

Recursion / Backtracking

arr → [3, 1, 2]

Print all Subsequence

→ A contiguous / non-contiguous sequence, which follows the order

arr → [3, 1, 2]

f(index, Σ)

{

if (index >= n)

{ print ([]);

return;

}

[].add(arr[i]);

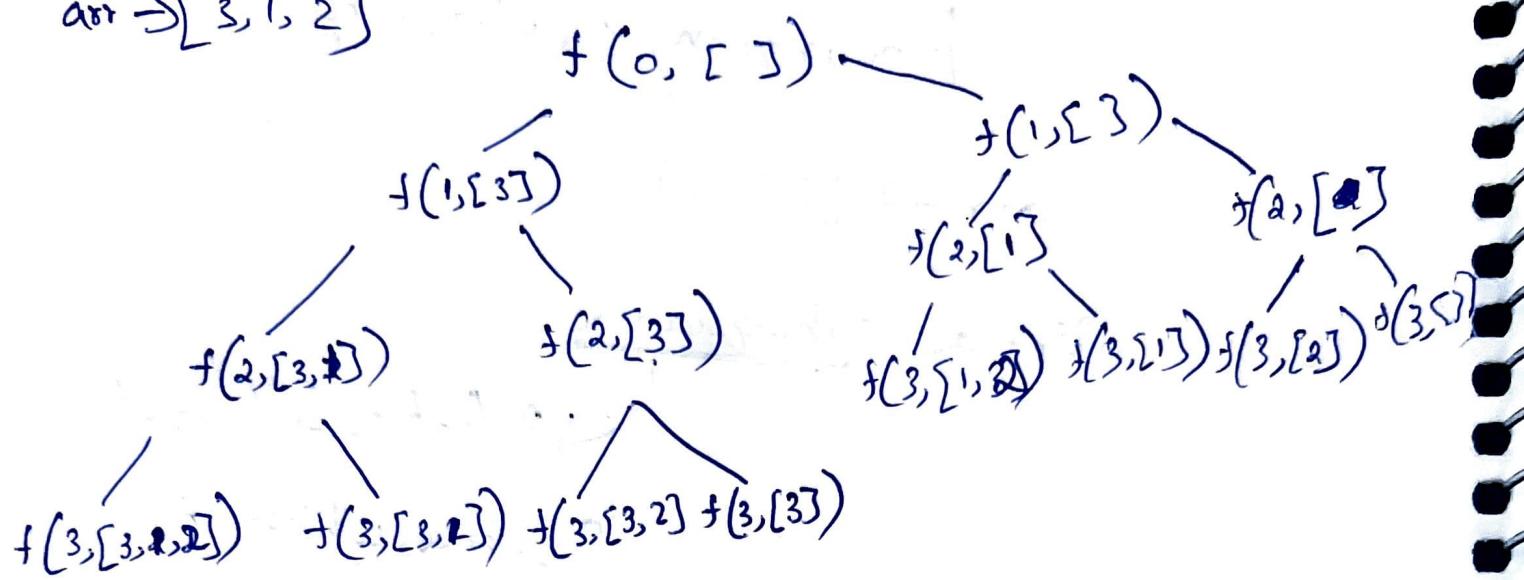
f(index + 1, []);

[].remove(arr[i]);

f(index + 1, [])

}

arr $\rightarrow [3, 1, 2]$



if $(index >= arr.length) \text{ print } C \}$

[3, 1, 2]	[1, 2]
[3, 1]	[1]
[3, 2]	[2]
[3]	[]



Java Code:

```
getsubsequence (index, arr, List<Integer> res) {  
    if (index >= arr.length)  
    {  
        System.out.println (res);  
        return;  
    }  
    res.add (arr [index]);  
    getsubsequence (res.size () + 1, arr, res);  
    res.remove (res.size () - 1);  
    getsubsequence (index + 1, arr, res);  
}
```

5

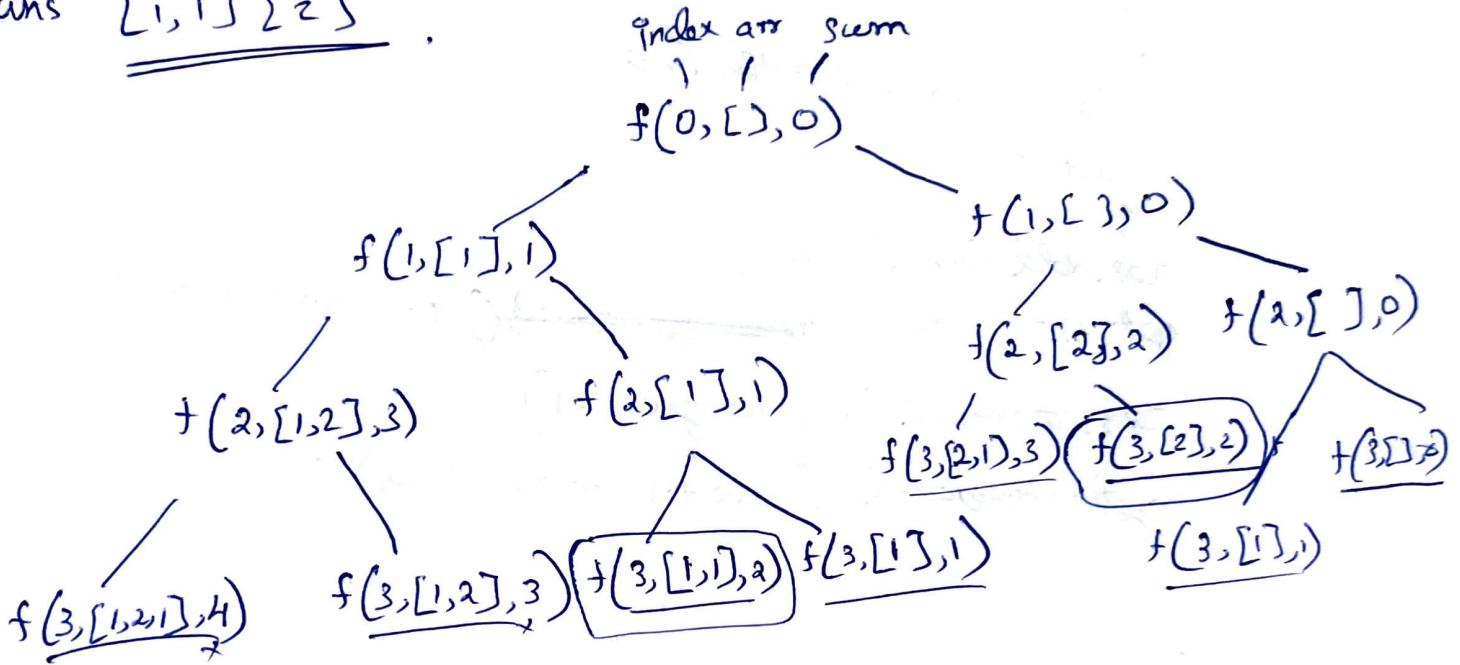


Print Subsequence whose sum is k

arr $\rightarrow [1, 2, 1]$ Sum = 2

take/not take.

ans $[1, 1] \{2\}$.



