rate of r for N periods, the total interest would be

$$I = PNr$$

The total amount available at the end of 'N' periods is

$$F = P + I = P + PNr = P(1 + Nr)$$

where F = Future sum of Money

compound interest:

:- When the interest earn in each period is calculated on the basis of the total amount at the end of the previous and this total amount includes the original principal plus the accumulated interest that has been left in the account is called compound interest.

: So, the total amount, at the end of N period when sum P is deposited at interest rate 'i' is

$F = P(1+i)^N$

Nominal interest Rate: (r)

: The nominal interest rate is ~~periodic interest rate~~ annualized percentage rate

:- It is stated or contract interest rate

:- It is the annual interest rate without considering the effect of any compounding

:-

Effective interest Rate: (i):

:- The actual rate of interest earned during one year is known as effective interest rate

:- It is the annual interest rate taking into account the effect of any compounding during the year.

:- It is expressed on the annual basis, unless specifically stated otherwise.

Relation between i and r

$$\boxed{i = \left(1+\frac{r}{m}\right)^m - 1}$$

where

i = effective interest rate

r = nominal " "

m = no. of compounding sub-periods per year.

m = 1 for yearly
= 2 for semiannualy
= 4 for quarterly
= 12 for monthly
= 52 for weekly
= 365 for daily
= ∞ for continuously

for continuously

$$\boxed{i = e^r - 1}$$

$$i_{eff} = (1+i_N)^m - 1$$

where

i_{eff} = effective interest rate

i_N = interest rate per compounding period

m = no. of compounding periods per year

$$i_e = \left(1+\frac{r}{m}\right)^m - 1$$

or $\left(1+\frac{r}{m}\right)^m = (1+i_e)$

or, $\left(1+\frac{r}{m}\right) = (1+i_e)^{1/m}$

or, $\frac{r}{m} = (1+i_e)^{1/m} - 1$

$\therefore\ i_{monthly} = (1+i_e)^{1/m} - 1 = (1+i_e)^{1/12} - 1$

Simillarly

$$i_e = (1+i_{monthly})^m - 1$$
$$= (1+i_{monthly})^{12} - 1$$

ठूलो निकाल्नु E_e direct (power मा m)

सानो ,, ,, ($\frac{1}{m}$ power मा राख्ने)

Question मा per year compounded monthly or quarterly or semiannu
भने E_e भने पहिला $i_e = \left(1+\frac{r}{m}\right)^m - 1$ वाला formula

राख्ने यो per year को term मा आउँछ ।

:- compounded भनेको छैन भने

$$i_{monthly} = (1+i_e)^{1/m} - 1$$
$$= (1+i_e)^{1/12} - 1$$

$i_{semi} = (1+i_{quarterly})^2 - 1$

$i_{monthly} = (1+i_{semi})^{1/6} - 1$

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Eg: Find effective interest rate when the nominal rate of interest is 9 % per year and compounding is (i) yearly (ii) Monthly (iii) Daily (iv) Hourly (v) quarterly (vi) semiannually.

Soln:

~~For compounding yearly~~

Here,

nominal interest rate $(r) = 9\% = 0.09$

effective interest rate $(i) = ?$

~~For compounding yearly~~

we have

$$i = \left(1+\frac{r}{m}\right)^m - 1$$

where m = no. of compounding periods per year

For compounding yearly; $m = 1$

$$i = \left(1+\frac{0.09}{1}\right)^1 - 1 = 0.09 = 9\%$$

For compounding semiannually; $m = 2$

$$i = \left(1+\frac{0.09}{2}\right)^2 - 1 =$$

For compounding quarterly; $m = 4$

$$i = \left(1+\frac{0.09}{4}\right)^4 - 1 = 0.09308 = 9.308\%$$

For compounding monthly; $m = 12$

$$i = \left(1+\frac{0.09}{12}\right)^{12} - 1 = 0.0938 = 9.38\%$$

For compounding daily; $m = 365$

$$i = \left(1+\frac{0.09}{365}\right)^{365} - 1 = 0.0941 = 9.41\%$$

For compounding hourly ; $m = 8760$

$$i = \left(1 + \frac{0.09}{8760}\right)^{8760} - 1$$

$$=$$

For compounding continuously; $m = \infty$

$$i = e^{r} - 1$$

$$= e^{0.09} - 1$$

$$= 9.417\%$$

eg: bank interest rate $(r) = 12\%$ compounded quarterly deposit monthly then

$$i = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$$= \left(1 + \frac{0.12}{4}\right)^{4} - 1$$

$$= 0.1255$$

$$= 12.55\% \text{ yearly}$$

$$i_{monthly} = \left(1 + i_{yearly}\right)^{1/12} - 1$$

$$= (1 + 0.1255)^{1/12} - 1$$

$$= 0.0099$$

$= \cancel{0.099\%}$

$$= 0.99\%$$

or $i_q = \frac{12\%}{4}$

$$= 3\%$$

$$i_m = (1 + i_q)^{1/3} - 1$$

$$= (1 + 0.03)^{1/3} - 1$$

$$= 0.0099$$

$= \cancel{0.099}\%$

$$= 0.99\%$$

eg: Monthly interest rate is 1.5%

$i_{eff} = ?$

$r = ?$

$$i_{eff} = (1 + i_{monthly})^{12} - 1$$

$$= (1 + 0.015)^{12} - 1 = 0.1956$$

$$= 19.56\% \text{ yearly}$$

$$i_{eff} = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$$0.1956 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$r = 0.18 = 18\%$$

or

r = monthly interest rate * Months in year

$= 1.5 \times 12$

$= 18\%$

Economic Equivalence:

:- The process of comparing two different cash amounts at different points in time is called economic equivalence.

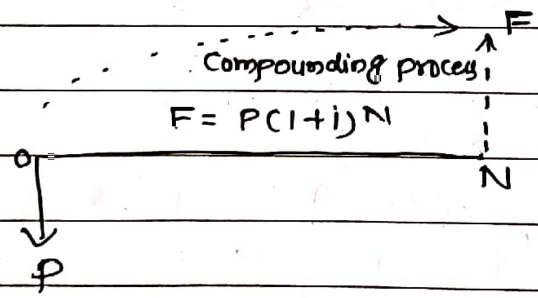
:- Different sums of money at different times are equal in economic value.

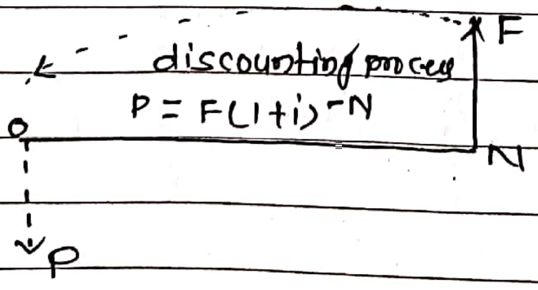
:- Economic equivalence refers to the fact that a cash flow whether a single payment or a series of payments can be converted to an equivalent cash flow at any point ~~of~~ in time.

* Present Worth (PW):

:- In this method, all the cash inflows and outflows are discounted to the beginning points at an interest rate for the economic study.

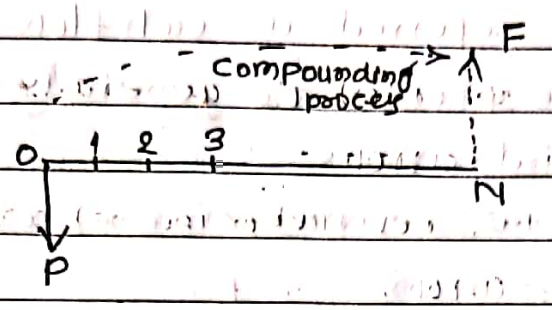
:- It is also known as Net present value (NPV)





* Future Worth (FW):

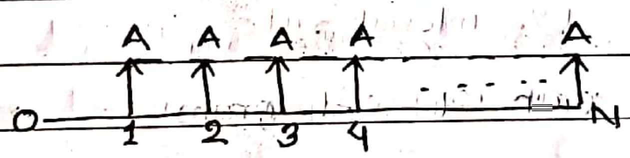
The future worth of money relates how much a current investment will be worth in the future.



$$F = P(1+i)^N$$

* Annual worth (AW):

It provides basis for measuring investment worth in a series of equal payments at the each period.



$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

Cash Flow Diagram (CFD):

:- The graphical representation of the cash flows i.e. both cash out flows and cash inflows with respect to a time scale is referred as cash flow diagram.

:- The cash outflows (i.e, expenses) are generally represented by vertical downward arrows

:- The cash inflows (i.e, revenue or income) are represented by vertical upward arrows.

:- The number of interest period is shown on the time scale.

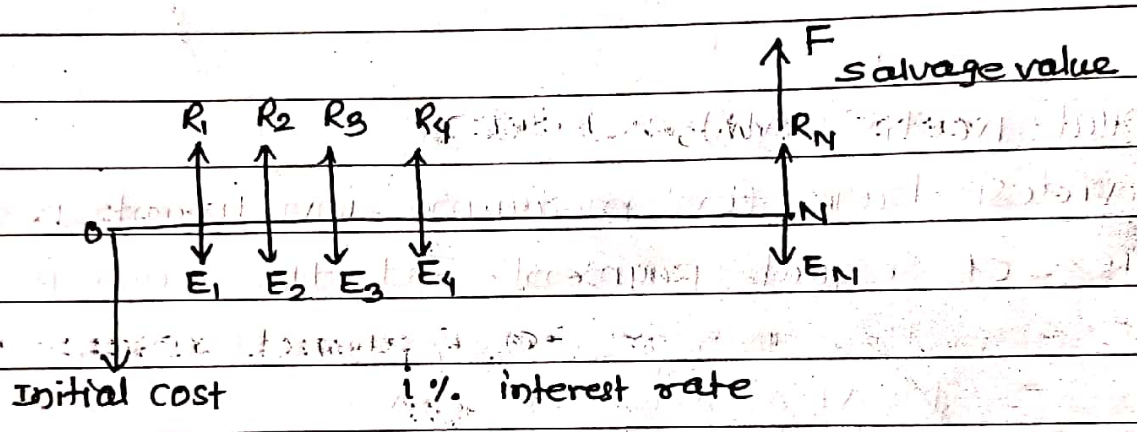
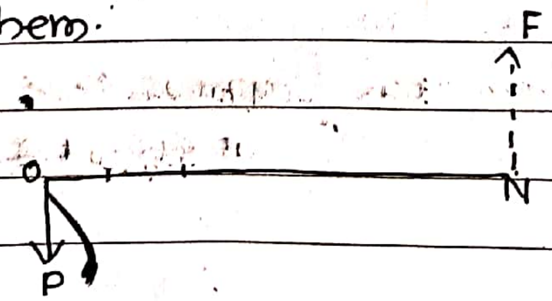


fig. cash flow diagram

Types of cash flow:

(i) Single cash flow:

: This type of cash flow involves the financial transaction only once in the cash flow diagram. the cash flow may be in initial point or at the end or somewhere in between them.



$$F = P(1+i)^N$$

$$F = P\left(\frac{F}{P}, i\%, N\right)$$

$$= P(1+i)^N$$

$\left(\frac{F}{P}, i\%, N\right) \rightarrow$ single payment compound amount factor

$$P = F(1+i)^{-N}$$

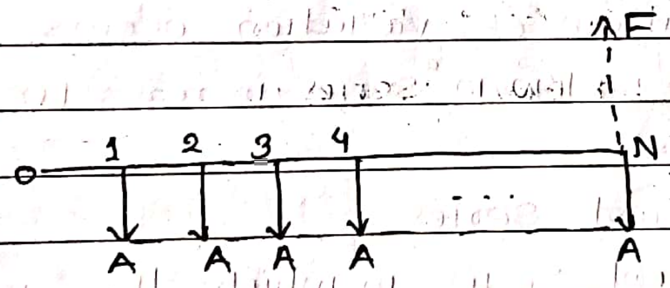
$$= F\left(\frac{P}{F}, i\%, N\right)$$

$\left(\frac{P}{F}, i\%, N\right) \rightarrow$ single payment present worth factor

(ii) Equal payment (Uniform) Series:

:- This type of cash involves the equal payments at the end of each interest period.

: Eg: Commercial installment, Rental payment, Insurance payment plans



$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$= A\left[F/A, i\%, N\right]$$

$\left(\frac{F}{A}, i\%, N\right) \rightarrow$ Uniform series compound amount factor
~~or Sinking fund factor~~

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

$$= F\left[\frac{A}{F}, i\%, N\right]$$

$\left(\frac{A}{F}, i\%, N\right) \rightarrow$ Uniform series sinking fund factor

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$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$
$$= A\left(\frac{P}{A}, i\%, N\right)$$

$\left(\frac{P}{A}, i\%, N\right) \rightarrow$ Uniform series present worth factor

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$
$$= P\left(\frac{A}{P}, i\%, N\right)$$

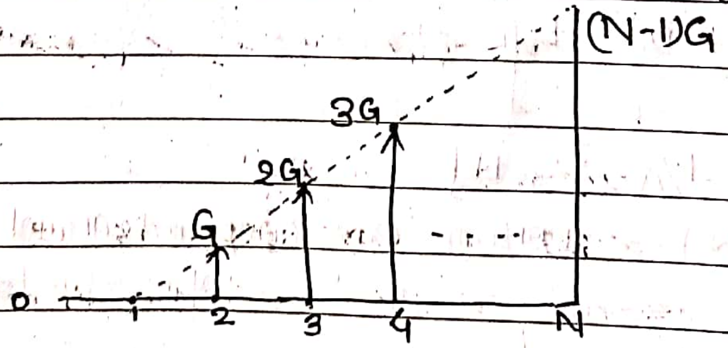
$\left(\frac{A}{P}, i\%, N\right) \rightarrow$ capital Recovery factor

(iii) Linear gradient series:

:- one common pattern of variation occurs when each other cash flow in a series increases (or decreases) by a fixed amount.

