

Graph

Graph Data Structure is a Collection of nodes Connected by edges. It's used to represent relationship between different entities.

- ① Graph data structure is a non-linear data structure.
- ② It is used in fields such as network analysis recommendation system, and Computer network.
- ③ In the field of sports data science, graph data structures can be used to analyze and understand the dynamics of team performance and player interactions on the field.

Components of Graph data Structure

① Vertices

② Edges

① Vertex -

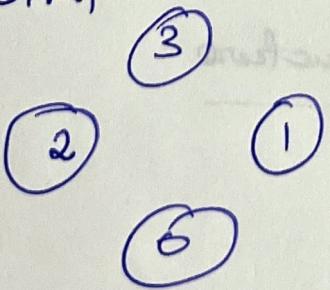
- * Vertices are the fundamental units of the graph
- * Vertices are also known as vertex or nodes
- * Every node / vertex can be labeled or unlabelled

② Edges -

- * Edges, are drawn or used to connect two nodes of the graph
- * It can be ordered pair of nodes in a directed graph
- * Edges are also known as arcs.

Types of Graph

① Null Graph



- * A graph is known as null graph if there are no edges in the graph.

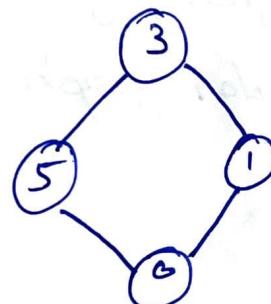
② Trivial Graph

Graph only have single vertex, it is also the smallest graph possible.

(2)

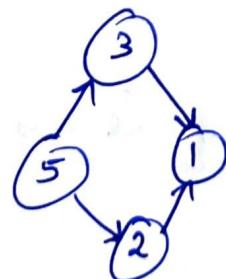
③ Undirected Graph.

Graph with edges don't have any directions.



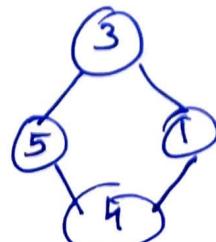
④ Directed Graph

A graph in which edge has a direction.



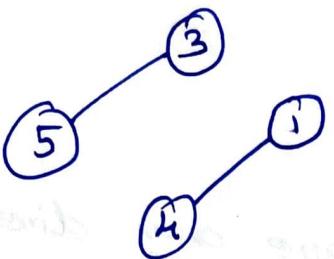
⑤ Connected graph

The graph in which from one node we can visit any other node in a graph is known as a connected graph.



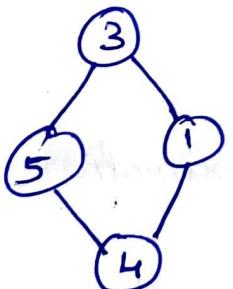
⑥ Disconnected Graph:-

The graph in which one node is not reachable from a node is known as a disconnected graph.



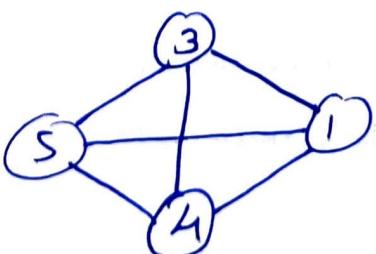
⑦ Regular Graph:-

The graph in which the degree of every vertex is equal to k is called k regular graph.



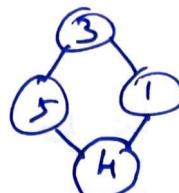
⑧ Complete Graph.

The graph in which each node is edge to each other node



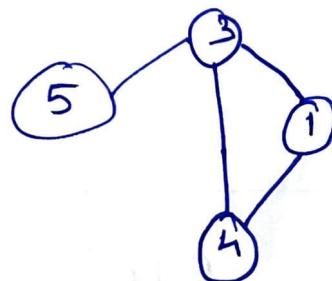
⑨ Cycle graph

The graph in which the graph is a Cycle in itself , the minimum value of degree of each vertex is 2.



⑩ Cyclic graph

A graph containing at least one cycle is known as a Cyclic graph

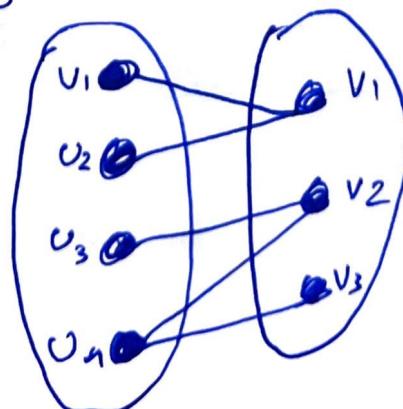


⑪ Directed Acyclic Graph:-

A directed graph that does not contain any cycle

⑫ Bipartite graph

A graph in which vertex can be divided into two sets such that vertex in each set does not contain any edge between them.



⑬ weighted graph

- * A graph in which the edges are already specified with suitable weight is known as a weighted graph.
- * Weighted graph can be further classified as directed weighted graph and undirected graph.

Representation Of graph Data Structure :-

- * Adjacency Matrix
- * Adjacency List.

Dfs - Depth First Search.

Adj List

Step① - Convert edges to adjacency list.

Step② - Create Visited array to mark the visited nodes.

Step③ - Do dfs.

```
f() {  
    v[cur] = 1;  
    for (n → adj[cur])  
        if (v[n] == 0)  
            f(n);  
}
```

y

bfs

Adj List

- ① Convert edges to adj list.
- ② Create visited array to mark visited nodes
- ③ do bfs.

bfs

- Create a queue
- add a first node to queue

Queue < > q;

```
q.offer(i); //  
if [c] = 1;  
while (q.isEmpty())
```

{

```
int curr = q.poll();  
print(curr);
```

```
for (int n → adj[curr])
```

```
{ if [visited[n]] == 0 }
```

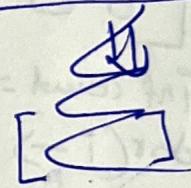
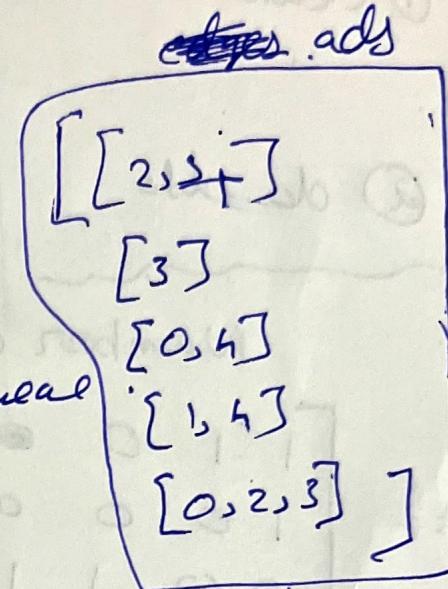
{

```
qu.offer(n);  
visited[n] = 1;
```

}

3

2



Adj matrix..

① Dfs

① Create a visited matrix \rightarrow store Visited cells.

② do dfs

Ex: Number of Islands.

$$\text{mat} = \begin{bmatrix} 1 & 1 & 0 & \bullet 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

int count = 0

for ($i \rightarrow \text{mat}.len$)

{ for ($j \rightarrow \text{mat}[i].len$)

{ $\text{if } (\text{v}[\text{i}][\text{j}] == 0)$

dfs(what, v, i, j)

} count ++

}

dfs(ads, v, row, col) {

if ($r < 0 || c < 0 || r \geq \text{adj.length} || c \geq \text{adj}[0].len$) return;

if ($\text{adj}[r][c] == 0 || \text{visited}[r][c] == \bullet 1$) return;

visited[r][c] = 1;

dfs(adj, v, r-1, c);

dfs(adj, v, r, c+1);

dfs(adj, v, r+1, c);

dfs(adj, v, r, c-1);

}

Adj matrix

② BFS

① Create a visited matrix to mark the visited cell

② Do bfs

Ex: rotten tomatoes:-

$$\text{arr}[3][3] = \begin{bmatrix} 2 & 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 & 1 \end{bmatrix}$$

① Create a cell datatype

```
public class cell {
```

```
    int row;
```

```
    int col;
```

```
    cell (int row, int col) {
```

```
        this.row = row;
```

```
        this.col = col;
```

```
    }
```

Create a queue.

② Put all ①'s in queue -

```
for (P → arr.len)
```

```
{ for (g → arr.len)
```

```
{ if (arr[i][j] == 2)
```

```
{ queue.offer(new cell(i, j));
```

```
}
```

③ do bfs

Count = 0;

while (queue.isEmpty()) {

n = queue.size();

for (int i → n)

{

cell curr = queue.poll();

visited[curr.row][curr.col] = 1;

// up

if (curr.row - 1 >= 0 && arr[r - 1][c] == 1 && v[r - 1][c])

{

visited[r - 1][c] = 1;

queue.offer(new cell(r - 1, c));

}

// right

if (row < arr[0].length && arr[r][c + 1] == 1 && v[r][c + 1])

{

visited[r][c + 1] = 1;

queue.offer(new cell(r, c + 1));

}

// down

if (row + 1 < arr.length && arr[r + 1][c] == 1 && v[r + 1][c])

{

visited[r + 1][c] = 1;

queue.offer(new cell(r + 1, c));

}

11/est.

if ($col - 1 \geq 0$ & $arr[r][c-1] == 1$ & $visited[r][c-1] == 0$)

{
 $visited[r][c-1] = 1$;
 queue.offer(new cell(r, c-1));

}

}

cout++;

y

Cycle Detection Directed (DFS)

① Convert edges to graph

② Create a visited array and path visited
array

③ do dfs.

