Artificial Intelligence
CE-417, Group 1
Computer Eng. Department
Sharif University of Technology

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Courtesy: Most slides are adopted from CSE-573 (Washington U.), original slides for the textbook, and CS-188 (UC. Berkeley).

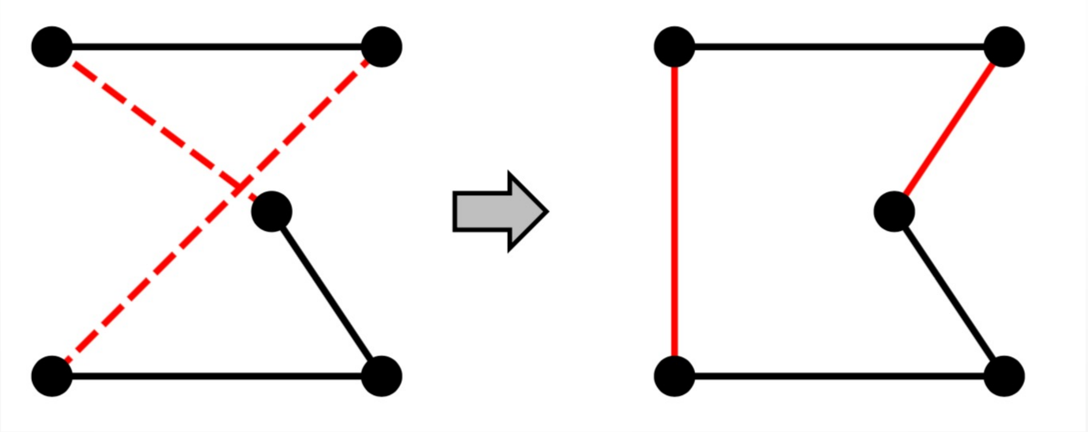
Local Search

Iterative improvement algorithms

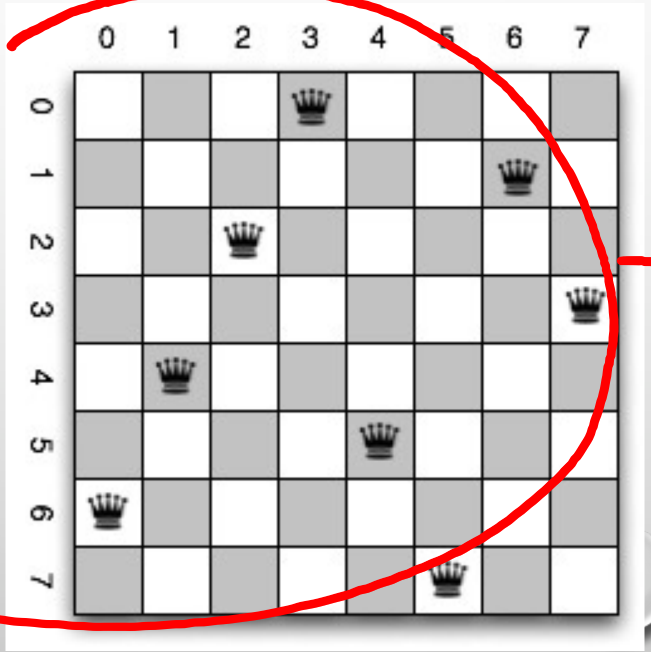
- Previously: Search to find best path to goal
  - Systematic exploration of search space.
- Today: a state is solution to problem
  - For some problems path is irrelevant.
  - e.g., 8-queens
- In such cases, can use iterative improvement algorithms;
  - **keep a single "current" state, try to improve it**