(a) Strict gradient series

:- The gradient series in which there is no cash flow in the first year is called strict gradient series.



$$F = G\left[\frac{(1+i)^N - iN - 1}{i^2}\right] = G\left(\frac{F}{G}, i\%, N\right)$$

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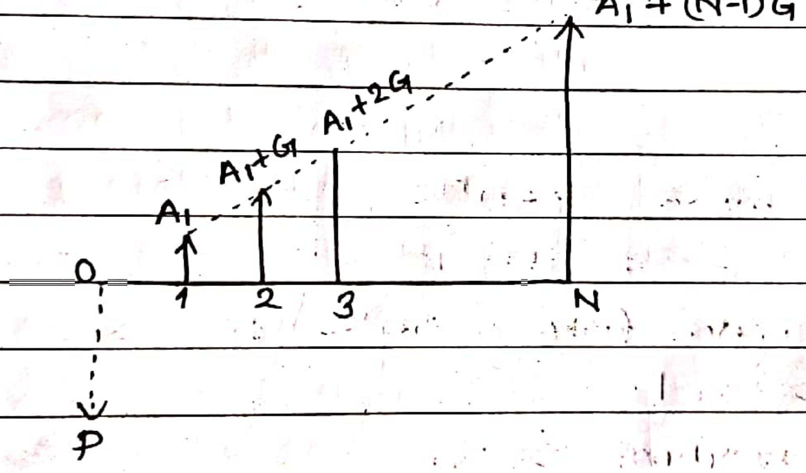
$$P = G\left[\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right] = G(P/G, i\%, N)$$

$$A = G\left[\frac{(1+i)^N - iN - 1}{i[(1+i)^N - 1]}\right] = G(A/G, i\%, N)$$

(b) Composite gradient series:

:- In this series, there is an initial payment in the first interest period and it increases by a gradient G over the rest of the interest periods.

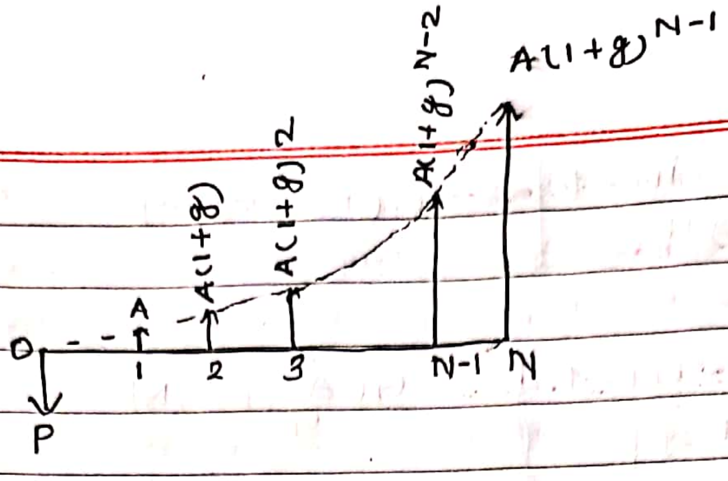


$$A = A_1 + G(A/G, i\%, N) \rightarrow \text{increasing series}$$

$$A = A_1 - G(A/G, i\%, N) \rightarrow \text{decreasing series}$$

(iv) Geometric gradient series:

:- When the series in cash flow is determined ~~not~~ by a fixed ~~amount~~ rate, then the series is called geometric gradient series.



If $i \neq g$

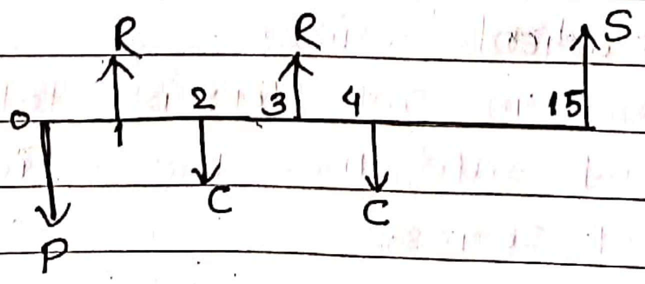
$$F = A_1 \left[\frac{(1+i)^N - (1+g)^N}{i-g} \right]$$

If $i = g$

$$F = N A_1 (1+i)^{N-1}$$

(V) Irregular Series:

: A series of cash flow which are irregular and doesn't exhibit an overall regular pattern.



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Eg: If a saving bank pays 2% interest every three months, what are the nominal and effective interest rate per year?

Soln:

$i_{quarterly} = 2\% = 0.02$

$r = ?$

$i_{effective} = ?$

$$i_{eff.} = (1 + i_{quat})^4 - 1$$
$$= (1 + 0.02)^4 - 1$$
$$= 0.0824$$
$$= 8.24\% \text{ per year}$$

We have

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$$0.0824 = \left(1 + \frac{r}{4}\right)^4 - 1$$

$$\left(1 + \frac{r}{4}\right)^4 = 1.0824$$

$$\left(1 + \frac{r}{4}\right) = (1.0824)^{1/4}$$

$$\left(1 + \frac{r}{4}\right) = 1.01999$$

$$\frac{r}{4} = 0.01999$$

$$r = 0.07996$$

$\therefore r \approx 8\%$ per year

If you ~~deposit~~ wish to draw Rs 10000 per month for 4 years. How much should you deposit at present for that when rate of interest is 7% per year?

Soln:-

$A =$ Rs 10000 per month

$N = 4$ years

$P = ?$

$i_{eff} = 7\%$ per year $= 0.07$

we have

$$i_{month} = (1 + i_{year})^{1/12} - 1$$

$$= (1 + 0.07)^{1/12} - 1$$

$$= 0.00565$$

we know,

$$P = A\,(P/A,\ i\%,\ N)$$

$$= 10000 \left[\frac{(1+i)^N - 1}{i(1+i)^N} \right]$$

$$= 10000 \left[\frac{(1+0.00565)^{48} - 1}{0.00565(1+0.00565)^{48}} \right]$$

$$= \text{Rs } 419387.3$$

If you have Rs 1000000 loan now from a bank, how much Rs. should you pay as installment per two month for 5 years if bank interest rate is 12% per year?

Soln:

$P =$ Rs 1000000

$N = 5$ years

$i = 12\%$ per year $= 0.12$

installment per two month

$A = ?$

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$$i_{2month} = (1+i)^{1/6} - 1$$

$$= (1+0.12)^{1/6} - 1$$

$$= 0.01907$$

$$= 1.907\%$$

we have

$$A = P\left(\frac{A}{P}, i\%, N\right)$$

$$= 1000000\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$= 1000000\left[\frac{0.01907(1+0.01907)^{30}}{(1+0.01907)^{30} - 1}\right]$$

$$= 44080.98$$

$$\simeq Rs\ 44081$$

Eg: How much money should you deposit now in a saving account earning 10% compounded annually so that you may make eight end of year withdrawals of Rs 2000 each?

Soln:

$P = ?$

$i = 10\%$

$N = 8$ year

$A =$ Rs 2000

we have

$$P = A\left(\frac{P}{A}, 10\%, 8\right)$$

$$= 2000 * 5.3349$$

$$= Rs\ 10669.8$$

$$\simeq Rs\ 10670$$

Eg: A person is planning for his retired life and has 10 more years of service. He would like to deposit 20% of his salary, which is Rs 10000 at the end of the first year and thereafter he wishes to deposit the same amount (Rs 10000) with an annual increase of Rs 2000 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10th years of the above services

Soln:-

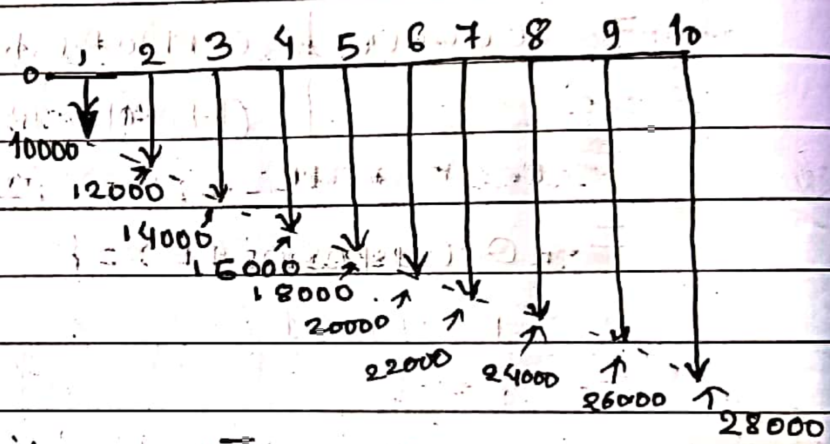
$N = 10$ years

$A_1 =$ Rs 10000

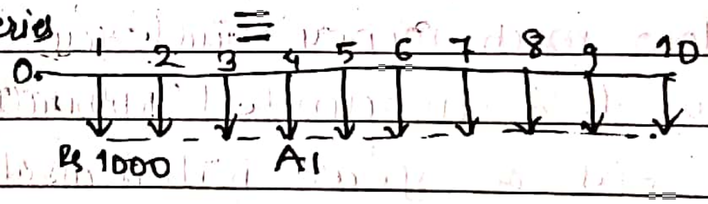
$G =$ Rs 2000

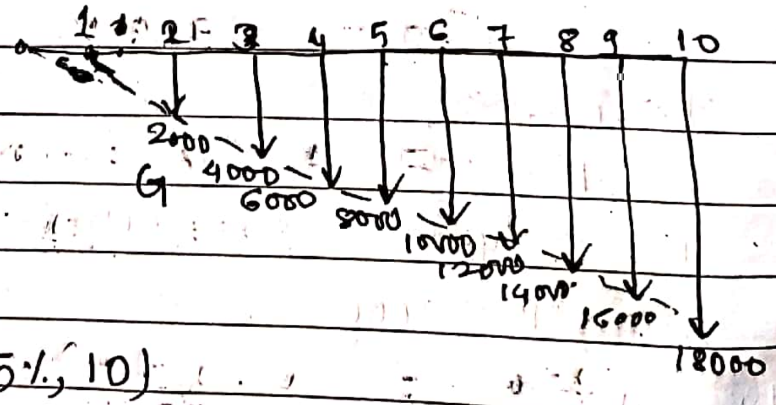
$i = 15\%$

$F = ?$



Linear composite gradient series ≡





$$A = A_1 + G\left(\frac{A}{G}, 15\%, 10\right)$$

$$= 10000 + 2000 \times 3.3832$$

$$= \text{Rs } 16766.4$$

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Now, Future amount, F, is calculated as follows

$F = A_1 (\frac{F}{A}, 15\%, 10)$

$= 16766.4 \times 20.3037$

$= 340419.9$

$\approx$ Rs 340420

Eg:- How much deposits of Rs 5000 each should make per month so that the final accumulation amount will be Rs 100000 if the bank interest rate is 12% per year?

Soln:

Monthly deposit (A) = Rs 5000

Future amount (F) = Rs 100000

interest rate (i_y) = 12% per year = 0.12

No. of deposits (N) = ? (month)

We have,

$i_{monthly} = (1+i_y)^{1/12} - 1$

$= (1+0.12)^{1/12} - 1$

$= 0.00948$

$i_m = 0.948\%$

We have

$F = A (F/A, i\%, N)$

$100000 = 5000 \boxed{\frac{(1+i)^N - 1}{i}}$

$20 = \frac{(1.00948)^N - 1}{0.00948}$

$0.1896 = (1.00948)^N - 1$

or, $1.1896 = (1.00948)^N$

taking log both sides

$\log 1.1896 = N \log(1.00948)$

$N = 18.4$ months

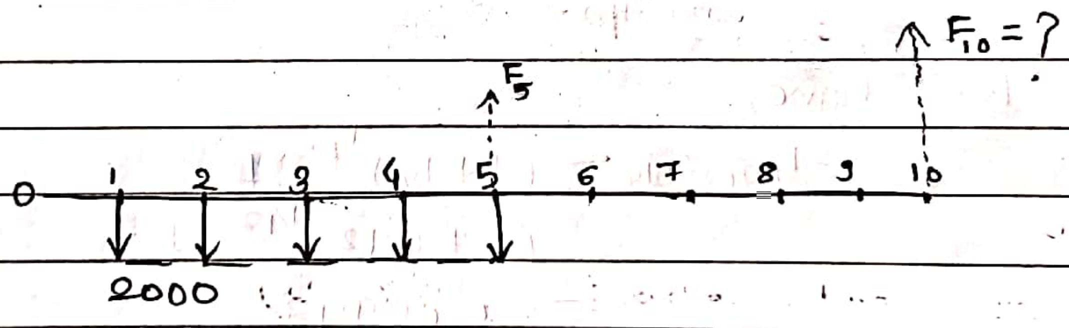
Eg: An individual makes five annual deposits of Rs 2000 in a saving account that pays interest at a rate of 4% interest rate, interest rate change to 6% per year. Five years after last deposit, the accumulated money is withdrawn from account. How much is withdrawn?

