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PRACTICAL - 1

Topic - Limits and continuity

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+2x} - 2\sqrt{x}} \right]$

$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \right]$

$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$= \frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{2a+a} + \sqrt{3a}}$

$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$$= \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

2.) $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$$

$$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$$

$$\lim_{y \to 0} = \frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$$

$$= \frac{1}{\sqrt{a}(2\sqrt{a})} = \frac{1}{2a}$$

3) $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

By substituting $x - \pi/6 = h$

$x = h + \pi/6$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/6) - \sqrt{3}\sin(h + \pi/6)}{\pi - 6(h + \pi/6)}$$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/6 - \sinh \sin \pi/6 - \sqrt{3}\,(\sinh \cos \pi/6 + \cosh \sin \pi/6)}{\cancel{\pi - 6h + \pi}\ \pi - 6\left(\frac{6h + \pi}{6}\right)}$$

$$\lim_{h \to 0} \frac{\cosh \cdot \frac{\sqrt{3}}{2} - \sinh \frac{1}{2} - \sqrt{3}\left(\sinh \frac{\sqrt{3}}{2} + \cosh \cdot \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h \to 0} \frac{\left(\cosh \frac{\sqrt{3}}{2} + \frac{\sinh}{2}\right) - \sqrt{3}\left(\frac{1}{2}\cosh - \frac{\sqrt{3}}{2}\sinh\right)}{6h}$$

$$\lim_{h \to 0} \frac{\cancel{\frac{\sqrt{3}}{2}\cosh} + \frac{\sinh}{2} - \frac{\sqrt{3}\cosh}{2} + \frac{3\sinh}{2}}{6h}$$

$$\lim_{h \to 0} \frac{4\sinh}{2(6h)}$$

$$\lim_{h \to 0} \frac{4\sinh}{12h}$$

$$= 1/3$$

4) $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

By rationalizing numerator & denominator both.

$$\lim_{\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{\infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{\infty} \frac{8}{2} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{\infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit we get

$= 4$

5) i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$ & at $x=\pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$

$$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \quad \therefore f(\pi/2) = 0$$

f at $x = \pi/2$ defined

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

by substituting method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\pi/2\right)}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$= 1/2$$

b) $\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to \pi/2^-} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$$

$$\lim_{x\to \pi/2^-} \frac{2 \cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to \pi/2^-} \cos x.$$

LHL $\neq$ RHL

$\therefore$ f is not continuous at $x = \pi/2$

5) ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \le x \le 6$
$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$
at $x=3$ and $x=6$

at ~~3~~ $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ defined

ii) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is defined at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3}$ $\quad \frac{(x-3)(x+3)}{x-3}$

$\therefore$ LHL = RHL

f is continous at $x=3$.

2) $\lim_{x \to 6^-} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^-} \frac{(x-3)(x+3)}{x+3}$

$\lim_{x \to 6^-} (x-3) = 6-3 = 3$

$\lim_{x \to 6^+} x+3 = 3+6 = 9$

LHL $\neq$ RHL

Function is not continuous

Q6)

i) $f(x) = \frac{1-\cos 4x}{x^2}, \; x \neq 0$
$\quad = k, \; x = 0$ } at $x=0$

f is continuous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$k = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}, \; x \neq 0$
$\quad = k, \; x = 0$ } $x=0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x \to 0} (1+\tan^2 x)^{1/\tan^2 x}$

We know that

$\lim_{x \to 0} (1+px)^{1/px} = e$

$\therefore e$

$\therefore k = e$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}, \; x \neq \pi/3$
$\quad = k, \; x = \pi/3$ } at $x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

Where $h \to 0$

$$f\left(\frac{\pi}{3}+h\right) = \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}+h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h\to0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}+h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h\to0} \frac{\dfrac{\sqrt{3} - \tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{\pi - \pi - 3h}$$

$$\lim_{h\to0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3}\tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3}\tan h}}{-3h}$$

$$\lim_{h\to0} \frac{\dfrac{\left(\sqrt{3} - \sqrt{3}\times\sqrt{3}\tan h\right) - \left(\sqrt{3} + \tan h\right)}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{-3h}$$

$$\lim_{h\to0} \frac{\dfrac{\left(\sqrt{3} - 3\cdot\tan h\right) - \left(\sqrt{3} + \tan h\right)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$= \lim_{h\to0} \frac{-4\tan h}{-3h\left(1 - \sqrt{3}\tan h\right)}$$

$$= \lim_{h\to0} \frac{4\tan h}{3h\left(1 - \sqrt{3}\tan h\right)}$$

$$= \frac{4}{3} \lim_{h\to0} \frac{\tan h}{h} \cdot \lim_{h\to0} \frac{1}{\left(1 - \sqrt{3}\tan h\right)}$$

$$= \frac{4}{3}\, \frac{1}{\left(1 - \sqrt{3}(0)\right)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

