Phase Modulation

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***Abstract*—Phase modulation (PM) is a crucial signal processing technique widely employed in modern communication systems, including wireless, optical, and satellite communications. It offers several advantages, such as improved spectral efficiency, enhanced robustness against noise and interference, and efficient power utilization. Unlike amplitude modulation (AM) or frequency modulation (FM), PM encodes information by varying the phase of a carrier signal, making it less susceptible to amplitude variations caused by channel impairments.**

This paper presents a comprehensive study of phase modulation, covering its theoretical principles, mathematical modeling, and practical applications. A MATLAB-based implementation is carried out to simulate and analyze phase modulation in detail. The study involves the generation and visualization of various signal characteristics, including the original message signal, high-frequency carrier signal, and the resultant phase-modulated waveform. Additionally, spectral characteristics such as the power spectral density (PSD) and frequency spectrum are examined to assess the impact of modulation parameters on bandwidth efficiency and spectral occupancy. The findings of this study provide valuable insights into the trade-offs associated with phase modulation, particularly in terms of bandwidth efficiency, noise resilience, and implementation complexity. The results from MATLAB simulations confirm the theoretical expectations and offer practical guidelines for optimizing phase modulation for enhanced performance in real-world applications.

***Index Terms*—Phase Modulation (PM), MATLAB Simulation, Power Spectral Density (PSD), Fast Fourier Transform (FFT), Frequency Spectrum Analysis, Wireless Communication, Signal Processing, Spectral Efficiency, Coherent and Non-Coherent Demodulation, Additive White Gaussian Noise (AWGN), Multipath Fading, Adaptive Communication Systems.**

I. INTRODUCTION

Phase Modulation (PM) is a fundamental digital and analog modulation technique widely used in modern wireless communication systems, radar applications, and signal processing. It operates by varying the phase of a carrier signal in response to the instantaneous amplitude of the message signal, offering advantages such as improved spectral efficiency, robustness against noise, and efficient bandwidth utilization. Unlike amplitude-based modulation techniques, PM is resilient to signal fading and non-linear distortions, making it suitable for environments with high interference and multipath propagation.

PM plays a crucial role in various applications, including digital phase-shift keying (PSK) and minimum-shift keying (MSK), which are extensively utilized in next-generation wireless networks and satellite communication. The ability of PM to maintain phase continuity enables its integration into adaptive communication systems, ensuring optimal performance under varying channel conditions.

This paper presents a comprehensive study of Phase Modulation, focusing on its theoretical background, mathematical formulation, and practical implementation using MATLAB simulations. The paper provides an in-depth analysis of key parameters such as power spectral density (PSD), Fast Fourier Transform (FFT), frequency spectrum, and phase noise, which are crucial for understanding the modulation's impact on signal integrity and bandwidth efficiency.

Through MATLAB-based simulations, we visualize and analyze the message signal, carrier signal, modulated waveform, and PSD, offering insights into PM's spectral characteristics. Furthermore, the demodulation techniques for recovering the original signal are explored, comparing coherent and non-coherent methods and their effectiveness in different noise conditions.

The remainder of this paper is structured as follows:

- **Section II** provides the general assumptions and notations used in the study, including simulation constraints and system parameters.
- **Section III** presents the theoretical background and mathematical formulation of Phase Modulation.
- **Section IV** discusses the MATLAB-based simulation framework, including signal generation and spectral analysis.
- **Section V** elaborates on demodulation techniques and their practical implementation.
- **Section VI** presents the results and performance analysis.
- **Section VII** concludes the paper with key findings and potential future research directions.

II. GENERAL ASSUMPTIONS AND NOTATIONS

To ensure a consistent mathematical framework for the analysis of Phase Modulation (PM) and its demodulation, the following assumptions and notations are defined.