Soln:–

$A = $ Rs 2000 annually

$i = 4\%$ for first 5 yrs

$N = $ ~~5+5 =~~ 5 years + 5 years



$F_5 = 2000\left(\frac{F}{A}, 4\%, 5\right)$

$= 2000 * 5.4163$

$=$ Rs 10832.6

$F_{10} = 10832.6\left(\frac{F}{P}, 6\%, 5\right)$

$= 10832.6 * 1.3382$

$= 14496.18$

$\approx$ Rs 14496 Ans

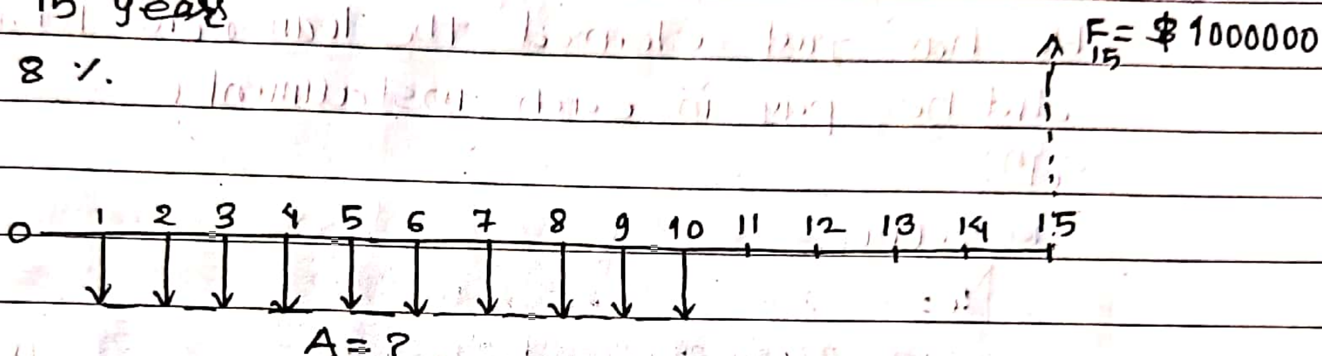
Eg: Mr. Pandey wants to have \$.1000000 for the studies of his daughter after period of 15 years. How much rupees does he has to deposit each year for 10 continuous years in a saving account that earns 8% interest annually.

Soln:

$F = \$ 1000000$

$N = 15$ years

$i = 8\%$



we have

$$F_{10} = A\left(\frac{F}{A}, 8\%, 10\right)$$

$$= A * 14.4866$$

Now

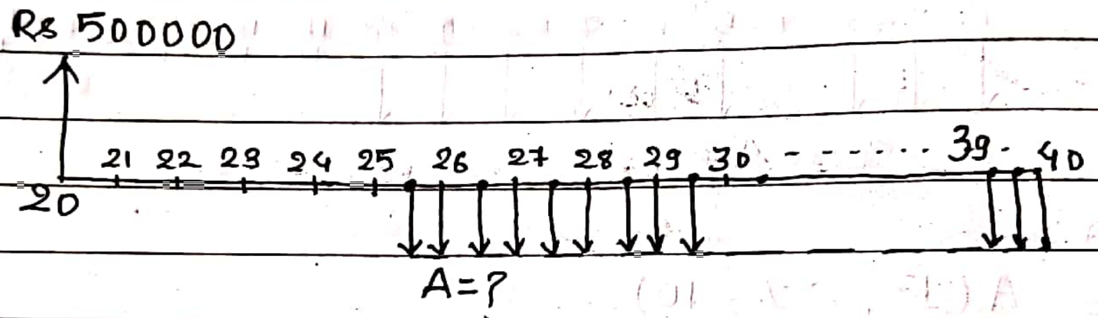
$$F_{15} = A * 14.4866 \left(\frac{F}{P}, 8\%, 5\right)$$

$$1000000 = A * 14.4866 * 1.4693$$

$\therefore A = \$ 46981$ Ans.

Eg: A man aged 40 years now had borrowed Rs 500 000 from a bank for his further studies at the age of 20 years. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just cleared the loan now. What amount did he pay in each installment?

Soln:



Given,

$P =$ Rs 500000

$i_{or(r)} = 11\%$ per year compounded quarterly

$N = 20$ years

$A = ?$ (Semi annual installments)

$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$

$= \left(1+\frac{0.11}{4}\right)^4 - 1$

$= 0.1146$

$i_{semiannual} = (1+i_e)^{1/2} - 1$

$= (1+0.1146)^{1/2} - 1$

$= 0.0557$

$= 5.57\%$

or $i_{quarterly} = \frac{11\%}{4}$

$= 2.75\%$

$i_{semiannual} = (1+i_q)^2 - 1$

$= (1+0.0275)^2 - 1$

$= 0.0557$

$= 5.57\%$

Using the single payment compound amount factor

$$F = 500000\,(F/P,\ 5.57\%,\ 40)$$
$$= 500000\,[1+0.0557]^{40}$$
$$= 4371101.251 \quad \text{---}\ (1)$$

Using the uniform series compound amount factor

$$F = A\,(F/A,\ 5.57\%,\ 30)$$
$$= A\left[\frac{(1+0.0557)^{30}-1}{0.0557}\right]$$
$$= 73.323A \quad \text{---}\ (2)$$

Equating eqn (1) & (2)

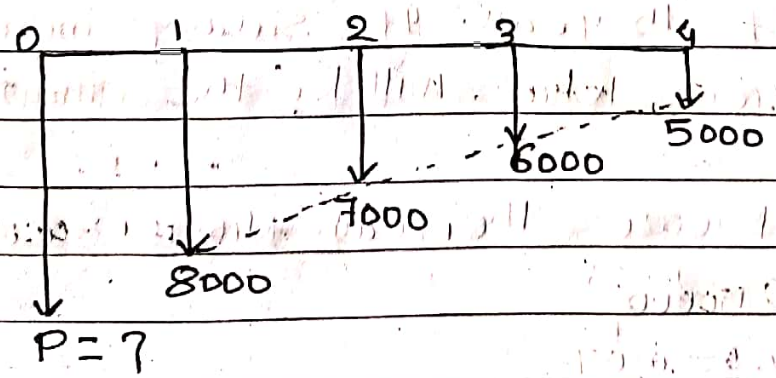
$$4371101.251 = 73.323A$$
$$\therefore A = 59614.32$$

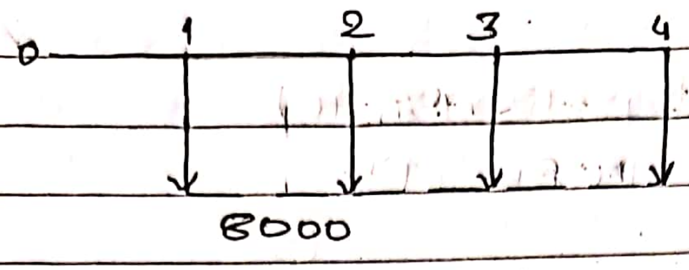
Eg:- Suppose that one has cash flows as follows

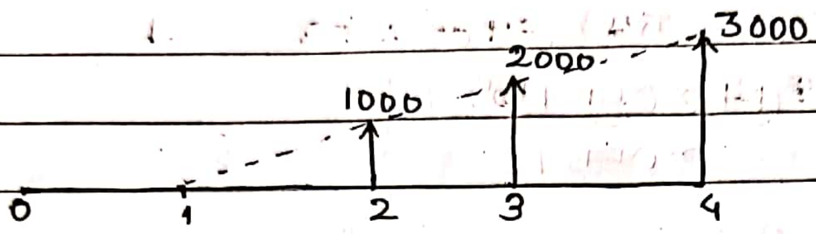
EOY	Net cash flow
1	8000
2	−7000
3	−6000
4	− 5000

calculate the present equivalent at $i = 15\%$.

Soln:







Using Gradient to present equivalent conversion factor

$$= 8000\left(\frac{P}{A}, 15\%, 4\right) - 1000\left(\frac{P}{G}, 15\%, 4\right)$$

$$= 8000 * 2.8550 - 1000 * 3.7864$$

$= Rs\ 19053.6$ Ans

Eg: Ramesh, a civil engineer is planning to placed a total of 20% of his salary, which is Rs 250000 per year. He expects 7% increase in salary for next 15 years if the mutual fund results in 10% annual return, what will be the amount at the end of 15 years. If salary increases by Rs 25000 per year, what will be the amount.

Soln:-

For First case, the cash flow is geometric

$A_1 =$ Rs 250000

$g = 7\% = 0.07$

$i = 10\% = 0.1$

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N = 15 years

We have

$$F = A_1 \left[\frac{(1+i)^N - (1+g)^N}{i-g} \right]$$

$$= 250000 \left[\frac{(1.1)^{15} - (1.07)^{15}}{0.1 - 0.07} \right]$$

$= 11818471.91$ Ans.

Second case:

$A_1 =$ Rs 250000

$G =$ Rs 25000

$F = ?$

$$F = 250000\left(\frac{F}{A}, 10\%, 15\right) + 25000\left(\frac{F}{G}, 10\%, 15\right)$$

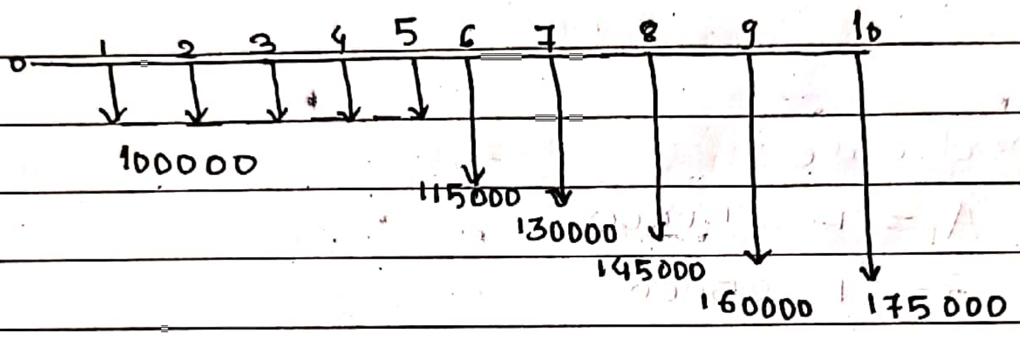
$$= 250000 * 31.7725 + 25000 * \cancel{5.27}\left(\frac{A}{G}, 10\%, 15\right)\left(\frac{F}{A}, 10\%, 15\right)$$

$$= 250000 * 31.7725 + 25000 * 5.2789 * 31.7725$$

$=$ Rs 12136221.26

Eg: Mr Kumar has inspected his yearly household expenses for the last 10 years. cost averages were steady at Rs 100000 per year for the first 5 years, but have increased consistently by Rs 15000 per year for each of the last 5 years. Calculate total present worth in year zero. Use gradient formula. (i = 10%)

soln :-



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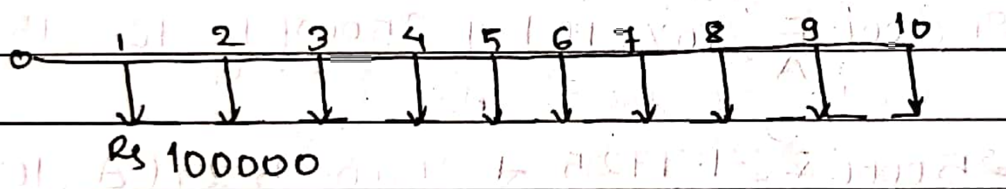

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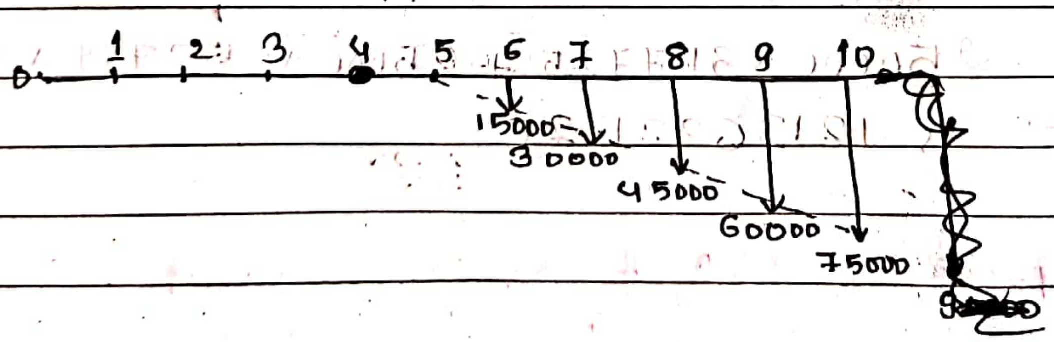


$$P = 100000\left(\frac{P}{A}, 10\%, 10\right) + \left\{15000\left(\frac{P}{G}, 10\%, 6\right)\right\} * \left(\frac{P}{F}, 10\%, 4\right)$$

$= 100000 * 6.1446 + 15000 * 9.6842 * 0.6830$

$=$ Rs. 713674.629 Ans.

Eg: What will be the amount of money at the end, if you deposit Rs 5000 per month for five years — continuously. If nominal interest rate is 10% & compounded quarterly.

Soln:

$A =$ Rs 5000 per month for 5 years.

$i_{nominal} = r = 10\%$ & compounded quarterly

$$i_{quart.} = \frac{10\%}{4} = 2.50\%$$

$$i_{monthly} = (1 + i_{quart})^{1/4} - 1$$

$$= (1 + 0.025)^{1/4} - 1$$

$$=$$

$$= 0.62\%$$

Now,

$$F = A\left(\frac{F}{A}, 0.62\%, 60\right)$$

$$= 5000 \times \left[\frac{(1+0.0062)^{60} - 1}{0.0062}\right]$$

$$\doteq 362070.93 \quad \underline{\underline{Ans.}}$$

Eg: Ram invested at high yield account aimed to get the double of his investment at the end of 10 years. Compute the effective interest rate per year he ~~the~~ received on the account.

