# Solving Master-mind game with the help of SAT solver

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## 1 Part 1

In this part all colors in the original color list are unique. There are 8 colors in the game and let's say they are $a, b, c, d, e, f, g, h$ and let the current guess by the code breaker is $a_1, b_2, c_3, d_4$, where $a_1$ means color $a$ is at position 1. Similarly for $b_2, c_3, d_4$ also. So, the remaining colors are $e, f, g, h$.

- Initialization

  Initial constraints are rules which are always applicable irrespective of the state of the game. In this case it has 3 initial constraints. they are,

  - First condition is if one color is present in any position then it will not present in other positions.
    Example: If color a is present in position 1, then it will not present in any other position.
    So the logical inference here is-
    $a_1 \rightarrow (\neg a_2 \wedge \neg a_3 \wedge \neg a_4)$
    $\implies (\neg a_1 \vee \neg a_2) \wedge (\neg a_1 \vee \neg a_3) \wedge (\neg a_1 \vee \neg a_4)$
    Similarly for position $a_2, a_3, a_4$ also.
    Similarly for color $b, c, d, e, f, g, h$ also.

  - Second condition is if any color is present in any position then other colors will not present in that position
    Example: If color $a$ is present in position 1 then any other colors cannot present in position 1.
    So, the logical inference here is -
    $a_1 \rightarrow (\neg b_1 \wedge \neg c_1 \wedge \neg d_1 \wedge \neg e_1 \wedge \neg f_1 \wedge \neg g_1 \wedge \neg h_1$
    $\implies (\neg a_1 \vee \neg b_1) \wedge (\neg a_1 \vee \neg c_1) \wedge (\neg a_1 \vee \neg d_1) \wedge (\neg a_1 \vee \neg e_1) \wedge (\neg a_1 \vee \neg f_1) \wedge$
    $(\neg a_1 \vee \neg g_1) \wedge (\neg a_1 \vee \neg h_1)$
    Similarly, for $a_2, a_3, a_4, b_1, b_2 \ldots h_2, h_3, h_4$ also.

  - Third condition is every position has a color in it
    Example: Position 1 has color-a or color-b or ... color-h in it.
    So, the logical inference here is -
    $(a_1 \vee b_1 \vee c_1 \vee \ldots \vee h_1) \wedge (a_2 \vee b_2 \vee c_2 \vee \ldots \vee h_2) \wedge (a_3 \vee b_3 \vee c_3 \vee \ldots \vee h_3) \wedge (a_4 \vee b_4 \vee c_4 \vee \ldots \vee h_4)$

- Condition: Black-0, White-0 :
  - No color in the present guess is present in the actual colors picked by code-maker. So, the logical inference is - $(e_1 \lor f_1 \lor g_1 \lor h_1) \land (e_2 \lor f_2 \lor g_2 \lor h_2) \land (e_3 \lor f_3 \lor g_3 \lor h_3) \land (e_4 \lor f_4 \lor g_4 \lor h_4)$

- Condition: Black-0, White-1 :
  - No color is in it's current position
    So, the logical inference is -
    $\neg a_1 \land \neg b_2 \land \neg c_3 \land \neg d_4$
  - At least one color is present in the next guess
    So, the logical inference is -
    $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$
  - If one color is present in any of the position then other guess colors will not present in the next guess.
    So, the logical inference is -
    $a_2 \rightarrow ((e_1 \lor f_1 \lor g_1 \lor h_1) \land (e_3 \lor f_3 \lor g_3 \lor h_3) \land (e_4 \lor f_4 \lor g_4 \lor h_4))$
    Similarly, for $a_3, a_4, b_1, b_3, b_4 \ldots d_2, d_3$ also.

- Condition: Black-0, White-2 :
  - No color is in it's current position
    So, the logical inference is -
    $\neg a_1 \land \neg b_2 \land \neg c_3 \land \neg d_4$
  - At least one color is present in in the next guess
    So, the logical inference is -
    $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$

- Condition: Black-0, White-3 :
  - No colors is in their current position
    So, the logical inference is -
    $\neg a_1 \land \neg b_2 \land \neg c_3 \land \neg d_4$
  - At least one color is present in in the next guess
    So, the logical inference is -
    $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$

- Condition: Black-0, White-4 :

- All colors are present but their position is different
  So, the logical inference is -
  $(b_1 \vee c_1 \vee d_1) \wedge (a_2 \vee c_2 \vee d_2) \wedge (a_3 \vee b_3 \vee d_3) \wedge (a_4 \vee b_4 \vee c_4)$