$[1, 1]$

$[2]$

$f(index, \{ \}, sum, k)$

? if (~~index~~ \geq index \geq arr.length)

{ if ($k == sum$) return $\{ \}$ }

}

{ } . add (arr[index]);
sum = ~~arr~~ [index];

$f(index + 1, \{ \}, sum, k)$

[] . remove (arr[index])

sum = ~~arr~~ [index];

$f(index + 1, \{ \}, sum, k)$

Print first subsequence whose sum is equals k.

```
getfirstsubsequence ( ) {
```

```
    if (index >= n) {
```

```
        if (k == sum)
```

```
            { cout (res); }
```

```
        return true;
```

```
    else
```

```
        { return false; }
```

```
}
```

```
// take //
```

```
    res.add (arr [index]);
```

```
    sum += arr [index];
```

```
    if (getfirstsubsequence ()) return true;
```

```
// no take //
```

```
    res.remove (res.size () - 1);
```

```
    sum -= arr [index];
```

```
    if (getfirst (index + 1)) return true;
```

```
    return false;
```

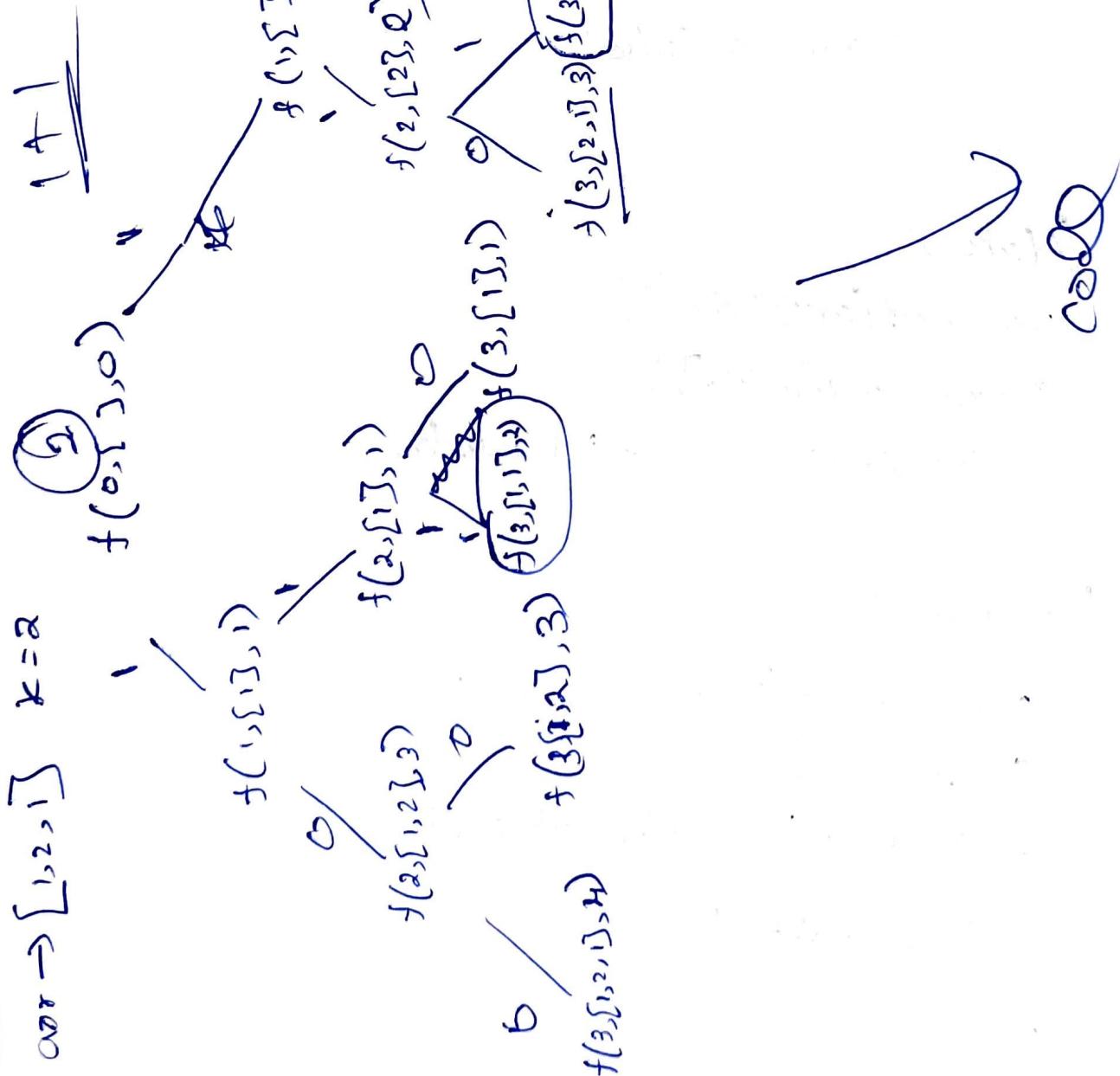
```
}
```

Count Subsequence sum = k

$[1, 2, 1]$

Output $\rightarrow 2$

$k = 2$.



Public static int getCount(int index, int[] arr, int k, int n, List<List<int>> res, int sum)

{ if(index >= n)

{ if(k == sum)

{ return 1;

else {

return 0;

}

}
res.add(arr[index]);

sum += arr[index];

int left = getCount(index+1, arr, k, n, res, sum);

res.remove(res.size()-1);

take

sum -= arr[index];

int right = getCount(index+1, arr, k, n, res, sum);

not take

return left + right;

y

Merge Sort

(1, 1, 2, 3, 4, 5, 6)

[3, 1, 2, 4, 1, 5, 2, 6, 4]

follows
some pattern

5 ↘ (1, 1, 2, 3, 4)

[3, 1, 2, 4, 1]

(2, 4, 5, 6) ↘

[5, 2, 6, 4]

(1, 2, 3) ↖

[3, 1, 2]

↘ (6, 4)

[4, 1]

(1, 3) ↖ (2)

[3, 1]

[2]

(4) ↖ (1)

[4]

[1]

(2, 5) ↖

[5, 2]

(5) ↖ (2)

[5]

(2) ↖ (3)

[2]

(4, 6) ↖

[6, 4]

(6) ↖ (4)

[6]

[4]

[3] [1]

{ } [1, 2]

Time Complexity : $N \log N$

Space Complexity : $\alpha(N)$

merge sort :- Pseudocode

mergeSort (arr, low, high)
{
 if (low == high) return;
 mid = (low + high)/2;

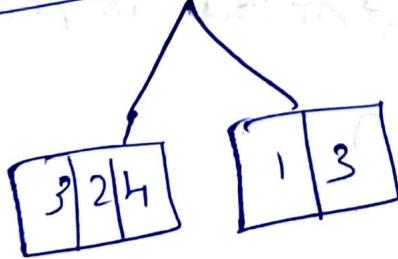
 mergeSort (arr, low, mid);

 mergeSort (arr, ~~high~~, mid+1, high);

 merge (arr, low, mid, high);