Examples

- TSP



- n-queens

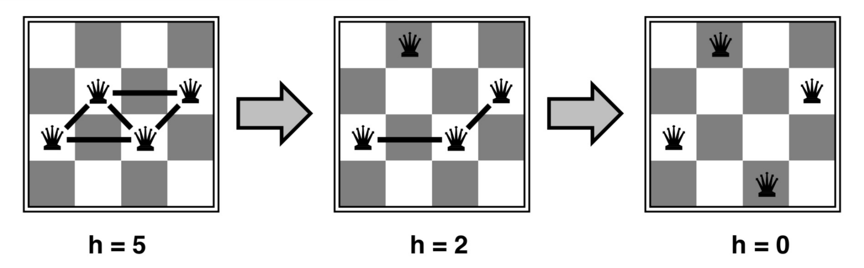


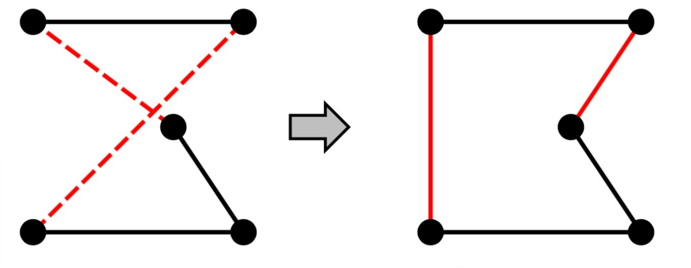
a_k

Local search algorithms

- State space = set of "complete" configurations
- Find configuration satisfying constraints,
  - e.g., all n-queens on board, no attacks
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it.
- Very memory efficient
  - *duh* - only remember current state

Constraint Satisfaction vs. Constraint Optimization





Goal Satisfaction

Constraint satisfaction
reach the goal node
guided by heuristic fn

Optimization

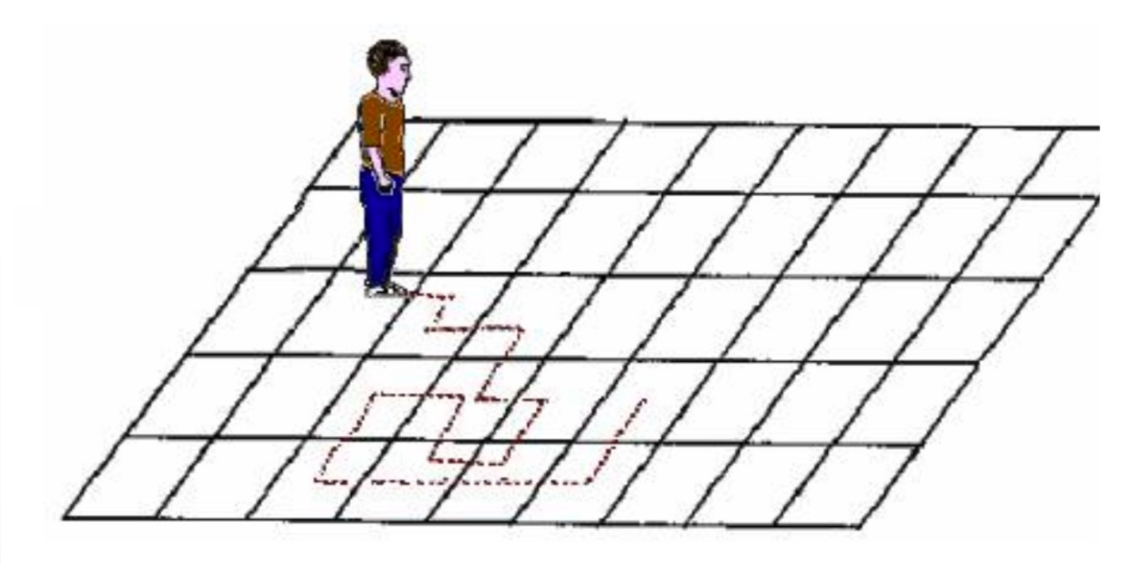
Constraint Optimization
optimize(objective fn)

You can go back and forth between the two problems. Typically in the same complexity class

Local Search and Optimization

- **Local search**:
  - Keep track of single current state
  - Move only to “neighboring” state (defined by operators)
  - Ignore previous states, path taken
- **Advantages**:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.
- **“Pure optimization” problems**
  - All states have an objective function
  - Goal is to find state with max (or min) objective value
  - Does not quite fit into path-cost/goal-state formulation
  - Local search can do quite well on these problems.

