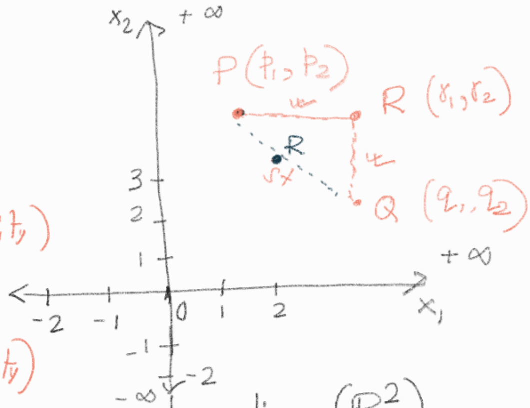
Oct 12 (First class)

- Metric Spaces

dist b/w P, Q

$$\sqrt{(p_1-q_1)^2+(p_2-q_2)^2}$$

- dist ≥ 0 (non-negativity)
- dist $= 0$
  $\iff P = Q$ (separability)
- $\text{dist}(P,R) + \text{dist}(R,Q) \geq \text{dist}(P,Q)$
  (triangle inequality)



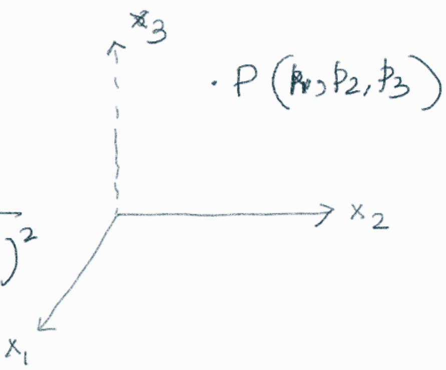
two-dim ($\mathbb{R}^2$)

$\mathbb{R}$: the real line

Qn: $\text{dist}(P,R) + \text{dist}(R,Q) = \text{dist}(P,Q)$?

Ans: R sits between P and Q.

Ex: $\mathbb{R}^n = \underbrace{\mathbb{R}\times\mathbb{R}\times\cdots\times\mathbb{R}}_{n \text{ copies}}$



$P = (p_1, p_2, \ldots, p_n)$

$Q = (q_1, q_2, \ldots, q_n)$

$$\|P-Q\|_2 = \sqrt{(p_1-q_1)^2+\cdots+(p_n-q_n)^2} = \sqrt{\sum_{i=1}^{n}(p_i-q_i)^2}$$

Euclidean dist   subscript

Qn: Show that $\| \quad \|_2$ has all the above properties.