3 Mathematically derive the average runtime Complexity of the non-random pivot version of quicksort.

Sol°-

$$T(n) = n + \frac{1}{n} \sum_{i=0}^{n-1} (T(i) + T(n-i-1))$$

$T(n)$ = Average time Complexity for array size n

Solve the above relation,

$$T(n) = n + \frac{2}{n} \sum_{i=0}^{n-1} T(i)$$

divide $T(i)$ in terms of n and i

$$T(n) = n + \frac{2}{n} \sum_{i=0}^{n-1} (i + T(i))$$

$$T(n) = n + \frac{2}{n} \sum_{i=0}^{n-1} i + \frac{2}{n} \sum_{i=0}^{n-1} T(i)$$

$$T(n) = n + \frac{2}{n} \times \frac{n(n-1)}{2} + \frac{2}{n} \sum_{i=0}^{n-1} T(i)$$

$$T(n) = n + n - 1 + \frac{2}{n} \sum_{i=0}^{n-1} T(i)$$

$$T(n) = 2n - 1 + \frac{2}{n} \sum_{i=0}^{n-1} T(i)$$

Hence the average time Complexity for the above expression is $O(n \log n)$.