↓ next page

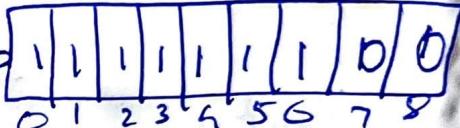
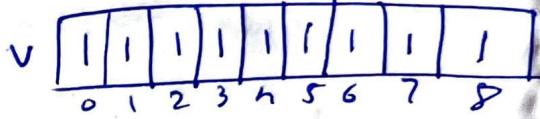
dfs (curr)

{

visited[curr] = 1;

pathvisited[curr] = 1;

for (int n → adj[curr]) {



if (visited[n] == 0) {

if (dfs)

{ return true

}

}

else if (pathvisited[n] == 1) {

return true;

}

}

pathvisited[curr] = 0;

return false;

}

Detect Cycle in undirected Graph. (Dfs)

- ① Convert edges to adj list.
- ② Create a visited array to mark visited nodes.
- ③ do dfs. | Intuition (Parent != neighbour) - Cycle detected

dfs(curr, p)

{
 visited[curr] = 1;

 for (int n → adj[curr])

 {
 if (visited[n] == 0)

 {
 if (dfs(ne, curr)) return true;

 }

 else if (Parent != neighbour)

 return true

 }

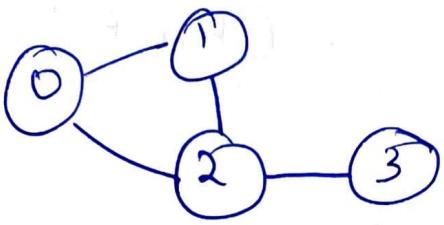
}

 return false;

}

Detect a Cycle in undirected (BFS).

- ① Convert edges to adj. list.
- ② Create a Visited array to ~~store~~ mark visited nodes
- ③ do BFS.



[1, 2]
[2]
[3]
[]



Queue $\leftarrow q$;

queue.offer(-1, start) \rightarrow (0, -1)

while (!q.isEmpty()) {

 Visited [child] = true;

 for (int n \rightarrow adj[curr])

 {
 if (visited[n] == 0) {

 qu.offer(c, n);

}

 else if (parent != neighbor)

{

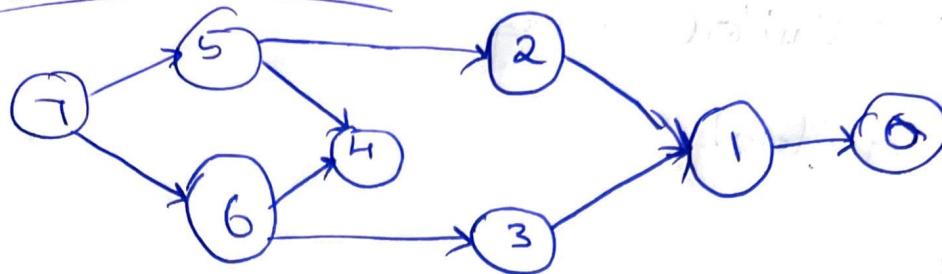
 return true;

 }

 return false;

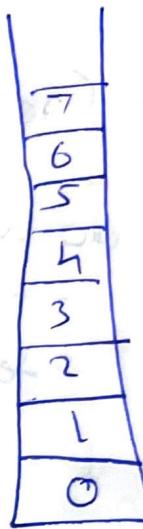
i, o
 $p = 0$
if ($n \neq p$)
{ return
true
}

Topological Sort - only work DAG (Directed Acyclic Graph)

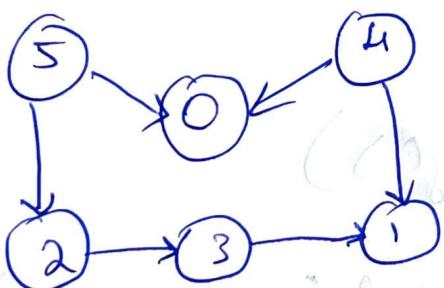


Pop

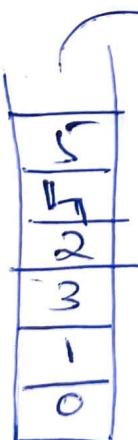
7 6 5 4 3 2 1 0



Topological sort DFS (using stack)



Linear ordering of vertices such that if there is an edge between v and u , v appears before u in the ordering.



5 4 2 3 1 0

Steps

① Create a visited array

② Create a stack

③ do dfs.

```
for(int i = 0 → adj)
```

```
{ if(visited[i] == 0)
```

```
{ dfs(adj, visited, stack, i)
```

```
}
```

```
}
```

```
while(!stack.isEmpty())
```

```
{ System.out.print(stack.pop());
```

```
}
```

```
dfs()
```

```
{ visited[curr] = 1;
```

```
for(int n → adj[curr]) {
```

```
if(visited[n] == 0)
```

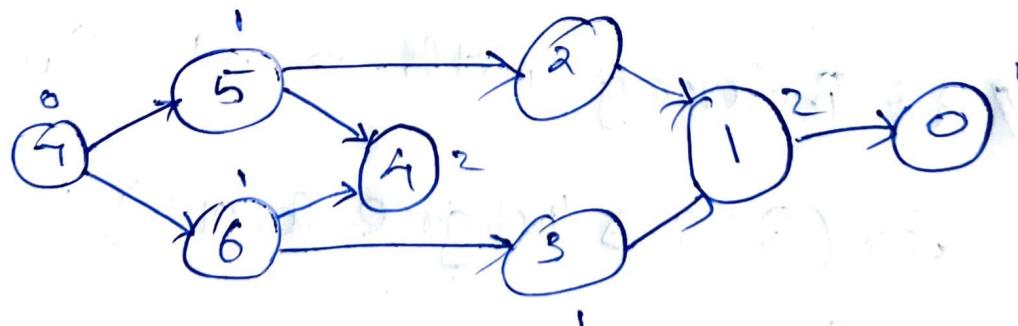
```
{ dfs()
```

```
}
```

```
stack.push(curr);
```

Topological sort (BFS) Kahn's Algorithm:-

- ① Convert edges to adj list.
- ② calculate indegree.
- ③ put nodes that has indegree 0 in to the queue
- ④ Do BFS
- ⑤ while visiting neighbour node from current node decrease indegree by one.



queue

7	8	6	4	2	3	1
---	---	---	---	---	---	---

result = 7 5 6 4 2 3 1

→ node

0	0	0	0	0	0	
X	0	0	X	0	0	
1	2	X	X	2	1	X
0	1	2	3	4	5	6

// calculate indegree.

int [] indegree = new int [adj.length];

for (i → adj.length)

{ for (n → adj[i])

{ indegree[n]++; }

bfs()

// Create queue

Queue q < >;

// Put indegree with 0 into Queue.

for (int i → indegree.length)

{

if (indegree[i] == 0)

{ queue.add(i); }

}

11

bfs

while ($\neg q.\text{isempty}()$)

{
int curr = queue.poll();

res.add(curr);

for (int n \rightarrow adj[curr])

{
indegree[n]--;

if (indegree[n] == 0)

{
q.offer(n);

}
}

}
}

}
}

}
}

}
}

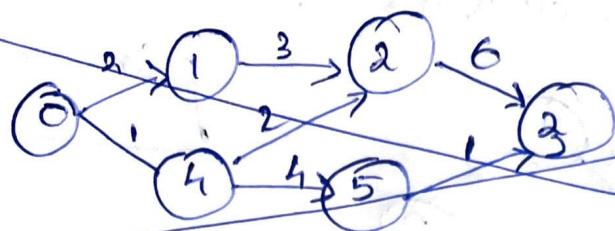
}
}

Detect a Cycle in directed Graph . (Topological)

- ①. Reverse the graph edges.
- ② calculate indegree
- ③ Create queue
- ④ put the node into queue with indegree 0
- ⑤ do Bfs - (kahn's algorithm).

Shortest Path DAG:- (topological sort)

- ① Only works in Directed Acyclic graph.
- ② first do dfs and get the result in stack.
- ③ Create a list array
- ④ fill dist array with infinity
- ⑤ mark last node to 0.
- ⑥ ~~Do not~~ Take a node out of the stack and check the neighbour nodes and update the dist array.