}

0	1	2	3	4
3	2	4	1	3

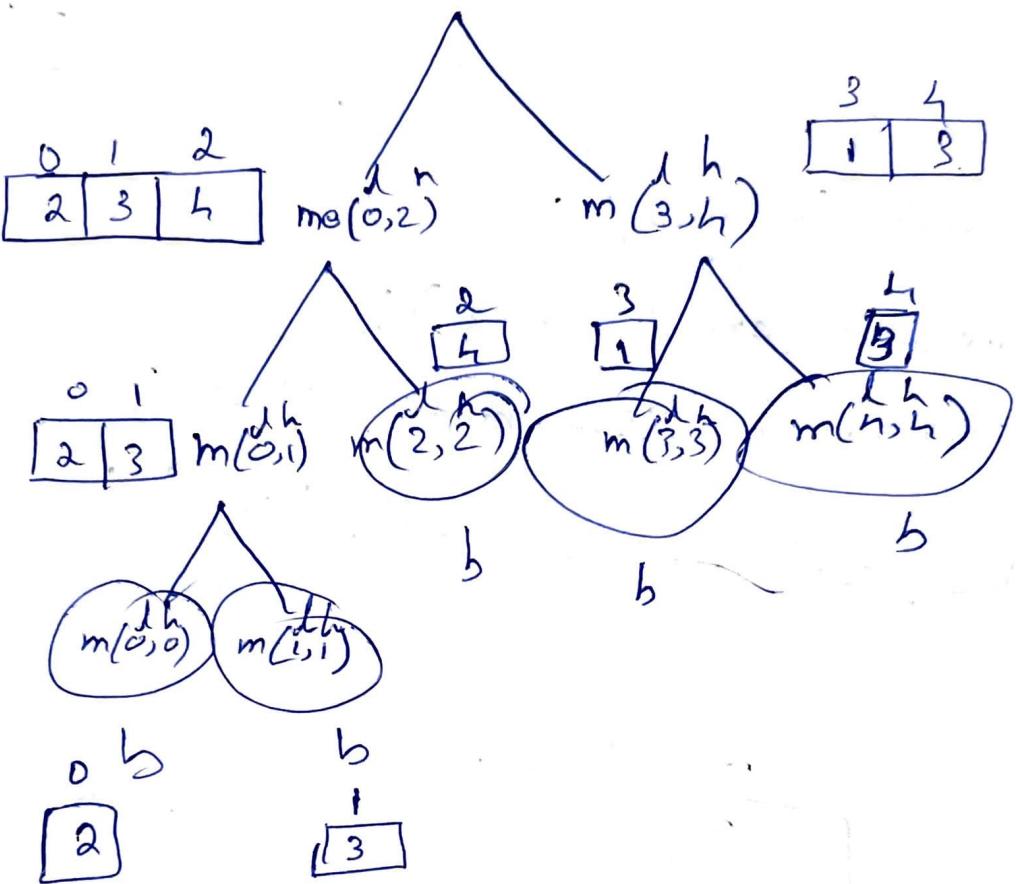


$$\begin{aligned}l &= 0 \\h &= 4 \\m &= 2\end{aligned}$$



0	1	2	3	4
2	3	4	1	3

mergesort($0, 4$) $\quad \begin{array}{|c|c|c|c|c|} \hline 0 & 1 & 2 & 3 & 4 \\ \hline 2 & 3 & 4 & 1 & 3 \\ \hline \end{array}$



merge ($\text{int } [] \text{ arr}, \text{int low, int mid, int high}$) {

$\text{int temp} = \text{new } []$

$i = 0$; p

~~$i = j = low$~~

~~$j = mid + 1$~~ : high

while ($p <= \text{mid}$ && $j <= \text{high}$) {

if ($\text{arr}[i] <= \text{arr}[j]$)

{ $\text{temp}[p] = \text{arr}[i]$ }

$i++$;

y

```
    }  
else {  
    temp[t++] = arr[i];  
    i++;  
}  
}  
  
while (i <= mid) {  
    temp[t++] = arr[i];  
    i++;  
}  
while (j <= high) {  
    temp[t++] = arr[j];  
    j++;  
}  
  
for (int k = 0; k < temp.length; k++) {  
    arr[low + k] = temp[k];  
}  
}
```

Java Code :

```
Public static void mergesort (int [ ] arr, int low, int high) {
```

```
if (low == high) return;
```

```
int mid = (low+high)/2;
```

```
mergeSort (arr, low, mid);
```

```
mergeSort (arr, mid+1, high);
```

```
merge (arr, low, mid, arr);
```

```
}
```



```
Public static void merge (int [ ] arr, int low, int mid,  
int high) {
```

```
int [ ] temp = new int [high - low + 1];
```

```
int t = 0;
```

```
int i = low;
```

```
int j = mid + 1;
```

```
while (i <= mid && j <= high) {
```

```
if (arr[i] <= arr[j]) {
```

```
temp[t++] = arr[i];
```

```
i++;
```

```
}
```

```
else {
```

```
temp[t++] = arr[j];
```

```
j++;
```

```
,
```

```
y
```

```
while ( $i \leq mid$ ) {
```

```
    temp [ $t++$ ] = arr [ $i$ ];
```

```
     $i++$ ;
```

```
}
```

```
while ( $j \leq high$ ) {
```

```
    temp [ $t++$ ] = arr [ $j$ ];
```

```
     $j++$ ;
```

```
}
```

```
for ( $int k=0; k < temp.length; k++$ ) {
```

```
    arr [ $(count+k)$ ] = temp [ $k$ ];
```

```
}
```

```
}
```

Quick Sort

Time Complexity: $O(N \log N)$
Space Complexity: $O(N)$

arr [] = $\left[\begin{matrix} P \\ 4, 6, 2, 5, 7, 9, 1, 3 \end{matrix} \right]$

QDR = $\left[\begin{matrix} 2, 3, 4 \end{matrix} \right]$

arr [] = $\left[\begin{matrix} P \\ 1, 6, 2, 5, 7, 9, 1, 3 \end{matrix} \right]$

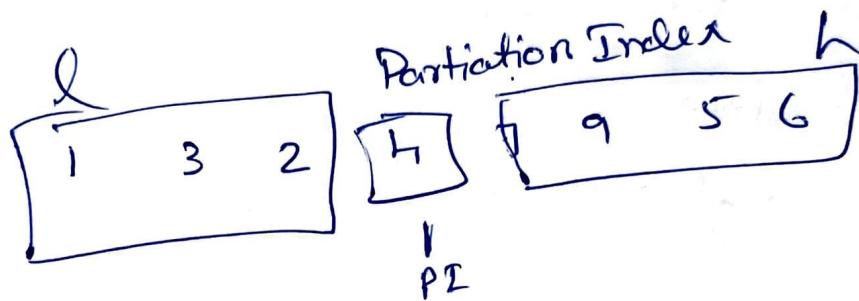
1. Take a pivot & place it in its correct place in its sorted array.

2. Smaller on the left and larger on the right



100
 |
 4 6 2 5 7 9 1 3 8
 ↓
 P=4

Pivot = arr[low]



Pseudocode:

quicksort(arr, low, high)

{ if (low < high)

{ PartitionIndex = f (arr, low, high);

quicksort (arr, low, partitionIndex);

quicksort (arr, partitionIndex + 1, high);

}
y
y



int f (arr, low, high)

{

1 3 2 4 7 9 5 6

pivot = arr[low],

i = low;

j = high;

while (i < j)

{

while (arr[i] <= arr[pivot] ~~&&~~ i <= high)

{

i++;

}

while (arr[j] >= arr[pivot] ~~&&~~ i >= low)

{

j--;

}

if (i < j) swap(arr[i], arr[j]);

}

swap(arr[low], arr[j]);

return j;