7) i) $f(x) = \begin{cases} \dfrac{1 - \cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

$$f(x) = \frac{1 - \cos 3x}{x\tan x}$$

$$\lim_{x\to0} \frac{2\sin^2 3/2x}{x\tan x}$$

$$\lim_{x\to0} \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2}\times x^2}{x\cdot\dfrac{\tan x}{x^2}\times x^2}$$

$$= 2\lim_{x\to0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= 2\times\frac{9}{4}$$

$$= \frac{9}{2}$$

$\lim_{x\to 0} f(x) = 9/2 \quad 9 \neq f(0)$

$\therefore f$ is not continous at $x=0$

Redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x \tan x} & x \neq 0 \\ 9/2 & x = 0 \end{cases}$$

now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

7) ii) $$f(x) = \begin{cases} \frac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases} \text{ at } x=0$$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3 \cdot e^{3x}-1}{\cancel{3x}} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$3 \log e \, \pi/180 = \pi/60 = f(0)$

f is continous at $x=0$

8) $f(x) = \frac{e^{x^2}-\cos x}{x^2}$; $x=0$

is continous at $x=0$

given

f is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + 2 \lim_{x\to 0} \left(\frac{\sin^2 x/2}{x}\right)^2$$

multiply with 2 in numerator and denominator

$= 1 + 2 \times 1/4 = 3/2 = f(0)$

9] $f(x) = \sqrt{2} - \sqrt{1+\sin x}$, $x \neq \pi/2$

$f(0)$ is continous at $x = \pi/2$

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$\lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{1-\sin^2 x\left(\sqrt{2}+\sqrt{1-\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1}{(1-\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$\lim_{x\to\pi/2} \frac{1}{(1-\sin x)\left(\sqrt{x}+\sqrt{1+\sin x}\right)}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$\therefore f(\pi/2) = 1/4\sqrt{2}$$

AK 5/12/19

PRACTICAL - 2

1) Show that the following function, defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

i) $\cot x$ ii) $\operatorname{cosec} x$ iii) $\sec x$.

~~2) Show that the following function defines from $\mathbb{R}$ to $\mathbb{R}$ are diff~~

2) If $f(x) = 4x+1, \quad x \le 2$
$= x^2+5, \quad x \geq 0$, of $x=2$, then finds its f is differentiable or not?

3) If $f(x) = 4x+7, \quad x < 3$
$= x^2+3x+1, \quad x \geq 3$ at $x=3$. then find its f is differentiable or not?

4) If $f(x) = 8x-5, \quad x \le 2$
$= 3x^2-4x+7, \quad x \geq 2$ at $x=2$ then find f is differentiable or not?

Solution

1) $\cos x$

$$f(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$$

put $x-a=h$

$x = a+h$

$x\to a \quad h\to a$

$$f(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \cancel{\lim_{h\to 0}} \frac{\tan a - \tan(a+h)}{h\cdot \tan(a+h)\tan a}$$

Formula $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1+\tan A\cdot \tan B)$

$$= \lim_{h\to x} \cdot \frac{\tan(a-a-h) - (1+\tan A) + \tan(a+h)}{h\cdot \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1-\tan a \; k\tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$= -\csc^2 a$

$= f(a) = -\csc^2 a$

$= f$ is differentiable $a \in R$.

ii) $\csc x$

$f(x) = \csc x$

$$f(a) = \cancel{\lim_{x\to a}} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\csc x - \csc a}{x-a}.$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}.$$

$$\lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin a \cdot \sin x}$$

put $x-a=h$

$x=a+h$

as $x \to a$, $h \to 0$

$$\therefore f(a) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(\cancel{a}+h-\cancel{a})\sin a \cdot \sin(a+h)}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)$

$$\lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \sin a \cdot \sin(a+h)}$$

$$\lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{\cos a}{\sin^2 a} = -\cot a \cdot \operatorname{cosec} a.$$

iii) $\sec x$

$f(x) = \sec x$

$$f(a) = \lim_{h\to 0} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x-a=h$

$x=a+h$

$x \to a \cdot h \to 0$

$$\text{Df}(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

formula: ~~$2\sin\left(\frac{a+a+h}{2}\right)$~~ $2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to a} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a+h}{a}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h\to a} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin h/2}{\cos a \cdot \cos(a+h) \times -h/2} \times -1/2$$

$$= 1/2 \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+h)}$$

$$= 1/2 \times \frac{-2\sin a}{\cos a \times \cos a}$$

$= \tan a \sec a$

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~~Q.2 If f(x)~~

2) LHD

$$f(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{x-2} = 4$$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$= 2+2$

$= 4$

RHD = LHD

f is differentiable at $x=2$

3) If $f(x) = 4x+7 \quad x<3$

$\quad = x^2+3x+1 \quad x\geq 3$ $\quad x=3$ then,

find f is differentiable or not ?

RHD

$$f(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+3-(3^2+3+3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+13x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-8x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$= 9$

LHD

$$f(3^-) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$f(3^-) = 4.$

$\therefore$ RHD $\neq$ LHD

f is not differentiable at $x=3$

Q4 RHD ~~fix~~

$$f(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= \cancel{3x}\ 3\times 2+2 = 8$

$f(2^+) = 8$

LHD =

$$f(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 8$

$Df(2^-) = 8$

LHD = RHD

f is differentiable at $x=3$

AV 19/12/19

PRATICAL -3,

AIM: Application of Derivatives

1] Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x + 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv) $f(x) = x^3 - 27x + 5$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2] Find the intervals in which function is concave upwards

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv) $y = 2x^3 + x^2 - 20x + 4$

v) $y = 69 - 24x - 9x^2 + 2x^3$

Solution:

Q1.)