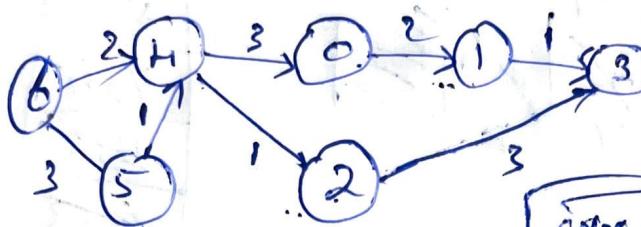
adjList

$$0 \rightarrow \{1, 2\}$$

$$1 \rightarrow \{3, 5\}$$

$$2 \rightarrow \{3, 4\}$$

$$4 \rightarrow \{5\}$$

$$5 \rightarrow \{3\}$$


adjList

$0 \rightarrow \{1, 2\}$

$1 \rightarrow \{3, 5\}$

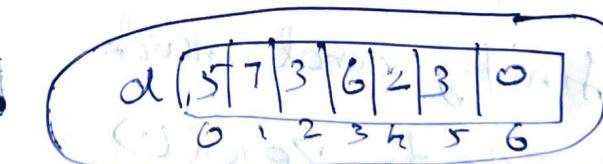
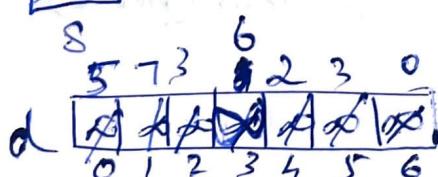
$2 \rightarrow \{3, 5\}$

$3 \rightarrow$

$4 \rightarrow \{0, 3\} \{2, 1\}$

$5 \rightarrow \{4, 1\}$

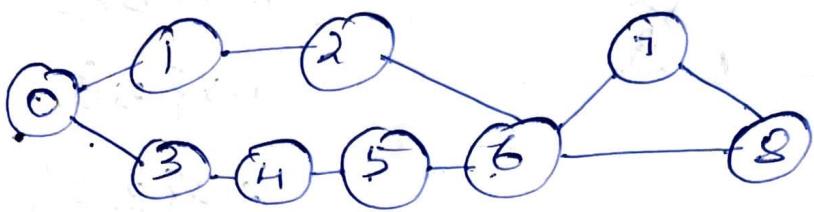
$6 \rightarrow \{1, 2\} \{5, 3\}$



step①. \rightarrow Do toposort on a graph [marks src nodes to zero]

step② \rightarrow Take a nodes out of a stack and check the neighbour nodes and update the list array.

Shortest path in undirected Graph with unite weight



Src = 0

dist	0	1	2	1	2	3	3	3	4	5
	0	1	2	3	4	5	6	7	8	



8,5
7,4
5,3
6,3
4,2
2,2
(3,1)
(4,1)
(5,0)

Open (node, wt)

- ① Convert edges to adj list
- ② Create a dist array and fill infinity and mark source node to zero(0)
- ③ Do bfs

important

* if the weight is unite weight

use Queue

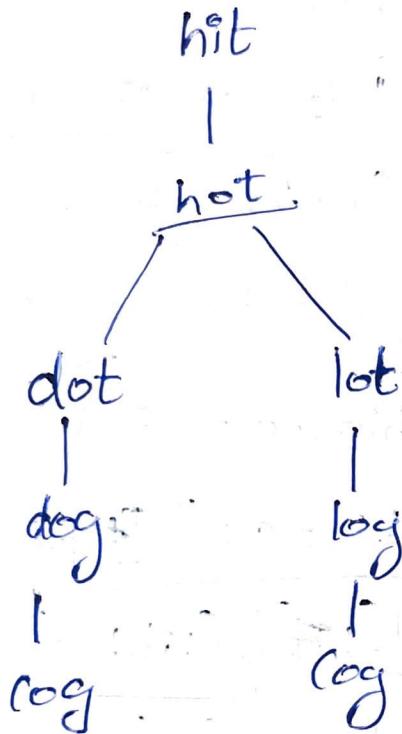
* if the weight is varying weight

use priorityqueue (Dijkstra's algorithm)

word ladder - I

begin word = "hit", end word = "cog"

wordlist = ["hot", "dot", "dog", "lot", "log", "cog"]



④.

(cog, 1)
(log, 2)
(dog, 3)
(lot, 2)
(dot, 2)
(hot, 1)
(hit, 0)

- ① Create a Set and put all words into the set.
- ② Create a Queue and put begin word into queue
(hit, 0) and length 0.

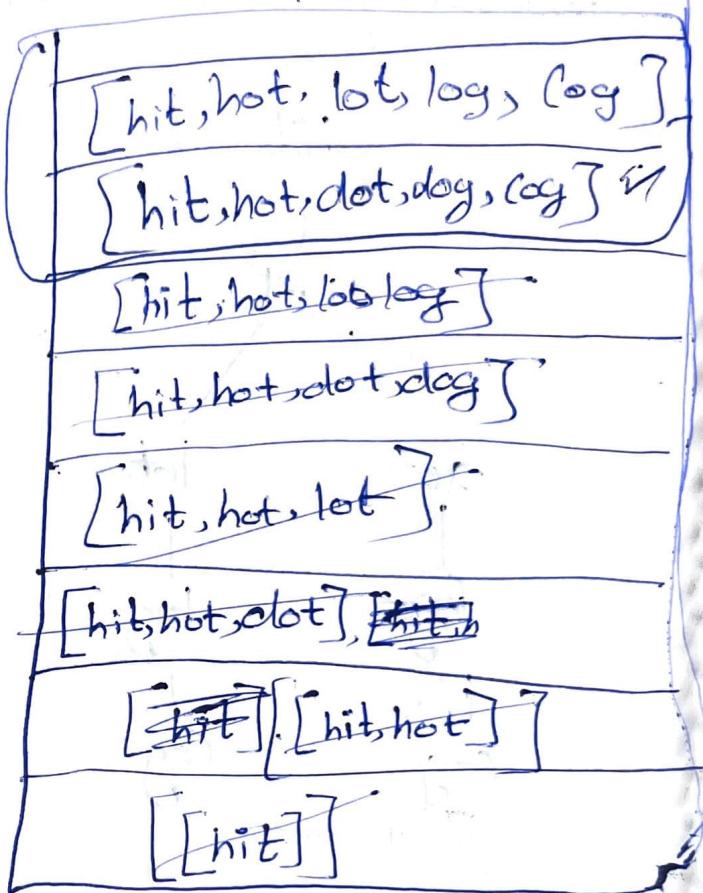
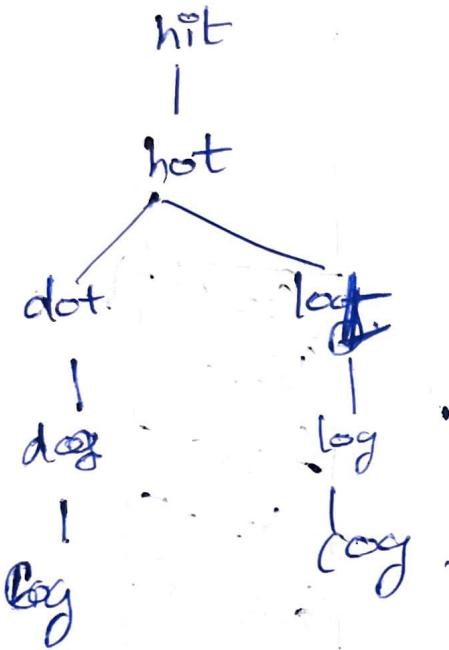
③ Do bfs.
Take each word and put all combinations and check if it is in set, if present put it into queue and delete word from set.

Do this until you get the end word.

word ladder 2

begin word = "hit", end word = "cog"

word list = ["hot", "dot", "dog", "lot", "log", "cog"]



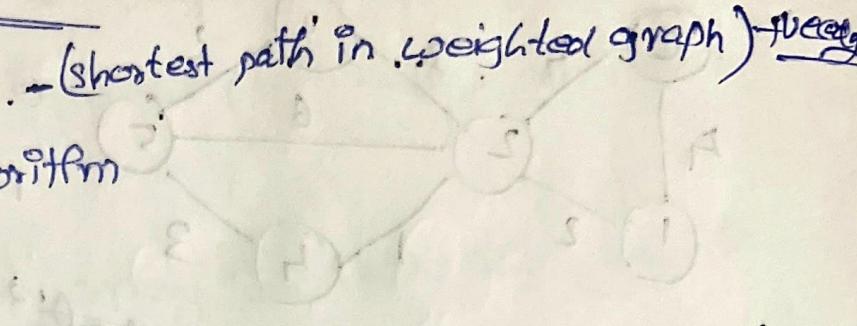
(~~list strings~~).

(~~list strings~~)

- ① Create a set and put all words into a set
- ② Create a Queue and put begin word in List and put the list to Queue

- ③ do bfs
 take the last ~~the~~ word in list and put all combination if its valid add to level visited node and ~~then~~ put the word into list and put the list back to Queue - do until getting result.

Shortest path Algorithms:

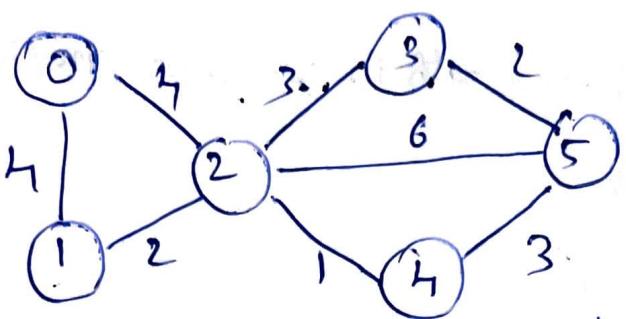
- + Dijkstra Algorithm. - (shortest path in weighted graph) 
- + Bellman - Ford Algorithm
- + Floyd - Warshall.

Dijkstra's Algorithm: (Priority Queue or Set)

- + Dijkstra's Algorithm is a graph search algorithm it is used to find the shortest path between nodes in a weighted graph
- + It was developed by Edsger W. Dijkstra in 1956

Purpose:

- ↳ Find the shortest path from a starting node to all other nodes in the graph
- + work only in non-negative edge weights (positive)
- * It is a greedy algorithm. - It always selects the smallest known distance that hasn't been visited yet.
- + Output = The shortest path from the source to each node



Adj list.