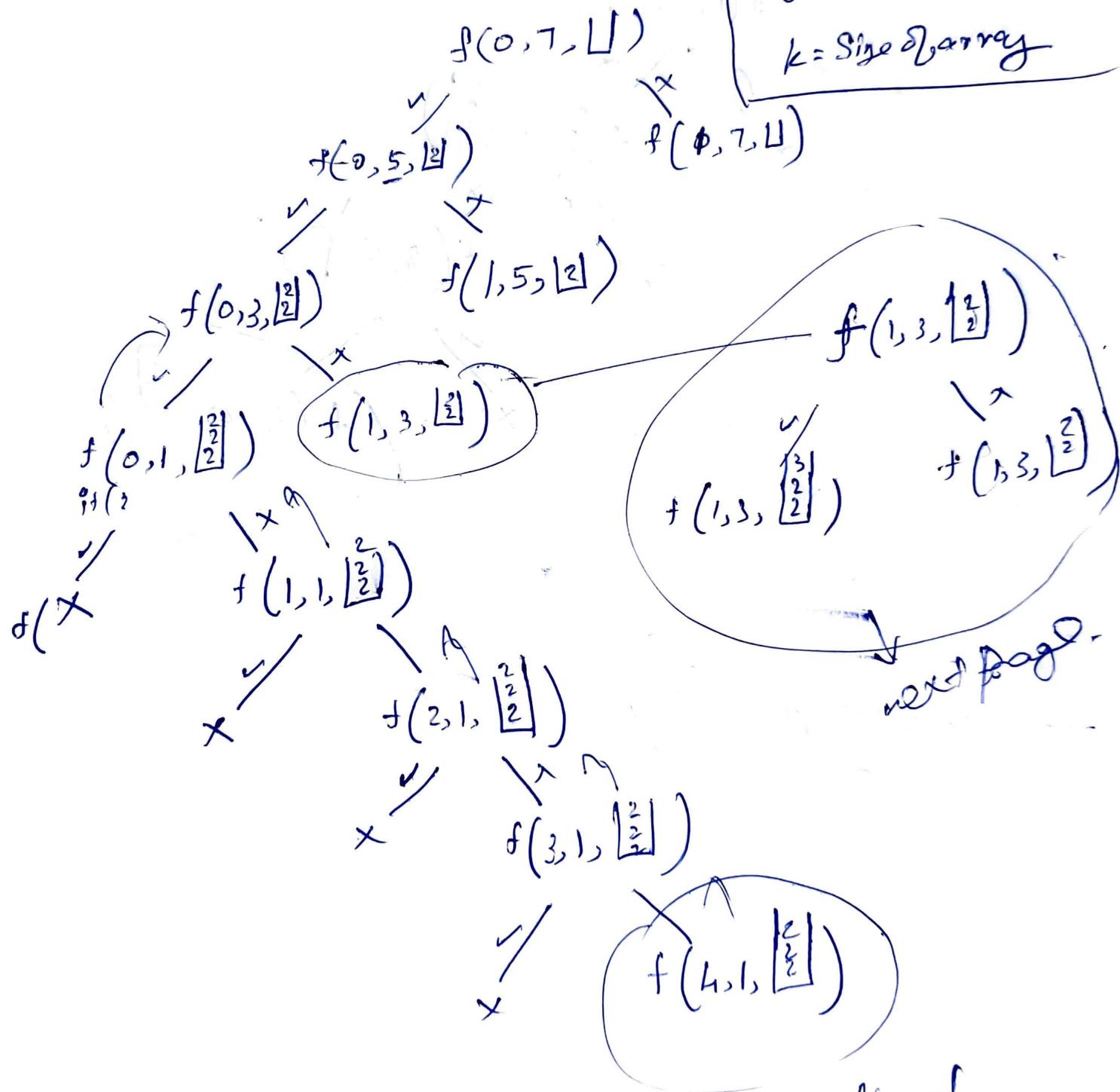
}

Combination Sum

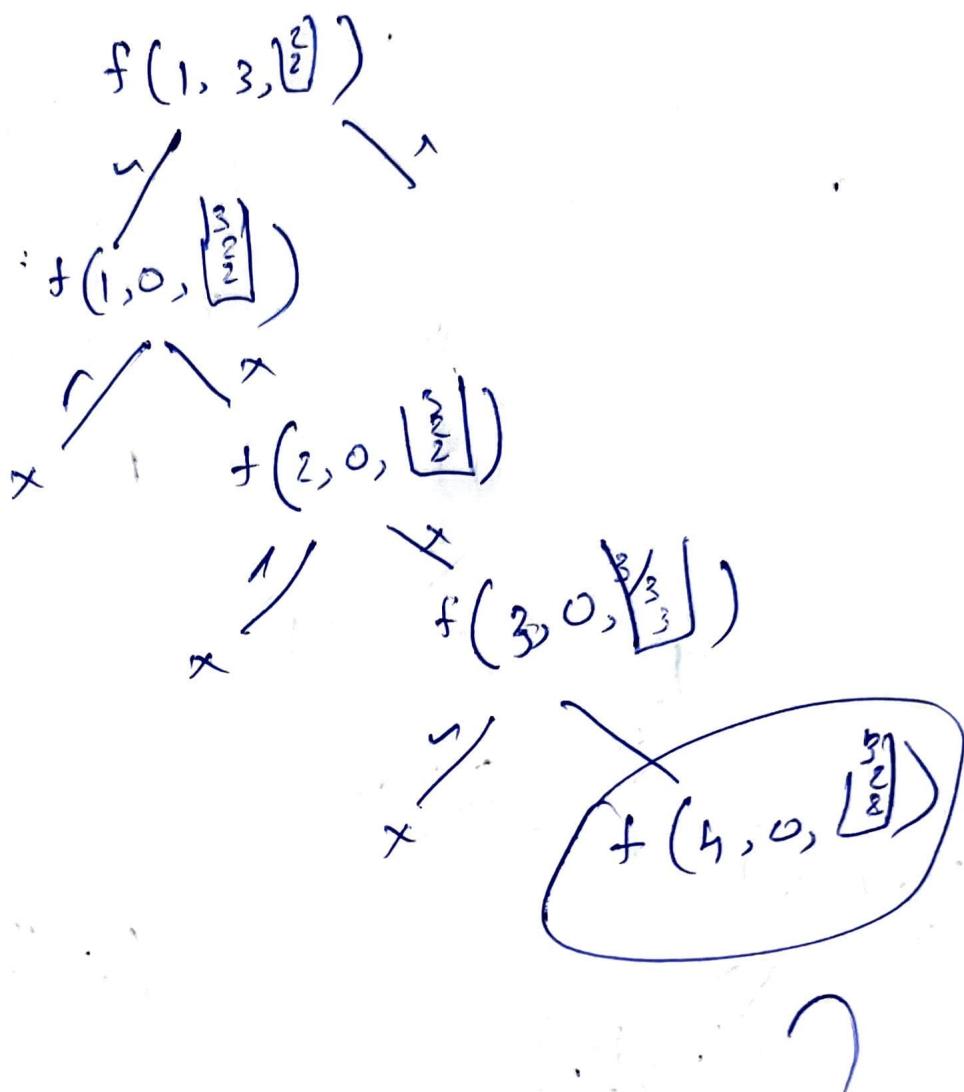
$$TC = 2^t \times k$$

$arr = \{2, 3, 6, 7\}$ target = 7

$t = \text{take/notake}$
 $k = \text{Size of array}$



This combination is not valid.



$$res = \left\{ \sum_{i=1}^3 \right\}$$

$f(index, target, ds)$

$ds.remove(ds.size() - 1)$

✓ $ds.add(a[index])$ ✗

$f(index, target - a[index], ds)$

$f(index + 1, target, ds)$



$\text{if } (a[index] \leq target)$

combinationsum(i, arr, target, list, res) {

 if (index >= arr.length) {

 if (target == 0)

 { res.add(new ArrayList<Integer>(list));

 return;

 }

}

// take

 if (arr[index] <= target) {

 list.add(arr[index]);

 getcombination(index, arr, target - arr[index], list, res);

 list.remove(list.size() - 1);