- Condition: Black-1, White-0 :
  - At least one of the colors is present in its own position
    So, the logical inference is -
    $a_1 \vee b_2 \vee c_3 \vee d_4$
  - If one color is present in its own place then the remaining colors will not present in the next guess
    So, the logical inference is -
    $(a_1 \rightarrow (e_2 \vee f_2 \vee g_2 \vee h_2) \wedge (e_3 \vee f_3 \vee g_3 \vee h_3) \wedge (e_4 \vee f_4 \vee g_4 \vee h_4) \wedge$
    $\implies ((\neg a_1 \vee e_2 \vee f_2 \vee g_2 \vee h_2) \wedge (\neg a_1 \vee e_3 \vee f_3 \vee g_3 \vee h_3) \wedge (\neg a_1 \vee e_4 \vee f_4 \vee g_4 \vee h_4))$
    Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-1 :
  - At least one one of the color is present in its own position
    So, the logical inference is -
    $a_1 \vee b_2 \vee c_3 \vee d_4$
  - If one colors is present in its own position then the remaining colors will not present in their own position also at least one of the remaining colors will present in the next guess.
    So, the logical inference is -
    $a_1 \rightarrow \neg b_2 \wedge \neg c_3 \wedge \neg d_4 \wedge (c_2 \vee d_2 \vee b_3 \vee d_3 \vee b_4 \vee c_4)$
    $\implies (\neg a_1 \vee \neg b_2) \wedge (\neg a_1 \vee \neg c_3) \wedge (\neg a_1 \vee \neg d_4) \wedge (\neg a_1 \vee c_2 \vee d_2 \vee b_3 \vee d_3 \vee b_4 \vee c_4)$
    Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-2 :
  - At least one one of the color is present in its own position
    So, the logical inference is -
    $a_1 \vee b_2 \vee c_3 \vee d_4$
  - If one colors is present in its own position then the remaining colors will not present in their own position also at least one of the remaining colors will present in the next guess.
    So, the logical inference is -
    $a_1 \rightarrow \neg b_2 \wedge \neg c_3 \wedge \neg d_4 \wedge (c_2 \vee d_2 \vee b_3 \vee d_3 \vee b_4 \vee c_4)$
    $\implies (\neg a_1 \vee \neg b_2) \wedge (\neg a_1 \vee \neg c_3) \wedge (\neg a_1 \vee \neg d_4) \wedge (\neg a_1 \vee c_2 \vee d_2 \vee b_3 \vee d_3 \vee b_4 \vee c_4)$

Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-3 :
  - At least one one of the colors is present in its own position
    So, the logical inference is -
    $a_1 \vee b_2 \vee c_3 \vee d_4$

  - If one colors is present in its own position then the remaining colors will not present in their own position, but they will present in the next guess.
    So, the logical inference is -
    $a_1 \rightarrow (c_2 \vee d_2) \wedge (b_3 \vee d_3) \wedge (b_4 \vee c_4)$
    $\implies (\neg a_1 \vee c_2 \vee d_2) \wedge (\neg a_1 \vee b_3 \vee d_3) \wedge (\neg a_1 \vee b_4 \vee c_4)$
    Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-2, White-0 :
  - Two color of the current guess must be in their own position in actual colors. So if we take any three colors in their own place then at least one of them must be true.
    So, the logical inference is -
    $(a_1 \vee b_2 \vee c_3) \wedge (a_1 \vee b_2 \vee d_4) \wedge (a_1 \vee c_3 \vee d_4) \wedge (b_2 \vee c_3 \vee d_4)$

  - If any two colors are present in their own place then the remaining colors will not present
    So, the logical inference is -
    $(a_1 \wedge b_2) \rightarrow (e_3 \vee f_3 \vee g_3 \vee h_3) \wedge (e_4 \vee f_4 \vee g_4 \vee h_4)$
    $\implies (\neg a_1 \vee \neg b_2 \vee e_3 \vee f_3 \vee g_3 \vee h_3) \wedge (\neg a_1 \vee \neg b_2 \vee e_4 \vee f_4 \vee g_4 \vee h_4)$
    Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-2, White-1 :
  - Two color of the current guess must be in their own position in actual colors.
    So, the logical inference is -
    $(a_1 \vee b_2 \vee c_3) \wedge (a_1 \vee b_2 \vee d_4) \wedge (a_1 \vee c_3 \vee d_4) \wedge (b_2 \vee c_3 \vee d_4)$