$0 \rightarrow \{1, 4\} \{2, 4\}$

$1 \rightarrow \{0, 4\} \{2, 2\}$

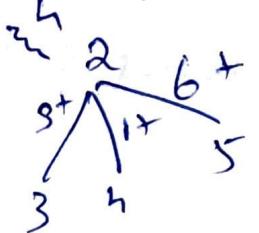
$2 \rightarrow \{0, 4\} \{1, 2\} \{3, 3\} \{4, 1\} \{5, 6\}$

$3 \rightarrow \{2, 3\} \{5, 2\}$

$4 \rightarrow \{2, 1\} \{5, 5\}$

$5 \rightarrow \{2, 6\} \{3, 2\} \{4, 3\}$

$\text{if } (\text{current} + \text{Adjwt} < \text{dist}[i])$
 $\quad \{$
 $\quad \quad \text{adjwt} = \text{dist}[i]$
 $\quad \quad \text{dist}[i] = \text{current} + \text{Adjwt};$
 $\quad \quad \text{spq.add}(i)$



(5, 8)
(5, 10)
(4, 5)
(8, 7)
(2, 5)
(1, 1)
(0, 0)
(red, wt)

9
5

0	1	2	3	4	5	8	10	red
✓	✓	✓	✓	✓	✓	✓	✓	dist[i]

step① - convert edges to adjlist

step② - Create a priority queue (min heap) and put a source to pq

step③ - Create a dist array to store the result

step④ - fill dist array with infinity and mark src to 0

step⑤ - do bfs

bfs

while (!pq.isEmpty())

{ Pair curr = pq.pop();

int currnode = curr.node;

int current = curr.wt;

for (int[] node : adj.get(currNode)) {

int adjnode = node[0];

int adjwt = node[1];

if (current + adjwt < dist[adjnode]) {

dist[adjnode] = current + adjwt;

pq.offer(new pair(adjnode, dist[adjnode]))

}

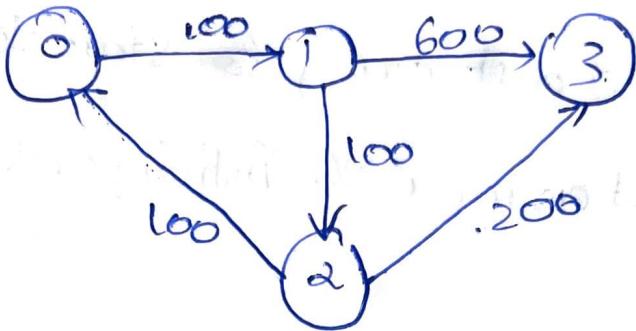
}

cheapest flight

$$s = 0$$

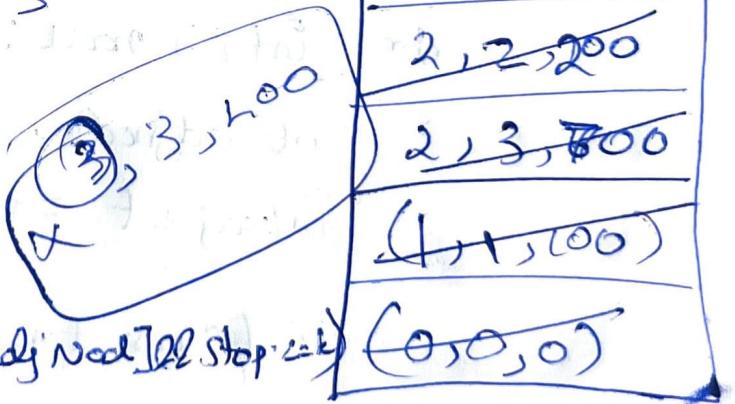
$$k = 2$$

$$d = 3$$



here our first priority is stops

0	100	200	700
0	1	2	3



if (CurrentDist + adjDist < dist[adjNode][2] stop == 1)

dist[adjNode] = CurrentDist + adjDist; (stops, node, dest)

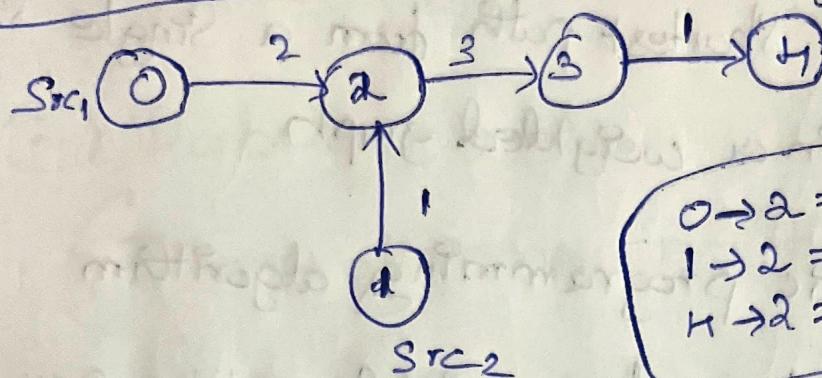
queue.offer(new pair(adjNode, adjDist + currentDist, stop + 1))

3

3

Minimum weighted Subgraph with the required path.

(Lowest Common Ancestor) Graph



$$\begin{aligned}0 \rightarrow 2 &= 2 \\1 \rightarrow 2 &= 1 \\4 \rightarrow 2 &= 4\end{aligned} \quad \rightarrow 7$$

(LCA)

① Convert edges to adj list - to calculate Src to all

② Convert edges to reverseadjlist - to calculate

dest to all other nodes.

③ Do dijkstra and get the shortest path

Src = 0 dist 1

0	∞	2	5	6
0	1	2	3	4

LCA = 2

Src = 1 dist 2

∞	∞	1	4	5
0	1	2	3	4

2+1+6	5+4+1	6+3+6
10	10	10

dest = 4 dist 3

7	8	4	1	0
0	1	2	3	4

↓
GOV

res = Inf;

for (int i = 0; i < n; i++)

{

 if (dist[i] == Inf || dist2[i] == Inf || dist3[i] == Inf) continue;

 int res = min(dist[i] + dist2[i] + dist3[i], res);

 res = min(res, dist[i] + dist2[i] + dist3[i], res);

return res;

Bellman Ford Algorithm:-

- * Used to find the shortest path from a single source to all other nodes in a weighted graph
- It is a Dynamic programming algorithm
- * It handles negative edges weight unlike Dijkstra
- * It can also detect negative weight cycle in the graph

Suitable when a Graph Contains negative weight and only need path from single source

Algo:

- ① The algorithm works by "relaxing all edges $n-1$ times", where n is number of vertices
- ② If any distance can still be relaxed after $n-1$ iteration, a negative cycle exists.
- ③ Time Complexity: $O(V \times E)$

V - Vertices
E - edges .

Steps ① - Create a dist array to store distance
fill infinity and mark src to 0

② Do bellman ford.

(Traverses all edges n-1 times) n-number of vertices.

for (int i → n-1)

{

for (int j → edges.length)

{

int u = edges[j][0]

int v = edges[j][1]

int wt = edges[j][2]

if ($dist[u] + wt < dist[v]$)

$dist[v] = dist[u] + wt;$

③ Detect negative cycle.

for (int i → edges.length)

u = edges[j][0]

v = - "

w = "

if ($dist[u] > (int)(i \oplus 1) \text{ and } dist[v] < dist[u]$)

return true

}

if result
improves
negative cycle
exists

$$\text{edges} = \begin{bmatrix} u & v & \text{wt} \\ \end{bmatrix}$$

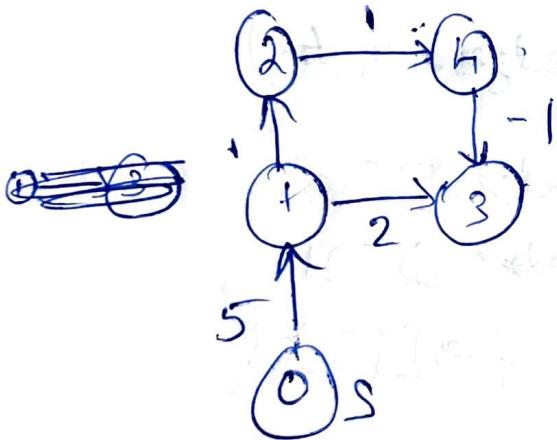
$[1, 3, 2]$

$[4, 3, -1]$

$[2, 4, 1]$

$[1, 2, 1]$

$[0, 1, 5]$



$$dist[u] + wt < dist[v]$$

$$dist[v] = dist[u] + wt$$

$$dist[u] + wt < dist[v]$$

$$dist[v] = dist[u] + wt$$

0	5	6	7	8
0	1	2	3	4

Floydwarshall Algorithm:-

- * It is used to find the shortest path between all pairs of vertices in a weighted graph
- * It works in both directed and undirected.
- * It is a Dynamic programming algorithm
- * It uses 2D matrix to store the shortest distance between each pairs of nodes
- * It works with negative edges as long as there is no negative cycle.