}

of Notake

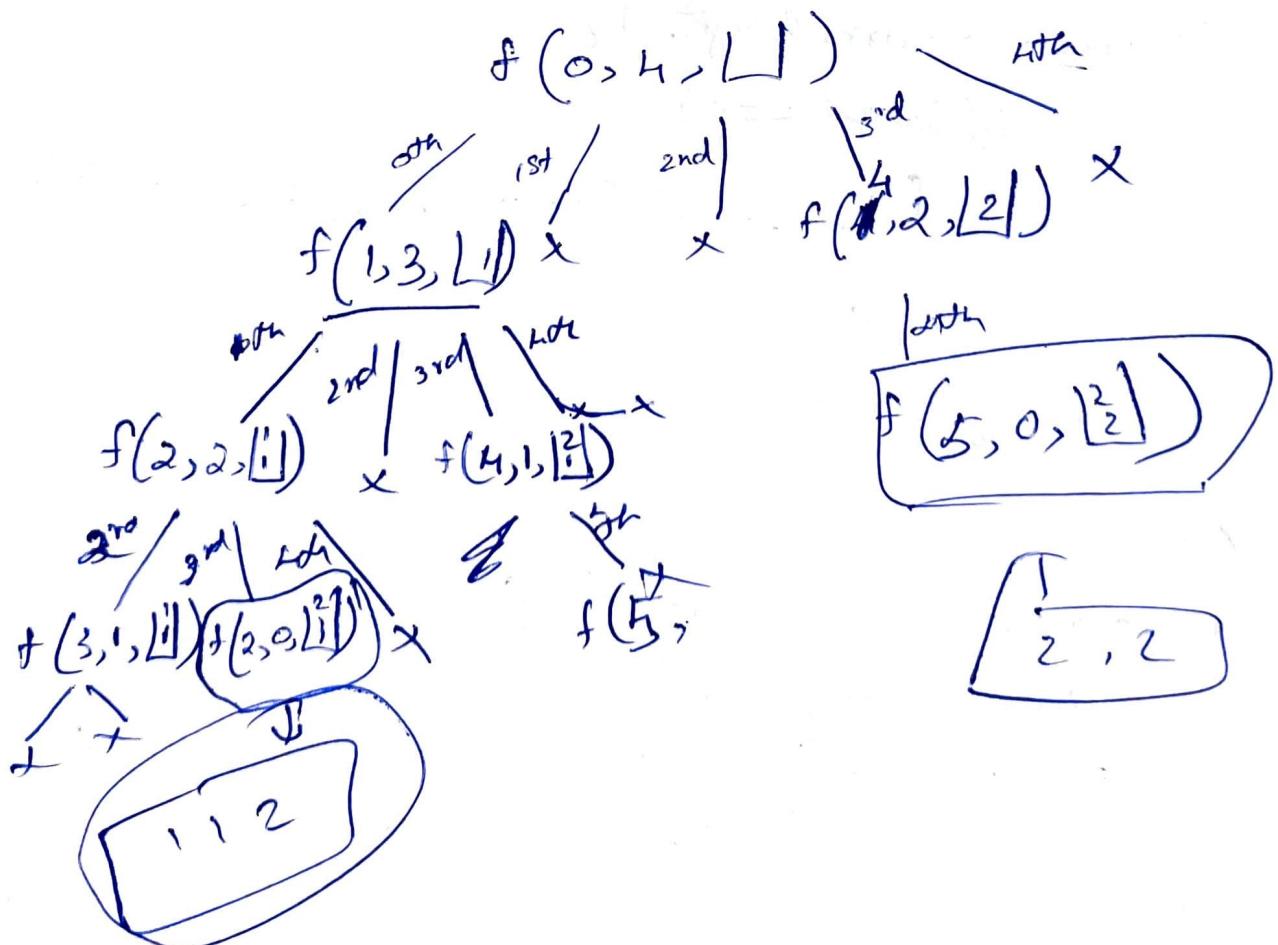
get Combination Sum (index+1, arr, target, last, res);

3

Combination Sum - II

arr: $\downarrow [1, 1, 1, 2, 2]$ target = 7

$\boxed{[1, 1, 2] \quad [2, 2]}$



getCombination (int i, int arr[], int target, List<List> res) {

if (target == 0)

{
res.add (new ArrayList<>(list));
return;
}

for (int i = index; i < arr.length; i++)

{
if (i > index && arr[i] == arr[i - 1]) continue;
if (target < 0 || arr[i] > target) break;

list.add (arr[i])

getCombination (i + 1, arr, target - arr[i], list, res)

list.remove (list.size() - 1);

}

}

Subset

Brute force approach - Power Set.

↓

Time Complexity $2^h \times n$

Print all subset

$n=3$ $\text{nums} = [1, 2, 3]$

$n=2^3 = 8$

$\boxed{[], [1], [2], [3], [1, 2], [1, 3], [2, 3], [1, 2, 3]}$

↓
 2^h

$\frac{2}{0} \quad \frac{1}{0} \quad \frac{0}{0}$ — $[]$

$0 \quad 0 \quad 1$ — $[1]$

$0 \quad 1 \quad 0$ — $[2]$

$0 \quad 1 \quad 1$ — $[1, 2]$

$1 \quad 0 \quad 0$ — $[3]$

$1 \quad 0 \quad 1$ — $[1, 3]$

$1 \quad 1 \quad 0$ — $[2, 3]$

$1 \quad 1 \quad 1$ — $[1, 2, 3]$

Solution

$$\boxed{n=3} \quad 2^n = 2^3 = 8 \quad \text{ans} = []$$

for ($\text{num} = 0 \rightarrow \cancel{\text{sum}}(n-1)$)
 $\quad \quad \quad \text{list} = []$

{
 for ($i = 0 \rightarrow n-1$)

{
 if ($\text{num} \& (1 << i)$)
 list \leftarrow $\text{add}(\text{num} \ll i)$

ans.add(list);

}
return ans.

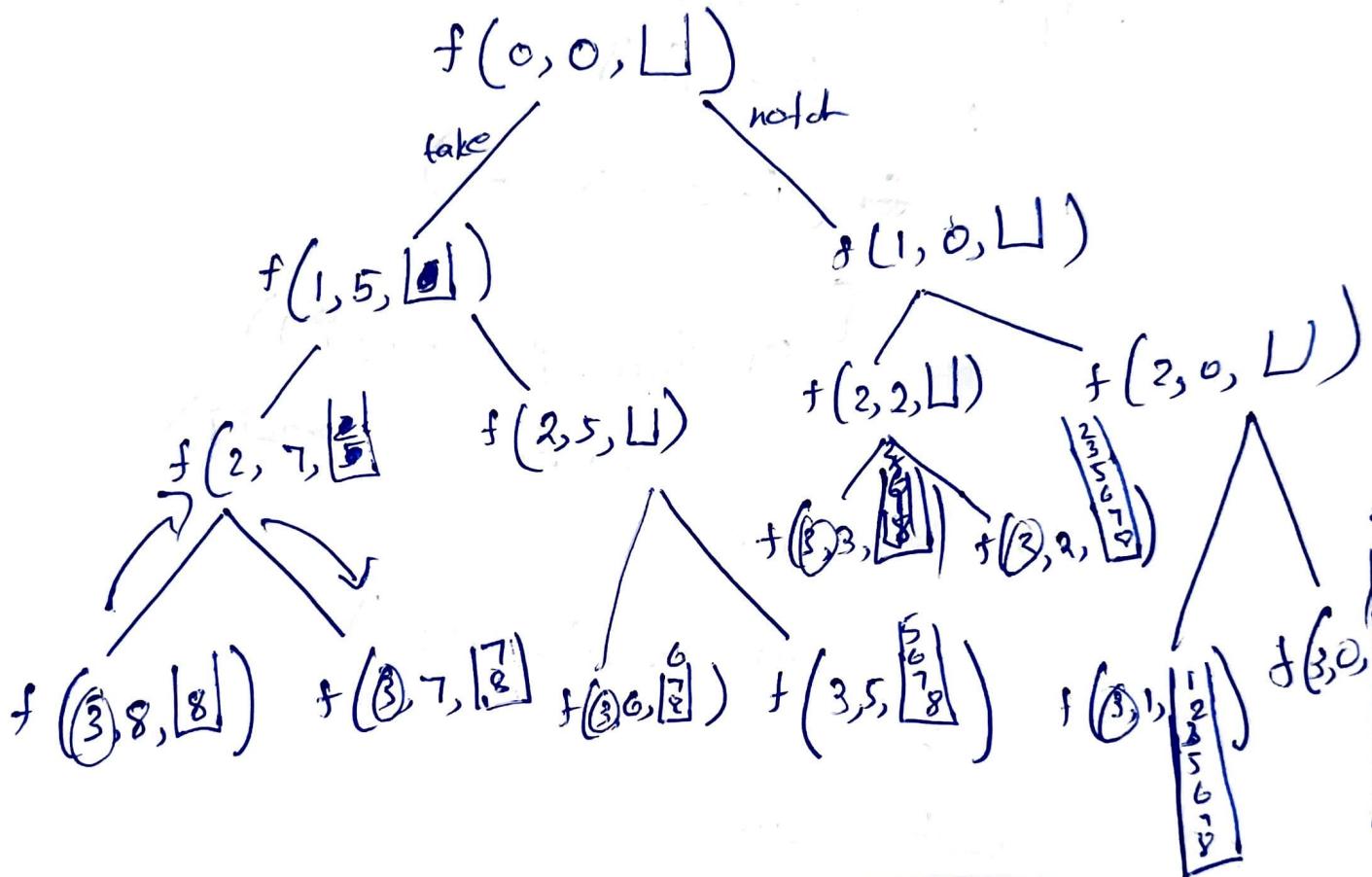
$$\text{Time Complexity} = 2^n \times n$$

$$\text{Space Complexity} = \approx 2^n \times n$$

Subset - I

num

$$\text{num} = [5, 2, 1]$$



Output: $[0, 1, 2, 3, 5, 6, 7, 8]$

↓
Subset Sum arr
Subtree

Public static getsumlist (index, nums, res, sum)

{

if (index >= num.length)

{

res.add (sum);

return

}

// take

SumList (index + 1, nums, res, sum + nums[index])

// no take

SumList (index + 1, nums, res, sum);

}.

