Mathematic Analysis with Matlab

Lecture 5: Integral and Plotting 3D figures

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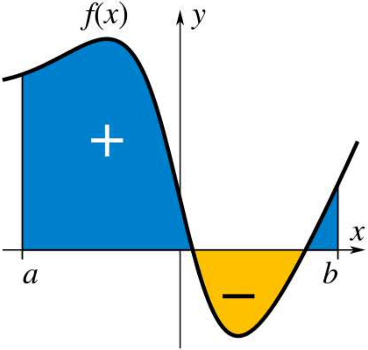
Outline

Major Matlab Commands: int(.)

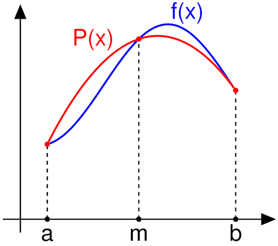
1. $int(f)$: indefinite integral for function f
2. $int(f, x)$: indefinite integral for function f with respect to x
3. $int(f, a, b)$: definite integral for function f from a to b
4. $int(f, x, a, b)$: definite integral for function f from a to b with respect to variable x



Major Matlab Commands: quad(.)

- Quadratic interpolation for function f(x)
- Given f(x), we want to know integral a to b for f(x)
- It is approximated by following rule:

$$\int_a^b f(x)dx = \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)] \tag{1}$$



Brief introduction about Simpson's rule

- Not all functions have their primitive functions
- $quad(inline('f(x)'), a, b)$: approximation of definite integral for function f, from a to b
- It is achieved via **Simpson's rule**

$$\int_a^b f(x)dx = \frac{b-a}{6}[f(a) + 4f(\frac{a+b}{2}) + f(b)]$$
$$= \frac{m-a}{6}[f(a) + 4f(\frac{a+m}{2}) + f(m)] + \frac{b-m}{6}[f(m) + 4f(\frac{b+m}{2}) + f(b$$

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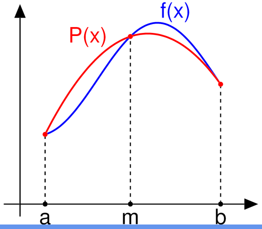


Numerical Integral

- Given following equation:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n-1} f(a+k*\frac{b-a}{n}) \\ = \lim_{n\to\infty} \frac{b-a}{n} \sum_{k=0}^{n} f(a+k*\frac{b-a}{n}) \quad (2)$$

- Approximate integral: $\int_0^1 x^2 dx$ by above method

1. n=128
2. x=0:1/n:1
3. left_sum=0; right_sum=0;
4. for i=1:n
    - left_sum=left_sum+x(i)^2*(1/n)
    - right_sum=right_sum+x(i+1)^2*(1/n)
5. end

Numerical Integral

- The actual integral should be in between 'left_sum' and 'right_sum'
- The larger of n, the more precise
  - Set n to 512, repeat the previous procedure
- Try the same way for $\int_0^1 \frac{sinx}{x} dx$
- Set n to 128
- See the results of 'left_sum' and 'right_sum'

Indefinite Integral

$$\int x^2(1-x^3)^5 dx \tag{3}$$

- Input following commands:
  1. syms x
  2. int('x^2*(1-x^3)^5', x)

Indefinite Integral

$$\int e^{-2x} sin3xdx \tag{4}$$

- Input following commands:
  1. syms x
  2. int('exp(-2*x)*sin(3*x)', x)

Indefinite Integral

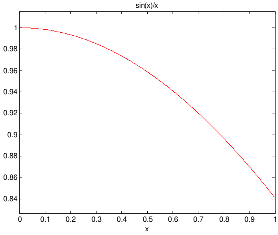
$$\int x^2 arctanx dx \tag{5}$$

- Input following commands:
  1. syms x
  2. int('atan(x)*x^2', x)

Indefinite Integral

$$\int \frac{sinx}{x} dx \tag{6}$$

- Input following commands:
  1. syms x
  2. int('sin(x)/x', x)
- The result cannot be represented by elementary function



Definite Integral

$$\int_0^1 (x - x^2)dx \tag{7}$$

- Input following commands:
  1. syms x
  2. int('(x-x^2)', x,0,1)