+

| It can detect size of negative cycle

| if $\text{dist}[i][i] < 0$, return true.

| Neg Cycle exist

0	2	-1	2
10	0	4	2
1	2	0	1
-8	7	2	0

no Cycl

0	2	1	-2
3	0	2	1
1	2	0	1
8	2	1	0

Cycle exist

Time Complexity $O(V^3)$ where V is the number of vertices}

Space Complexity $\{O(V^2)\}$. due to distance matrix

$$\text{dist}[p][j] = \min(\text{dist}[i][j], \text{dist}[i][k] + \text{dist}[k][j])$$

steps ① - Convert the edges to adj. matrix

fill infinity except diagonal matrix

step ② - Set edges to matrix

for ($i \rightarrow$ edges.length)

{
 for ($j \rightarrow$ edges[i].length)

 int v = edges[p][o];

 int w = edges[i][j];

 int wt = edges[i][j][2];

 if matrix[u][v] >= wt;

}

step (3) - Do Floyd-Warshall algorithm.

for (via \rightarrow n)

 for (i \rightarrow matrix.length)

 for (j \rightarrow matrix[i].length)

{

 dist[i][j] = Math.min (dist[i][j], dist[i][via] + dist[via][j])

}

} // loop via

step (4) Detect Negative Cycle

for (i \rightarrow matrix.length)

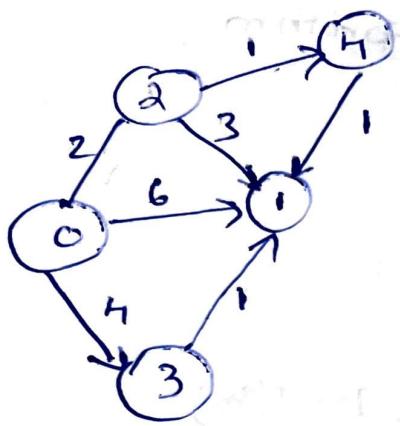
{ if (matrix[i][i] < 0)

{

 return true;

}

y



(Multisource shortest path Algorithm)
+ help you detect negative Cycles as well

Note: Go via every vertex/node.

$$d[0][1] = 0 \rightarrow 1 \quad 6$$

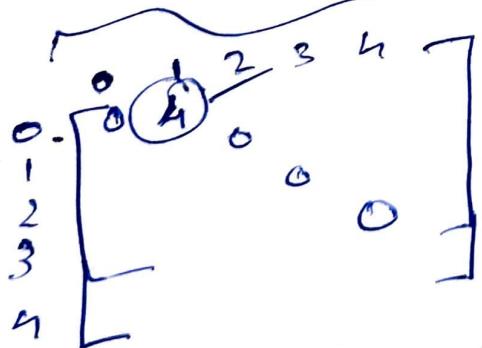
$$\text{via } 2 \quad d[0][1] = d[0][2] + d[2][1] \rightarrow 5$$

$$0 \rightarrow 2 \rightarrow 1$$

$$\text{via } 3 \quad d[0][1] = d[0][3] + d[3][1] = 5$$

$$\text{via } 4 \quad d[0][1] = d[0][4] + d[4][1] = 4$$

Via (4) $d[0][1] = d[0][4] + d[4][1] = 4$



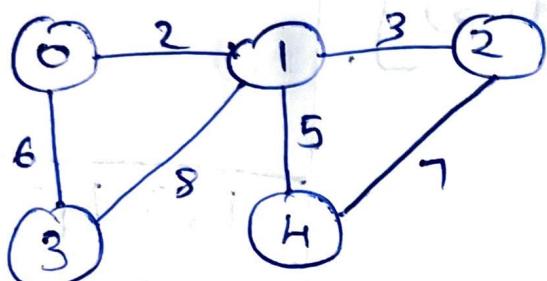
close the same for all
the edges

~~Prims algorithm~~

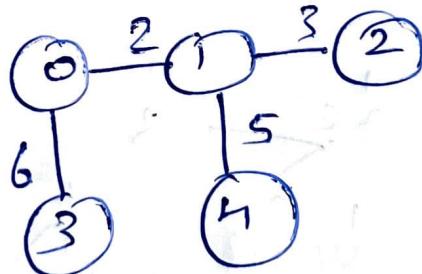
Minimum Spanning Tree

* Prims algorithm

* Kruskal's Algorithm



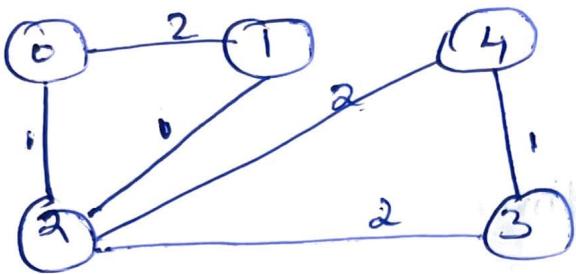
- MST.



$$\text{sum} = 18$$



Prims Algorithm



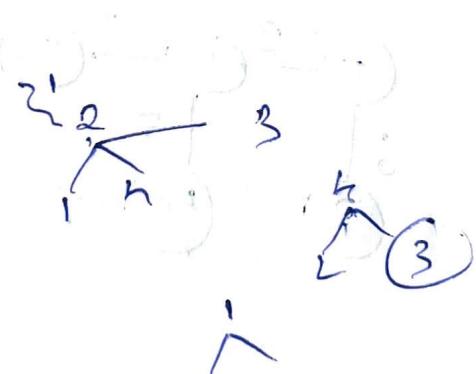
$$\text{sum} = \cancel{0} + \cancel{2} + \cancel{5}$$

MST $\Rightarrow [(0, 2), (2, 1), (4, 2), (4, 3)]$.

Visited

1	*	1	1	1
0	1	2	3	4

~~(1, 2)~~
~~(0, 2)~~
~~(0, 3)~~



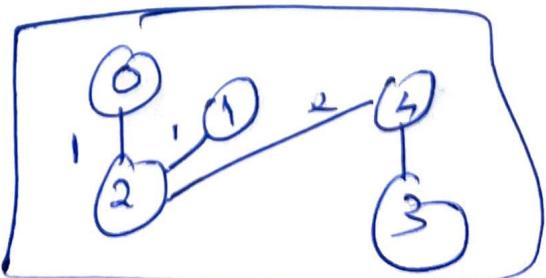
1, 3, 4
2, 3, 2
2, 4, 2
1, 1, 2
1, 2, 0
2, 1, 0
(0, 0, -1)

(wt, node, parent)

(min heap)

if (parent == -1)

don't add edge to MST or ~~wt do sum~~



Step(1) - Convert edges to adjlist

Step(2) - Create a Visited array.

Step(3) - Create mst array to store list of mst edges

Step(4) - Create a sum variable to calculate the weight

Step(5) - Create a priority Queue (min-heap)

Step(6) ~

① Start from any node.

② Maintain a minheap (Priority Queue) to

Pick the edge with the smallest weight

that connects the new vertex.

③ Group the the mst one vertex at time

④ Time Complexity $O(E \log V)$

It is a greedy algorithm

Convert edges to adj

List<int[]> mst = new ArrayList<>();

int visited[] = new int[n];

int sum = 0;

PQ <

PQ.offer(n, wt, p)

while (!PQ.isEmpty())

{

: if (visited[current] == 1) continue

visited[current] = 1

if (current != -1) {

mst.add(new int[] {current, current})

sum = sum + currwt;

}

for (int node : adj.get(current)) {

int adjn = nod[0];

int adjw = nod[1];

{ if (visited[adjnode] == 0) {

PQ.offer(new pair (adjnode, adjw))

5.

only works in
undirected graph

Disjoint Set (undirected graph)

(find Union)

Properties

Solve graph
Probs in O(1)

methods.

→ find parent (C)

→ Union (C) $\begin{cases} \text{by rank} \\ \text{by size} \end{cases}$

not actually
for intution!

A disjoint set Union (DSU) or Union find data structure keeps track of a partition of elements into disjoint (non-overlapping subset). It supports two main operations

find(x) → find the leader or root of a set containing x

Union(x,y) → merge the set containing x and y

Efficient implementation uses

- * Path Compression (in find)
- * Union by Rank / size (in union).

Time Complexity: $O(\alpha(n))$ per operation, where

$\alpha(n)$ is the inverse Ackermann function -

(extremely fast)

Use Cases

① Kruskal's Algorithm (Minimum Spanning Tree)

② Cycle Detection In Undirected Graph

If you try to union two vertices already

connected (same root), a cycle exist.

③ Connected Components

• "How many distinct components are there?"

Just Count how many unique parent exists after
Unionsing connected nodes.

④ Dynamic Connectivity

→ you can dynamically merge component
and check whether two nodes are in the same
Component - useful in real time graph processing.

⑤ Network Connectivity

used in problems like
→ Are all computers in network connected.
→ After adding/removing cables, is the
network still connected.

⑥ Percolation Theory / Maze Generation

→ used in simulations to track connected
open space in grid or graph.

⑦ Game Logic

Used in :

→ Region merging

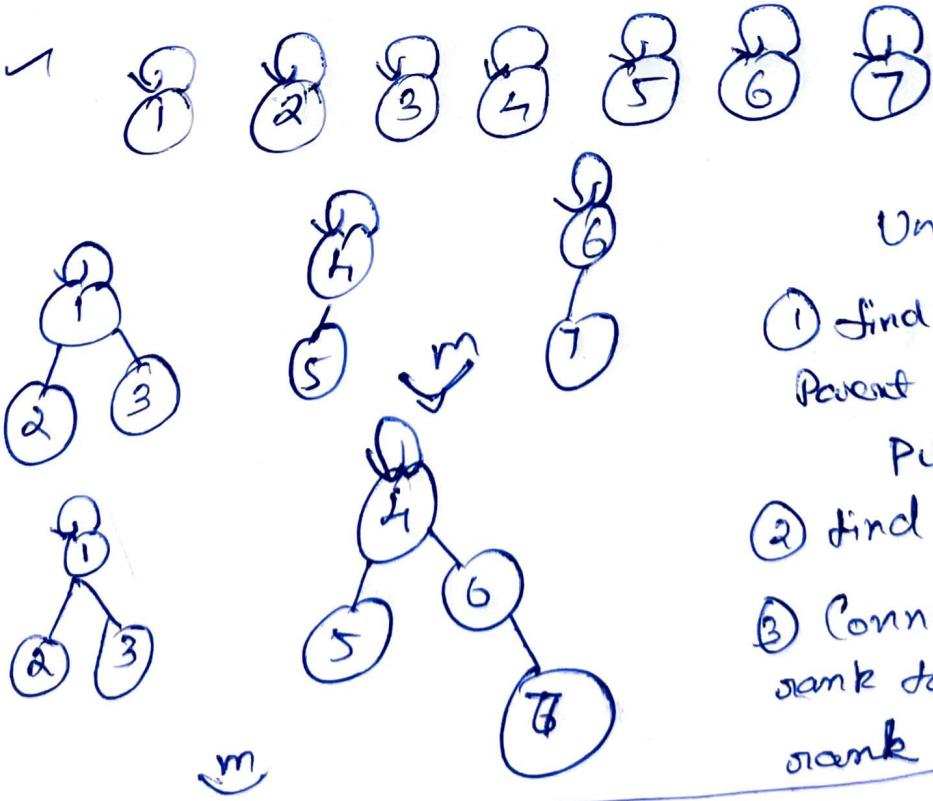
→ Area Clamping

→ Team or alliance Tracking.

