

1. Given a dynamic table (see section 17.4) that doubles in size when it needs more space. Find the amortized runtime for inserting n elements.

a) use the aggregate method.

Sol:- Here by using the aggregate method we can calculate total cost performing n insertions divide it with the number of insertions so that we can get the average cost per insertion.

Now assume the initial size for the table is 1.

Then the 2nd insertion cost is 2.

And the 3rd insertion cost is 1.

And the 4th insertion cost is 4.

And the 5th insertion cost is 1.

And the 6th insertion cost is 8.

Hence the Total cost calculated as
 $1 + 2 + 1 + 4 + 1 + 8 + 1 + \dots + 2^{(\log(n)-1)}.$

so here $2^{(\log(n)-1)}$ is represented for cost of resizing the table.

Now, by dividing total by n to get amortized cost per insertion.

$$O(\log n) - 1 = \frac{(1+2+4+1+8)}{2} \dots \text{is the formula.}$$

b) Use the Accounting method -

So here in the accounting method for every insertion action we will use credit or debit method. So for amortized cost it contains the additional credit or debit cost and the operation cost. The table is double in size, each element is given credit 2, and each insertion operation costs 2. At first the table is 1.

So every insertion process will cost 2, then the n insertions will cost $2n$ at last. So for resizing the table we need to multiply by $\log_2(n)$, it doubles with each of $\log_2(n)$ increases the table size. So at the end the credit of $2n$ will be applied to the each element in the table, which results $2+4+8+8+2+2^k$. So the total credit for the each insertion is $2n-2=2n-2$.

So for the Accounting method the amortized cost for the each insertion in the table is $O(1)$.