Soln:- $N = 10$

$i_{eff} = ?$

We have $F = P(1+i)^N$

$$2\cancel{P} = \cancel{P}(1+i)^N$$

$$2 = (1+i)^{10}$$

$$2^{1/10} = 1+i$$

$$i = 0.0718$$

$$= 7.18\% \text{ per year.}$$

Eg: calculate the future worth of the following cash flows deposited at 8% compounded continuously for 5 years.

(i) Rs 50000 at the beginning of each year

(ii) Rs 50000 at the end of each year

Soln:

N = 5 years

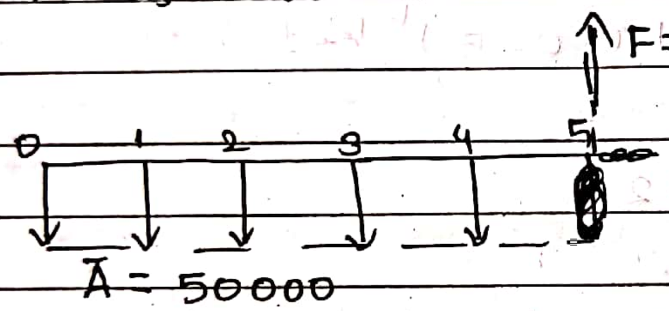
r = 8% compounded continuously

(i) $\bar{A}$ = 50000 at the beginning of each year

F = ?

(ii) A = Rs 50000 at the end of each year

(i)



$i = e^{r} - 1$

$= e^{0.08} - 1$

$= 0.0832$

$= 8.32\%$

$F = \bar{A}_0 \left(\frac{F}{P}, 8.32\%, 5\right) + \bar{A} \left(\frac{F}{A}, 8.32\%, 4\right)\left(\frac{F}{P}, 8.32\%, 1\right)$

$= 50000(1.0832)^5 + 50000 \left[\frac{(1.0832)^4 - 1}{0.0832}\right] * (1.0832)^1$

$= 74561 + 50000 * ~~5.9041~~ \; 4.9041$

$= ~~369768.49~~$

$=$ Rs 319768.49

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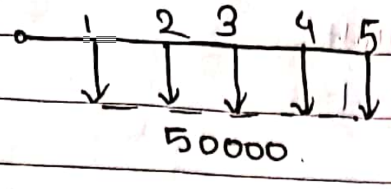
(ii)



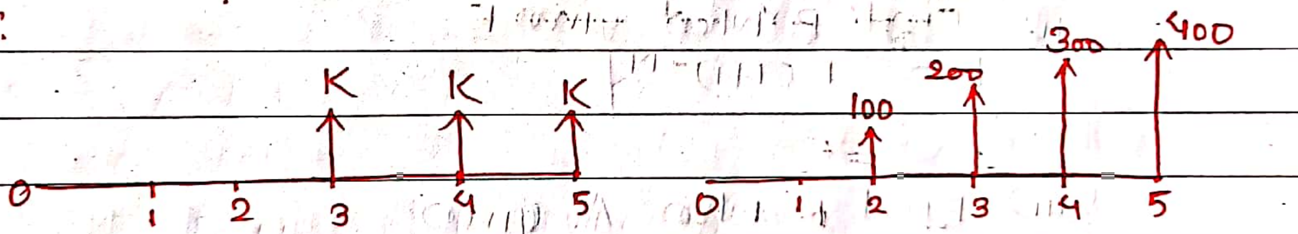
$$F = 50000\left(\frac{F}{A}, 8.32\%, 5\right)$$

$$= 50000\left[\frac{(1.0832)^5 - 1}{0.0832}\right]$$

$$= Rs\ 295207.4$$

Eg: For the cash flow diagram given below, what should be the value of K on left hand ~~side~~ cash flow diagram to be equal to right hand cash flow diagram if $i = 12\%$?

Soln:



Soln:

$i = 12\%$

$K = ?$

We have

For 1st figure

$$F = K\left(\frac{F}{A}, 12\%, 3\right) = K * 3.3744 \quad \text{--- (i)}$$

For 2nd figure

$$F = G\left(\frac{F}{G}, 12\%, 5\right) = 100\left[\frac{(1+0.12)^5 - 0.12\times 5 - 1}{(0.12)^2}\right]$$

$$= 1127.37 \quad \text{- (ii)}$$

~~Equating (i) & (ii)~~

$K = 334.09$ Ans.

Development of formulas

(i) Finding F when given P

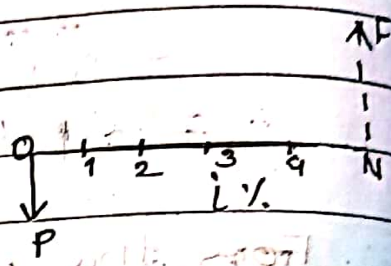
$$F_1 = P + Pi = P(1+i) \qquad (\because F_1 = P + I_1)$$

$$F_2 = F_1 + P(1+i)*i \qquad (\because F_2 = F_1 + I_2)$$

$$= P(1+i) + P(1+i)*i$$

$$= P(1+i)(1+i)$$

$$= P(1+i)^2$$



$$F_3 = F_2 + I_3$$

$$= P(1+i)^2 + P(1+i)^2 * i$$

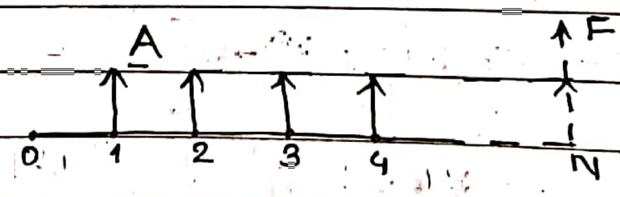
$$= P(1+i)^3$$

$$\boxed{F_N = P(1+i)^N}$$

(ii) Find P when given F

$$\boxed{P = F(1+i)^{-N}}$$

(iii) Find F when A given



$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A(1+i)^1 + A$$

$$= A\left[(1+i)^{N-1} + (1+i)^{N-2} + \cdots + (1+i) + 1\right]$$

Gemetric sequence

common ratio $(r) = (1+i)^{-1}$

First term $(a) = (1+i)^{N-1}$

Last term $(b) = 1$

$$S_N = \frac{a - rb}{1-r} = \frac{(1+i)^{N-1} - (1+i)^{-1}}{1-(1+i)^{-1}}$$

Page:

$$= \frac{(1+i)^{-1}\left[(1+i)^N + 1\right]}{1 - \frac{1}{(1+i)}}$$

$$= \frac{\left[(1+i)^N - 1\right]}{\cancel{(1+i)}\ \frac{\left[\cancel{1}+i-\cancel{1}\right]}{\cancel{(1+i)}}}$$

$$= \frac{(1+i)^N - 1}{i}$$

$$\therefore F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

(iv) Find A when F Given

we have

$$F = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$\therefore A = \frac{i}{(1+i)^N - 1}$$

(v) Finding A when Given P

we have

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] \quad \text{--- (1)}$$

Also, $F = P(1+i)^N$ --- (2)

Equating (1) & (2)

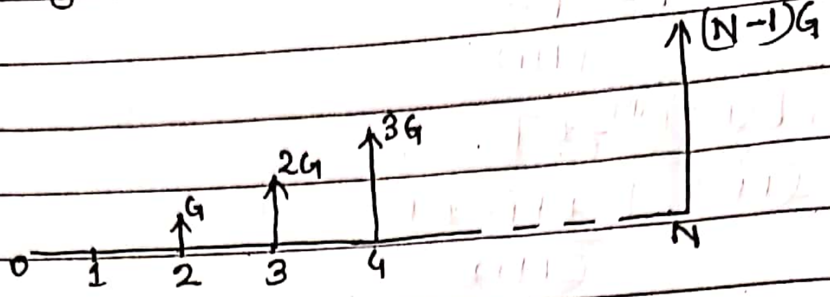
$$P(1+i)^N = A\left[\frac{(1+i)^N - 1}{i}\right]$$

$$\therefore A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

(vi) Find P when A Given

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

(vii) Finding F when Given G



$$F = G(1+i)^{N-2} + 2G(1+i)^{N-3} + 3G(1+i)^{N-4} + \cdots (N-1)G$$

$$= G\left[(1+i)^{N-2} + 2(1+i)^{N-3} + 3(1+i)^{N-4} + \cdots + (N-1)\right] \quad \text{--- (1)}$$

Multiply eqn (1) by $(1+i)$ to obtain

$$(1+i)F = G\left[(1+i)^{N-1} + 2(1+i)^{N-2} + 3(1+i)^{N-3} + \cdots + (N-1)(1+i)\right] \quad \text{-(2)}$$

Subtracting eqn (1) from eqn (2) to yield

$$iF = G\left[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \cdots - (N-1)\right]$$

$$= G\left[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \cdots + (1+i) - N + 1\right]$$

$$= G\left[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \cdots + (1+i) + 1\right] - NG$$

$$= G\left[\frac{(1+i)^{N-1} - (1+i)^{-1} \cdot 1}{1 - (1+i)^{-1}}\right] - NG$$

$$= G\left[\frac{(1+i)^{N-1} - \frac{1}{(1+i)}}{1 - \frac{1}{(1+i)}}\right] - NG$$

$$= G\left[\frac{(1+i)^{N} - 1}{(\cancel{1}+i) - \cancel{1}}\right] - NG$$

$$= G\left[\frac{(1+i)^{N} - 1 - iN}{i}\right]$$

$$\therefore F = G\left[\frac{(1+i)^{N} - iN - 1}{i^2}\right]$$

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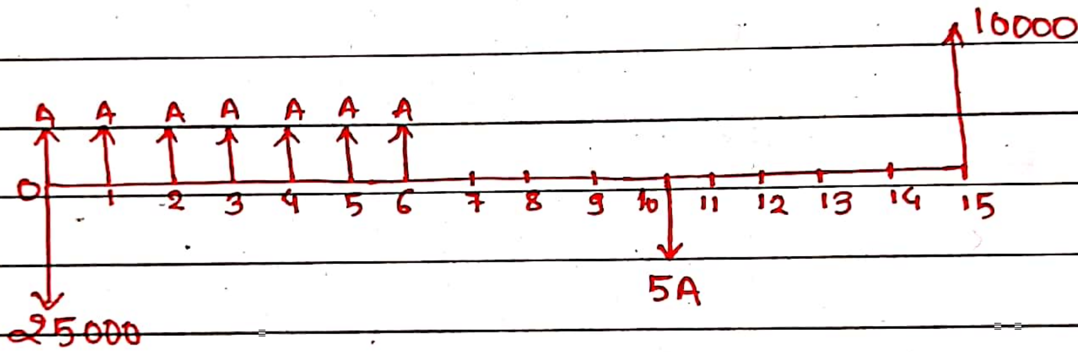
Eg: Find the value of A if i = 15%



Soln:-

Using concept of Equivalance

Converting all cash ~~inflows~~ into present value

$$\cancel{25000 + 5A(1+0.15)^{-10} = A + A(P/A, 15\%, 6) + 10000(}$$

$$25000 + 5A\left(\frac{P}{F}, 15\%, 10\right) = A + A\left(\frac{P}{A}, 15\%, 6\right) + 10000\left(\frac{P}{F}, 15\%, 15\right)$$

or, $25000 + 5A * 0.2472 = A + A * 3.7845 + 10000 * 0.1229$

or, $\cancel{5A - 4.7845A = 1229}$

or, $25000 - 1229 = 4.7845A - 1.236A$

$\therefore A = 6698.88$ Ans.

CH-4 Basic Methods of Engineering Economic Studies

1. Minimum attractive Rate of Return (MARR) Method:

:- MARR is the minimum interest rate that encourages the investor to invest in financial projects.

:- MARR is the interest rate at which a firm can always earn or borrow money.

:- MARR is the interest rate used in the time value of money calculation.

:- MARR is generally determined by top management from policy level, so it may be different from time to time and firm to firm

:- MARR is determined from the opportunity cost viewpoint.