1) $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

f is Increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5}/3 \quad \sqrt{5}/3$

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$

and f is decreasing iff $f(x) < 0$

$3x^2 - 5 < 0$

$3(x^2 - \sqrt{5}/3) < 0$

$(x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$

$+ \quad - \quad +$

$-\sqrt{5}/3 \quad \sqrt{5}/3$

$x \in (-\sqrt{5}/3, \sqrt{5}/3)$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0 \qquad x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$-2 \quad 5/3$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) \quad 3x^2 - 27$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

f is increasing iff $f'(x) > 0$

$6x^2 - 18 - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 - 18 - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x+4) < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$

$-1 \quad 4$

$\therefore x \in (-1, 4)$

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3) $f(x) = 2x^3 + x^2 - 20x + 4$

∴ $f'(x) = 6x^2 + 2x - 20$

∴ f is increasing iff $f'(x) > 0$

∴ $6x^2 + 2x - 20 > 0$

∴ $2(3x^2 + x - 10) > 0$

∴ $3x^2 + x - 10 > 0$

∴ $3x^2 + 6x - 5x - 10 > 0$

∴ $3x(x+2) - 5(x+2) > 0$

∴ $(x+2)(3x-5) > 0$

$+ \quad - \quad +$

$-2 \qquad 5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$-2 \qquad 5/3$

$+ \quad - \quad +$

$-2 \qquad 5/3$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

∴ f is increasing iff $f'(x) > 0$

∴ $3(x^2 - 9) > 0$

∴ $(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \qquad 3$

∴ $x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

∴ $3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

f is increasing iff $f'(x) > 0$

$6x^2 - 18 - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \qquad 4$

∴ $x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 - 18 - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x+4) < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$

$-1 \qquad 9$

∴ $x \in (-1, 4)$

Q2)

1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$\therefore x - 1/2 > 0$

$\therefore x > 1/2$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 9x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(2) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$.

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2 \quad \therefore x \in (3/2, \infty)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6$

$\because f''(x) \not> 0$

$\therefore$ There exist interval $(-1/6, \infty)$

PRACTICAL – 4.

AIM: Application of derivatives & newton's method

Q1) Find maximum & minimum value of following.

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2) Find the root of the following equation by newton's (Take 4 iteration only correct upto 4 decimal)

i) $f(x) = x^3 - 3x^2 - 55x + 95$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$

Solution

Q1)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4$

$= 8$

$\therefore$ $f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has maximum value at $x = 2$

Function reaches minimum value at $x = 2$, and $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$

$\therefore$ $15x^2 + 15x^4 = 0$

$\therefore$ $15x^4 = 15x^2$

$\therefore$ $x^2 = 1$

$\therefore$ $x = \pm 1$

$\therefore$ $f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has maximum

$f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5 + 1$

$\therefore f''(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$= 3+5-3 = 5$

$\therefore$ f has maximum value 5 at $x=-1$ and has minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$3x = 0$ or $x-2=0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x-6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0 \therefore$ f has maximum value at $x=0$

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$f''(2) = 6(2) - 6$
$= 12-6$
$= 6 > 0$

$\therefore$ f has minimum value at $x=2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$
$= 8 - 3(4) + 1$
$= 9 - 12$
$= -3$

$\therefore$ f has maximum value 1 at $x=0$ and f has minimum value -3 at $x=2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$
$= 24 - 6$
$= 18 > 0$

$\therefore$ f has minimum value at $x = +2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$
$= 2(8) - 3(4) - 24 + 1$
$= 16 - 12 - 24 + 1$
$= -19$

$\therefore f''(-1) = 12(-1) - 6$
$= -12 - 6$
$= -18 < 0$

$\therefore$ f has maximum value at $x=-1$

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$
$= -2 - 3 + 12 + 1$
$= 8$

$\therefore$ f has maximum value 8 at $x=-1$ and

$\therefore$ f has minimum value -19 at $x = 2$

Q2

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$, $x_0 = 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1725) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(\cancel{60}.0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829 / 55.9467$

$= \cancel{0.1727}\ 0.1712$

$\therefore f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2) / f'(x_2)$

$= \cancel{0.1712}\ 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

$\therefore$ The root of the eqn is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(13) = 33 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

let $x_0 = 3$ be the initial approximation by

$\therefore$ by newton's method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 6.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8886 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$$2.7071 - \frac{0.0102}{17.9851}$$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7)58 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 2.18943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= 2.7065$

3) $f(x) = x^3 - 18x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(2)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17 = -2.2$

let $x_0 = 2$ be initial approximation by Newton's method.

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$f(x_2) = (1.6592)^3 - 18(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.6592 / 0.0204 / 7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.8618 + \frac{0.0004}{7.6977}$

$= \underline{1.6618}$

PRACTICAL - 5

Aim Integration

a) Solve the following integration

i) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x+1})\,dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

iv) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^7 \sin(2t^4)\,dx$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii) $\int \dfrac{\cos x}{3\sqrt{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x} \sin 2x\,dx$

x) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right) dx$

i) $\int \dfrac{1}{x^2+2x-3}\,dx$

$= \int \dfrac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \dfrac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \dfrac{1}{\sqrt{(x+1)^2-4}}\,dx$

Substitute put $x+1=t$

$dx = 1/t \times dt$ where $t = 1$, $t = x+1$

$\int \dfrac{1}{\sqrt{t^2-4}}\,dt$

using

$\int \dfrac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(|x+\sqrt{x^2-a^2}|\right)$

$= \ln\left(|t+\sqrt{t^2-4}|\right)$

$= \ln\left(|x+1\sqrt{(x+1)^2-4}|\right)$

$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)$

$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)+C$

2) $\int (4e^{3x+1})\, dx$

$= \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + \int 1\, dx$ # $\int e^{dx}\, dx = 1/x * e^{dx}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \cancel{\frac{3x}{x^{1/2}}} + 4/x^{1/2}\, dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\, dx$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int 4/x^{1/2}\, dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3}\, du$

