FORMULAE

UNIT-I

1. Iterative formula for Regula Falsi Method

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

2. In Regula Falsi method to reduce the number of iterations if we start with a smaller interval.

3. Iterative formula for Newton-Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

4. The rate of convergence in N.R. Method is of order 2.

5. N.R. Method is convergent quadratically.

6. Condition for convergence of N.R. method is $|f(x)\, f''(x)| < |f'(x)|^2$

7. Newton's method is useful when the graph of the function crosses the x-axis is nearly vertical.

8. Condition for the convergence of iteration method is $|g'(x)| < 1$.

9. The two types of pivoting are 1. Partial pivoting and 2. Complete pivoting

10. The numerical methods of solving linear equations are of two types : **One is direct and the other is indirect or iterative.**

11. In Gauss-Jordan method the coefficient matrix is transformed into **diagonal matrix.**

12. In Gaussian-elimination method the coefficient matrix is transformed into **upper triangular matrix.**

13. Condition for Gauss-Jacobi method to converge is "The coefficient of matrix should be diagonally dominant."

14. Gauss elimination and Gauss-Jordan are direct methods while Gauss-Seidal and Gauss-Jacobi are iterative methods.

15. The convergence in the Gauss-Seidel method is thrice as fast as in Jacobi's method.

16. The power method will work satisfactorily only if A has a **Dominant** eigenvalue.

UNIT-II

1. Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots\dots + \dots\dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

2. Inverse of Lagrange's interpolation formula is

$$x = f(y) = \frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)} . x_0 + \frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)} . x_1 + \dots\dots + \dots\dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})} . x_n$$

3. Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0) f(x_0, x_1, x_2) + \dots + [(x-x_0)(x-x_1)\dots\dots(x-x_{n-1})] f(x_0, x_1, \dots x_n)$$

4. Newton's forward difference formula is

$$P_n(x) = 1 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots$$

where $u = \dfrac{x - x_0}{h}$

5. Newton's backward difference formula is

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \ldots$$

where $v = \dfrac{x - x_n}{h}$

6. Newton's forward interpolation formula used only for equal intervals.

Unit-III

1. Newton's formula to find $f'(x)$ is

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2 - 6u + 2)}{3!}\Delta^3 y_0 + \ldots\right]$$

where $u = \dfrac{x - x_0}{h}$

2.

AT A GLANCE

Rule	Degree of $y(x)$	No. of intervals	Error	Order
Trapezoidal rule	One	any	$\lvert E \rvert < \frac{(b-a)h^2}{12} M$	h^2
Simson's 1/3 rule	Two	even	$\lvert E \rvert < \frac{(b-a)h^4}{180} M$	h^4
Simson's 3/8 rule	Three	Multiple of 3		

3. The trapezoidal rule is so called, because it approximates the integral by the sum of n trapezoids.

4. Trapezoidal rule

$$\int_a^b y\,dx = \frac{h}{2}\,[(\text{sum of the first and last ordinate}$$
$$+\ 2(\text{remaining ordinates})]$$

5. Simpson's one third rule

$$\int_a^b y\,dx = \frac{h}{3}\,[\text{Sum of the first and last ordinates}$$
$$+\ 2(\text{sum of remaining odd ordinates})$$
$$+\ 4(\text{sum of even ordinates})]$$

6. Simpson's three-eight rule

If n is a multiple of 3,

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}[(y_0+y_n)+3(y_1+y_2+y_4+y_5+\ldots+y_{n-1})$$
$$+\ 2\,(y_3+y_6+y_9+\ldots+y_n)]$$

7. Simpson's rule will give exact result, if the entire curve $y=f(x)$ is itself a parabola.

8. The general Newton-Cotes quadrature formula is

$$\int_a^{a+nh} f(x)\,dx = nh\left[1+\frac{n\Delta}{2}+\frac{n(2n-3)}{12}\Delta^2+\frac{n(n-2)^2}{24}\Delta^3+\ldots\right]f(a)$$

9. Error in the Trapezoidal formula is of the order h^2.

10. Error in the Simpson formula is of the order h^4.

11. Two points Gaussian Quadrature.
If the interval given is -1 to 1 then the apply

$$\int_{-1}^{1} f(x)\,dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right).$$ This formula is exact for polynomials upto degree 3.