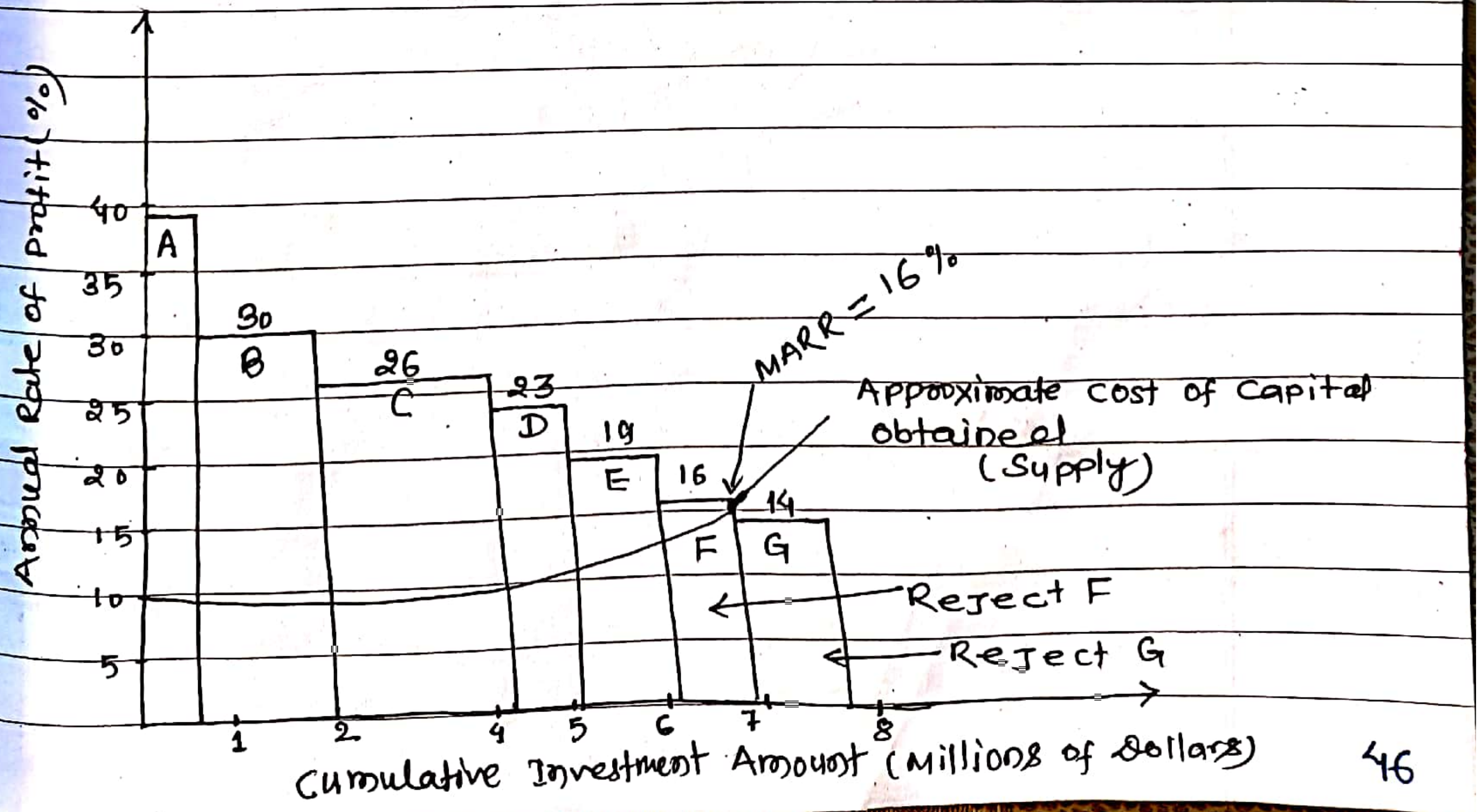
* Factors influence the determination of MARR:-

(1) The amount of fund available for investment and its Source.

(2) The nature of investment alternatives.

(3) The amount of risk perceived in the investment.

(4) The type of organization involved. (government, public, private)



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(2) Equivalent worth method :

:- This method converts all cash flows into equivalent worth at present or Future or Annual time by using an interest rate equal to MARR.

(i) Present worth (PW) Method:

:- present worth method discounts future amounts to the present by using the interest rate over the appropriate study period as

$$PW(i\%) = F_0(1+i)^0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} + \cdots + F_N(1+i)^N$$

$$= \sum_{K=0}^{N} F_K (1+i)^{-K}$$

Where, i = effective interest rate or MARR

K = index for each compounding period

F_K = Future cash flows at the end of period K.

N = No. of compounding periods in study period.

Decision criteria :

If $PW(i\%) > 0$, accept the project

If $PW(i\%) = 0$, remain indifferent

If $PW(i\%) < 0$, reject the project

(ii) Future worth (FW) method :

:- Future worth of a project is the equivalent worth of all cash flows at the end of its investment period.

$$\cancel{FWT}\ FW(i\%) = F_0(1+i)^N + F_1(1+i)^{N-1} + \cdots + F_N(1+i)^0$$

$$= \sum_{K=0}^{N} F_K (1+i)^{N-K}$$

Decision Rule:

$FW(i\%) > 0$, accept the project

$FW(i) = 0$, remain indifferent

$FW(i\%) < 0$, reject the project

(iii) Annual Worth (AW) Method :

: Annual worth is the equivalent worth of a lump-sum amount converted into a series of equal payments at the end of each period and is calculated as,

$$AW(i\%) = R - E - CR(i\%)$$

where,

R = Annual Equivalent revenues

E = " " Expenses

$CR(i\%)$ = annual equivalent capital recovery amount

$$= I\left(\frac{A}{P}, i\%, N\right) - S\left(\frac{A}{F}, i\%, N\right)$$

Where I = initial investment

S = salvage value.

Decision Rule:

$AW(i\%) > 0$, Accept the project

$AW(i\%) = 0$, remain indifferent

$AW(i\%) < 0$, reject the project

Eg: From the following cash flow information, calculate PW, AW and FW by assuming rate of interest is 7% per year that compounded semi annually.

EOY	0	1	2	3	4	5
Cash flow	-300000	80000	90000	90000	90000	120000

Soln:-

$r = 7\%$ per year compounded semiannually

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.07}{2}\right)^2 - 1$$

$$= 0.0712$$

$$= 7.12\% \text{ per year}$$

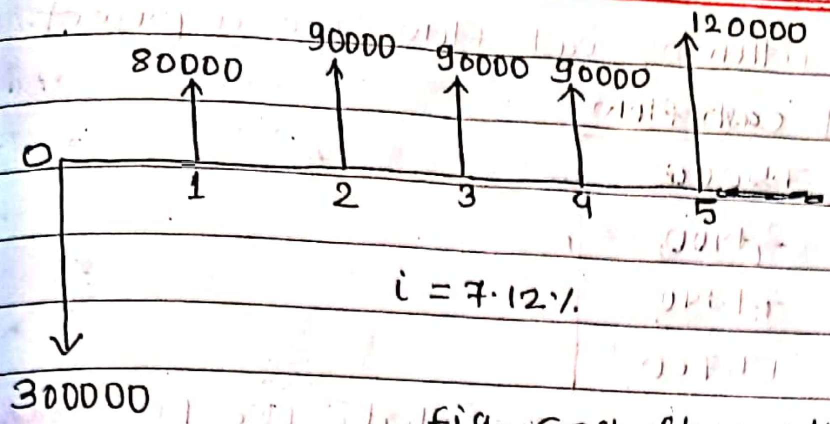


fig. cash flow diagram

$$PW(7.12\%) = -300000 + 80000(1.0712)^{-1} + 90000(1.0712)^{-2} + 90000(1.0712)^{-3} + 90000(1.0712)^{-4} + 120000(1.0712)^{-5}$$

$$= Rs\ 79769.85$$

Now,

$$AW(7.12\%) = PW(7.12\%)\left[\frac{A}{P}, 7.12\%, 5\right]$$

$$= 79769.85 * \left[\frac{0.0712(1.0712)^5}{(1.0712)^5 - 1}\right]$$

$$= Rs\ 19517.67$$

$$FW(7.12\%) = PW(7.12\%)\left(\frac{F}{P}, 7.12\%, 5\right)$$

$$= 79769.85 * (1.0712)^5$$

$$= Rs\ 112510.12$$

Ans.

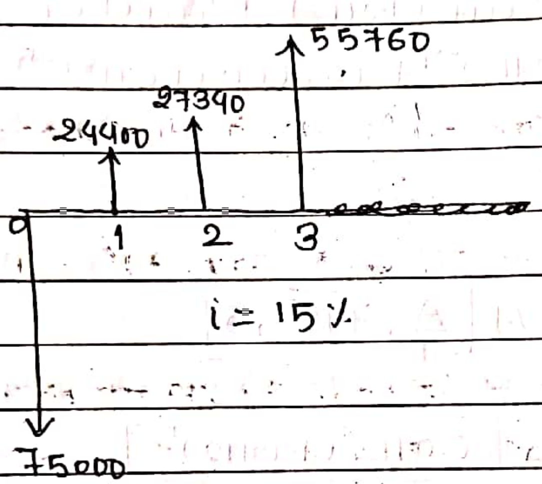
Here, PW(7.12%), FW(7.12%) & AW(7.12%) is greater than 0 so the investment is acceptable.

Eg: Consider the following cash flow for a project.

EOY	Net cash flow
0	- Rs 75000
1	Rs 24400
2	Rs 27340
3	Rs 55760

MARR=15%. Is the investment accepted? Use FW formulation.

Soln:-

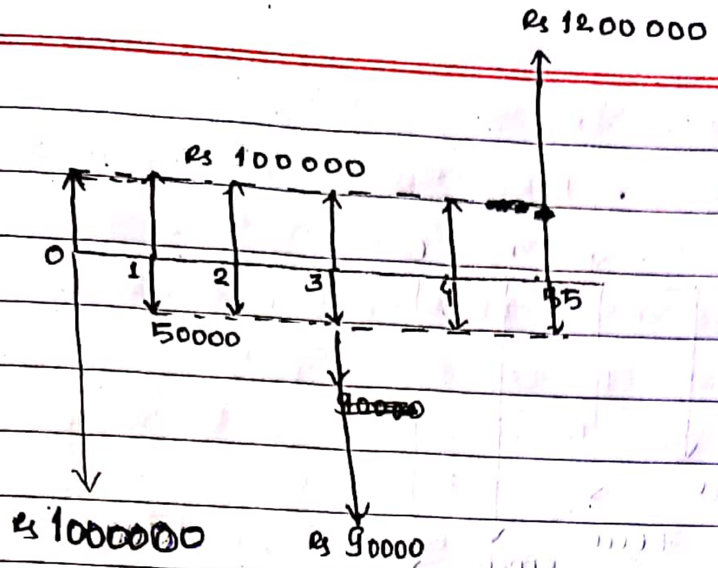


$$FW(15\%) = -75000(1.15)^3 + 24400(1.15)^2 + 27340(1.15)^1 + 55760$$
$$= Rs\ 5404.37$$

Here, FW(15%) > 0, the investment is economically accepted.

Eg: You purchased a building 5 years ago for Rs 10 00 000. Annual maintenance cost is Rs 50000 per year. At the end of 3 years, Rs 90000 was spent on roof repairs. At the end of 5 years you sell a building for Rs 1200000. During the period of ownership, you rented the building for Rs 1 00 000 per year paid at the beginning of each year. Use AW method to evaluate the investment if MARR = 12%.

Soln:-



$i = 12\%$

$$AW(12\%) = (-1000000 + 100000)\left(\frac{A}{P}, 12\%, 5\right) + 100000\left(\frac{P}{A}, 12\%, 4\right)\left(\frac{A}{P}, 12\%, 5\right)$$

$$- 50000 - 90000\left(\frac{P}{F}, 12\%, 3\right)\left(\frac{A}{P}, 12\%, 5\right) + 1200000\left(\frac{A}{F}, 12\%, 5\right)$$

$$= -900000 * 0.2774 + 100000 * 3.0373 * 0.2774 - 50000 - 90000 * 1.4049 * 0.2774 + 1200000 * 0.1574$$

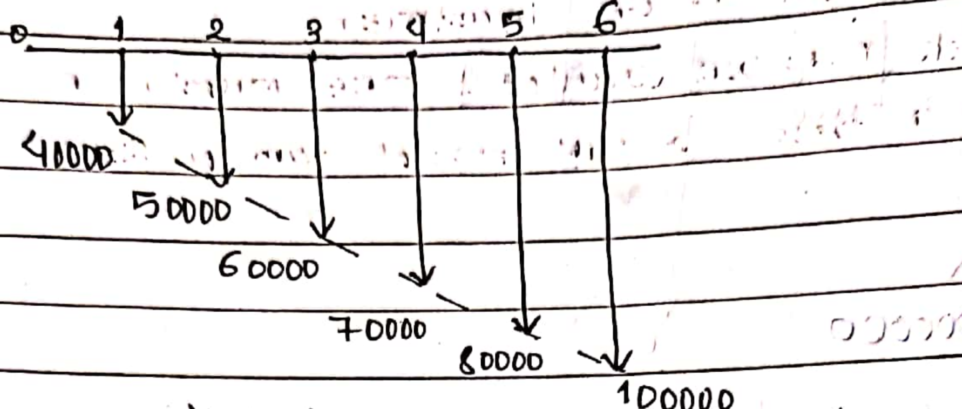
$$= Rs\ -61600.03$$

Since, $AW(12\%) < 0$, the investment is rejected.

Eg: Find the present equivalent from the cash flow given if interest rate is 11% per year using uniform gradient method.

EOY	Cash flow (Rs)
1	−40000
2	−50000
3	−60000
4	−70000
5	−80000
6	−100000

Soln:



$i = 11\%$

$A_1 = -40000$

$G = -10000$

$N = 6$ years

$i = 11\%$ per year $= 0.11$

$A = A_1 + G(A/G, 11\%, 6)$

$= -40000 + (-10000) * 2.1976$

$= -61976$

$P = A(P/A, 11\%, 6) + (-10000)(P/F, 11\%, 6)$

$= -61976 * \cancel{7.9129}\ 4.2305 - 10000 * 0.5346$

$= \cancel{-495755.89}\ - 267535.468$

$\approx Rs\ \cancel{-495758}\ \approx Rs - 267535$

or

$P = -40000(P/A, 11\%, 6) - 10000(P/G, 11\%, 6) - 10000(P/F, 11\%, 6)$

$= -40000 * 4.2305 - 10000 * 9.2972 - 10000 * 0.5346$

$= -Rs\ 267538$ Ans.

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(3) Payback period Method:

:- The payback period is the length of time required to recover the cost of an investment.

:- The payback method calculates the number of years required for cash inflows to just equal cash outflows.

:-

(i) Simple payback period:

:- Simple payback period denotes the time period required to breakeven on an investment without considering time value of money.

:- It measures a projects liquidity

$$\text{simple payback period} = \frac{\text{Initial investment}}{\text{Annual Savings or Net cashinflows per year}}$$

In other way,

$$\sum_{K=1}^{\theta} (R_K - E_K) - I \geq 0$$

Where, I = capital investment

R_K = Revenues at the end of year K

E_K = Expenses " " " " " K

θ = simple payback period

(ii) Discounted payback period

: It is defined as the number of years required to recover the investment from discounted cash flows i.e. considering time value of money.