$= \int t^4 \sin(2t^4) \times 1/2\times4\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du = \frac{t^4 \times \sin(2t^4)}{8}\, du$

substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8}\, du$

$= \int \cancel{\frac{u \times \sin(u)}{2}} / 8\, du$

$= \int \frac{u \times \sin(u)}{16}\, du$

$= 1/16 \int u \times \sin(u)\, du$

2) $\int (4e^{3x+1})\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$ # $\int e^{dx}\,dx = 1/x \times e^{dx}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + C$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + 4/x^{1/2}\,dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int 4/x^{1/2}\,dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\,dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$

$\int t^4 \sin(2t^4) \times 1/2\times 4\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du = \frac{t^4 \times \sin(2t^4)}{8}\,du$

substitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8}\,du$

$= \int \frac{u \times \sin(u)}{2} / 8\,du$

$= \int \frac{u \times \sin(u)}{16}\,du$

$= 1/16 \int u \times \sin(u)\,du$

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$\int u\,dv = uv - \int v\,du$

where $u = 4$

$dv = \sin(4) \times du$

$du = 1\,du \quad V = -\cos(4)$

$= \frac{1}{16}(u \times (-\cos(u)) - \int -\cos(u)\,dx$

$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\,du$

$\int \cos x\,dx = \sin(x)$

$= \frac{1}{16} \times (4x(-\cos(4)) + \sin(v))$

Return the substitution $4 = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

vi) $\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2}\,\cancel{x^2} - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/5}\,dx$

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$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C$

viii) $\int \frac{\cos x}{3\sqrt{\sin(x)^2}}\,dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\,dt$

$= \frac{1}{\sin x^{3/2}}\,dt$

$= \frac{1}{t^{2/3}}\,dt$

$I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{(2/3^{-1})t^{2/3-1}} = \frac{-1}{-1/3\,t^{2/3-1}}$

$= \frac{1}{1/3\,t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

Return substitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1} dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times2x} dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)} dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} dt$

$= \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$

$= \frac{1}{3}\int \frac{1}{t} dt \qquad \int \frac{1}{x} dx = \ln(|x|)$

$= \frac{1}{3} \times \ln|t| + C$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + C$

(xi) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$.

$I = \int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$.

$= \frac{\cos x}{\sin x^{2/3}} dx$.

put $t = \sin x$, $dt = \cos x \, dx$

$= \frac{1}{(t)^{2/3}} dt$

$= t^{-2/3} dt$

$\frac{t^{-2/3+1}}{-2/3+1}$

$= \frac{t^{1/2}}{1/2}$

$= 2\sqrt{t} + C$

Resubstituting $t = \sin x$.

$= \underline{3\sqrt{\sin x} + C}$

AV 02/01/2020

PRACTICAL-6.

AIM: Application of integration & Numerical integrations.

Q.1 Find the length of the following curve.

1. $x = t\sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$
2) $y = \sqrt{4-x^2}$ $x \in [-2, 2]$
3. $y = x^{3/2}$ in $[0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$
5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

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Q.2 Using Simpson's rule solve the following

1) $\int_0^2 e^{x^2} \, dx$ with $n = 4$

2) $\int_0^4 x^2 \, dx$ with $n = 4$

iii) $\int_0^{\pi/3} \sqrt{\sin x} \, \cancel{dx}$ sin $n = 6$

Q.1

2) $y = \sqrt{4-x^2}$ $x \in [2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$$

$$= \frac{-2x}{\sqrt{4-x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2} \, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}} \, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}} \, dx$$

$$= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}} \, dx$$

$$= \cancel{2}\int_{-2}^{2} \frac{1}{\sqrt{(2)^2 - (x)^2}} \, dx$$

$$= 2 \left[\sin^{-1}\left(x/2\right)\right]_{-2}^{2}$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= 2\left[\pi/2 - (-\pi/2)\right]$$

$$= 2\left[\pi/2 + \pi/2\right]$$

$$= 2\pi$$

3) $y = x^{3/2}$ in $[0,4]$

$\frac{dy}{dx} = \frac{3}{2} \cdot x^{3/2-1}$

$l = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4 \sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2} dx$

$= \int_0^4 \sqrt{\left(1+\frac{9x}{4}\right)}\, dx$

$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$

$= \frac{1}{2} \int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{27} (4+0)^{3/2} - (4+36)^{3/2}]$

$= 1/27 \left[(4)^{3/2} - (40)^{3/2}\right]$

$= \frac{1}{27}\left[4 - \cancel{40}\right]^{3/2}$

$= \cancel{\frac{1}{27} \times (3\cancel{6})^{3/2}}$

$\frac{1}{27}\left[(4)^{3/2} - 8\right]$ units.