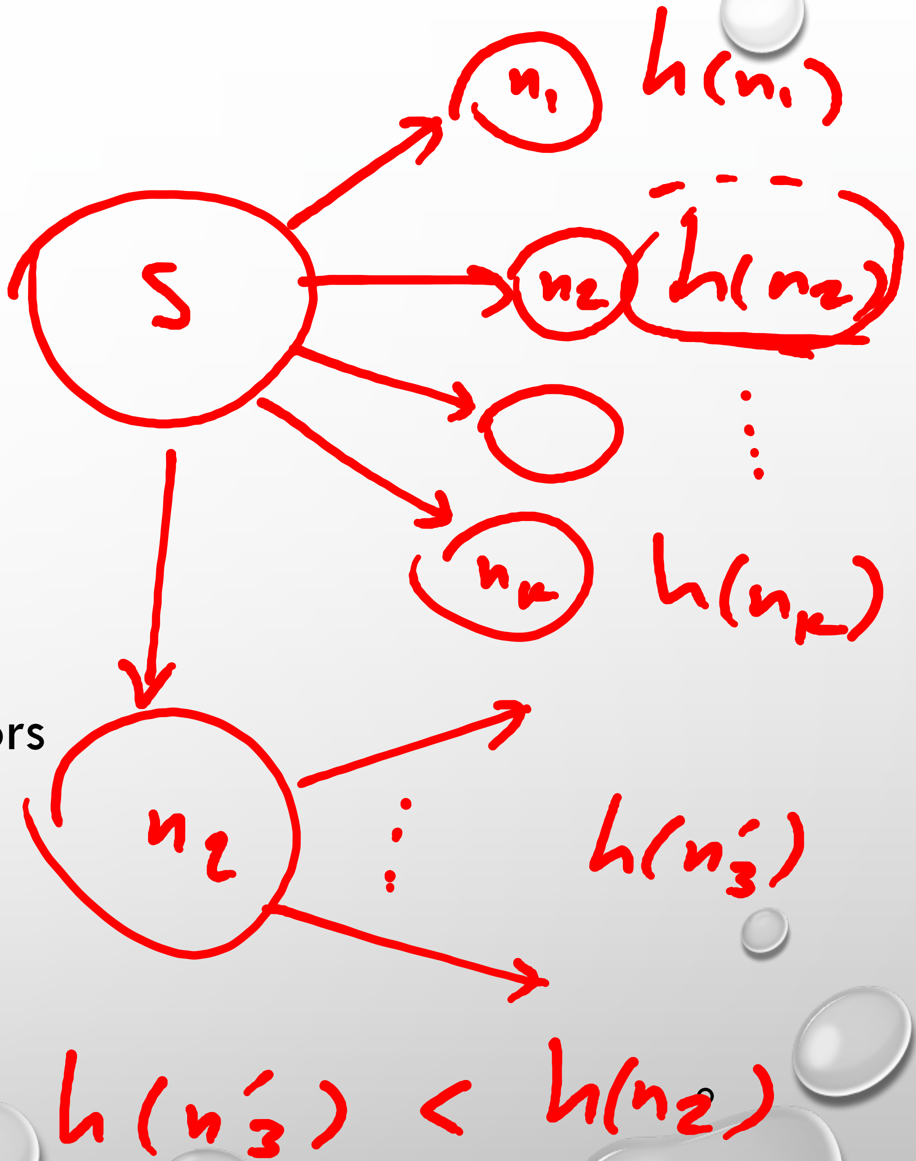
Trivial Algorithms



- Random Sampling
  - Generate a state randomly
- Random Walk
  - Randomly pick a neighbor of the current state
- Why even mention these?
  - Both algorithms are asymptotically complete.
    - If the state space is finite, each state is visited at a fixed rate asymptotically.

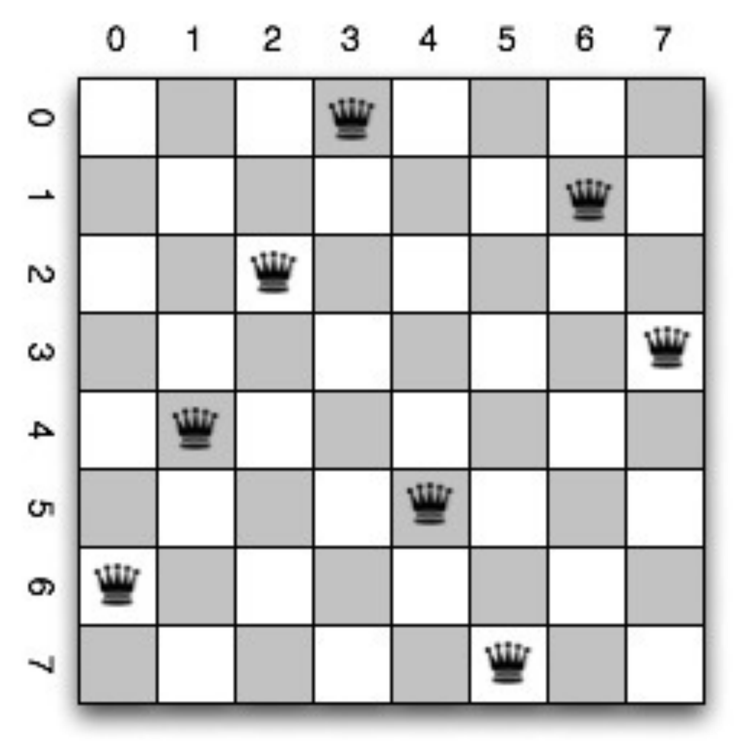
Hill-climbing search

- "a loop that continuously moves towards increasing value"
  - terminates when a peak is reached
  - Aka greedy local search
- Value can be either
  - Objective function value
  - Heuristic function value (minimized)
- Hill climbing does not look ahead of the immediate neighbors
- Can randomly choose among the set of best successors
  - if multiple have the best value
- **"climbing Mount Everest in a thick fog with amnesia"**