:- Discounted payback period for a project having one time investment at time zero can be computed as

$$\sum_{K=1}^{\theta'} (R_K - E_K)\left(\frac{P}{F}, i\%, K\right) - I \geq 0$$

Where

I = Capital investment

R_K = Revenue at the end of year K

E_K = Expenses " " " " " K

i = MARR

θ' = Discounted payback period.

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Eg:- Calculate both type of payback period for the given cash flow of the project:

Period	Net cash flow (Rs)
0	-25000
1	8000
2	8,000
3	8,000
4	8.000
5	13000

MARR = 20%

Soln: (i) simple payback period:

Period	Net cash flow (Rs)	cumulative cash flow (Rs)
0	-25000	-25000
1	8000	-17000
2	8000	- 9000
3	8000	- 1000
4	8000	7000
5	13000	20000

Here the cumulative cash flow turns to positive in year 4. therefore payback period lies between 3 and 4. By interpolating, we get the payback period $= 3 + \frac{1000}{8000}$

$= 3.125$ years

(ii) Discounted payback period:

Period	Net Cash flow (Rs)	PW(20%) of Net Cash flow	cumulative cash flow
			-25000
0	-25000	-25000	-18333
1	8000	6667	-12777
2	8000	5556	-8147
3	8000	4630	-4289
4	8000	3858	934
5	13000	5223	

Here the cumulative cash flow turns to positive in year 5. Therefore payback period lie between year 4 and 5. By interpolating, we get

$$\text{the payback period} = 4 + \frac{4289}{5223}$$

$$= 4.82 \text{ years}$$

Eg:

Initial investment = Rs 6000

Annual Benefits = Rs 3000

Annual O&M cost = Rs 1000

Salvage value = Rs 1500

MARR = 10%

Evaluate both type of payback period. If useful life = 5 years. (Take standard payback period = 3 years)

Solution:

(i) Simple payback Period Method

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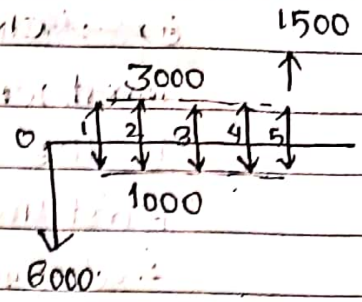
Period	Net Cash flow (Rs)	Cumulative cash flow
0	-6000	-6000
1	3000-1000=2000	-4000
2	2000	-2000
3	2000	0
4	2000	2000
5	1500 + 2000 = 3500	5500



simple payback period = 3 years

(ii) Discounted payback period:

Period	Net cash flow (Rs)	PW(10%) of Net cashflow	Cumulative cash flow
0	-6000	-6000	-6000
1	2000	1818.18	-4181.82
2	2000	1652.89	-2528.99
3	2000	1562.63	-1026.3
4	2000	1366.26	339.96
5	1500+2000 = 3500	2173.22	2513.18

Discounted payback period $= 3 + \frac{1026.3}{1366.26}$

$= 3.75 > 3$ years

This project is economically not justified.

Eg:- Check the feasibility of the following investment by using PW, FW, AW method. What is the capital Recovery amount of this project?

First investment (Rs) = 100000

Salvage value (Rs) = 25000

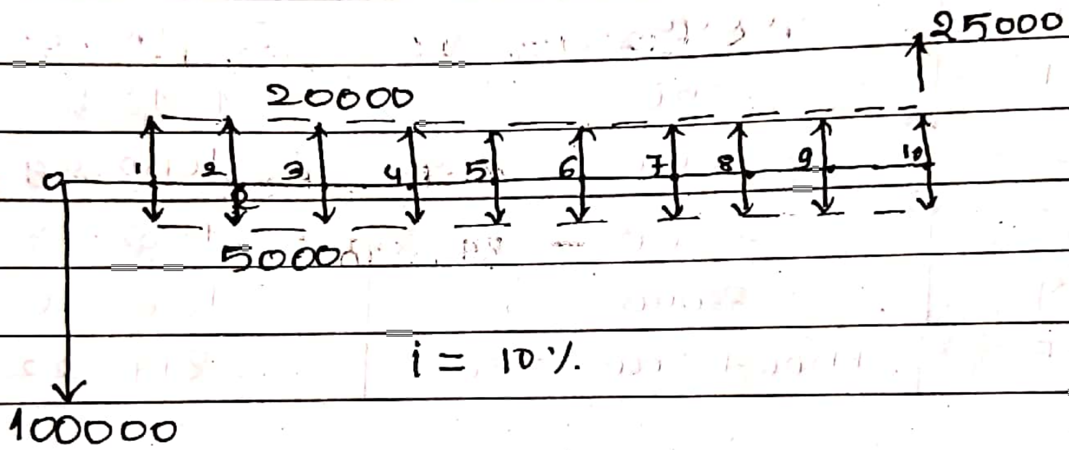
Project period (years) = 10

Annual Revenue (Rs) = 20000

Annual cost (Rs) = 5000

MARR = 10% per years

Soln:



Present worth method

$$PW(10\%) = -100000 + (20000-5000)(P/A, 10\%, 10) + 25000 (P/F, 10\%, 10)$$

$$= -100000 + 15000 \times 6.1446 + 25000 \times 0.3855$$

$$= Rs\ 1806.5$$

Since $PW(10\%) > 0$, the project is accepted

Future worth method

$$FW(10\%) = -100000(F/P, 10\%, 10) + (20000-5000)(F/A, 10\%, 10) + 25000$$

$$= -100000 \times 2.5937 + 15000 \times 15.9374 + 25000$$

= ~~416153.5~~

= Rs 4691

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Since FW (10%) > 0, the project is accepted.

Annual worth Method

$AW(10\%) = -100000\left(\frac{A}{P}, 10\%, 10\right) + (20000 - 5000) + 25000\left(\frac{A}{F}, 10\%, 10\right)$

$= -100000 * 0.1627 + 15000 + 25000 * 0.0627$

$=$ Rs 297.5

Since AW(10%) > 0, the project is accepted.

Capital Recovery amount (CR)

$CR(10\%) = I\left(\frac{A}{P}, 10\%, 10\right) - S\left(\frac{A}{F}, 10\%, 10\right)$

$= 100000 * 0.1627 - 25000 * 0.0627$

$=$ Rs 14702.5

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(4) Rate of Return (RoR) method:

:- Rate of Return is defined as an annual rate of profit resulting from an investment.

:- It is especially relevant to evaluate mutually exclusive alternatives.

:- RoR is defined as the interest rate earned on the unpaid balance of an amortized (installment) loan.

(i) Internal Rate of Return (IRR) method:

:- IRR is the breakeven interest rate at which equivalent worth of a project's cash flow is zero.

:- IRR is that interest rate (return rate) which equates the equivalent worth of an alternative's cash inflows to the equivalent worth of cash outflows.

Mathematical Relation:

Based on PW formulation

$$PW(i'\%) = 0$$

$$PW(i'\%)_{inflow} = PW(i'\%)_{outflow}$$

or, $$PW(i'\%)_{inflow} - PW(i'\%)_{outflow} = 0$$

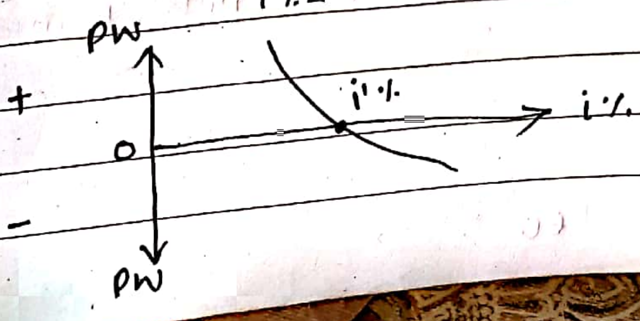
$$PW(i'\%) = \sum_{K=0}^{N} R_K (P/F, i'\%, K) + \sum_{K=0}^{N} E_K (P/F, i'\%, K) = 0$$

where, R_K = revenues for the K^{th} year

E_K = expenses for the K^{th} year

N = project life or study period

$i'\%$ = IRR



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Decision Rule

IRR > MARR, accept the project

IRR < MARR, Reject " "

IRR = MARR, remain indifferent

* ## Drawbacks of IRR

(1) When the algebric sign of the cash flow changes more than one in the series, it is possible to obtain multiple rate of return.

(2) When mutually exclusive projects are considered it can recommend the wrong investment and does not consider the scale of the investment. Under this circumstance, incremental analysis is necessary.

(3) IRR method involves linear interpolation of non-linear function and when solved manually by trial and error method, may not give accurate result and it is more time consuming.

(4) There are situations in which its iterative calculation process fails to produce a solution.

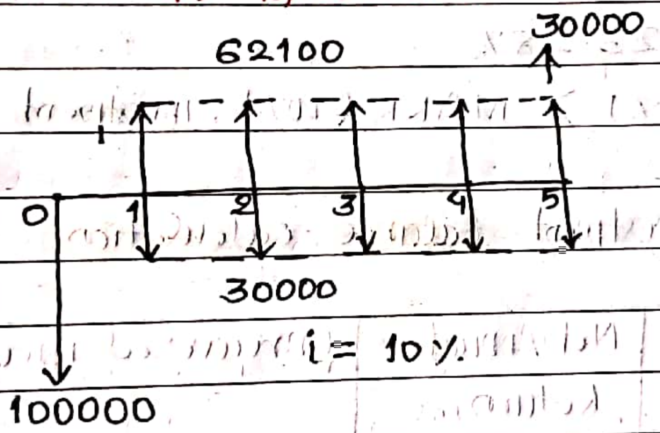
(5) The IRR method is based on the assumption that the recovered funds, if not consumed in each time period, are reinvested at i% rather than at MARR. This ~~is~~ not always practical.

Eg: Evaluate the IRR for the following project and decide whether the project is accepted or not? Also draw the investment balance diagram.

Initial investment = Rs 100000
Annual Revenues = Rs. 62100
Annual expenses = Rs 30000
Salvage value = Rs 30000
Project life = 5 year
MARR = 10%.

~~Is the investment good? use IRR method~~
~~Also draw an investment~~

Soln:-



Using PW Formulation

$$PW(i'\%) = -100000 + (62100-30000)\left(\frac{P}{A}, i'\%, 5\right) + 30000\left(\frac{P}{F}, i'\%, 5\right) = 0$$

$$\text{or,} \quad -100000 + 32100\left[\frac{(1+i')^5-1}{i'(1+i')^5}\right] + 30000(1+i')^{-5} = 0$$

By trial and error

At $i_1' = 23\% = 0.23\ (x_1)$; $PW_1(i_1') = 647.52\ (y_1)$ (+ve)

$i_2' = 24\% = 0.24\ (x_2)$; $PW_2(i_2') = -1639.92\ (y_2)$ (−ve)

Hence, IRR lies between 23% and 24%.

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Using Linear interpolation

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

or, $x = x_1 + \frac{(y-y_1)(x_2-x_1)}{(y_2-y_1)}$

or, $x = 0.23 + \frac{0-647.52}{-1639.92-647.52}(0.24-0.23)$

($\because y=0$, for IRR, PW=0)

$\therefore x = 0.2328$

$= 23.28\%$

i.e IRR = 23.28%

Here, IRR(23.28%) > MARR (10%), investment is accepted.

Unrecovered Investment Balance calculation

Year	Investment	Net Annual Return	Unrecovered Investment
0	−100000	~~32100~~ −	−100000
1	−123280	32100	−91180
2	−112406.7	32100	−80306.7
3	−99002	32100	−66902
4	−82476.91	32100	−50376.91
5	−62104.65	32100+3000	−4.65 ≈ 0