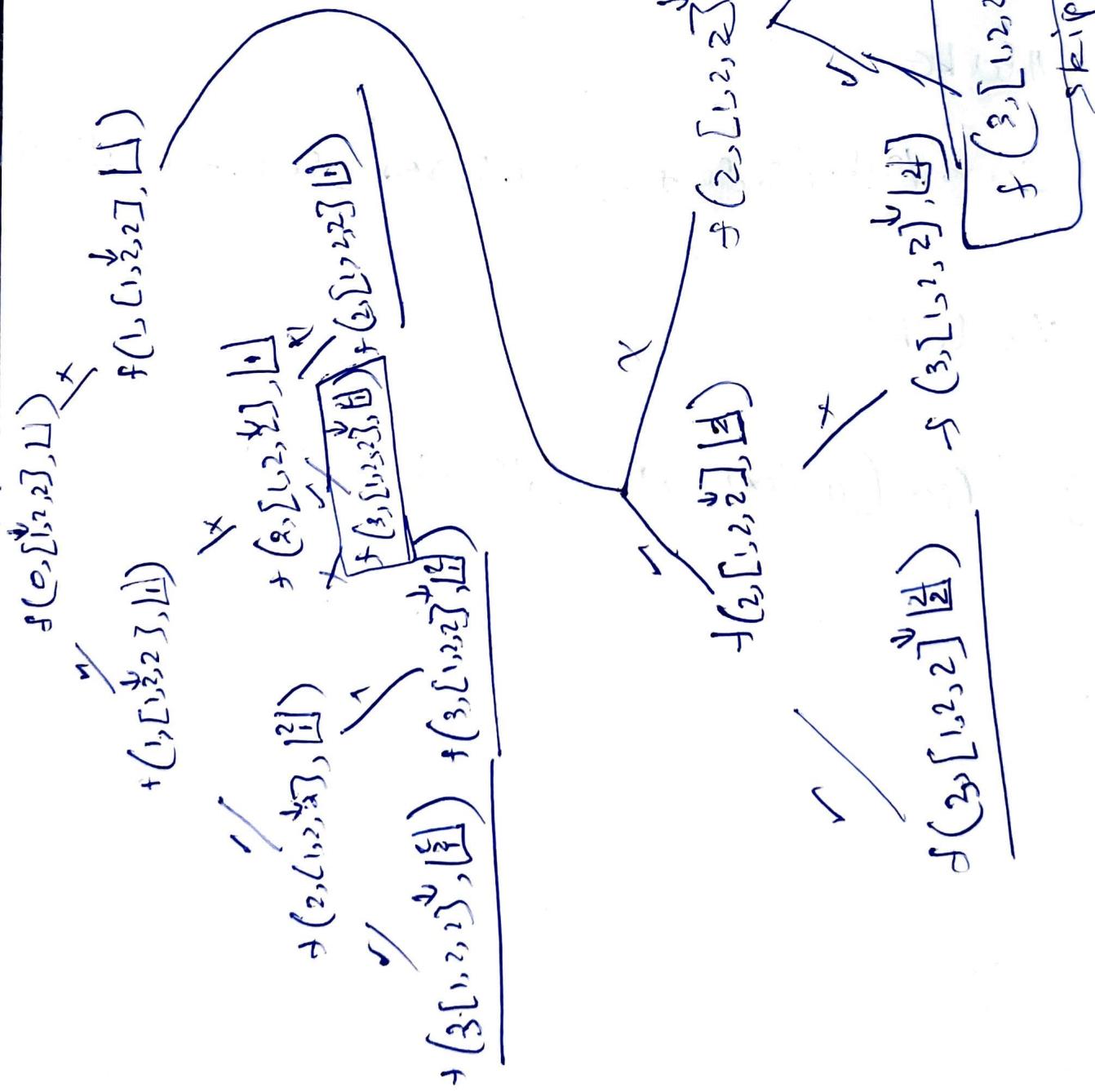


Subset - II

$\text{arr} = [1, 2, 2]$

Brute force - $T.C. = 2^n \times n \rightarrow$ storing subsets in a list
 for all generating subsets

Use set to avoid duplicates



Set res = $\left[[1, 2, 2], [1, 2], [1], [2, 2], [2], [] \right]$

to avoid duplicates.

+ In this approach first we are generating all the subsets in them we are skipping the duplicates.

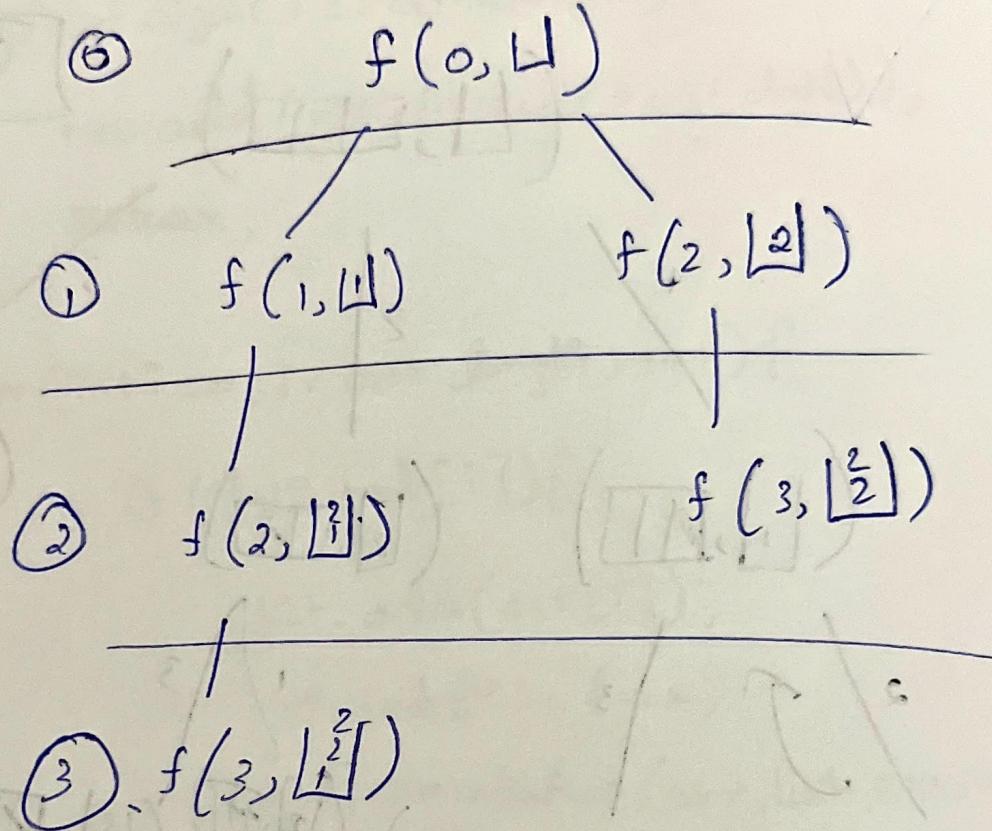
+ After that converting Set <list> to List<List>

```
getsubset (i, [], list, res) {
    if (i >= nums.length) {
        res.add (new ArrayList<List> (list));
        return;
    }
    // take
    list.add ();
    get (i+1, (i), list, res);
    list.remove ();
    // not take
    get (i+1, (i), list, res);
```

Optimal approach

$$arr = [1, 2, 2] \text{ res} = [3,$$

Code:



Code

```
getsubsetSum (index, nums, list, res) {  
    res.add (new ArrayList<> (list));  
    for (int i = index; i < nums.length; i++) {  
        if (i > index && nums[i] == nums[i - 1]) continue;  
        list.add (nums[index]);  
        getsubset (i + 1, nums, list, res);  
        list.remove (list.size() - 1);  
    }  
}
```

Permutation:

number of permutation

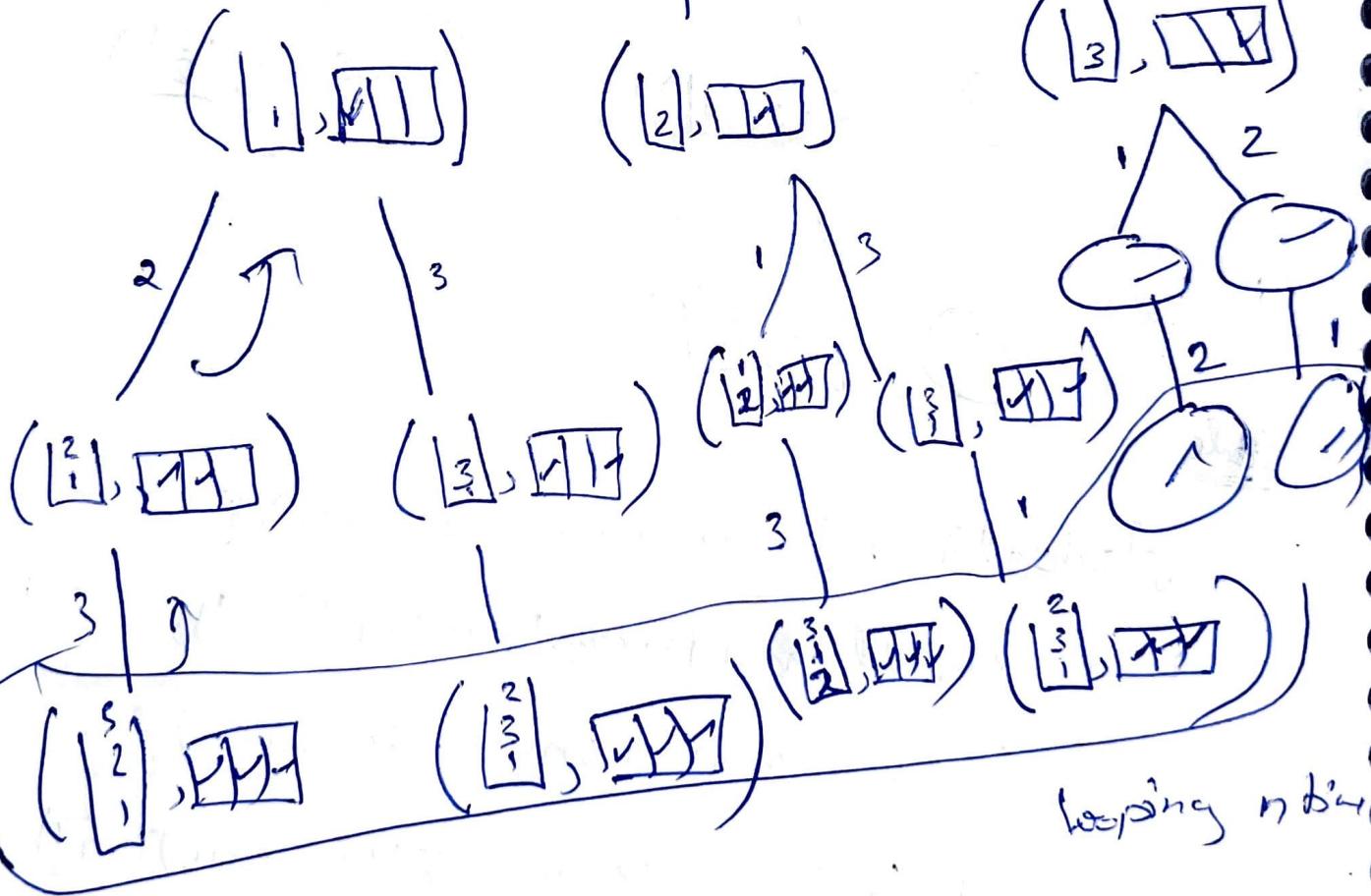
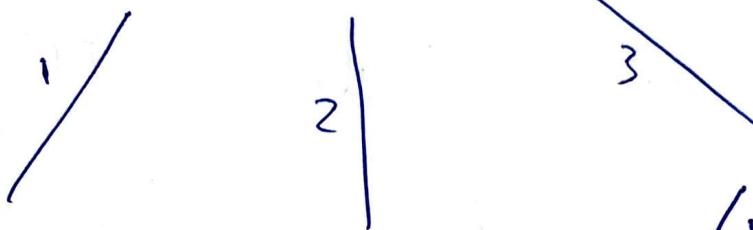
Brute force

$[1, 2, 3]$

$n!$

6 Perm

$([1], \boxed{\quad \quad})$



keeping n^b

Time Complexity = $n! \times n$

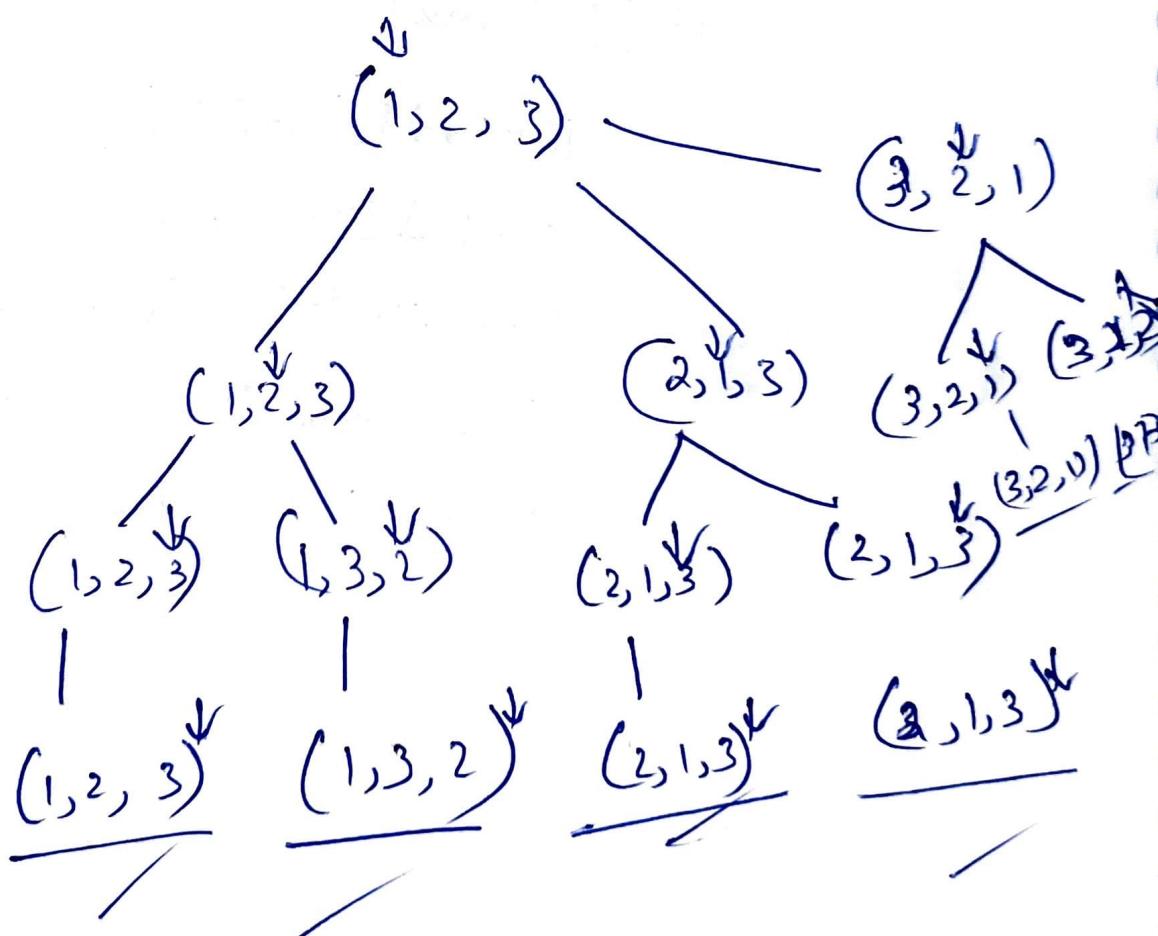
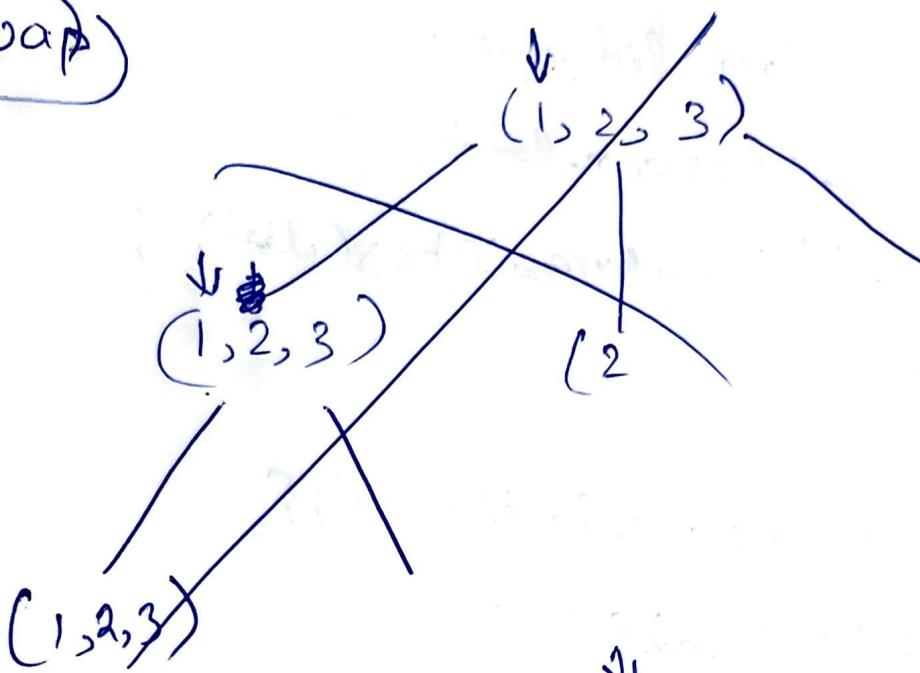
Space Complexity = $O(n) + O(n)$

