Mathematic Analysis with Matlab

Lecture 4: Applications about Function Derivative

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Fall Semester 2019

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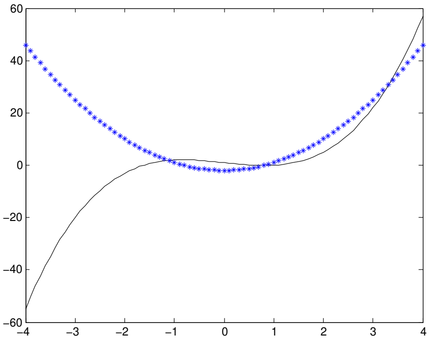
Outline

1. Extreme Points of a Function
2. Minimum and Maximum Values of a Function
3. Prove inequations with monotonic properties

Find out the extreme points of a function

$$f(x) = x^3 - 2x + 1 \qquad (1)$$

- Step 1:
  1. syms x
  2. diff('x^3-2*x+1')
- Step 2:
  1. x=-4:0.1:4;
  2. y1=x.^3-2*x+1;
  3. y2=3*x.^2-2;
  4. plot(x,y1,'k-',x,y2,'b*')
- Step 3:
  1. c=roots([3,0,-2])
  2. Output: c=0.8165,-0.8165



Outline

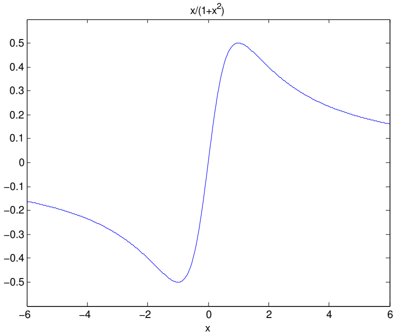
1. Extreme Points of a Function
2. Minimum and Maximum Values of a Function
3. Prove inequations with monotonic properties

- Given f(x)
- Search minimum value for f(x),
  1. **x**=fminbnd(f,x1,x2)
  2. [**x**,**fval0**]=fminbnd(f,x1,x2)
  3. [**x**, **fval0**, **exitflag**, **output**]=fminbnd(f,x1,x2)

Minimum values of a Function

$$f(x) = \frac{x}{1+x^2} \qquad (2)$$

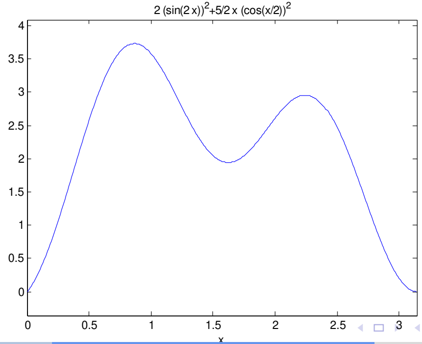
- Step 1:
  1. ezplot('x/(1+x^2)', [-6, 6])
- Step 2:
  1. clear;syms x;
  2. f='x/(1+x^2)';
  3. [xmin,ymin]=fminbnd(f,-10,10)
- Step 3:
  1. clear;syms x;
  2. f='-x/(1+x^2)';
  3. [xmax,ymax]=fminbnd(f,-10,10)



Minimum and Maximum values of a Function (1)

$$f(x) = 2sin^2(2x) + \frac{5}{2}xcos^2(\frac{x}{2}),$$
$$\text{where } x \in (0, \pi)$$

- Step 1:
  1. ezplot('2*(sin(2*x))^2+5/2*x*(cos(x/2))^2', [0, pi])



Minimum and Maximum values of a Function (2)

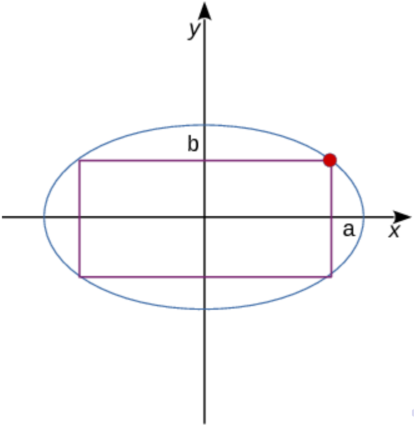
$$f(x) = 2sin^2(2x) + \frac{5}{2}xcos^2(\frac{x}{2}),$$
$$\text{where } x \in (0, \pi)$$