$U \cup V$
 $(1, 2) \leftarrow$
 $(2, 3) \leftarrow$
 $(4, 5) \leftarrow$
 $(6, 7) \leftarrow$
 $(5, 6) \leftarrow$
 $(3, 7)$

rank	1	2	3	4	5	6	7
	∅	∅	∅	∅	∅	∅	∅

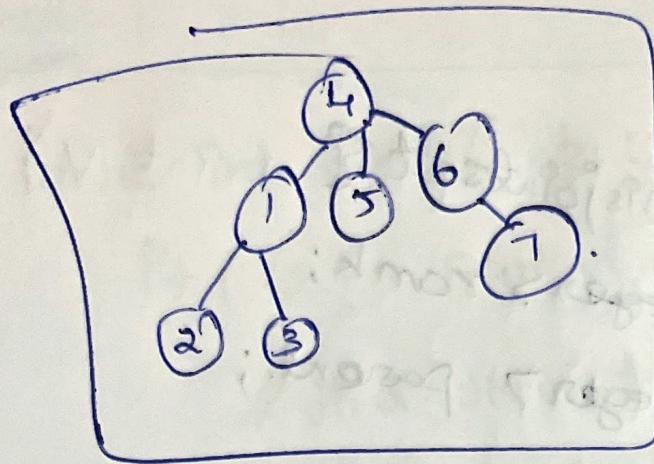
Parent	1	2	3	4	5	6	7
	1	2	3	4	5	6	7



Union(U, V)

- ① find the ultimate Parent of $U \& V$
 P_U, P_V
- ② find rank $P_U \& P_V$
- ③ Connect smaller rank to larger rank

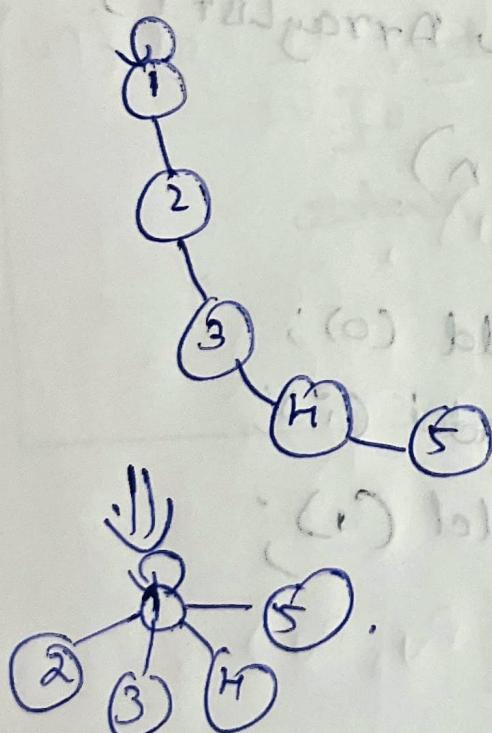
while merging parent with same rank increase rank



find()

Path Compression

find(5)



Path Compression.

function (0)

```
{ if (p[u] == Parent.get(u)) {
```

}

// find ultimate parent
root = find(Parent.get(u));

// path compression

```
Parent.set(root, u);
```

return root;

```
} // returns Parent.get(u);
```

}

Disjointset

- Public static class Disjointset {
 List<Integer> rank;
 List<Integer> parent;
 List<Integer> size;

 Disjointset(int n) {

 this.rank = new ArrayList<()>;
 this.parent = new ArrayList<()>;
 this.size = new ArrayList<()>;

 for (int i = 0; i < n; i++)

 {

 rank.add(0);
 parent.add(i);
 size.add(1);

 }

}

// find

```
3. Public int find (int u) {
```

```
    if (u != parent.get(u)) {
```

```
        // find ultimate parent
```

```
        int root = find (parent.get(u));
```

```
        // path compression
```

```
        parent.set(u, root);
```

```
        // set new root
```

```
        return root;
```

```
}
```

```
return parent.get(u);
```

```
}
```

Union by rank

```
Public void UnionbyRank (int u, int v) {  
    int pu = find (u);  
    int pv = find (v);  
    If (pu == pv) return ;  
    If (rank.get (pu) < rank.get (pv)) {  
        Parent.set (pu, pv);  
    }  
    else if (rank.get (pv) < rank.get (pu)) {  
        Parent.set (pv, pu);  
    }  
    else {  
        Parent.set (pv, pu);  
        rank.set (pu, rank.get (pu) + 1);  
    }  
}
```

Union by Size.

```
Public Void UnionbySize (int u, int v) {
```

```
    int pu = find(u);
```

```
    int pv = find(v);
```

```
    If (pu == pv) return;
```

```
    If (size.get(pu) < size.get(pv)) {
```

```
        parent.set(pu, pv);
```

```
        size.set(pv, size.get(pu) + size.get(pu));
```

```
}
```

```
Else If (size.get(pv) < size.get(pu)) {
```

```
{
```

```
    parent.set(pv, pu);
```

```
    size.set(pu, size.get(pu) + size.get(pv));
```

```
}
```

```
Else {
```

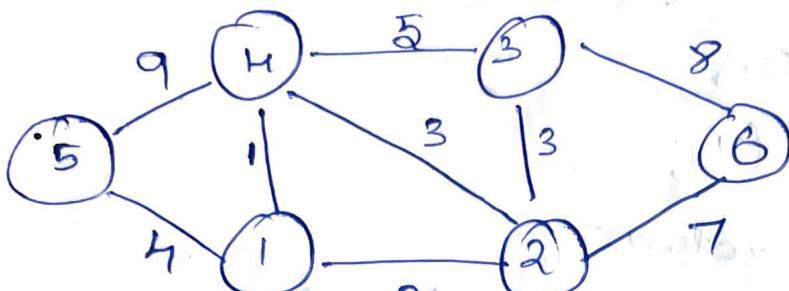
```
    parent.set(pv, pu);
```

```
    size.set(pv, size.get(pu) + size.get(pv));
```

```
}.
```

```
}
```

Kruskal's Algorithm → find MST



Step ① - Sort all the edges according to weight. (1 to 9) to get

Step ② - Do union for all edges.

(not $v \cup v$)

1, 1, 4 ↗

2, 1, 2 ↗

3, 2, 3 ↗

3, 2, 4 ✗

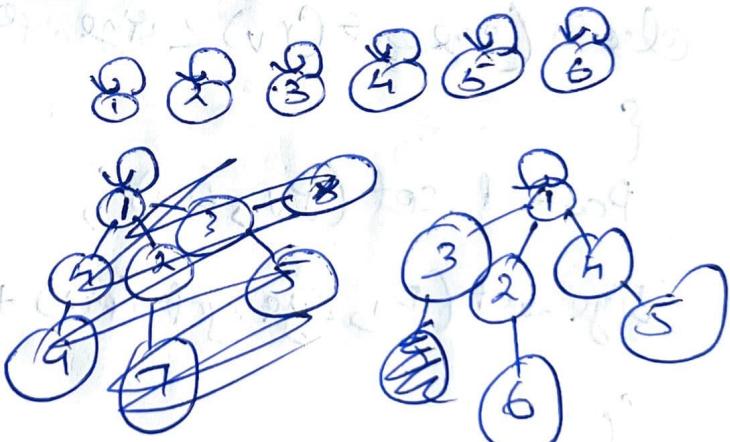
4, 1, 5 ✗

5, 3, 4 ↗

7, 2, 6 ↗

8, 3, 6 ↗

9, 4, 5 ↗



Public static void Krushals(int[][] edges, int n)

{

DisjointSet ds = new DisjointSet(n);

int sum = 0;

List<int> mst = new ArrayList<int>();

Array.sort(edges, (a, b) → a[2] - b[2]);

for (int i = 0; i < edges.length; i++)

{

int u = edges[i][0];

int v = edges[i][1];

int wt = edges[i][2];

If (ds.find(u) != ds.find(v)) {

ds.union(u, v);

sum = sum + wt;

mst.add(new int[]{u, v});

}

System.out.print(sum);

}

y.