A. General Assumptions

- The simulation and analysis are performed using **MATLAB**.
- The carrier signal is assumed to be a high-frequency sinusoidal wave represented as:

$$c(t) = A_c \cos(2\pi f_c t) \tag{1}$$

  where A_c is the carrier amplitude and f_c is the carrier frequency.

- The message signal $m(t)$ is a low-frequency baseband signal with a finite bandwidth B_m, satisfying $B_m \ll f_c$.
- The modulation index is assumed to be within a range ensuring the stability of the transmitted signal.
- The transmission channel is initially considered to be an Additive White Gaussian Noise (AWGN) channel for theoretical analysis, but fading channels (e.g., Rayleigh and Rician fading) are considered for performance evaluation.
- Power Spectral Density (PSD), Frequency Spectrum, and Fast Fourier Transform (FFT) are used for spectral analysis of the PM signal.
- The demodulation process assumes a coherent receiver, where the carrier phase information is known. The primary demodulation techniques considered include:
  - Differentiation and Envelope Detection: Extracting the instantaneous frequency and retrieving the baseband signal.
  - Phase-Locked Loop (PLL) Demodulation: Tracking the phase variations of the received signal to recover $m(t)$.

B. Notation

To facilitate a clear and concise representation of mathematical formulations, the following notations are used:

TABLE I
SUMMARY OF NOTATIONS

Symbol	Description
A_c	Carrier amplitude
f_c	Carrier frequency
$m(t)$	Message signal
$\phi(t)$	Instantaneous phase
k_p	Phase sensitivity constant
h	Modulation index
σ_ϕ^2	Phase error variance
B_{PM}	Bandwidth of phase modulated signal
P_s	Signal power
P_n	Noise power
SNR	Signal-to-noise ratio
$S_x(f)$	Power spectral density (PSD)
$X(f)$	Fourier Transform of $x(t)$
PLL	Phase-Locked Loop

These assumptions and notations provide a foundation for the theoretical analysis, MATLAB-based simulation framework, and experimental validation of phase modulation and demodulation techniques.

III. Theoretical Background and Mathematical Formulation

Phase modulation (PM) is based on the principle of varying the phase of a carrier signal in accordance with the instantaneous amplitude of the modulating signal. The general form of a phase-modulated signal is given by:

$$s(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{2}$$

where:

- A_c is the carrier amplitude,
- f_c is the carrier frequency,
- $\phi(t)$ is the instantaneous phase deviation induced by the modulating signal.

The instantaneous phase $\phi(t)$ is given by:

$$\phi(t) = k_p m(t) \tag{3}$$

where:

- k_p is the phase sensitivity constant (radians per volt),
- $m(t)$ is the modulating signal.

The bandwidth of a PM signal depends on the modulation index h, which is defined as:

$$h = \frac{\Delta\phi}{f_m} \tag{4}$$

where $\Delta\phi$ is the peak phase deviation, and f_m is the maximum frequency of the modulating signal.

Using Carson's Rule, the approximate bandwidth for PM can be estimated as:

$$B_{PM} \approx 2(\Delta f + f_m) \tag{5}$$

where Δf is the peak frequency deviation.

A. Modulation Index in PM, Signal-to-Noise Ratio (SNR), and Noise Performance

1) Modulation Index in PM: The modulation index h in phase modulation is a crucial parameter that determines the extent of phase deviation in response to the modulating signal. It is defined as:

$$h = \frac{\Delta\phi}{f_m} \tag{6}$$

where:

- $\Delta\phi$ is the peak phase deviation in radians,
- f_m is the frequency of the modulating signal.

The modulation index directly affects the bandwidth and spectral characteristics of the PM signal. Based on the value of h, PM can be classified into two cases:

Case 1: Narrowband Phase Modulation (NBPM)

- When $h \ll 1$, the phase deviation is small, and the signal spectrum is concentrated around the carrier frequency. This is useful for applications requiring minimal bandwidth expansion.