4) $x = 3\sin t$, $y = 3\cos t$

$\frac{dx}{dy} = 3\cos t$, $\frac{dy}{dx} = -3\sin t$

$= \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9(\sin^2 t + \cos^2 t)}\, dt$

$= \int_0^{2\pi} \sqrt{9(1)}\, dt$

$= \int_0^{2\pi} 3\, dt$

$= 3\int_0^{2\pi} dt$

$= 3[x]_0^{2\pi}$

$= 3(2\pi - 0)$

$= 6\pi$

5.) $\frac{dx}{dy} = \frac{1}{6}\frac{d}{dy}(y^3) + \frac{1}{2}\frac{dx}{dy}(1/y)$

$= \frac{1}{6}3y^2 + \frac{1}{2}(-1/y^2)$

$= \frac{y^2}{2} - 1/2y^2$

$= \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}$

$= \int_1^2 \sqrt{1+\left(\frac{y^4-1}{2y^2}\right)^2}\,dy$

$= \int_1^2 \sqrt{1+\frac{(y^4-1)^2}{4y^4}}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{4y^4}}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\,dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\,dy$

$= \int_1^2 \frac{y^4}{2y^2}\,dy + \int_1^2 1/2y^2\,dy$

$= 1/2 \int_1^2 y\,dy + 1/2 \int \frac{1}{y^2}\,dy$

$= 1/2 \left[\frac{y^3}{3} - y^{-2}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - 1/2 - 1/3 + 1\right]$

$= 1/2\left[\frac{7}{3} + 1/2\right]$

$= 1/2\left[\frac{17}{6}\right]$

$= \frac{17}{12}$

i) Curve length : $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1-2\cos t + \cos^2 t + \sin^2 t + dt} =$$

$$= \sqrt{1-2\cos t + \sin^2 t}$$

$$= \int_0^{2\pi} \sqrt{2-2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\sin \tfrac{1}{2}\, dt$$

$$= \left[- 9\cos \tfrac{1}{2} \right]_0^{2\pi}$$

$$= (-4\cos\pi) + 4\cos 0$$

$$= 8 \text{ units}$$

Q.2

i) $\int_0^2 e^{x^2}\, dx$ with $n = 4$

$\cancel{h = \frac{b-a}{n}}$ $a = 0,\ b = 2;\ n = 4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} = 0.5$$

x	0	0.5	1	1.5	2
	1	1.284	2.718	9.487	54.598

$$\int_0^2 e^{x^2} = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + (x_4) \right]$$

$$= \frac{0.5}{3}\left[(1) + 4(1.284) + 2(2.718) + 4(9.487) + 54.598 \right]$$

$$= \frac{0.5}{3}\left[104.118 \right]$$

$$= \underline{\underline{17.353}}$$

ii)

$$\int_0^4 x^2\,dx \quad \text{with} \quad n=4$$

$a=0$, $b=4$, $n=4$

$$h = \frac{b-a}{n} = \frac{4-0}{4} = \frac{4}{4} = 1$$

x	0	1	2	3	4
	0	1	4	9	16
~~0~~	~~0~~	~~1.3862~~	~~2.1972~~	~~2.772~~	

$$\cancel{\int_0^4 x^2\,dx = \frac{h}{3}\left[(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + x_4\right]}$$

$$\cancel{= \frac{1}{3}\left[f(0) + 4(0) + 2(1.3862) + 4(2.1972) + 2.772\right]}$$

$$\cancel{= \frac{1}{3}(14.3332)}$$

$$\cancel{\approx 4.7773}$$

$$\int_0^4 x^2\,dx = \frac{h}{3}\left[x_0 + 4f(x_1) + 2f(x_2) + 4f(x_3) + x_4\right]$$

$$= \frac{1}{3}\left[0 + 4(1) + 2(4) + 4(9) + 16\right]$$

$$= \frac{1}{3}\left[0 + 4 + 8 + 36 + 16\right]$$

$$= \frac{1}{3}\left[12 + 36 + 16\right]$$

$$= \frac{1}{3}\left[64\right]$$

$$\approx \underline{21.333}$$

iii) $$\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad \text{with } n=6.$$

$$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi/3}{6} = \frac{\pi}{18}$$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$ $(\pi/3)$
y	0	0.4167	0.4385	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{\pi/18}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.9752) + 2(0.5848 + 0.8017)\right]$$

$$= \pi/54 \left[1.3473 + 4(1.999) + 2(1.3865)\right]$$

$$= \pi/54 \left[1.3473 + 7.996 + 2.773\right]$$

$$= \pi/54 \times 12.1163$$

$$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049.$$

AK 09/01/2020

PRACTICAL- 7.

Topic - Differential Equation.

Q.1 Solve the following differential equation

1. $x \frac{dy}{dx} + y = e^x$

Dividing by x

$$\frac{dy}{dx} + y/x = \frac{e^x}{x}$$

Comparing with $\frac{dy}{dx}$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F. = e^{\int p\, dx}$$
$$= e^{\int 1/x\, dx}$$
$$= \int d. e^{\log x} dx = x$$

$$y(I.F.) = \int Q(I.F.)\, x\, dx + C$$
$$y(x) = \int \frac{e^x}{x} \cdot x\, dx + C$$
$$y(x) = \int e^x + C$$
$$y(x) = e^x + C$$

2 $e^x \frac{dy}{dx} + 2y = 1$

Dividing by e^x

$\frac{dy}{dx} + 2y = 1/e^x$

By comparing with

$\frac{dy}{dx} + p(x)y = Q(x)$

$I.f = e\int p\,dx$

$= e\int 2\,dx$

$= e^{2x}$

$y(I.f) = \int Q(I.f) \times dx + C$

$y(e^{2x}) = \int 1/e^x \cdot e^{2x} \times dx + C$

$= \int e^{2x} - e^x dx + C$

$= \int e^x\,dx + c$

$y e^{2x} = e^x + c$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\cancel{\frac{x\,dy}{dx} = \frac{\cos x}{x} - 2y}$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