12. Three points Gaussian quadrature.

$$\int_{-1}^{1} f(x)\,dx = \frac{5}{9}\left[f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)\right] + \frac{8}{9}f(0)$$

This formula is exact for polynomials upto degree 5.

13. If the range is not (–1, 1) then the idea to solve the Gaussian quadrature problems is

$$x = \frac{b-a}{2}z + \frac{b+a}{2}$$

14. Trapezoidal rule for [Double intgrals]

$I = \frac{hk}{4}$ [(Sum of values of f at the four corners
+ 2 (Sum of values of f at the remaining nodes on the boundary) + 4 (Sum of the values of f at the interior nodes)]

15. Simpson's rule for double integration

$I = \frac{hk}{9}$ [(Sum of the values of f at the four corners)
+ 2 (Sum of the values of f at the odd positions on the boundary except the corners)
+ 4 (Sum of the values of f at the even positions on the boundary)
+ {4 (Sum of the values of f at odd positions)
+ 8 (Sum of the values of f at even positions) on the odd row of the matrix except boundary rows}
+ {8 (Sum of the values of f at the odd positions)
+ 16 (Sum of the values of f at the even positions) on the even rows of the matrix)]

UNIT-IV

1. Taylor's Series expansion of $y(x)$ about $x = x_0$ is given by $y(x) = y_0 + (x - x_0)\, y_0' + \frac{(x - x_0)^2}{2!} y_0'' + \ldots$

2. The fourth order Taylor Algorithm

$$y_{m+1} = y_m + h y_m' + \frac{h^2}{2!} h_m'' + \frac{h^3}{3!} y_m''' + \frac{h^4}{4!} y_m^{iv}$$

3. Euler Algorithm

$$y_{n+1} = y_n + h f(x_n, y_n), \; n = 0, 1, 2, 3, \ldots$$

4. Modified Euler's formula

$$y_{n+1} = y_n + h \left[f\left(x_n + \frac{1}{2} h, y_n + \frac{1}{2} h f(x_n, y_n)\right) \right]$$

(or) $$y(x + h) = y(x) + h \left[f\left(x + \frac{1}{2} h, y + \frac{1}{2} h f(x, y) \right) \right]$$

5. The modified Euler method is based on the average of points.

6. Algorithm for Second Order R.K Method

If the initial values of (x, y) for the differential equation $\frac{dy}{dx} = f(x, y)$ then the first increment in y namely Δy is calculated from the formula.

$$k_1 = hf(x, y)$$

$$k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2} \right]$$

$$\Delta y = k_2 \text{ where } h = \Delta x$$

7. Algorithm for Third Order R.K Method

The algorithm for this method is given below :

$$k_1 = hf(x, y)$$

$$k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2}\right]$$

$$k_3 = hf[x + h, y + 2k_2 - k_1]$$

and $\Delta y = \frac{1}{6}(k_1 + 4k_2 + k_3)$

8. Algorithm for Fourth Order R.K Method

The algorithm for this method is given below :

$$k_1 = hf(x, y) \qquad k_2 = hf\left[x + \frac{h}{2}, y + \frac{k_1}{2}\right]$$

$$k_3 = hf\left[x + \frac{h}{2}, y + \frac{k_2}{2}\right] \qquad k_4 = hf[x + h, y + k_3]$$

and $\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$$y(x + h) = y(x) + \Delta y$$

9. Milne's Predictor formula

Solution : Milne's predictor formula is

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) + \frac{14h^5}{45} y^5(\varepsilon_1)$$

where ε_1 lies between x_{n-3} and x_{n+1}

10. Milne's corrector formula

$$y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1}) - \frac{h^5}{90} y^5(\varepsilon_2)$$

where ε_2 lies between x_{n-1} and x_{n+1}.