Permutation - code - Approach ①.

```
getpermutation (arr, list, res, visited) {  
    if (list.size() == arr.length) {  
        res.add (new ArrayList<T>(list));  
        return;  
    }  
    for (int i = 0; i < arr.length; i++) {  
        if (!visited[i]) {  
            list.add (arr[i]);  
            visited[i] = true;  
            getpermutation (arr, list, res, visited);  
            list.remove (list.size() - 1);  
            visited[i] = false;  
        }  
    }  
}
```

~~Permutation~~ Permutation - II Approach ②.

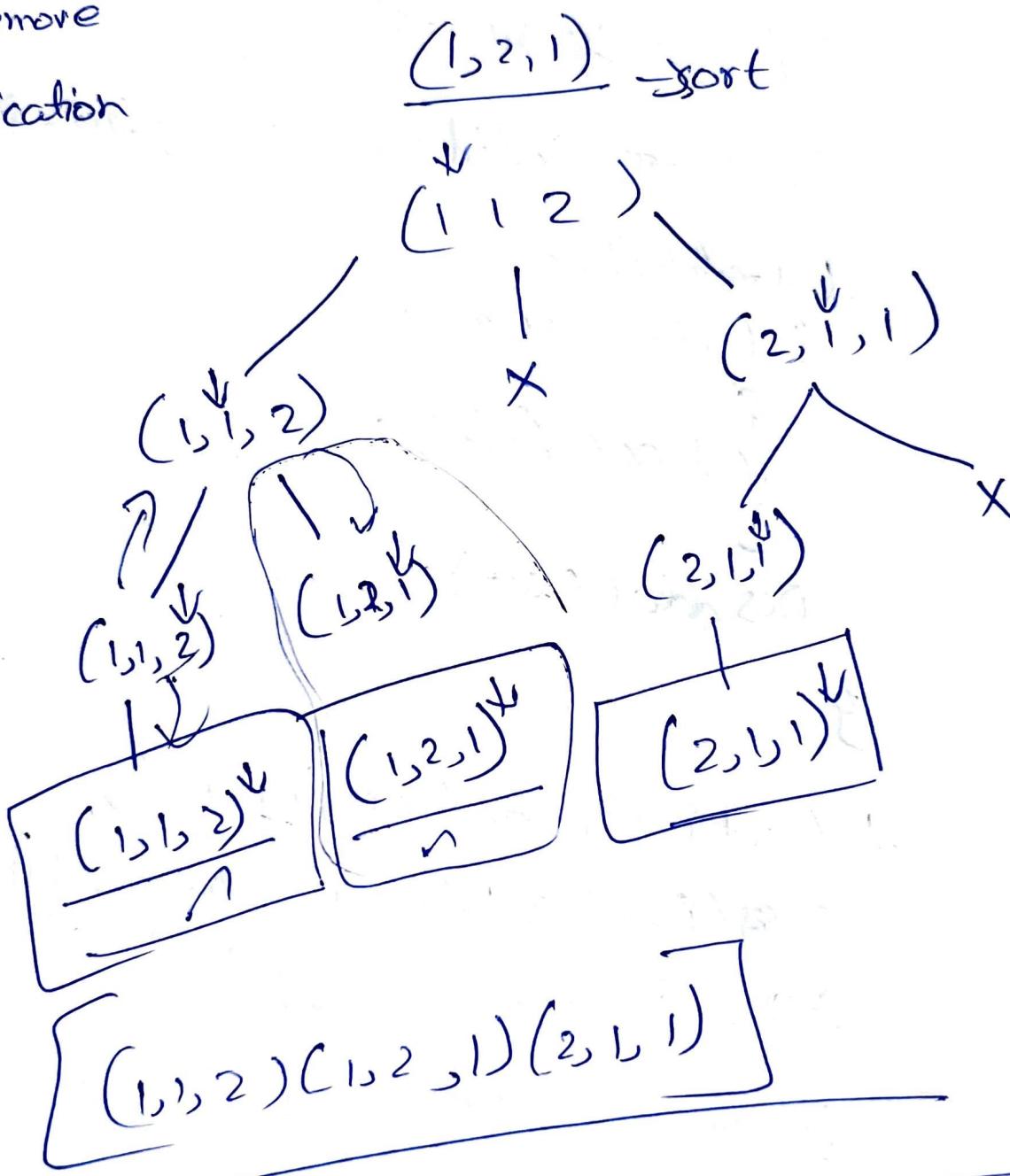
Swap



Permutation/Combination - II

~~(1, 2, 1)~~ ~~(1, 1, 2)~~ $\text{arr} = [1, 2, 1]$

To remove
duplication



If $(\text{arr}[i] > \text{index}$ & $\text{arr}[i] \in \text{arr}[0:i])$ = arr[0:i] (Container)

Arry. Sort(num);

get Combination (index, nums, res);

{ n = len(nums)

if (index >= len(nums)) {

List<int> list = new ArrayList<int>();

list.add(Ans)

for (int ele : nums)

{ list.add(ele) ;

}

res.addAll(list);

}

for (int i = index; i < n; i++) {

if (i > index && nums[i] == nums[i - 1]) continue;

Scalp(index + i, nums);

get Combination(index + i, nums, res);

Scalp(index, i, nums);

}

}

N - Queens

$n = 4$