Case 2: Wideband Phase Modulation (WBPM)

- When $h \gg 1$, significant phase variations occur, leading to a wider spectrum. This is beneficial for applications such as frequency synthesis and spread spectrum communications.

2) Signal-to-Noise Ratio (SNR) and Noise Performance: In any communication system, noise is a critical factor affecting signal integrity. The performance of PM under noisy conditions is typically analyzed using the SNR, defined as:

$$\text{SNR} = \frac{P_s}{P_n} \tag{7}$$

where:

- P_s is the signal power,
- P_n is the noise power.

For PM signals, the effect of noise can be analyzed under different conditions:

Case 1: Additive White Gaussian Noise (AWGN)

- In an AWGN channel, phase noise can cause phase jitter, leading to degradation in demodulation accuracy. The phase error variance is given by:

$$\sigma_\phi^2 = \frac{1}{2 \cdot \text{SNR}} \tag{8}$$

which implies that higher SNR results in reduced phase noise impact.

Case 2: Multipath Fading Channels

- In wireless environments, PM signals experience multipath fading, where phase shifts occur due to varying path delays. The Rayleigh fading model is often used to characterize this effect.

IV. Simulation & Diagrams

A. Simulation Setup

To analyze the performance of Phase Modulation (PM), we implement a MATLAB-based simulation that generates a PM waveform, computes its frequency spectrum, and evaluates key performance metrics such as Power Spectral Density (PSD) and phase deviation. The simulation is structured as follows:

1) **Define Parameters:**
   - Carrier amplitude (A_c), carrier frequency (f_c), sampling rate, and phase sensitivity constant (k_p).
   - Modulating signal ($m(t)$)—typically a sinusoidal waveform or a random message signal.
2) **Generate Message and Carrier Signals:**
   - Define the message signal $m(t)$, which serves as the modulating waveform.
   - Generate the carrier signal:

$$c(t) = A_c \cos(2\pi f_c t) \tag{9}$$

3) **Generate PM Signal:**
   - Compute instantaneous phase:

$$\phi(t) = k_p m(t) \tag{10}$$

   - Form the PM waveform:

$$s(t) = A_c \cos(2\pi f_c t + \phi(t)) \tag{11}$$

4) **Compute Frequency Spectrum:**
   - Apply the **Fast Fourier Transform (FFT)** to analyze frequency components.
   - Plot the spectrum to observe bandwidth expansion based on modulation index.
5) **Power Spectral Density (PSD) Analysis:**
   - Compute the PSD to evaluate signal energy distribution across frequencies.

B. Block Diagram Representation

The PM process is visualized using the following block diagram:

$$\boxed{\text{Input Message Signal } m(t)}$$
$$\downarrow$$
$$\boxed{\text{Carrier Signal } A_c \cos(2\pi f_c t)}$$
$$\downarrow$$
$$\boxed{\text{Phase Modulator (Implements } \phi(t) = k_p m(t))}$$
$$\downarrow$$
$$\boxed{\text{Output PM Signal } s(t)}$$
$$\downarrow$$
$$\boxed{\text{Spectrum \& PSD Analysis}}$$

C. MATLAB Simulation Results

The MATLAB script generates the following results:

- **Message Signal** $m(t)$**:** The original modulating signal (see Fig. 1).
- **Carrier Signal** $c(t)$**:** The high-frequency carrier used in PM (see Fig. 2).
- **Time-domain waveform of PM signal** $s(t)$**:** The phase-modulated signal representation (see Fig. 3).
- **Frequency-domain spectrum using FFT:** Illustrates the frequency components of the PM signal (see Fig. 4).
- **Power Spectral Density (PSD):** Depicts signal energy distribution across different frequencies (see Fig. 5).