Merge accounts:-

$\left[\begin{array}{l} ["John", "Johnsmith@gmail.com", "John_newyork@gmail.com"] \\ ["John", "Johnsmith@gmail.com", "Johnoo@gmail.com"], \\ ["John", "Johnsmith@gmail.com", "Johnoo@gmail.com"], \\ ["Mary", "mary@gmail.com"], ["Jhon", "Johnnybravo@gmail.com"] \end{array} \right]$

\downarrow

① Create a map $\langle \text{String}, \text{Integer} \rangle$

Eg: $\text{Johnsmith@gmail.com} = 0$

$\text{John_newyork@gmail.com} = 0$

$\text{Johnsmith@gmail.com} = 1$

$\text{Johnoo@gmail.com} = 1$

$\text{mary@gmail.com} = 2$

$\text{Johnnybravo@gmail.com} = 3$

\downarrow

② Unionmap $\text{map}\langle \text{Integer}, \text{List}\langle \text{Strings} \rangle \rangle$

$\circ = \left[\begin{array}{l} \text{Johnsmith@gmail.com}, \text{John_newyork@gmail.com} \\ \text{Johnoo@gmail.com} \end{array} \right]$

~~#~~ 2 \Rightarrow marrg@gmail.com

3 \Rightarrow Johnngbravo@gmail.com

Number of Islands = II

0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

0	1	2	3	4
5	6	7	8	9
10	11	12	13	14

- 1 - (0,0)
1 - (0,0)
2 - (0,1)
1 - (1,0)
1 - (0,1)
2 - (0,3)
2 - (1,3)
2 - (0,4)
3 - (3,2)
3 - (2,2)
1 - (1,2)
① - (0,2)

get node (r, c)

return p[n+i];

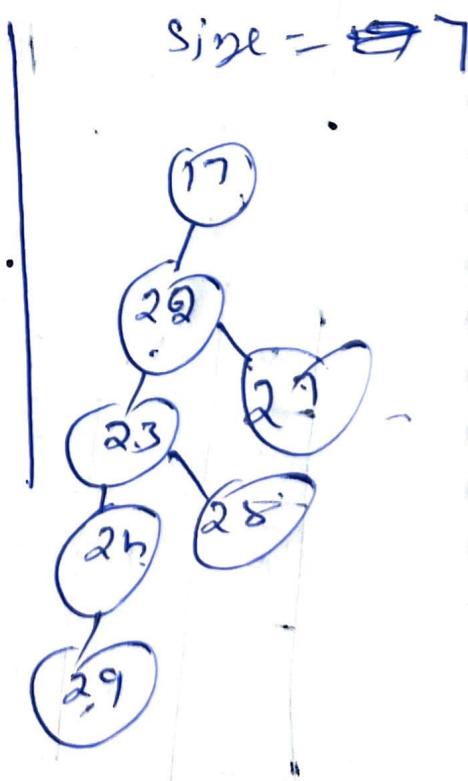
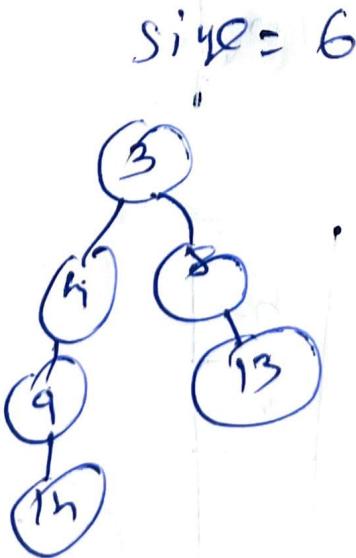
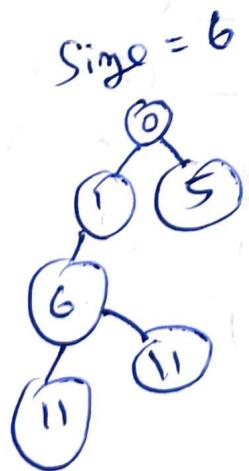
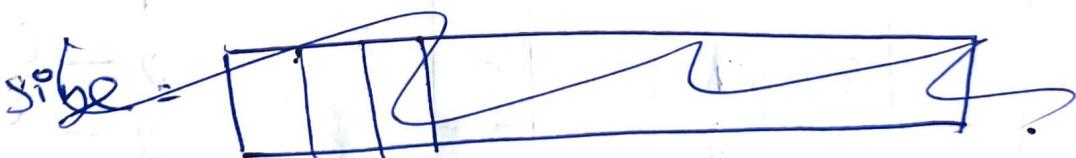
Making Large Island:-

Col

$i \neq n + j$	$n = \text{grid}[0].length$
10	0
6	1
0	2
5	3
10	4
15	5
20	6
25	7
11	8
16	9
21	10
26	11
12	12
17	13
22	14
18	15
23	16
28	17
24	18
29	19

0	1	2	3	4	
0	1	1	0	1	1
1	1	1	0	1	1
2	1	1	0	1	1
3	0	0	1	0	0
4	0	0	1	1	1
5	0	0	1	1	1

- first traverse the grid, and connect the components.



② do a traversal of grid [row][col] = 0
use set to avoid duplication, by adding size

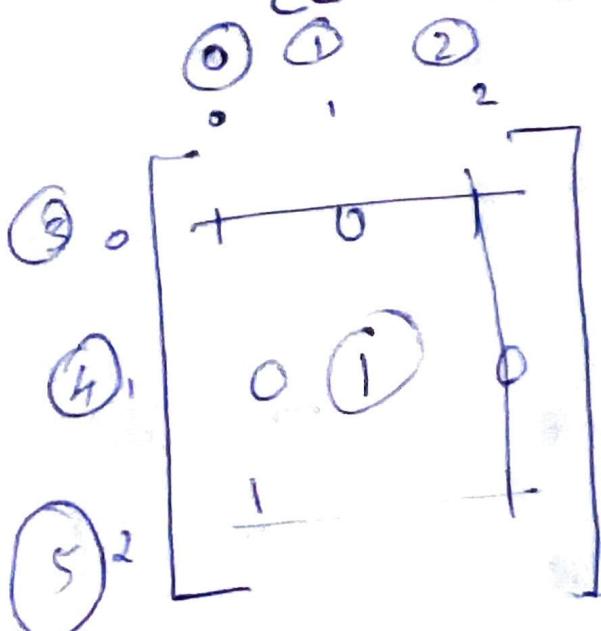
for example

$$6 - \begin{matrix} 0 \\ | \\ 1 \\ | \\ 2 \end{matrix} - 6 = \boxed{19}$$

③ most stones removed with same row or column

Input =

$$\text{stones} = [[0,0], [0,2], [1,1], [2,0], [2,2]]$$

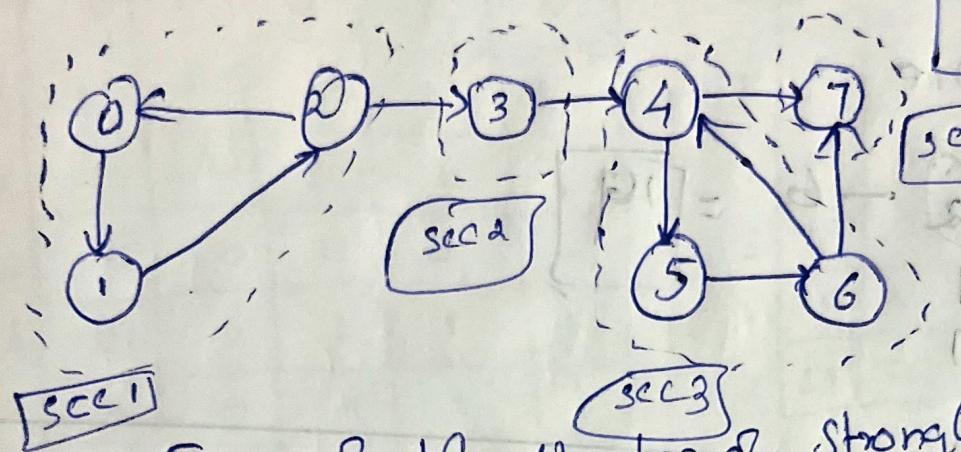


$r = 1$
 $\text{col} = \text{row} + n$
 $n = \text{number of rows}$

formula
Total number of stones -
number of components

Strongly Connected Components

Kosaraju's Algorithm



→ Figure 8 of the Number of Strongly Connected Components.

→ Print the Strongly Connected Components.

Strongly Connected Components are only valid for Directed graphs.

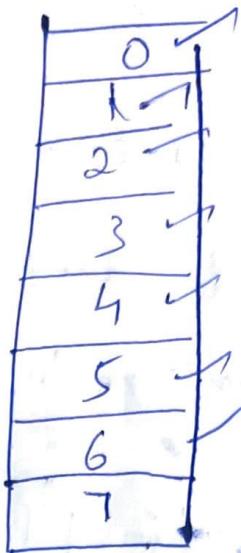
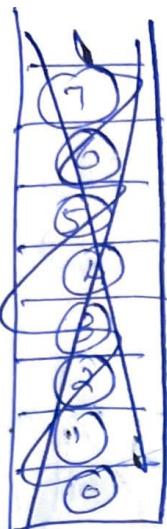
Implementation Steps -