  - If any two colors are present in their own place then the remaining colors will not present in their own position
    So, the logical inference is -
    $(a_1 \wedge b_2) \rightarrow \neg c_3 \wedge \neg d_4$
    $\implies (\neg a_1 \vee \neg b_2 \vee \neg c_3) \wedge (\neg a_1 \vee \neg b_2 \vee \neg d_4)$
    Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-2, White-2 :
  - Two color of the current guess must be in their own position in actual colors.
    So, the logical inference is -
    $(a_1 \lor b_2 \lor c_3) \land (a_1 \lor b_2 \lor d_4) \land (a_1 \lor c_3 \lor d_4) \land (b_2 \lor c_3 \lor d_4)$

  - If any two colors are present in their own place then the remaining two colors positions are interchanged.
    So, the logical inference is -
    $(a_1 \land b_2) \rightarrow c_4 \land d_3$
    $\implies (\neg a_1 \lor \neg b_2 \lor c_4) \land (\neg a_1 \lor \neg b_2 \lor d_3)$
    Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-3, White-0 :
  - Three color must be present in the original colors list and they will be in their own place. So, if we take any two colors in the guess, at least one of them must be in its own place.
    So, the logical inference is -
    $(a_1 \lor b_2) \land (a_1 \lor c_3) \land (a_1 \lor d_4) \land (b_2 \lor c_3) \land (b_2 \lor d_4) \land (c_3 \lor d_4)$

  - If any three colors are present in their own place then the remaining one color is not present.
    So, the logical inference is -
    $(a_1 \land b_2 \land c_3) \rightarrow \neg d_4$
    $\implies (\neg a_1 \lor \neg b_2 \lor \neg c_3 \lor \neg d_4)$
    Similarly for $(a_1, b_2, d_4), (a_1, c_3, d_4), (b_2, c_3, d_4)$ triplet also. I can see that they all will produce same clause so, they can be skipped, although if not skipped it will not create any problem.

- Condition: Black-3, White-1 :
  - This condition will not appear, as if three colors is in their correct position and the fourth color is present , then it must be present in it's correct position

- Condition: Black-4, White-0 :
  - This condition will also not arise as the code-maker will not call code-breaker if this condition happen and return to the main function.

# 2 Part 2

This is the part 2 of the assignment where all the logical constraints of the SAT solver with duplicate color is given. There are 8 colors in the game let's say $i, j, k, l, m, n, o, p$ and let the current guess be $a_1, b_2, c_3, d_4$ where $a_1$ means color $a$ is at position 1 and $a, b, c, d \in i, j, k, l, m, n, o, p$ and $a, b, c, d$ may not unique. For this selection, the number of remaining colors which are not in the current guess is $\geq 4$ and let the number be $\beta$. Let the remaining colors be $r_1, r_2, r_3 \ldots r_\beta$. Here $r_{ij}$ means remaining color $i$ is at position $j$. Also the number of unique colors present at the current guess be $\alpha = 8 - \beta$. Let the present colors be $p_1, p_2 \ldots p_\alpha$. Here $p_{ij}$ means present color $i$ is at position $j$.

- Initialization

  Initial constraints are rules which are always applicable irrespective of the state of the game. In this case it has 2 initial constraints. they are,

  - First condition is if any color is present in any position then other colors will not present in that position
    Example: if color $a$ is present in position 1 then any other colors cannot present in position 1.
    So, the logical inference here is -
    $a_1 \rightarrow (\neg b_1 \wedge \neg c_1 \wedge \neg d_1 \wedge \neg e_1 \wedge \neg f_1 \wedge \neg g_1 \wedge \neg h_1$
    $\implies (\neg a_1 \vee \neg b_1) \wedge (\neg a_1 \vee \neg c_1) \wedge (\neg a_1 \vee \neg d_1) \wedge (\neg a_1 \vee \neg e_1) \wedge (\neg a_1 \vee \neg f_1) \wedge$
    $(\neg a_1 \vee \neg g_1) \wedge (\neg a_1 \vee \neg h_1)$
    Similarly, for $a_2, a_3, a_4, b_1, b_2 \ldots h - 2, h_3, h_4$ also.