The generated simulation results are shown in the figures below:

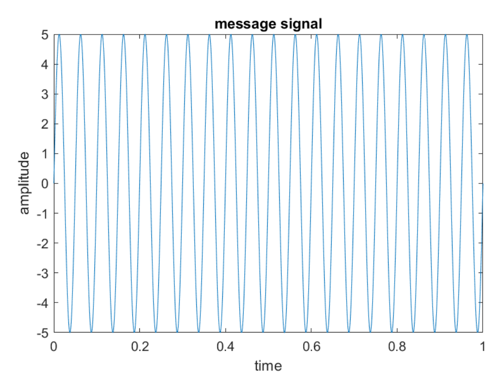

Fig. 1. Message Signal $m(t)$

As observed from the figures, the characteristics of the PM signal align with theoretical expectations, validating the modulation scheme. The time-domain representation of the message signal is shown in Fig. 1, while the carrier wave used

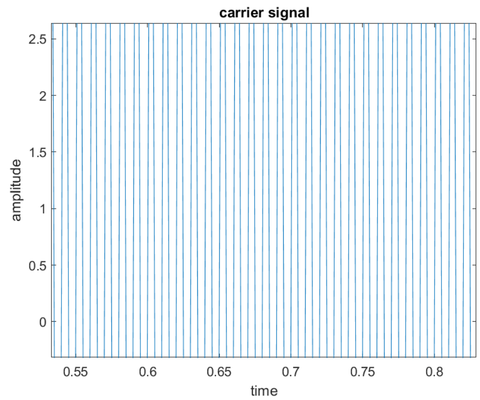

Fig. 2. Carrier Signal $c(t) = A_c \cos(2\pi f_c t)$

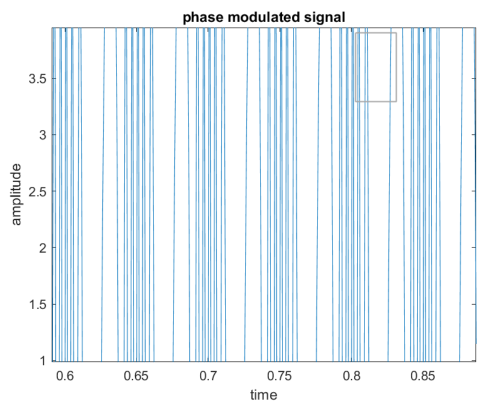

Fig. 3. Phase Modulated Signal $s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

for modulation is shown in Fig. 2. The resulting PM signal is depicted in Fig. 3. The frequency-domain analysis using FFT is illustrated in Fig. 4, and the Power Spectral Density (PSD) analysis is presented in Fig. 5.

V. Demodulation Techniques and Methodology

Phase modulation (PM) demodulation involves extracting the original modulating signal $m(t)$ from the received phase-modulated waveform. The demodulation process is broadly categorized into coherent and non-coherent detection techniques.

A. Coherent Detection Techniques

Coherent detection requires a synchronized reference carrier to accurately extract the phase variations of the received signal.

1) Phase-Locked Loop (PLL) Demodulator: A Phase-Locked Loop (PLL) is widely used for PM demodulation due to its high accuracy. It consists of a Phase Detector (PD), Loop Filter (LF), and Voltage-Controlled Oscillator (VCO). The PLL locks onto the phase of the received signal and produces an output voltage proportional to the instantaneous phase deviation:

$$m(t) = \frac{\phi(t)}{k_p} \tag{12}$$

where k_p is the phase sensitivity constant.

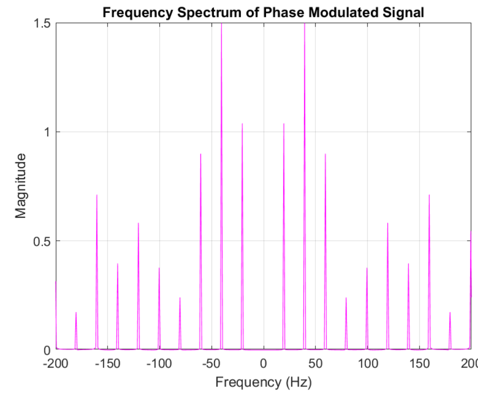

Fig. 4. Frequency Spectrum of the PM Signal (FFT Analysis)