Comparing with

$\frac{dy}{dx} + p(x)y = Q(x)$

$I.f = e\int p\,dx$

$= e\int 2/x\,dx$

$= e^{2/x}x = \ln x^2 = x^2$

$y(I.f) = \int Q(x)\,(I.f)\,dx + c$

$= \int \frac{\cos x}{x^2} - x^2 dx + c.$

$y e^{2x} = e^x + c.$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

Comparing with

$\frac{dy}{dx} + p(x)y = Q(x)$

$I.f = e\int p\,dx$

$= e\int 2/x\,dx$

$= e^{2/x}x. = \ln x^2 = x^2$

$y(I.f) = \cancel{\int Q(x)}\,(I.f)\,dx + c$

$= \int \frac{\cos x}{x^2} - x^2 dx + c.$

$= \int \cos x + c$

$= x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

Divide by x on bothsides

$p(x) = 3/x \quad Q(x) = \sin x/x^3$

$If = e^{\int p\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3/x\,x}$

$= e^{\ln x^3}$

$If = x^3$

$y(I.f) = \int Q(x)(If)\,dx + c$

$= \int \frac{\sin x}{x^3} x^3\,dx + C$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + C$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} \quad = 2xe^{-2x}$

$I.f = e^{\int p(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(I.f) = \int Q(x)(I.f)\,dx + c$

$= \int 2xe^{-2x}e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c.$

$\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\frac{\sec^2 x\,dx}{\tan x} = -\frac{\sec^2 y\,dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

differentiating on both sides

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v\, dv = \int dx$$
$$\tan v = x + c$$
$$\tan(x+y-1) = x + c$$

e) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+7y+6}$$

put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$\int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = 3x$$

$$v + \log(x) = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c.$$

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Practical- 8

Q1 Using Euler's Method for the following.

① $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ find $y(2)$

$f(x) = y + e^x - 2$, $x_0 = 0$

$y(0) =$

$y(0.2) = ?$

n	x_n	$y(0)$	$f(x_n - y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14787	3.5743
2	1	3.5443	4.2925	5.7205
3	1.5	5.7208	8.2021	9.8215

$y(2) = 9.8215$

② $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

$y_2 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$2(x_n - y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.404	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.411	0.9234
4	0.8	0.9234	1.8526	61.2939
5	1	1.2939		

$y(1) = 1.2937$

3) $\frac{dy}{dx} = \sqrt{xy + 2}$, $y(1) = 1$ find $y(1.2)$ with

$h = 0.2$

$y(0) = 1$ $x(0) = 1$ $n = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1) = 3.6$

ca

(4) $\frac{dy}{dx} = 3x^2+1$, $y(1)=2$, find $y(2)$

for $h=0.5$ & $h=0.25$

$h=0.5$ & $h=0.25$, $y_0=2$, $x_0=1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	2	4
1	1.5	1.5	4	7.875
2	2	2	7.875	

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	~~59.6~~ 122.6482	299.9966
4	2	299.9966		

$y(1) = 299.69966.$

(5) $\frac{dy}{dx} = \sqrt{x/y}$, $y(0)=1$, $h=0.2$, find $y(1)$

$y(0)=1$, $x_0=0$, $n=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2104	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

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$y(1) = 1.6051$

Practical-9

Limit & partial Order Derivative.

① Evaluate the following limits.

i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

Applying limits.

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

Applying limit

$$\frac{(0+1)(2)^2+(0)^2-4(2)}{2+3(0)}$$

$$\frac{1(4-8)}{2} = -4/2$$

$= -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{(x)^2-(yz)^2}{x^2(x-yz)}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{(x+yz)(x-yz)}{x^2(x-yz)}$

$[\because (a)^2 - b^2 = (a+b)(a-b)]$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x}$

$\dfrac{1+(1)(1)}{(1)^2} = 2$

2) Find f_x, f_y, for each of the following

i) $f(x,y) = xy\,e^{x^2} + y^2$

$f(x) = \dfrac{\partial f}{\partial x}$

$= \dfrac{\partial(xye^{x^2}+y^2)}{\partial x}$

$= y\,\dfrac{\partial(x.e^{x^2}+y^2)}{\partial x}$

$= y\left[x.\dfrac{d}{dy}(e^{x^2}+y^2 + e^{x}+y\cdot\dfrac{d}{dx}(x)\right]$

$\left[\because \frac{d}{dx}(uv) = u.v' + v.u'\right]$

$= y\left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1)\right]$

$= y.e^{x^2+y^2}[2x+1]$

Now $f(y) = \frac{\partial f}{\partial y}$

$= \frac{\partial(xy e^{x^2+y^2})}{\partial y}$

$= x.\frac{\partial}{\partial y}(y.e^{x^2+y^2})$

$= x\left[y.\frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}.\frac{d}{dy}(y)\right]$

$\left[\frac{d}{dx}(uv) = u.v' + v.u'\right]$

$= x.\left[2y^2.e^{x^2+y^2} + e^{x^2+y^2}\right]$

$= x.e^{x^2+y^2}[2y^2+1]$

ii) $f(x,y) = e^x \cos y$

$f(x) = e^x.\cos y$

$f(y) = e^x . \frac{1}{\partial y}(\cos y)$

$= e^x(-\sin y)$

$= e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{f}{dx} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$= x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

3) Using defination find values of $fx \cdot fy$ at $(0,0)$

for $f(x,y) = \frac{2x}{1+y^2}$

~~$f(x,y) = (a,b)$~~

~~$f(a,b) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$~~

$fx(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

where $(a,b) = (0,0)$

$fx(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

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$$\lim_{h\to 0} \frac{2h-0}{2} = 2$$