  - Second condition is every position has a color in it
    Example: position-1 has color-a or color-b or ... color-h in it.
    So, the logical inference here is -
    $(a_1 \vee b_1 \vee \ldots \vee h_1) \wedge (a_2 \vee b_2 \vee \ldots \vee h_2) \wedge (a_3 \vee b_3 \vee \ldots \vee h_3) \wedge (a_4 \vee b_4 \vee \ldots \vee h_4)$

- Condition: Black-0, White-0 :

  - No color in the present guess is present in the actual colors picked by code-maker. So, every position has color which is present in the remaining colors list ($r_i$).
    So, the logical inference is -
    $(r_{11} \vee r_{21} \vee \ldots \vee r_{\beta 1}) \wedge (r_{12} \vee r_{22} \vee \ldots \vee r_{\beta 2}) \wedge (r_{13} \vee r_{23} \vee \ldots \vee r_{\beta 3}) \wedge (r_{14} \vee r_{24} \vee \ldots \vee r_{\beta 4})$

- Condition: Black-0, White-1 :

  - No color is in it's current position
    So, the logical inference is -
    $\neg a_1 \wedge \neg b_2 \wedge \neg c_3 \wedge \neg d_4$

- At least one color is present in in the next guess
  So, the logical inference is -
  $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$

- If one color is present in any of the position then other guess colors will not present in the next guess.
  So, the logical inference is -
  $a_2 \rightarrow ((a_1 \lor r_{11} \lor r_{21} \lor \ldots \lor r_{\beta 1}) \land (a_3 \lor r_{13} \lor r_{23} \lor \ldots \lor r_{\beta 3}) \land (a_4 \lor r_{14} \lor r_{24} \lor \ldots \lor r_{\beta 4}))$
  Similarly, for $a_3, a_4, b_1, b_3, b_4 \ldots d_2, d_3$ also.

- Condition: Black-0, White-2 :
  - No color is in it's current position
    So, the logical inference is -
    $\neg a_1 \land \neg b_2 \land \neg c_3 \land \neg d_4$
  - At least one color is present in in the next guess
    So, the logical inference is -
    $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$

- Condition: Black-0, White-3 :
  - No colors is in their current position
    So, the logical inference is -
    $\neg a_1 \land \neg b_2 \land \neg c_3 \land \neg d_4$
  - At least one color is present in in the next guess
    So, the logical inference is -
    $(a_2 \lor a_3 \lor a_4 \lor b_1 \lor b_3 \lor b_4 \lor c_1 \lor c_2 \lor c_4 \lor d_1 \lor d_2 \lor d_3)$

- Condition: Black-0, White-4 :
  - All colors are present but their position is different
    So, the logical inference is -
    $(b_1 \lor c_1 \lor d_1) \land (a_2 \lor c_2 \lor d_2) \land (a_3 \lor b_3 \lor d_3) \land (a_4 \lor b_4 \lor c_4)$

- Condition: Black-1, White-0 :
  - At least one of the color is present in its own position
    So, the logical inference is -
    $a_1 \lor b_2 \lor c_3 \lor d_4$

– If one color is present in it's own place, then the remaining colors will not present in the next guess
So, the logical inference is -
$(a_1 \rightarrow (r_{12} \vee r_{22} \vee \ldots \vee r_{\beta 2}) \wedge (r_{13} \vee r_{23} \vee \ldots \vee r_{\beta 3}) \wedge (r_{14} \vee r_{24} \vee \ldots \vee r_{\beta 4}) \wedge$
$\implies ((\neg a_1 \vee r_{12} \vee r_{22} \vee \ldots \vee r_{\beta 2}) \wedge (\neg a_1 \vee r_{13} \vee r_{23} \vee \ldots \vee r_{\beta 3}) \wedge (\neg a_1 \vee r_{14} \vee r_{24} \vee \ldots \vee r_{\beta 4}))$
Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-1 :
  – At least one of the color is present in its own position
  So, the logical inference is -
  $a_1 \vee b_2 \vee c_3 \vee d_4$

  – If one colors is present in its own position then the remaining colors(in guess) will not present in their own position also at least one of the guess color will present in the next guess.
  So, the logical inference is -
  $a_1 \rightarrow \neg b_2 \wedge \neg c_3 \wedge \neg d_4 \wedge (a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
  $\implies (\neg a_1 \vee \neg b_2) \wedge (\neg a_1 \vee \neg c_3) \wedge (\neg a_1 \vee \neg d_4) \wedge (\neg a_1 \vee a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
  Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-2 :
  – At least one of the color is present in its own position
  So, the logical inference is -
  $a_1 \vee b_2 \vee c_3 \vee d_4$