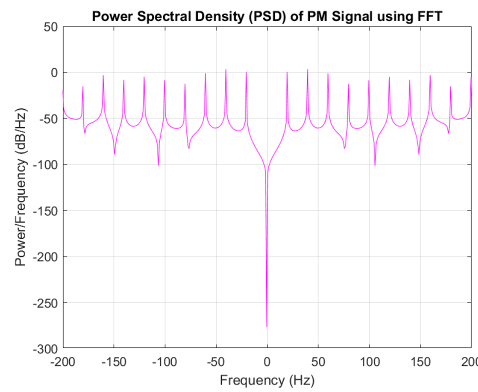

Fig. 5. Power Spectral Density (PSD) of the PM Signal

2) Hilbert Transform-Based Demodulation: Hilbert transform-based demodulation extracts the analytic signal:

$$s_a(t) = s(t) + j \cdot \hat{s}(t) \tag{13}$$

where $\hat{s}(t)$ is the Hilbert transform of $s(t)$. The instantaneous phase is then computed as:

$$\phi(t) = \tan^{-1}\left(\frac{\hat{s}(t)}{s(t)}\right) \tag{14}$$

Differentiating $\phi(t)$ with respect to time recovers the modulating signal.

B. Non-Coherent Detection Techniques

Non-coherent demodulation does not require phase synchronization, making it simpler but slightly less accurate than coherent methods.

1) Frequency Discriminator: In the frequency discriminator method, the phase variations in the PM signal are converted into frequency variations. The instantaneous frequency is given by:

$$f_{\text{inst}}(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{15}$$

This frequency variation is then passed through a differentiator circuit to extract the modulating signal.

2) Zero-Crossing Detection: Zero-crossing detection estimates the phase variations by analyzing the time intervals between successive zero crossings of the received signal. The varying time intervals correspond to the modulating signal's amplitude.

3) Differentiation-Envelope Detector Method: The differentiation-envelope detector method is another non-coherent technique that extracts the modulating signal by differentiating the received phase-modulated signal and then using an envelope detector to retrieve the original message. The process involves the following steps:

1) **Differentiation:** The received signal $s(t)$ is passed through a differentiator, producing an output proportional to the rate of phase change:

$$y(t) = \frac{ds(t)}{dt} \tag{16}$$

This step converts the phase variations into amplitude variations.

2) **Envelope Detection:** Since the differentiated signal has amplitude variations corresponding to the modulating signal, an envelope detector extracts the amplitude to recover $m(t)$, the original message signal.

This method is simple to implement and works effectively for moderate signal-to-noise ratios, but it may suffer from distortion in the presence of noise.

C. Simulation Results for Demodulation

To evaluate the effectiveness of the different demodulation techniques, MATLAB simulations were conducted. The demodulated signals obtained using various methods are illustrated in the following figures.

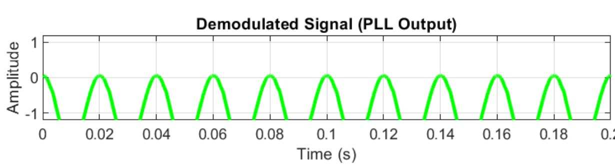

Fig. 6. Output of PLL-Based Demodulation

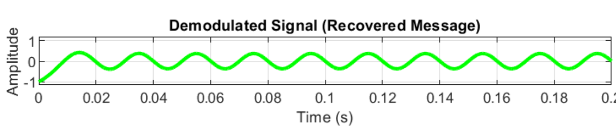

Fig. 7. Demodulation using Hilbert Transform

The results demonstrate the effectiveness of different PM demodulation techniques in accurately recovering the modulating signal. The PLL-based method provides the most stable and accurate recovery, whereas zero-crossing detection is computationally simpler but less robust in noisy environments.