Similarly,

$$f_y(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$f_x = 2$, $f_y = 0$

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{2h-0}{h} = 2$$

Q4.) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$\because f(x) = \dfrac{\partial f}{\partial x} = \dfrac{\partial (y^2 - xy)}{x^2}$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\cdot \frac{d}{dx}(x)^2}{(x^2)^2}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}\right]$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$$

$$= f(x) = \frac{xy - xy^2}{x^3}$$

$$= f(y) = \frac{\partial f}{\partial y} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \frac{\partial \left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$$

$$= \partial\left(\frac{y^2}{x^2} - y/x\right) / \partial y$$

$$= 1/x^2 \; 2y - 1/x$$

$$f(y) = \frac{2y - x}{x^2}$$

$$f(x,x) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy - 2y^2)(3x^2)}{6}$$

$$= \frac{x^3 y - 3x^3 y + 6x^2 y^2}{6}$$

$$= \frac{6x^2 y^2 - 2x^3 y}{x^6} = \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$f(y,y) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial y}$$

$$= \frac{\frac{1}{x^2}\partial(2y - x)}{\partial y} = 1/x\,(2) = 2/x^2$$

$$f(x,y) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(y/x^2 - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= 1/x^2 - 1/x^3 \; 2\,(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4yx^2}{x^6}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - 1/x\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + 1/x^2$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$\therefore f(xy) = f(yx) = \frac{x - 4y}{x^4}$$

Hence verified

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$f(x) = \frac{f}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$

$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$

$\therefore f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial y}$

$= 0 + 3(2y)(x^2) + 0$

$f(y) = 6x^2y$

$f(x,x) = \frac{\partial}{\partial x} f_x = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$

$\left[\frac{d}{dx} \frac{u}{v} = \frac{v.u' - u.v'}{v^2}\right]$

$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$

$f(y,y) = \frac{\partial f_y}{\partial y} = \frac{\partial(6x^2 y)}{6\,\partial y}$

$= 6x^2(1) = 6x^2$

$f(x,y) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$

$= 0 + 6x(2y)$

$= 12xy$

$f(y\,x) = \frac{\partial f_y}{\partial x}$

$= \frac{\partial(6x^2y)}{\partial x}$

$= f(xy) = f(yx) = 12xy$

Hence verified

iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(x\,x) = \frac{\partial f_x}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y \cdot \cos xy\,(y) + e^{x+y}(1)$

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$= y^2 \cos xy + e^{x+y}$

$f(y,y) = \frac{\partial fy}{\partial y} = \frac{\partial (x\cos xy + e^{x+y})}{\partial y}$

$= x \cos xy \, (x) + e^{x+y}(1)$

$= x^2 \cos xy + e^{x+y}$

$f(x,y) = \frac{\partial fx}{\partial y} = \frac{\partial (y\cos xy + e^{x+y})}{\partial y}$

$= y[-\sin(xy)(x) + \cos xy (1)] + e^{x+y}(1)$

$[\because \frac{d}{dx}(uv) = u \cdot v' + v \cdot u']$

$= -xy \sin(xy) + \cos xy + e^{x+y}$

$f(y,x) = \frac{\partial fy}{\partial x} = \frac{\partial (x\cos xy + e^{x+y})}{\partial x}$

$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$

$= -xy \sin(xy) + \cos(xy) + e^{x+y}$

$f(xy) = f(yx) = -xy \sin(xy) + \cos(xy) + e^{x+y}$

Q5 Find the linearization of $f(x,y)$ at given point.

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$.

$f(1,1) = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$fx(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$fy(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = 1/\sqrt{2}$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= \sqrt{2} + 1/\sqrt{2}(x-1) + 1/\sqrt{2}(y-1)$

$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$

$= \frac{2 + x + y - 2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y \sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0(\sin(\pi/2))$

$= 1 - \pi/2$

$f(x) = -1 + y \cos x$

$f(y) = 1$

$f\{x\}(\pi/2, 0) = -1 + 0 \cos \pi/2$

$= -1$

$fy(\pi/2, 0) = 1$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + y(a,b)(y-b)$

$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$

$$\text{or} \quad = 1 - \pi/2 - x + \pi/2 + y$$
$$= y - x + 1$$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1)$
$= 0 + 0$
$= 0$

$f(x) = 1/x \qquad f(y) = 1/y$

$fx(1,1) = 1 \qquad f(1,1) = 1$

$f(x,y) = f(a,b) + f(a,b)(x-a) + fy(a-b)(y-b)$
$= 0 + 1(x-1) + 1(y-1)$
$= x - 1 + y - 1$
$= x + y - 2$

PRACTICAL - 10

Q.1) Find the directional derivative of the following func^n at the given points of the direction of given vector

1) $f(x,y) = x + 2y - 3 \qquad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = 1/\sqrt{10} \cdot (3,-1)$
$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = 1/\sqrt{10}$

ii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4)$ $\quad u = i + 5j$

Here $u = i + 5j$ is not unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, 5/\sqrt{26}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right) / \cancel{h}$

$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x + 3y$ $\quad a(1,2)$, $u(3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = 1/5\,(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, 4/5\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$

$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(\frac{2 + 4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h}$

$= \frac{18}{5}$

Q.2 Find the gradient vector for the following func^n

i). $f(x,y) = x^y + y^x$, $a(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$f(x,y) = f_x, f_y$