- Step 2:
  1. clear;
  2. f='2*(sin(2*x))^2+5/2*x*(cos(x/2))^2';
  3. f1='-2*(sin(2*x))^2-5/2*x*(cos(x/2))^2';
  4. [xmin,ymin]=fminbnd(f,0,pi)
  5. [xmax1,ymax1]=fminbnd(f1,0,1)
  6. [xmax2,ymax2]=fminbnd(f1,1.5,pi)
- Step 3:
  1. clear;
  2. f1='-2*(sin(2*x))^2-5/2*x*(cos(x/2))^2';
  3. [xmax,ymax]=fminbnd(f1,0,pi)

Exercise 1 (1)

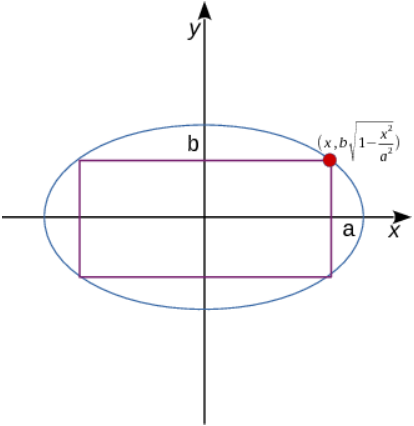
- Find out the inscribed rectangular of an ellipse that holds maximum area

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Exercise 1 (2)

- **Step 1.** Work out the area function
- **Step 2.** Find out the maximum point of the area function



Exercise 1 (3)

$$A(x) = 4{\cdot}x{\cdot}b{\cdot}\sqrt{1 - \frac{x^2}{a^2}}$$
$$(0 < x < a)$$

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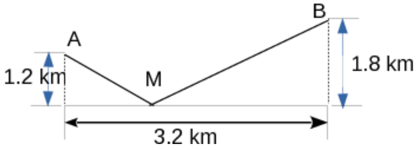
$$A(x) = 4 \cdot x \cdot b \cdot \sqrt{1 - \frac{x^2}{a^2}}$$
$$(0 < x < a)$$

- Answer:

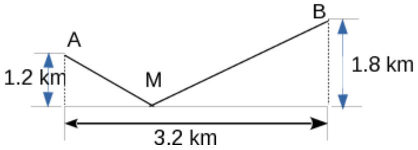
$$x = \frac{\sqrt{2}}{2} \cdot a \tag{3}$$
$$A = 2 \cdot a \cdot b \tag{4}$$

Exercise 2 (1)

- Power station project, supply electricities to A and B
- The cost of wires between A and M is a
- The cost of wires between B and M is b
- Problem: minimize the cost



Exercise 2 (2)

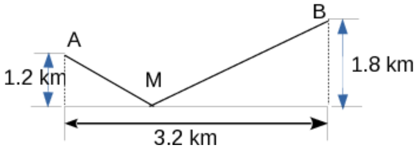


- Given x is the location of point M along x-axis

$$
\begin{aligned}
C(x) &= a \cdot dist(A, M) + b \cdot dist(B, M) \\
&= a \cdot \sqrt{x^2 + 1.44} + b \cdot \sqrt{(3.2 - x)^2 + 3.24}
\end{aligned}
$$

- Calculate minimum value for C(x), given a=20, b=10

Exercise 2 (3)



- Given x is the location of point M along x-axis

$$\begin{aligned} C(x) &= a \cdot dist(A, M) + b \cdot dist(B, M) \\ &= a \cdot \sqrt{x^2 + 1.44} + b \cdot \sqrt{(3.2 - x)^2 + 3.24} \end{aligned}$$

- Given a=20, b=10
  - x=**3.0573**, mVal=**171.9871**

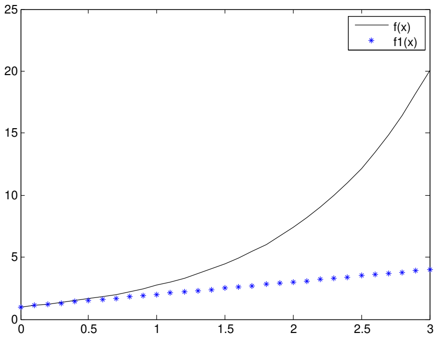
Outline

Proof about inequation

- Given inequation:

$$e^x > 1 + x, \quad \text{when } x > 0 \qquad (5)$$

- Step 1:
  1. clear;
  2. x=0:0.1:3;
  3. f1=exp(x);
  4. f2=1+x;
  5. plot(x,f1,'k-',x,f2,'b*')
- Step 2:
  1. clear;syms x;
  2. f=exp(x)-x-1
  3. df=diff(f,x);
  4. c=fzero('exp(x)-x-1',0);
- Output: df=exp(x)-1, c=0



Q & A

Thanks for your attention!