Definite Integral

$$\int_0^4 |x-2|dx \tag{8}$$

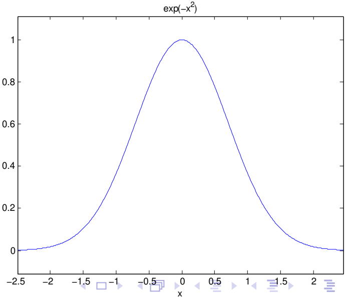
- Input following commands:
  1. syms x
  2. int('abs(x-2)', x,0,4)

Definite Integral

$$\int_0^1 \frac{sinx}{x} dx \tag{9}$$

$$\int_0^1 e^{-x^2} dx \tag{10}$$

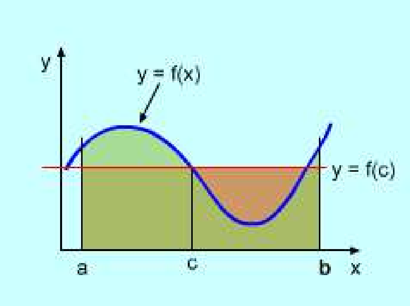
- Input following commands:
  1. quad(inline('sin(x)./x','x'), 0, 1)
- Alternative way:
  1. quad(@(x)sin(x)./x, 0, 1)



Exercise 1 (1)

- Given following function
- According to **Mean value theorem of integrals**, $\xi \in (2, 6)$
- That $f(\xi) = \frac{1}{(6-2)}\int_2^6 f(x)dx$
- Solve ξ out

$$f(x) = x^2 - 3x + 4$$



Outline

Drawing 3D figures: the commands

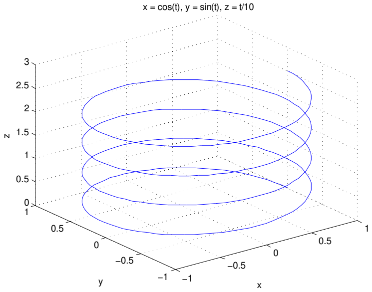
1. plot3(x, y, z, 's')
2. ezplot3('x(t)','y(t)','z(t)',[t1,t2])

- Given:

$$\begin{aligned} x &= cos(t) \\ y &= sin(t) \\ z &= \frac{t}{10}, \quad 0 \leq t \leq 8\pi \end{aligned}$$

- Input: fig1=ezplot3('cos(t)','sin(t)','t/10',[0,8*pi])
- Change color: set(fig1,'Color','r')

Drawing 3D figures: the commands

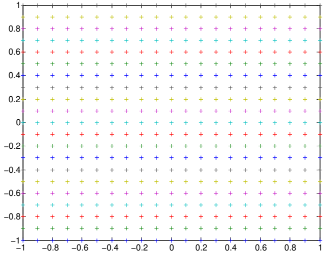


Drawing 3D figures: the commands

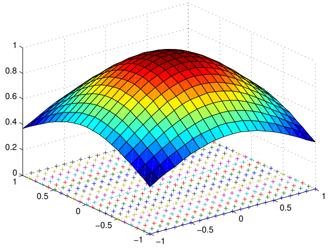
$$z = e^{-(x^2+y^2)} \tag{11}$$

$$z = xe^{-(x^2+y^2)} \tag{12}$$

1. x=-1:0.05:1;y=-1:0.05:1;
2. [X,Y]=meshgrid(x,y)
3. Z=f(X,Y)
4. mesh(X,Y,Z)
5. surf(X,Y,Z)



(a) meshgrid(x,y)



(b) surf(X,Y,Z)

Drawing 3D figures: the commands

- Try to draw following function:

$$\begin{aligned} x &= 2sin(\varphi)cos(\theta) \\ y &= 2sin(\varphi)sin(\theta) \\ z &= 2cos(\varphi) \\ 0 \le \varphi \le \pi, &\quad 0 \le \theta \le 2\pi \end{aligned} \tag{13}$$

1. [X,Y]=meshgrid(x,y)
2. Z=f(x,y)
3. mesh(X,Y,Z)
4. surf(X,Y,Z)