Step ①

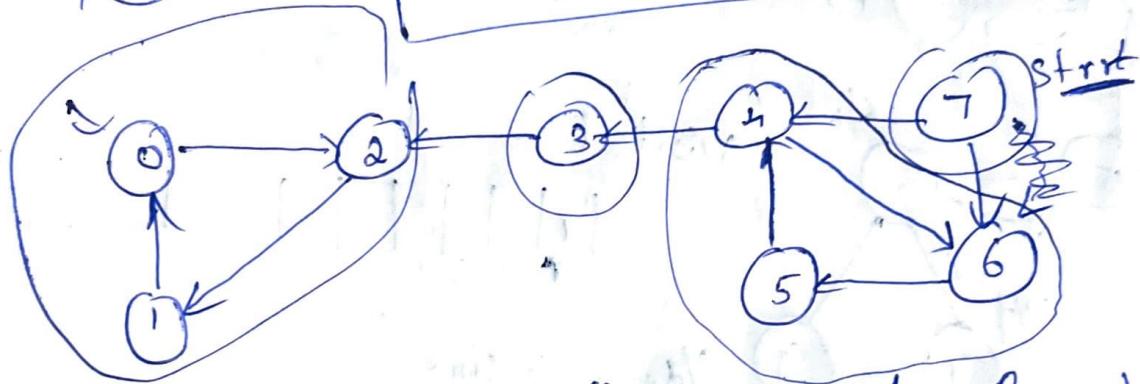
Do dfs and get the priority

Sort all the edges according to the finishing time

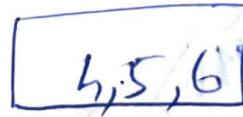
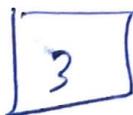
Stack



Step ② - ~~Reverses~~ Reverses the graph edges



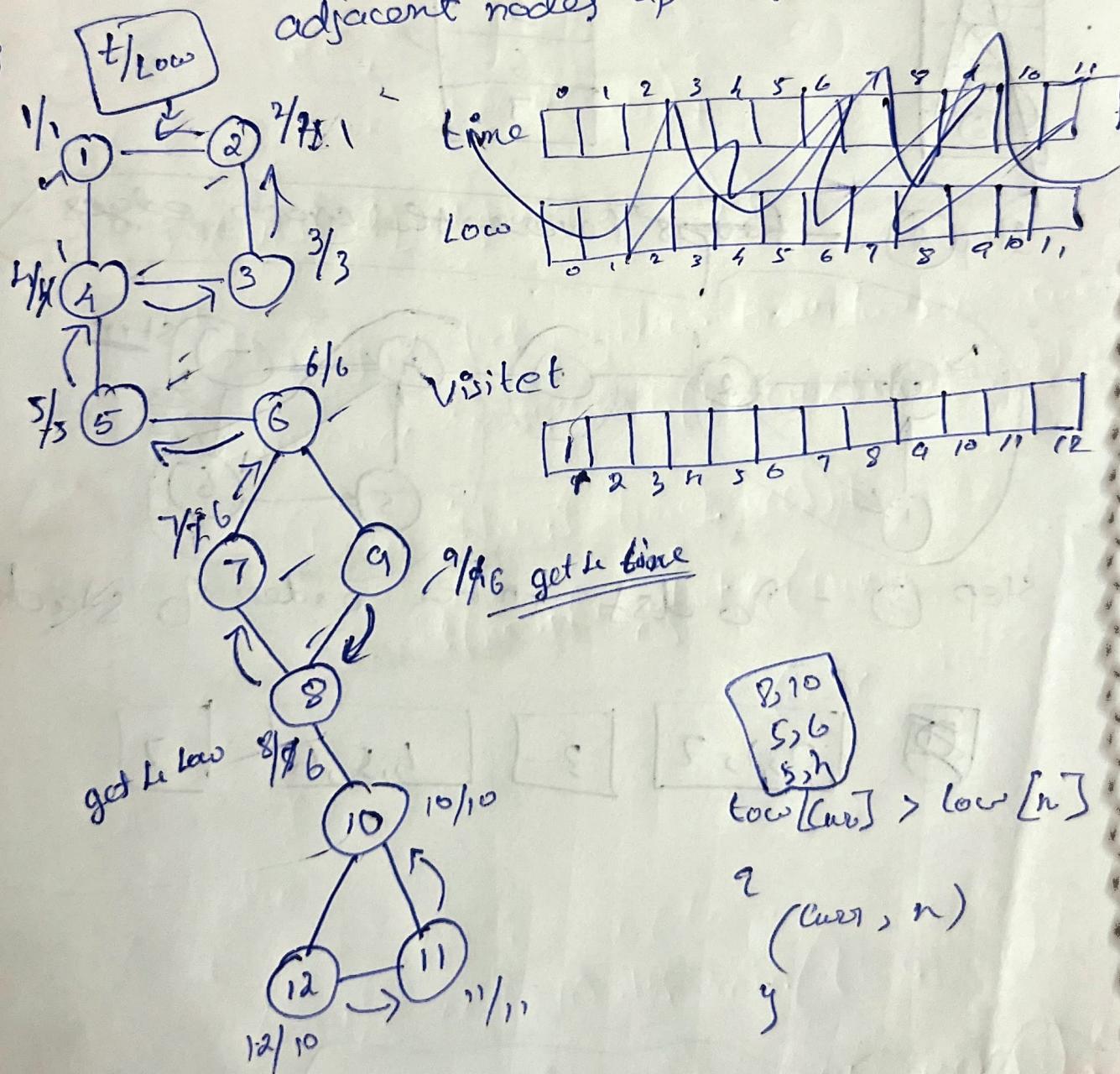
Step ③ - Do dfs in the order of stack

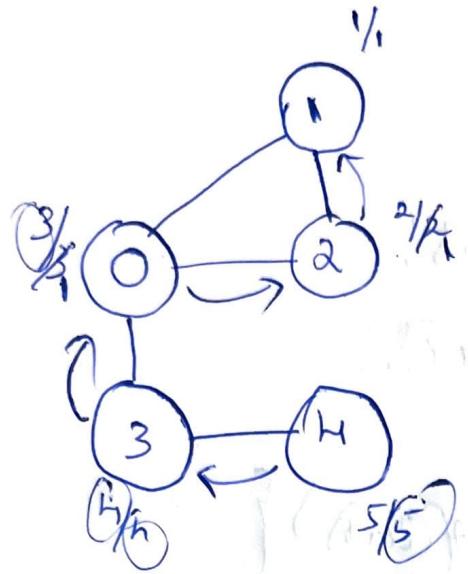


Tarjan's Algorithm

time[] - DFS time insertion

low[] - min lowest time insertion of all adjacent nodes apart from parent





$\text{if } (\text{low}[\text{neighbour}] > \text{time}[\text{curr}])$
 {
 res.add [curr, neighbour];
 $y = \dots$

step(1) - Convert edges to adjacent list

Step(2) - Create time, low, visited stores to store result.

Step(3) - do dss

~~dss()~~ {
~~time[curr] = timer[0];~~
~~low[curr] = timer[0];~~
~~timer[0]++;~~
~~for (in~~

next page

```
dfs() {
```

```
    visited[curr] = 1
```

```
    timer[curr] = timer[0];
```

```
    low[curr] = time[curr];
```

```
    timer[0]++;
```

```
    for (int neighbour : adj.get(curr)) {
```

```
        if (visited[neighbour] == 0) {
```

```
            dfs(adj, visited, neighbour, timer, low, res, curr)
```

```
            low[curr] = Math.min(low[curr], low[neighbour]);
```

```
        } else if (low[neighbour] > time[curr]) {
```

```
            res.add(new ArrayList<>(Arrays.asList(  
                curr, neighbour)));
```

```
}
```

```
else
```

```
{
```

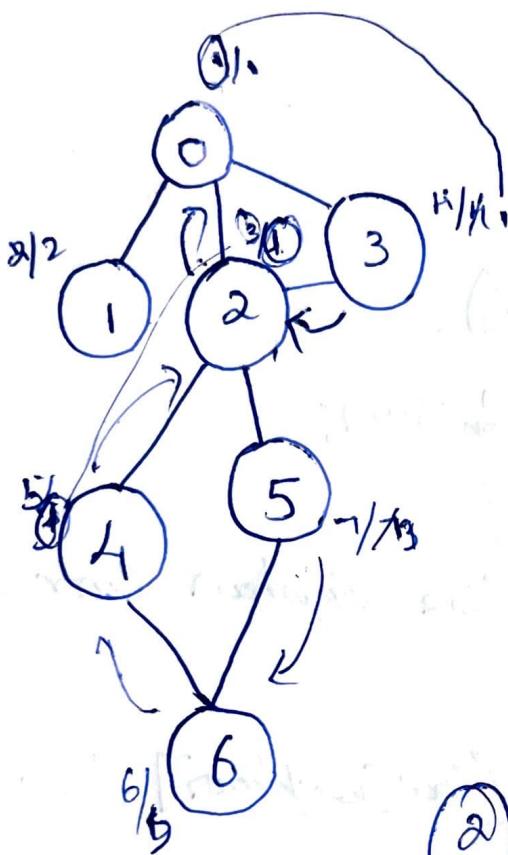
```
    low[curr] = Math.min(time[neighbour],  
                         low[curr]);
```

```
}
```

```
3
```

```
4
```

Articulation point:



Nodes on whose removal in the graph brakes into multiple Components.

time[i] - store the time of insertion
on doing DFS

Low[i] - min of all adjacent nodes apart from Parent & visited nodes.

if (Low[neighbours] >= time[curr] && parent != -1)

{



dfs()

{ visited[curr] = true;

low[curr] = timer[0];

time[curr] = timer[0];

timer[0]++;

for (int neighbour : adj.get(curr)) {

if (neighbour == parent) continue;

if (visited[neighbour] == false) {

dfs(adj, visited, low, time, neighbour, curr, timer,
v0s);

low[curr] = Math.min(low[neighbour], low[curr]);

if (low[neighbour] >= time[curr] && parent != -1) {

res.add(curr);

y

child++;

y

else {

low[curr] = Math.min(time[neighbour], low[curr]);

if (child > 1 && parent == -1)

{ res.add(curr);

y y