Mathematic Analysis with Matlab

Lecture 3: Applications of Function Derivative

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Outline

Lagrange Mean Value Theorem

- The definition
  - Given f(x) is **continuous** on the closed interval [a, b],
  - f(x) is differentiable on open interval (a, b),
  - there exists some **c** in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}. \tag{1}$$

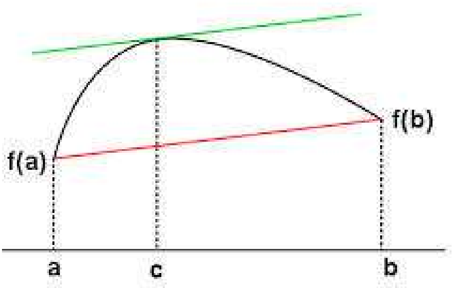


Figure: Demo of Lagrange Mean Value Theorem.

Lagrange Mean Value Theorem: an example (1)

- Given function $f(x) = 1/x^4$ is defined in the range [1, 2],
- it satisifies the conditions in Lagrange Mean Value Theorem,
- There must be ξ that makes $f'(\xi) = (f(2) - f(1))/(2 - 1)$
- Stage 1:
  1. syms x
  2. diff('1/x^4')
- ans: -4/x^5
- Stage 2:
  1. f=inline('-4/x^5-1/16+1');
  2. c=fzero(f, [1,2])
- ans: 1.3367

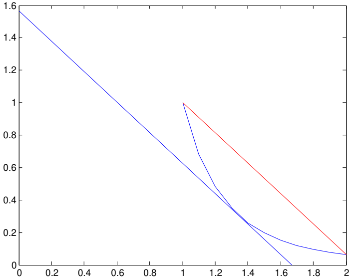
Lagrange Mean Value Theorem: an example (2)

- Show f(x)=1/x^4 along with the line defined by f'(ξ)

1. g =(f(2)-f(1))*x+b
2. g =-0.9375*x+b
3. **b** is unknown
4. plug x=1.3367 into f(x)
5. we have g(1.3367)=f(1.3367)=0.3132
6. 0.3132=-0.9375*1.3367+b

Lagrange Mean Value Theorem: an example (3)

4. plug x=1.3367 into f(x)
5. we have g(1.3367)=f(1.3367)=0.3132
6. 0.3132=-0.9375*1.3367+b
7. solve('0.3132=-0.9375*1.3367+b',b)
8. g=-0.9375*x+1.5664
9. Two points: (0, g(0)); (g^{-1}(0), 0)
10. u=1:0.1:2;
11. z=1./u.^4;
12. plot(u,z);hold on;
13. plot([0, 1.6708],[1.5664, 0])
14. plot([?, ?],[?, ?],'r')



Outline

Difference between Symbolic Expression, Equations and String Expression

$$f(x) = x^2 - 4x + 1 \tag{2}$$

- Symbolic Expr.
  1. syms x;
  2. f=x^2-4*x+1
  3. limit(f,x,1);
  4. diff(f)
  5. x=2
  6. val(f)
  7. ezplot(f,[0,10])
- String Expr.
  1. clear;
  2. f='x^2-4*x+1'
  3. limit(f,x,1);
  4. syms x;
  5. limit(f,x,1);
  6. clear;
  7. f='x^2-4*x+1'
  8. syms x;
  9. diff(f,x);
- Equations
  1. clear;
  2. x=[-1:0.1:2]
  3. f=x.^2-4*x+1
  4. Pay attention x is a matrix of numbers
  5. syms x;
  6. f=x.^2-4*x+1
  7. It is symbolic expression

- Try 'whos' whenever you are uncertain
- Convert symbolic expression to string: char(x^2+x)
- Convert string to symbolic expression: sym('x^2+x')

Difference between inline function and anonymous function

$$f(x) = x^2 - 4x + 1 \tag{3}$$

- Inline function
  1. f=inline('x^2-4*x+1','x')
  2. limit(f, x, 1);
  3. diff(f)
- Anonymous function (Matlab V9.0 or later)
  1. g=@(x) x^2-4*x+1;
  2. limit(g,x,1);
  3. diff(g,x)
  4. syms x;
  5. limit(g, x, 1);
  6. diff(g, x)
- function
  1. function val=f(x)
  2.   val=x^2-4*x+1
  3. end
  4. whos: inline
- Anonymous function
  1. g=x^2-4*x+1
  2. Comparable to '#define g(x) x^2-4*x+1' in C
  3. whos: function_handle
- Try 'whos' whenever you are uncertain