1. t=0:0.1:pi;r=-1:0.1:2*pi;
2. [R,T]=meshgrid(r,t);
3. x=2*sin(T).*cos(R);
4. y=2*sin(T).*sin(R);
5. z=cos(T);
6. surf(x,y,z)

Drawing 3D figures: display 3D space plane

- Try to draw following function:

$$z = 6 - 2x - 3y$$

$$where 0 \leq x \leq 3, 0 \leq y \leq 2$$

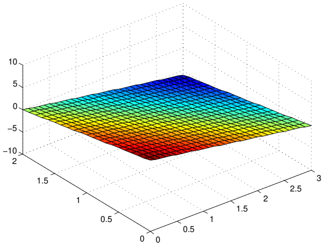
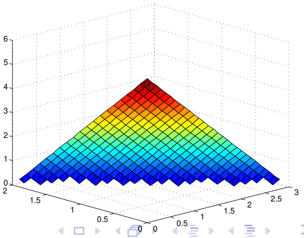


Figure: Full plane.

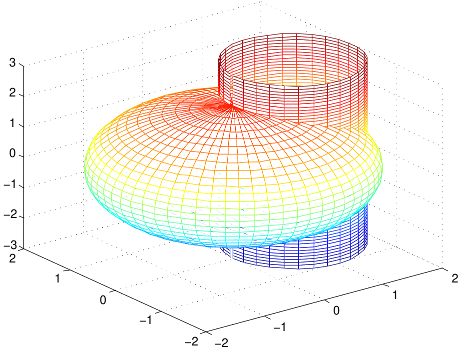
1. x=0:0.1:3.0; y=0:0.1:2;
2. [X,Y]=meshgrid(x,y);
3. z=6-2*X-3*Y;
4. surf(X,Y,z)
5. clf;idx=find($(z < 0)$);
6. z(idx)=NaN;
7. surf(X,Y,z);



Drawing 3D figures: display two surfaces (1)

- Try to draw following function:

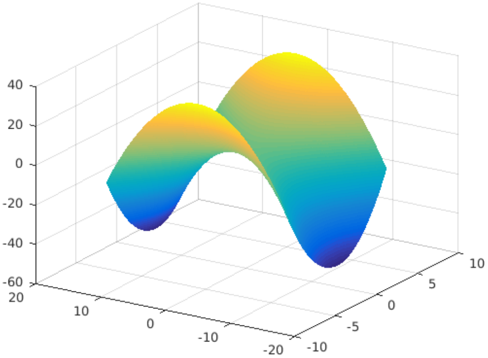
$$x^2 + y^2 + z^2 = 2^2$$
$$(x-1)^2 + y^2 = 1$$



Exercise 2 (1): Drawing 3D figures

- Try to draw following function:
- $-6 \leq x \leq 6$, $-14 \leq y \leq 14$

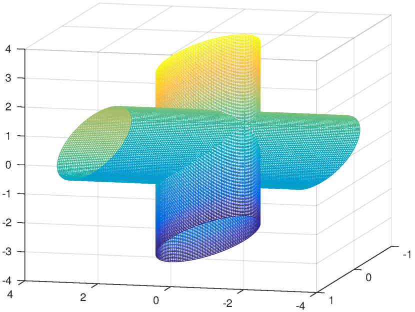
$$z = \frac{x^2}{1} - \frac{y^2}{4} \tag{14}$$



Exercise 3 (1): Drawing 3D figures

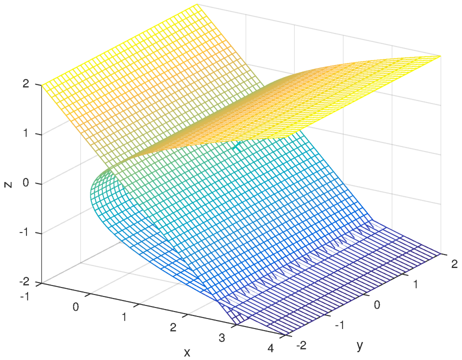
- Try to draw following functions:

$$x^2 + y^2 = 1$$
$$x^2 + z^2 = 1$$



Exercise 4 (1): plot surface and plane

$$x = y^2$$
$$x + z = 1$$



Q & A

Thanks for your attention!