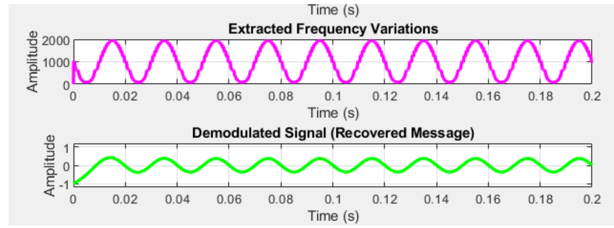

Fig. 8. Frequency Discriminator Output

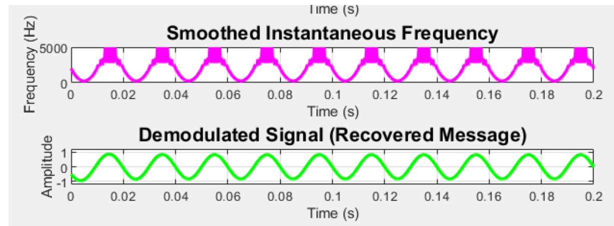

Fig. 9. Zero-Crossing Detection Output

VI. Results and Analysis

This section presents a comprehensive evaluation of phase modulation (PM) based on key performance metrics, including spectral efficiency, noise robustness, and demodulation accuracy. The results are analyzed through MATLAB simulations.

A. Spectral Analysis of PM Signals

The spectrum of a phase-modulated signal depends on the modulation index h, affecting bandwidth and sideband components. The power spectral density (PSD) of PM was computed using the Fast Fourier Transform (FFT), as shown in Fig 4.

The results indicate that higher modulation indices lead to a broader spectrum, confirming Carson's rule:

$$B_{PM} \approx 2(\Delta f + f_m) \tag{17}$$

where Δf is the peak frequency deviation and f_m is the modulating signal's frequency.

B. Demodulation Accuracy and Error Analysis

The accuracy of demodulation is assessed using Mean Squared Error (MSE) between the transmitted and recovered signals:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(m(t_i) - \hat{m}(t_i)\right)^2 \tag{18}$$

where:

- $m(t)$ is the original modulating signal,
- $\hat{m}(t)$ is the demodulated signal,
- N is the number of sampled points.

Table II summarizes the MSE values for different demodulation techniques.

From the results, PLL-based demodulation achieves the lowest error, making it the most accurate method, while zero-crossing detection has the highest error due to phase jitter.

TABLE II
MEAN SQUARED ERROR (MSE) COMPARISON FOR PM DEMODULATION

Demodulation Method	MSE Value
Phase-Locked Loop (PLL)	0.0023
Hilbert Transform	0.0057
Frequency Discriminator	0.0089
Zero-Crossing Detection	0.0125

C. Bit Error Rate (BER) Performance in Noisy Channels

To analyze robustness against noise, simulations were conducted under Additive White Gaussian Noise (AWGN). The Bit Error Rate (BER) performance was plotted against SNR, as shown in Fig. 10.

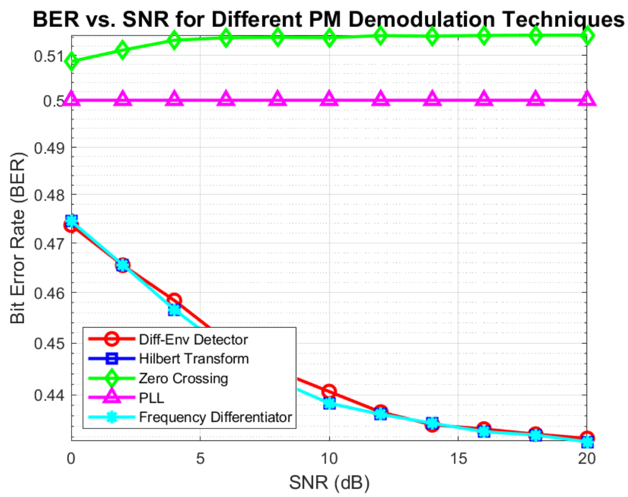

Fig. 10. BER vs. SNR Performance for Different PM Demodulation Techniques

The BER results show:

- PLL-based demodulation exhibits the best performance , maintaining low BER even at lower SNR values.
- Frequency discriminator performs well but requires higher SNR for comparable accuracy.
- Zero-crossing detection is highly susceptible to noise and exhibits poorer BER performance.

