Mathematic Analysis with Matlab

Lecture 9: Matrix Decomposition

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Outline

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1. Matrix Decomposition
   - Eigenvalue Decomposition
   - Singular Value Decomposition (SVD)

Ways of Matrix Decomposition and the Matlab commands

- [P,X]=eig(A): eigenvalue decomposition;
- [P,X]=eigs(A): only returns the most significant 6 eigenvalues and their corresponding eigenvectors
- [P,J]=jordan(A): returns Jordan standard form
- [P U V]=qr(A): returns QR decompostion of matrix A
- [U D L]=svd(A): returns Singular Value Decomposition of matrix A

Eigen value decomposition

- Given **square** matrix A, looking for λ, such that

$$|\lambda E - A| = 0$$

$$\Rightarrow \quad A = P * diag(\lambda) P^{-1}, \quad \text{where } P * P^{-1} = E$$

- Or looking for λ and v

$$A * v = \lambda v \tag{1}$$

Eigenvalue decomposition (1)

- Get the eigenvalues for following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 6 \end{bmatrix}$$

- Steps:
  1. clear;
  2. syms l;
  3. A=[1 2 3; 2 1 3;3 3 6];
  4. L=l*eye(3,3);
  5. D=det(L-A);
  6. e=solve(D)

Eigenvalue decomposition (2)

- Get the eigenvalues and eigenvectors for following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 6 \end{bmatrix}$$

- Steps:
  1. clear;
  2. A=[1 2 3; 2 1 3;3 3 6];
  3. v=eig(A)
  4. [P, X]=eig(A)
- Switch to use simblic operations:
  1. A=sym(A);
  2. v=eig(A)
  3. [P, X]=eig(A)

Eigenvalue decomposition (3)

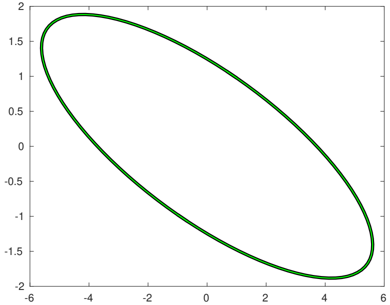
- Get the eigenvalues and eigenvectors for following matrix:

$$A = \begin{bmatrix} 1/3 & 1/3 & -1/2 \\ 1/5 & 1 & -1/3 \\ 6 & 1 & -2 \end{bmatrix}$$

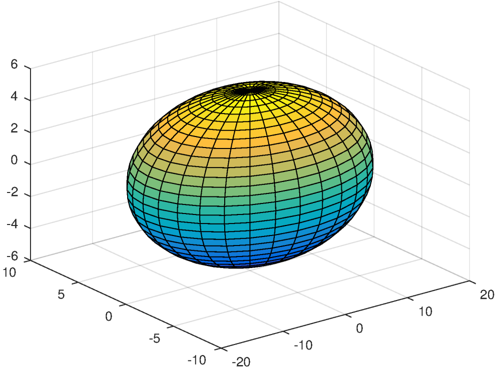
- Steps:
  1. clear;
  2. A=[1/3 1/3 -1/2;1/5 1 -1/3;6 1 -2];
  3. [P, X]=eigs(A)
- Switch to use 'eig':
  1. clear;
  2. A=[1/3 1/3 -1/2;1/5 1 -1/3;6 1 -2];
  3. [P, X]=eig(A)

What is Eigenvalue for? (1)

$$[x\ y] \times \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$



(a)



(b)

What is Eigenvalue for? (2)

```
function drawEllipse(Mi,i,j)
    clf;
    [v e]=eigs(Mi);
    l1 = (e(1)); l2 = (e(4));
    alpha=atan2(v(4),v(3));
    t = 0:pi/50:2*pi;
    x = (l1*cos(t));
    y = (l2*sin(t));
    xbar=x*cos(alpha) - y*sin(alpha);
    ybar=y*cos(alpha) + x*sin(alpha);
    plot(xbar+i,ybar+j,'-k','LineWidth',3);hold on
    plot(xbar+i,ybar+j,'-g','LineWidth',1);
end
```

```
    a=[10, 2 3; 4 10 2; 7 7 9];
    [v e]=eig(a);
    ellipsoid(0,0,0,e(1,1),e(2,2),e(3,3),30);
```

Quandratic Form

- Find out the standard quandratic form for:

$$f = 2x_1x_2 + 2x_1x_3 + 2x_2x_3 + 2x_4^2$$

- Matrix for the original quadratic form:

Outline

Singular Value Decomposition

- Given matrix $A \in R^{m\times n}$, it can be factorized as $A = U\Sigma V^T$
- $UU^T = E$ and $VV^T = E$, $U \in R^{m\times m}$ and $V \in R^{n\times n}$
- Now we have

$$
\begin{aligned}
A^TA = (U\Sigma V^T)^T U\Sigma V^T &\quad \Rightarrow \quad V\Sigma^2 V^T \\
AA^T = U\Sigma V^T (U\Sigma V^T)^T &\quad \Rightarrow \quad U\Sigma^2 U^T
\end{aligned}
$$

- So we know that $V\Sigma^2 V^T$ is eigenvalue decomposition of A^TA
- So we know that $U\Sigma^2 U^T$ is eigenvalue decomposition of AA^T

Singular Value Decomposition

- Given matrix A:

$$A = U\Sigma V^*,$$

where Σ is diagonal matrix $UU^T = I \quad and \ V^* = conj(V), \ VV^T = I(2)$

- SVD is applicable to any shape of matrix
- Given following matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$$

- Commands:
  1. A=[1 1 2 2; 2 3 1 5];
  2. [u dg v]=svd(A)
- Notice that the eigenvalues are sorted in descending order

Q & A

Thanks for your attention!