$= (y x^{y-1} + y^x \log y,\ x^y \log x + x y^{x-1})$

$f(1,1) = (1+0,\ 1+0)$
$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x) \cdot y^2$ $\quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1}x$

$f(x,y) = (f_x, f_y)$
$= \left(\frac{y^2}{1+x}, 2y\tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(2)\right)$
$= \left(\frac{1}{2},\ \frac{\pi}{4}\frac{(-2)}{2}\right)$
$= \left(\frac{1}{2},\ -\frac{\pi}{2}\right)$

$f(x,y,z) = xyz - e^{x+y+z}$; $a(1,-1,0)$
$f_x = yz - e^{x+y+z}$
$f_y = xz - e^{x+y+z}$
$f_z = xy - e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$
$= yz - e^{x+y+z},\ xz - e^{x+y+z},\ xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{1+(-1)+0},\ (1)(0) - e^{1+(-1)+0},\ (1)(-1) - e^{1+(-1)+0}\right)$
$= (0-e^0,\ 0-e^0,\ -1-e^0)$
$= (-1,\ -1,\ -2)$

Q.3 Find the equation of tangent & normal to each of the following using curves at given point.

i) $x^2\cos y + e^{2y} = 2$ $\quad$ at $(1,0)$
$f_x = \cos y\ 2x + e^{xy}y$
$f_y = x^2(-\sin y) + e^{xy} \cdot y$

$(x_0, y_0) = (1,0)$ $\quad \therefore x_0 = 1 \quad \therefore y_0 = 0$

eqn of tangent
$f_x(x-x_0) + f_y(y-y_0) = 0$
$f_x(x_0,y_0) = \cos 0\ 2(1) + e^0\ 0$
$= 1(2) + 0$
$= 2$

$f_y(x_0,y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$
$= 0 + 1 \cdot 1$
$= 1$

$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$ $\quad \rightarrow$ It is required eqn of tangent.

eqn of normal
$ax + by + c = 0$
$= bx + ay + d = 0$
$(1)(1) + 2(y) + d = 0$
$1 + 2(y) + d = 0$ $\quad$ at $(1,0)$
$1 + 2(0) + d = 0$
$d + 1 = 0$
$\therefore d = -1$

i) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$2x+0-2+0+0$

$f_x = \cancel{2x+0-2+0}$
$= 2x-2$

$f_y = 0+2y-0+3+0$
$= 2y+3$

$(x_0,y_0) = (2,-2)$ $\therefore x_0=2$; $y_0=-2$

$f_x(x_0,y_0) = 2(2)-2=2$

$f_y(x_0,y_0) = 2(-2)+3=-1$

eqn of tangent

$f_x(x-x_0)+f_y(y-y_0)=0$

$2(x-2)+(-1(y+2))=0$

$2x-2-y-2=0$

$2x-y-4=0$ → It is required eqn of tangent

eqn of normal

$= ax+by+c=0$

$bx+ay+d=0$

(-1)

$-1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$-2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore d=6$

Q.4 Find the eqn of tangent & normal line to each of following surface.

i) $x^2-2yx+3y+xz=7$ at $(2,1,0)$

$f_x = 2x-0+0+z$

$f_x = 2x-2$

$f_y = 0-2z+3+0$
$= 2z+3$

$f_z = 0-2y+0+x$
$= -2y+x$

$(x_0,y_0,z_0) = (2,1,0)$ $\therefore$ $x_0=2$, $y_0=1$; $z_0=0$

$f_x(x_0,y_0,z_0) = 2(2)+0=4$

$f_y(x_0,y_0,z_0) = 2(0)+3=3$

$f_z(x_0,y_0,z_0) = -2(1)+2=0$

eqn of tangent

$f_x(x_0-x_0)+f_y(y_0-y_0)+f_z(z_0-z_0)=0$

$= 4(x-2)+3(y-1)+0(z-0)=0$

$= 4x-8+3y-3=0$

$4x+3y-11=0$ → This is required eqn of tangent.

Eqn of normal at $(4,3,-11)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}\ 4$$

ii) $3xyz-x-y+z=-4$ at $(1,-1,2)$

$3xyz-x-y+z+4=0$ at $(1,-1,2)$

$f_x = 3yz-1-0+0+0$
$= 3yz-1$

$f_y = 3xz-0-1+0+0$
$= 3xz-1$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2) \therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqⁿ of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ → This is required eqn of tangent

Eqn of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q.5) Find the local maxima & minima for the following.

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6xy - 4y$

$f_x = 6x + 0 - 3y + 6 = 0$
$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$2x - y = -2 \rightarrow ①$

$f_y = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4 \quad \leftarrow ②$

multiply eqn ① with ②

$4x - \cancel{2y} = -4$
$\cancel{2y} - 3x = 4$
$x = 0$

substitute value of x in eqn ①

$2(0) - y = -2$
$\cancel{-}y = \cancel{-}2 \qquad y = 2$

Critical points are $(0, 2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$12 - 9$
$3 > 0$

f has maximum at ~~(2, 0)~~ $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8$
$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply eqn (1) with 3

(2) with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

substitute value of y in eqn (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical pts is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (s)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= 0$

(nothing to say)

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = -2/2 = -1$

$f_y = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} = 4 \quad \therefore y = 4$

Critical pt is $(-1,4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= \cancel{37}\ 47 - 70$

$= \cancel{-43}\ -33$

06/02/2020