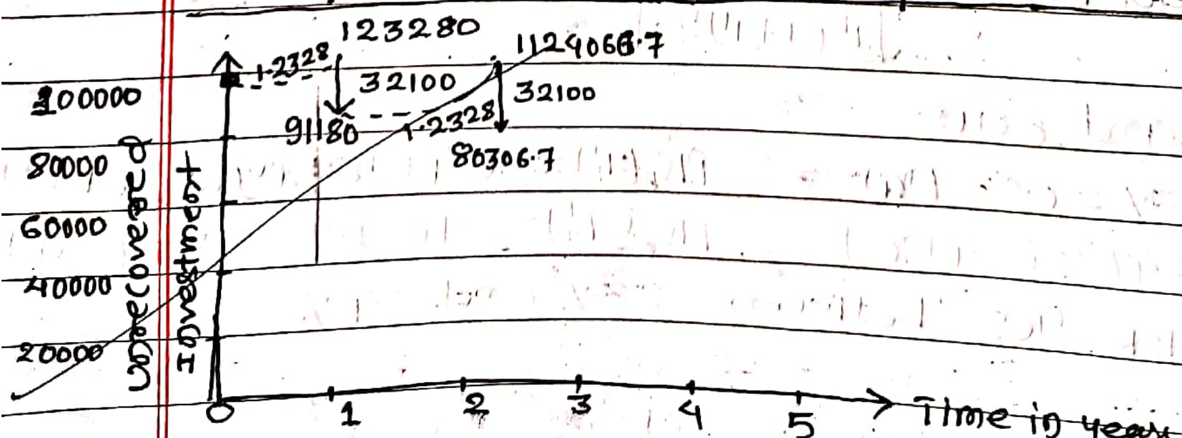


fig. Investment Balance diagram

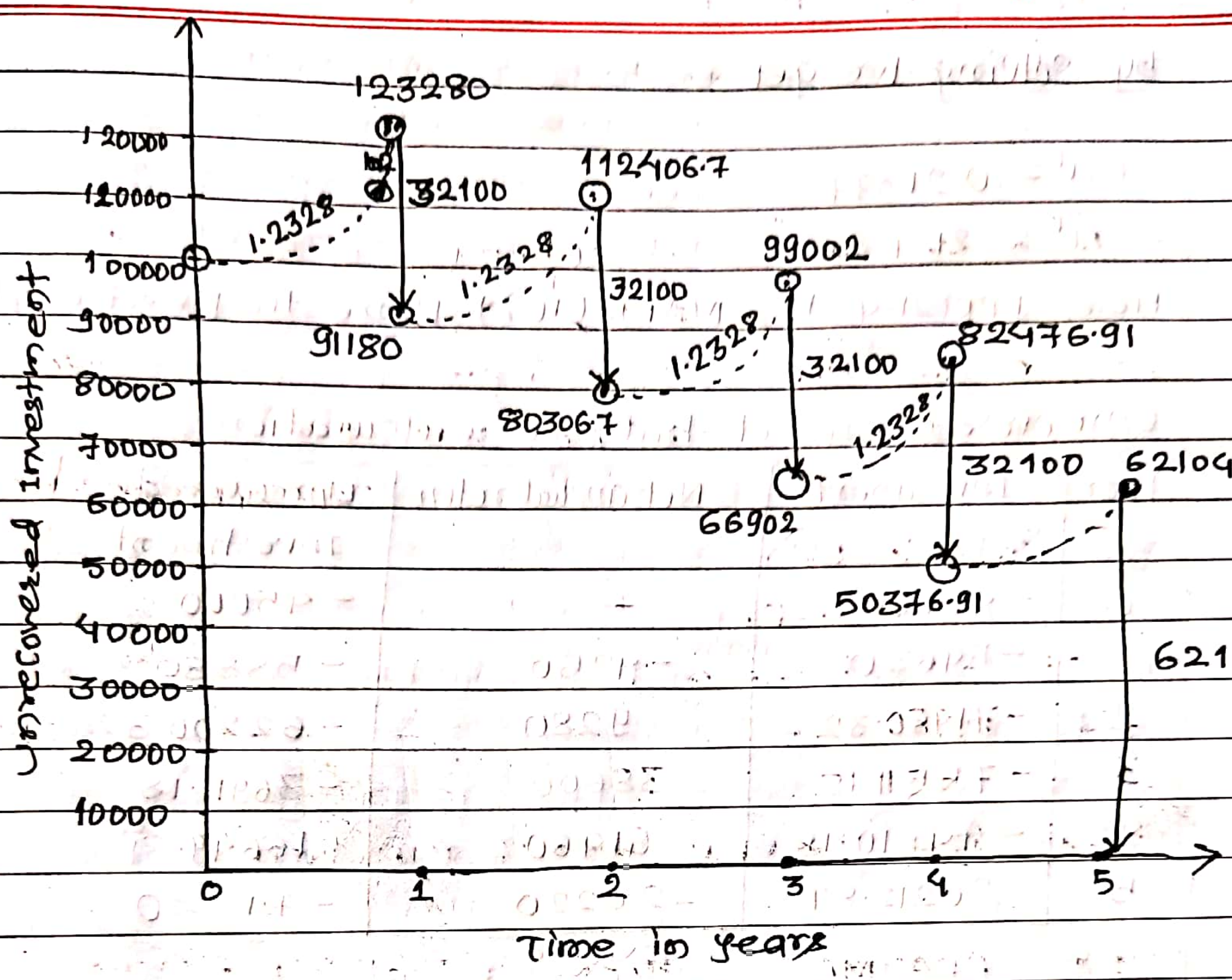


fig: Investment Balance Diagram

Eg: consider the investment project which has a project cash ~~cash~~ flow as follows. Is the project feasible if MARR = 10%. Use IRR method and draw investment balance diagram

EOY	0	1	2	3	4	5
Net cashflow	-45000	-4250	9280	38600	61460	-20220

Soln:

Using PW formulation

$$PW(i'\%) = -45000 - 4250(P/F, i', 1) + 9280(P/F, i', 2) + 38600(P/F, i', 3) + 61460(P/F, i', 4) - 20220(P/F, i', 5)$$

$$0 = -45000 - 4250(1+i')^{-1} + 9280(1+i')^{-2} + 38600(1+i')^{-3} + 61460(1+i')^{-4} - 20220(1+i')^{-5}$$

By solving we get,

$i' = 0.21397$

$i' \approx 21.4\ \%$

Here, IRR(21.4%) > MARR (10%), Hence the project accepted.

Unrecovered project balance calculation

Year	Investment	Net annual return	Unrecovered Investment
			−45000
0	−45000	—	−58880
1	−54630	−4250	−62200.32
2	−71480.32	9280	−36911.18
3	−75511.18	38600	16649.81
4	−44810.18	61460	−7.12 ≈ 0
5	20212.87	−20220	

Unrecovered Investment

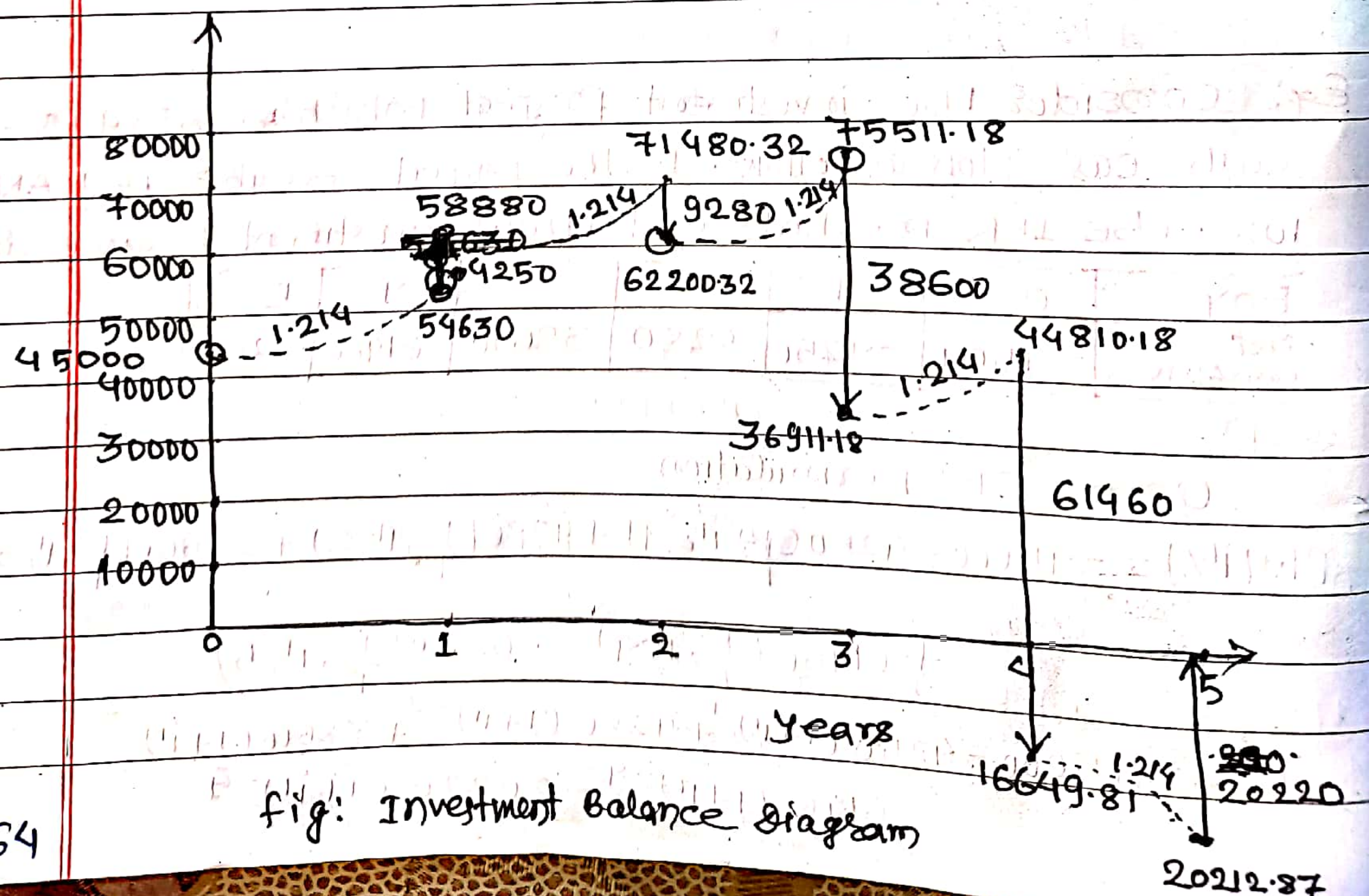


fig: Investment Balance diagram

External Rate of Return (ERR)
or
Modified Rate of Return (MIRR)

: The external rate of return (i^*) is the unique rate of return for a project that assumes that net positive cash flows, which represent money not immediately needed by the project, are reinvested at the reinvestment rate ε%. The reinvestment rate depends upon the market rate available for investments.

:- Advantages of ERR

(1) It does not need trial and error process to solve for i%.

(2) There is no possibility of multiple rate of return

(3) Eliminated the drawback of reinvestment assumption to ~~some extent~~.

Decision Rule:

If ERR > MARR, accept the project

If ERR < MARR, reject " "

If ERR = MARR, remain indifferent

Steps for ERR calculation:

Step1: Discount all cash outflows to the present at the external reinvestment rate, ϵ % per compounding period

Step2: All cash inflows are compounded to the future (Period N) at ϵ%.

Step3: ERR, i^*% is the interest rate that establishes equivalence between these terms

Mathematically

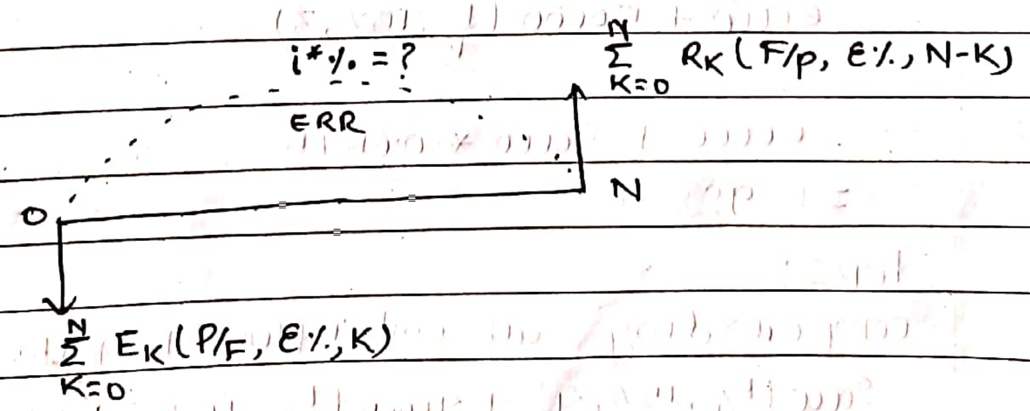
Step1: $$\sum_{K=0}^{N} E_K (P/F, \epsilon\%, K)$$

Step2: $$\sum_{K=0}^{N} R_K (F/P, \epsilon\%, N-K)$$

Step3:
$$\sum_{K=0}^{N} E_K (P/F, \epsilon\%, K)(F/P, i^*\%, N) = \sum_{K=0}^{N} R_K \left(\frac{F}{P}, \epsilon\%, N-K\right)$$

where,

R_K = net revenues in period K
E_K = " expenses " "
N = project life
ϵ% = external reinvestment rate per period
i^*% = ERR

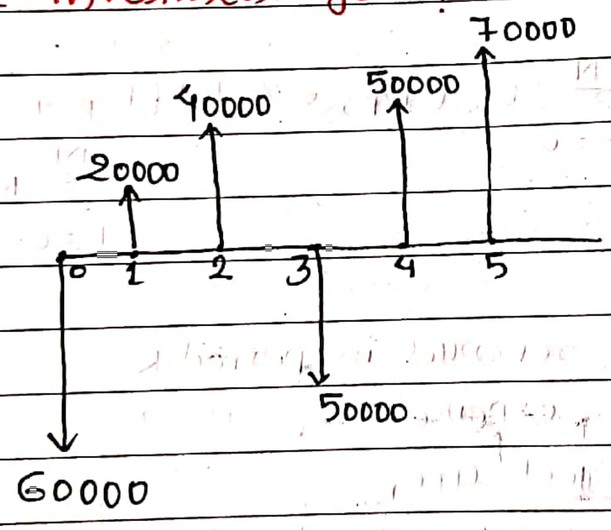


Eg: Find the ERR or MIRR when external reinvestment rate is 15% (= ∈) and MARR = 18%

EOY (Years)	Annual cash flow (Rs)
0	− 60000
1	+ 20000
2	+ 40000
3	− 50000
4	+ 50000
5	+ 70000

Is the investment good?

Soln:



Step 1:

Discounting all cash outflows to present time at ∈ = 15%.

$60000 + 50000\,(P/F, 15\%, 3)$

$= 60000 + 50000 * 0.6575$

$=$ Rs 92875

Step 2:

compounding all cash inflows to Future at ∈ = 15%.