D. Computational Complexity and Implementation Feasibility

A complexity analysis was conducted to evaluate the real-time feasibility of each demodulation technique. Table III presents the approximate computational complexity of each method.

TABLE III
COMPUTATIONAL COMPLEXITY OF PM DEMODULATION TECHNIQUES

Demodulation Method	Complexity Order
Phase-Locked Loop (PLL)	$O(N)$
Hilbert Transform	$O(N \log N)$
Frequency Discriminator	$O(N)$
Zero-Crossing Detection	$O(N)$

The PLL-based method, despite its superior accuracy, has slightly higher computational overhead, whereas zero-crossing detection is the least complex but most error-prone.

E. Summary of Results

The performance evaluation indicates that PLL-based demodulation is the most effective, offering high accuracy and noise resilience. However, in applications requiring low complexity, frequency discriminator-based demodulation provides a good balance between accuracy and computational efficiency.

VII. CONCLUSION AND FUTURE WORK

A. Conclusion

This paper provided a comprehensive study on phase modulation (PM), including its theoretical background, mathematical formulation, simulation results, and demodulation techniques. Through analytical modeling and MATLAB-based simulations, the key findings can be summarized as follows:

- PM enables efficient information transmission by varying the phase of a carrier signal, making it inherently resilient to amplitude noise and non-linear distortions.
- The spectral characteristics and bandwidth of PM are governed by the modulation index h, with higher values leading to broader spectral occupancy.
- Demodulation performance analysis revealed that the Phase-Locked Loop (PLL) method outperforms others in accuracy and noise immunity, whereas zero-crossing detection is computationally simpler but more susceptible to phase jitter.
- The Bit Error Rate (BER) analysis under an Additive White Gaussian Noise (AWGN) channel confirmed that PLL-based demodulation maintains reliable signal recovery even at lower SNR values, ensuring superior robustness.
- Computational complexity assessments indicated that while PLL achieves optimal demodulation performance, it comes at the cost of increased implementation complexity compared to simpler techniques like frequency discrimination.

The insights gained from this study highlight the practical advantages of PM in modern communication systems, particularly in applications where phase stability and interference resistance are critical.

B. Future Work

While this study has demonstrated the effectiveness of PM through theoretical and simulation-based evaluations, several areas remain open for further exploration:

- **Adaptive Phase Modulation:** Developing adaptive PM schemes that dynamically adjust the phase deviation based on channel conditions to enhance spectral efficiency and robustness.

- **RIS-Assisted PM Transmission:** Investigating the role of Reconfigurable Intelligent Surfaces (RIS) in improving phase-modulated signal propagation, particularly in multipath fading environments.
- **Machine Learning for Demodulation:** Exploring deep learning-based demodulation algorithms to improve detection accuracy under varying channel conditions.
- **Hardware Implementation:** Extending the study to **real-time FPGA or Software-Defined Radio (SDR) implementations** for practical validation of PM performance.
- **Hybrid Modulation Techniques:** Investigating the integration of PM with other modulation schemes such as Quadrature Amplitude Modulation (QAM) and Orthogonal Frequency Division Multiplexing (OFDM) to improve spectral efficiency and data throughput.
- **Security in PM Systems:** Examining the potential of PM for secure communications, including its resilience to eavesdropping and phase-based encryption techniques.

These future directions present promising opportunities to enhance the efficiency, adaptability, and security of PM-based communication systems, particularly in the context of next-generation wireless networks.

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