  – If one colors is present in its own position then the remaining colors(in guess) will not present in their own position also at least one of the guess color will present in the next guess.
  So, the logical inference is -
  $a_1 \rightarrow \neg b_2 \wedge \neg c_3 \wedge \neg d_4 \wedge (a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
  $\implies (\neg a_1 \vee \neg b_2) \wedge (\neg a_1 \vee \neg c_3) \wedge (\neg a_1 \vee \neg d_4) \wedge (\neg a_1 \vee a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
  Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-1, White-3 :
  – At least one of the color is present in its own position
  So, the logical inference is -
  $a_1 \vee b_2 \vee c_3 \vee d_4$

  – If one colors is present in its own position then the remaining colors(in guess) will not present in their own position, also at least one of the guess color will present

in the next guess.
So, the logical inference is -
$a_1 \rightarrow \neg b_2 \wedge \neg c_3 \wedge \neg d_4 \wedge (a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
$\implies (\neg a_1 \vee \neg b_2) \wedge (\neg a_1 \vee \neg c_3) \wedge (\neg a_1 \vee \neg d_4) \wedge (\neg a_1 \vee a_2 \vee c_2 \vee d_2 \vee a_3 \vee b_3 \vee d_3 \vee a_4 \vee b_4 \vee c_4)$
Similarly for $b_2, c_3, d_4$ also.

- Condition: Black-2, White-0 :
  - Two colors in the current guess must be in their own position in actual colors.
    So, the logical inference is -
    $(a_1 \vee b_2 \vee c_3) \wedge (a_1 \vee b_2 \vee d_4) \wedge (a_1 \vee c_3 \vee d_4) \wedge (b_2 \vee c_3 \vee d_4)$
  - If any two colors are present in their own place then the remaining colors will not present
    So, the logical inference is -
    $(a_1 \wedge b_2) \rightarrow (\neg c_3) \wedge (\neg d_4)$
    $\implies (\neg a_1 \vee \neg b_2 \vee \neg c_3) \wedge (\neg a_1 \vee \neg b_2 \vee \neg d_4)$
    Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-2, White-1 :
  - Two colors in the current guess must be in their own position in actual colors.
    So, the logical inference is -
    $(a_1 \vee b_2 \vee c_3) \wedge (a_1 \vee b_2 \vee d_4) \wedge (a_1 \vee c_3 \vee d_4) \wedge (b_2 \vee c_3 \vee d_4)$
  - If any two colors are present in their own place then the remaining colors will not present in their own position
    So, the logical inference is -
    $(a_1 \wedge b_2) \rightarrow \neg c_3 \wedge \neg d_4$
    $\implies (\neg a_1 \vee \neg b_2 \vee \neg c_3) \wedge (\neg a_1 \vee \neg b_2 \vee \neg d_4)$
    Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-2, White-2 :
  - Two colors of the current guess must be in their own position in actual colors.
    So, the logical inference is -
    $(a_1 \vee b_2 \vee c_3) \wedge (a_1 \vee b_2 \vee d_4) \wedge (a_1 \vee c_3 \vee d_4) \wedge (b_2 \vee c_3 \vee d_4)$
  - If any two colors are present in their own place then the remaining two colors positions are interchanged.
    So, the logical inference is -
    $(a_1 \wedge b_2) \rightarrow c_4 \wedge d_3$
    $\implies (\neg a_1 \vee \neg b_2 \vee c_4) \wedge (\neg a_1 \vee \neg b_2 \vee d_3)$

Similarly for $(a_1, c_3), (a_1, d_4), (b_2, c_3), (b_2, d_4), (c_3, d_4)$ pairs also.

- Condition: Black-3, White-0 :
  - Three colors must be present in the original colors list and they will be in their own place. So, if we take any two colors in the guess, at least one of them must be in its own place.
    So, the logical inference is -
    $(a_1 \lor b_2) \land (a_1 \lor c_3) \land (a_1 \lor d_4) \land (b_2 \lor c_3) \land (b_2 \lor d_4) \land (c_3 \lor d_4)$
  - If any three colors are present in their own place then the remaining one color is not present.
    So, the logical inference is -
    $(a_1 \land b_2 \land c_3) \rightarrow (\neg d_4)$
    $\implies (\neg a_1 \lor \neg b_2 \lor \neg c_3 \lor \neg d_4)$
    Similarly for $(a_1, b_2, d_4), (a_1, c_3, d_4), (b_2, c_3, d_4)$ triplet also. I can see that they all will produce same clause so I can skip them although if not skipped it will create any problem.
- Condition: Black-3, White-1 :
  - This condition will not arise, as if three colors in their correct position and the fourth color is present then it must be present in its correct position.
- Condition: Black-4, White-0 :
  - This condition will also not arise as the code-maker will not call code-breaker if this condition happen and return to the main function.