11. Four Prior values are required to predict the next value in Milne's method.

12. The error term in Milne's corrector formula is $-\frac{h}{90}\Delta^4 y_0'$

13. Milne's method is not a self starting method.

14. Predictor corrector methods are not self starting methods.

15. Adams-Bashforth Predictor formula

$$y_{k+1,p} = y_k + \frac{h}{24}\left[55\, y_k' - 59\, y_{k-1}' + 37\, y_{k-2}' - 9 y_{k-3}'\right]$$

16. Adams-Bashforth corrector formula

$$y_{k+1,c} = y_k + \frac{h}{24}\left[9 y_{k+1}' + 19\, y_k' - 5 y_{k-1}' + y_{k-2}'\right]$$

17. Four prior values are required to predict the next value in Adam's method.

18. Adams bashforth method is not a self starting method.

UNIT-V

1. **Classification of partial differential equation of second order**

The most general linear partial differential equation of second order can be written as

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x\,\partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = 0$$

i.e., $A\,u_{xx} + B\,u_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = 0$ ... (1)

where A, B, C, D, E, F are in general functions of x and y.

The above equation of second order (linear) (1) is said to

(i) elliptic if $B^2 - 4AC < 0$

(ii) parabolic if $B^2 - 4AC = 0$

(iii) hyperbolic if $B^2 - 4AC > 0$

Examples

	Elliptic Type	Parabolic Type	Hyperbolic Type
1.	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (Laplace Equation in two dimension)	$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$ (One dimensional heat equation)	$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial^2 u}{\partial t^2}$ (One dimensional wave equation)
2.	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ (Poisson's equation)		

2. Bender-Schmidt recurrence scheme is useful to solve one dimensional heat equation.

3. Implicit formula [Crank-Nicolson formula] to solve one dimensional heat flow equation.

$$\frac{1}{2}\lambda u_{i+1,j+1} + \frac{1}{2}\lambda u_{i-1,j+1} - (\lambda + 1) u_{i,j+1}$$
$$= -\frac{1}{2}\lambda u_{i+1,j} - \frac{1}{2}\lambda u_{i-1,j} + (\lambda - 1) u_{i,j}$$

(or) $\lambda (u_{i+1,j+1} + u_{i-1,j+1}) - 2(\lambda + 1) u_{i,j+1}$
$$= 2(\lambda - 1) u_{i,j} - \lambda (u_{i+1,j} + u_{i-1,j})$$

4. Crank-Nickholson's difference formula is used to solve parabolic equations.

5. Explicit scheme formula to solve the wave equation is

$$u_{i,j+1} = 2(1-\lambda^2 a^2) u_{i,j} + \lambda^2 a^2 (u_{i+1,j} + u_{i-1,j}) - u_{i,j-1}$$

6. Diagonal five-point [DFPF] formula to solve the Laplace equation $[u_{xx} + u_{yy} = 0]$ is

$$u_{i,j} = \frac{1}{4}[u_{i-1,j-1} + u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1}]$$

7. Standard five point formula [SFPF] to solve the Laplace equation is

$$u_{i,j} = \frac{1}{4}[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}]$$

8. Four conditions required to solve the Laplace equation.

9. The Laplace equations in difference quotient is

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = 0$$

10. Liebmann's iteration process formula

$$u_{i,j}^{(n+1)} = \frac{1}{4}\left[u_{i-1,j}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j-1}^{(n)} + u_{i,j+1}^{(n+1)}\right]$$

11 The finite difference form of the Possion equation is

$$u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j} = h^2 f(ih, jh)$$

12. Difference scheme for $\nabla^2 u = f(x,y)$ is

$$u_{i-1,j} + u_{i+1,j} + u_{i,j+1} - 4u_{i,j} = h^2 f(ih, jh)$$

13. The general form of Poisson's equation is partial derivatives is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$

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Name	Another name
1. Bisection method	Bolzano's method (or) Interval halving method
2. Regula Falsi method	The method of false position
3. Newton's method	Newton Raphson method (or) Method of tangents
4. Fixed point iteration method	Method of successive approximation
5. Newton-Cotes quadrature formula	Gauss-Lagendra integration formula
6. R.K. Second order	Modified Euler