2. (10 points) Use a Monte Carlo approach estimate the value of π based on 10,000 replications. [Ignorable hint: First, get a relation between π and the probability that a randomly selected point in a unit square with coordinates (0, 0), (0, 1), (1, 0) and (1, 1) falls in a circle with centre (0.5, 0.5) inscribed in the square. Then, estimate this probability, and go from there.]

Answer: We can consider a circle of radius 0.5, centred at (0.5, 0.5) inscribed in the unit square with coordinates (0, 0), (0, 1), (1, 0) and (1, 1). The Area of the square is 1 and the area of the circle will be πr2 which will be 0.25π or π/4.

Π = 4\*

We have generated random points that are uniformly distributed within the given area. We can check the number of points that lie within the circle and the total number of points. The ratio of the number of points within the circle and within the square will be approximately equal to the ratio of the areas of the circle and the square.

Π = 4\*

With a large number of points, we can have an estimate that is closer to the actual value of Pi.

R-Code

1. > x=runif(10000) #randomly generates 10000 X-coordinates
2. > y=runif(10000) #randomly generates 10000 X-coordinates
3. > distance=sqrt((x-0.5)^2+(y-0.5)^2) #generates the distance of point (x,y) from centre(0.5,0.5)
4. > ratio = length(which(distance<=0.5))/length(distance) #finds the proportion of points inside the circle to the total number of points or points inside square
5. > val=ratio\*4 #the ratio is multiplied by 4 to get the value of pi as per the equation stated above
6. > val
7. [1] 3.144

Alternate Code

d<- replicate(10000, sqrt((runif(1)-0.5)^2+(runif(1)-0.5)^2))

> length(which(d<=0.5))\*4/length(d)

[1] 3.1452