$20000\,(F/P, 15\%, 4) + 40000\,(F/P, 15\%, 3) + 50000\,(F/P, 15\%, 1)$

$+ 70000$

$= 20000 * 1.7490 + 40000 * 1.5209 + 50000 * 1.15 + 70000$
$=$ Rs 223316

Step3: Establishing the equivalance of the above two equation

$$92875 (\frac{F}{P}, i^*\%, 5) = 223316$$

$$\text{or, } (1+i^*)^5 = 2.4044$$

$$\text{or, } (1+i^*) = 1.1918$$

$$\text{or, } i^* = 0.1918$$

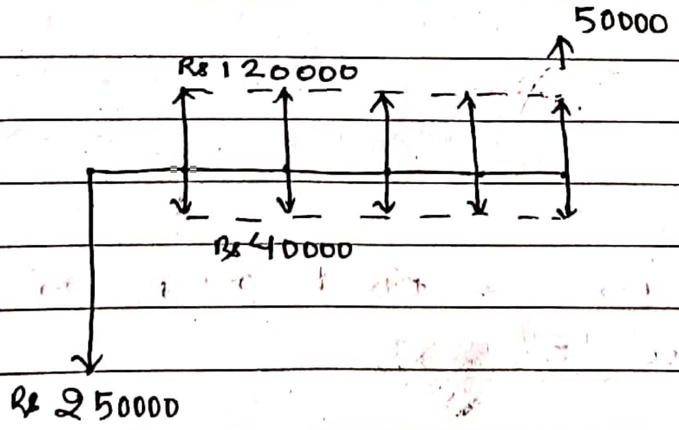
$$= 19.18\%$$

∴ The External Rate of Return (ERR) is 19.18%.

Since ERR(19.18%) > MARR(18%), the project is accepted.

Eg:- Equipment costs Rs 250000, and has salvage value of Rs 50000 at the end of its expected life 5 years. Annual expenses will be Rs 40000. It will produce revenue of Rs 120000 per year. MARR = 20% = ϵ. Find ERR.

Soln:-



Step 1: Discounting all cash outflows to present time at $\epsilon = 20\%$.

$250000 + 40000 (P/A, 20\%, 5)$

$= 250000 + 40000 * 2.9906$

$=$ Rs 369624

Step 2:

Compounding all cash inflows to Future at 20%.

$120000\,(\frac{F}{A}, 20\%, 5) + 50000$

$= 120000 * 7.4416 + 50000$

$=$ Rs 942992

Step 3:

Establishing the equivalance between two equations

$369624\,(\frac{F}{P}, i^*\%, 5) = 942992$

or, $(1+i^*)^5 = 2.5512$

or, $1+i^* = 1.2060$

$\therefore i^* = 0.2060$

$= 20.60\%$

Hence, ERR = 20.60 % Ans.

Benefit Cost Ratio (BCR or B/C) Method!

:- BCR defined as the ratio of equivalent worth of benefits to the equivalent worth of costs.
:- It is also called "savings investment ratio"
:- It is used to evaluate public Sector projects

Two commonly used formulation of BCR ratio are as follows

(i) conventional BCR :

: It is the ratio of gross benefits to costs and expressed as

(a) with PW formulation ::

$$BCR = \frac{PW(B)}{PW(I) - PW(S) + PW(O\&M)}$$

(b) with FW formulation

$$BCR = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$

(c) with AW formulation

$$BCR = \frac{AW(B)}{AW(I) - AW(S) + AW(O\&M)}$$

(ii) Modified BCR:

:- It is the ratio of net benefits to costs and expressed as

(a) with PW formulation

$$BCR = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(S)}$$

(b) with FW Formulation

$$BCR = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$

(c) with AW Formulation

$$BCR = \frac{AW(B) - AW(O\&M)}{AW(I) - AW(S)}$$

Decision Rule:

If $BCR > 1$, Accept the project

If $BCR < 1$, Reject " "

If $BCR = 1$, remain indifferent

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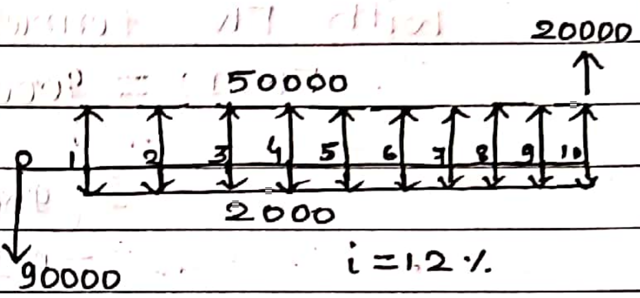
Eg: Find out the both types of B/C ratio using PW, FW and AW Formulation where

Initial investment = Rs 90000
Annual Revenue = Rs 50,000
Annual cost = Rs 2000
Salvage value = Rs 20000
MARR = 12%
Useful life = 10 years

Soln:–



With PW formulation

$PW(I) = Rs\ 90000$

$PW(B) = 50000\left(\frac{P}{A}, 12\%, 10\right)$

$= 50000 * 5.6502$
$= Rs\ 282510$

$PW(O\&M) = 2000\left(\frac{P}{A}, 12\%, 10\right)$

$= 2000 * 5.6502$
$= Rs\ 11300.4$

$PW(S) = 20000\left(\frac{P}{F}, 12\%, 10\right)$

$= 20000 * 0.3220$
$= Rs\ 6440$

$$\text{Conventional B/C ratio} = \frac{PW(B)}{PW(I) - PW(S) + PW(O\&M)}$$

$$= \frac{282510}{90000 - 6440 + 11300.4}$$

$= 2.978 > 1$, project is accepted

Modified B/C ratio $= \dfrac{PW(B) - PW(O\&M)}{PW(I) - PW(S)}$

$= \dfrac{282510 - 11300.4}{90000 - 6440}$

$= 3.2456 > 1$, project is accepted

with FW formulation

$FW(I) = 90000\left(\frac{F}{P}, 12\%, 10\right)$

$= 90000 * 3.1058$

$=$ Rs. 279522

$FW(B) = 50000 * \left(\frac{F}{A}, 12\%, 10\right)$

$= 50000 * 17.5487$

$=$ Rs 877435

$FW(O\&M) = 2000 * \left(\frac{F}{A}, 12\%, 10\right)$

$= 2000 * 17.5487$

$=$ Rs 35097.4

$FW(S) =$ Rs 20000

conventional B/C ratio $= \dfrac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$

$= \dfrac{877435}{279522 - 20000 + 35097.4}$

$= 2.978 > 1$

Modified B/C ratio $= \dfrac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$

$= \dfrac{877435 - 35097.4}{279522 - 20000}$

$= 3.2456 > 1$

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With AW Formulation

$AW(I) = 90000\left(\frac{A}{P}, 12\%, 10\right)$

$= 90000 * 0.1770$

$= Rs\ \cancel{8850}\ 15930$

$AW(B) = Rs\ 50000$

$AW(O\&M) = Rs\ 2000$

$AW(S) = 20000\left(\frac{A}{F}, 12\%, 10\right)$

$= 20000 * 0.0570$

$= 1140$

Conventional B/C ratio $= \dfrac{AW(B)}{AW(I) - AW(S) + AW(O\&M)}$

$= \dfrac{50000}{\cancel{8850}\ 15930 - 1140 + 2000}$

$= 2.978 > 1$

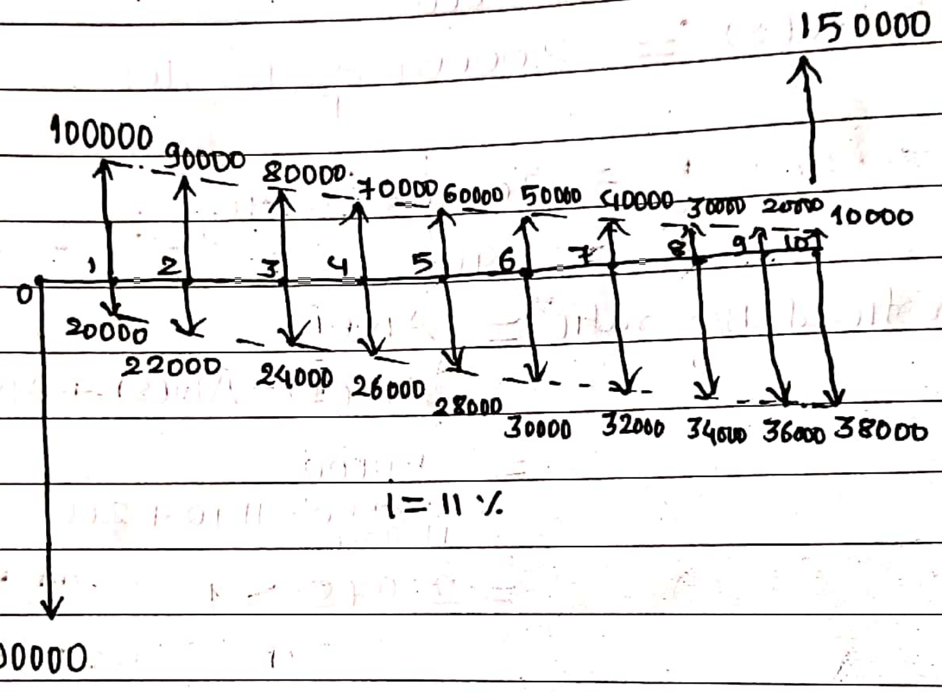
Modified B/C ratio $= \dfrac{AW(B) - AW(O\&M)}{AW(I) - AW(S)}$

$= \dfrac{50000 - 2000}{15930 - 1140}$

$= 3.2456 > 1$, project is accepted

Eg: Find both types of BCR using FW formulation where initial investment is Rs 500000 annual income is Rs 100000 and decreases by Rs 10000 per year; annual cost is Rs 20000 and increases by Rs 2000 per year and salvage value is Rs 150000 useful life = 10 years and MARR = 11 %.

Soln:-



Using FW Formulation

$FW(I) = 500000 (F/P, 11\%, 10)$

$= 500000 * 2.8394$

$=$ Rs 1419700

$FW(B) = 100000 (F/A, 11\%, 10) - 10000 (F/G, 11\%, 10)$

$= 100000 * 16.722 - 10000 * (A/G, 11\%, 10) * (F/A, 11\%, 10)$

$= 1672200 - 10000 * 3.6544 * 16.722$

$=$ Rs 1061111

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$FW(O\&M) = 20000(\frac{F}{A}, 11\%, 10) + 2000(\frac{F}{G}, 11\%, 10)$

$= 20000 * 16.722 + 2000 (\frac{A}{G}, 11\%, 10) * (\frac{F}{A}, 11\%, 10)$

$= 334440 + 2000 * 3.6544 * 16.722$

$=$ Rs 456658

$FW(S) =$ Rs 150000

Conventional B/c ratio $= \dfrac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$

$= \dfrac{1061111}{1419700 - 150000 + 456658}$

$= 0.614$

Modified B/c ratio $= \dfrac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$

$= \dfrac{1061111 - 456658}{1419700 - 150000}$

$= 0.476$

Eg: Calculate both types of BCR of a project with following details: MARR = 12%

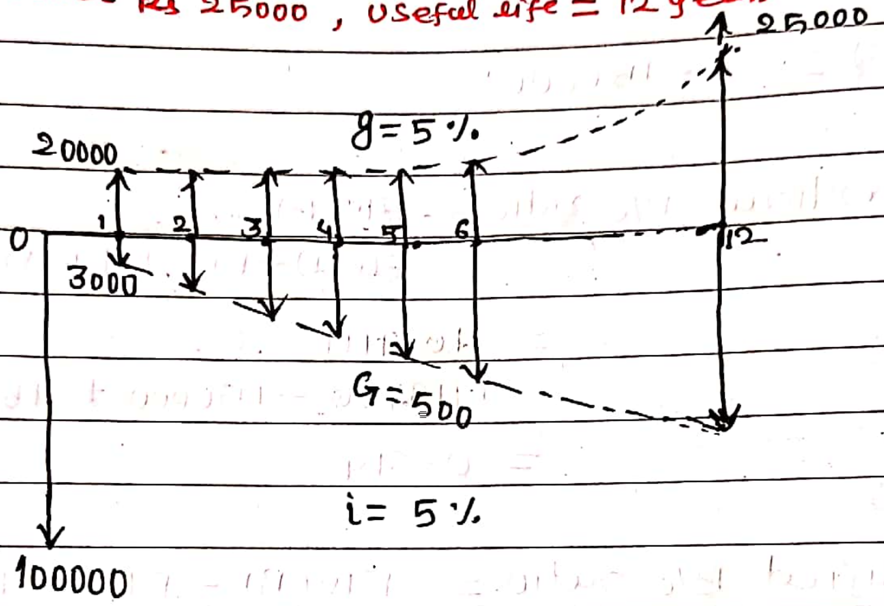
Initial investment = Rs 100000

Annual Income = Rs 20000 at the end of first year and increased by 5% per year

Annual cost = Rs 3000 at the end of first year and increased by Rs 500 per year.

Salvage value = Rs 25000, Useful life = 12 years.

Soln:



Using FW Formulation

$FW(I) = Rs\ 100000 = 100000 * (F/P, \cancel{5\%}\ 12\%, 12) = 100000 * \cancel{1.7959}\ 3.8960$

$= 389600$

$$FW(B) = 20000 \left[\frac{(1+i)^N - (1+g)^N}{i-g} \right]$$

$$= 20000 \left[\frac{(1.12)^{12} - (1.05)^{12}}{0.12 - 0.05} \right]$$

$= Rs\ 600034.19$

$FW(O\&M) = \cancel{3000 + 5000}($

$= 3000(F/A, 12\%, 12) + 500(F/G, 12\%, 12)$

$= 3000 * \cancel{3.8960}\ 24.1331 + 500 * (A/G, 12\%, 12) * (F/A, 12\%, 12)$

$= 11688 + 500 * 4.1897 * \cancel{0.0411}\ 24.1331$

$= Rs\ \cancel{622943.22}\ 122954.52$

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$FW(S) = Rs\ 25000$

Conventional B/c ratio $= \dfrac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$

$= \dfrac{600034.19}{389600 - 25000 + 122954.52}$

$= 1.23 > 1$

Modified B/c ratio $= \dfrac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$

$= \dfrac{600034.19 - 122954.52}{389600 - 25000}$

$= 1.30 > 1$