Outline

Commands for solving polynomial equations

- Given equation:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$$

$$c = [a_n, a_{n-1}, \cdots, a_1, a_0]$$

- Given f(x)=0, solve it by ‘fzero’
  1. f='x^2+2*x+4'
  2. r=fzero(f,[a,b]) returns roots in range [a,b]
  3. r=fzero(f,x_0) returns root close to x_0
- solve f(x)=0 by 'solve'
  1. f='x^2+2*x+4'
  2. r=solve(f);
- solve f(x)=0 by 'roots'
  1. c=[$a_n, a_{n-1}, \cdots, a_1, a_0$]
  2. r=roots(c);

(a) Evariste Galois (1811-1832)

(b) Niels Henrik Abel (1802-1829)

Figure: Scientists who found the solution for high order equations.

Commands for solving polynomial equations (example)

- Given equation:

$$x^3 + 6x^2 + 29 = 0 \tag{4}$$

$$e^x - 6x^2 + 9 = 0 \tag{5}$$

- Solve it by ‘fzero’
  1. fzero(??????)
  2. Attention: input range must be finite and real
  3. Applies to all types of equations
- solve it by ‘solve’
  1. solve(?????)
  2. Applies to all types of equations
- solve it by ‘roots’
  1. roots([????]);
  2. Only polynomial

Commands for solving polynomial equations (example)

- Given equation:

$$x^3 + 6x^2 + 29 = 0 \tag{1}$$

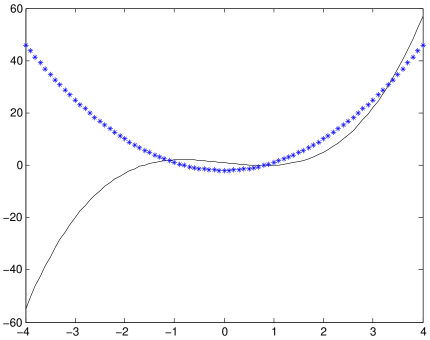
$$e^x - 6x^2 + 9 = 0 \tag{2}$$

- Solve it by 'fzero'
  1. Equation 1: fzero('x^3+6*x^2+29',[-10,10])
  2. Equation 2: fzero('exp(x)-6*x^2+9',0)
- solve it by 'solve'
  1. Equation 1: solve('x^3+6*x^2+29')
  2. Equation 2: solve('exp(x)-6*x^2+9')
- solve it by 'roots'
  1. Equation 1: roots([1, 6, 0, 29]);
  2. Equation 2: INAPPLICABLE

Take derivative of higher order

$$f(x) = x^3 - 2x + 1 \qquad (3)$$

- Step 1:
  1. syms x
  2. diff('x^3-2*x+1')
- Step 2:
  1. x=-4:0.1:4;
  2. y1=x.^3-2*x+1;
  3. y2=x.^2-2;
  4. plot(x,y1,'k-',x,y2,'b*')
- Step 3:
  1. c=roots([3,0,-2])
  2. Output: c=0.8165,-0.8165
  3. Consider following ranges: $(-\infty,-0.8165]$,



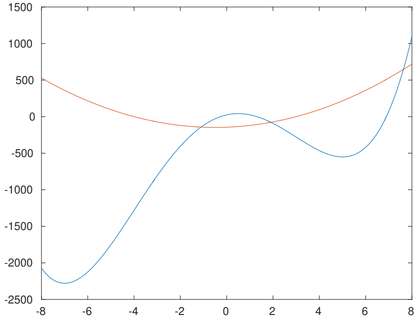
Exercise 1 (1)

- Plot figure for following function and its second order derivative in [-8, 7]
- Work out its concave and convex ranges and corresponding inflection point

$$f(x) = x^4 + 2x^3 - 72x^2 + 70x + 24 \tag{4}$$

Exercise 1 (2)

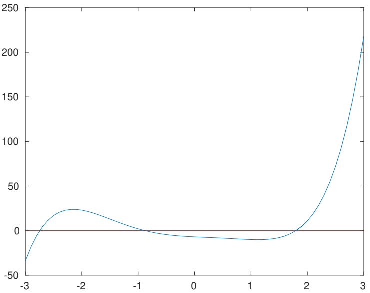
$$f(x) = x^4 + 2x^3 - 72x^2 + 70x + 24 \tag{5}$$



Exercise 2 (1)

- Work out the roots for following function with fzero and roots

$$f(x) = x^5 + x^4 - 4x^3 + 2x^2 - 3x - 7 \tag{6}$$



Q & A

Thanks for your attention!