Example: *n*-Queens

- State
  - All n queens on the board in some configuration
  - But each in a different column
- Successor function
  - Move single queen to another square in same column.
- How to convert this into an optimization problem?



Hill-climbing search: 8-queens

- Result of hill-climbing in this case...

Bummer

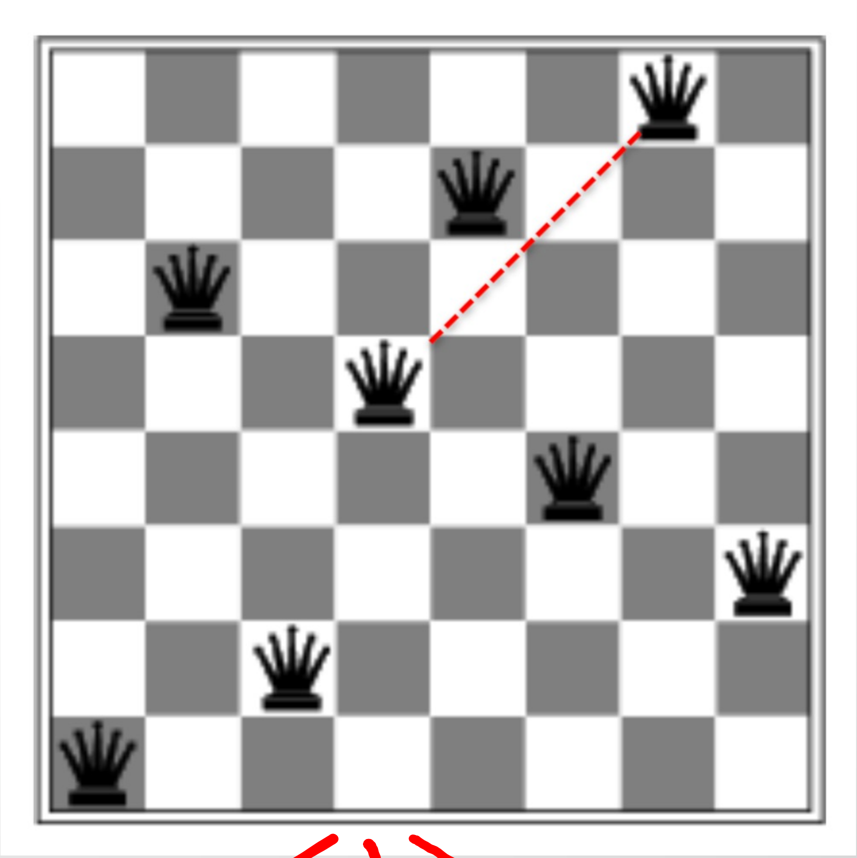
Local minimum

S_1 . . . .

A local minimum with $h = 1$ $\{\underbrace{FFF\ldots FS}_{1/p}\}$

p : Sucess

1-p : Failure



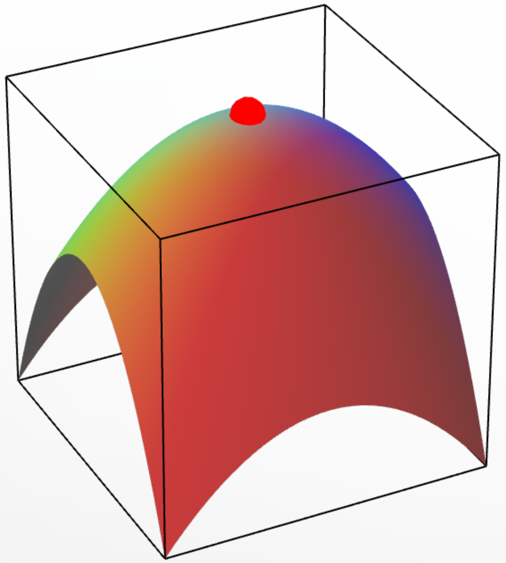
Hill-climbing performance on n-queens

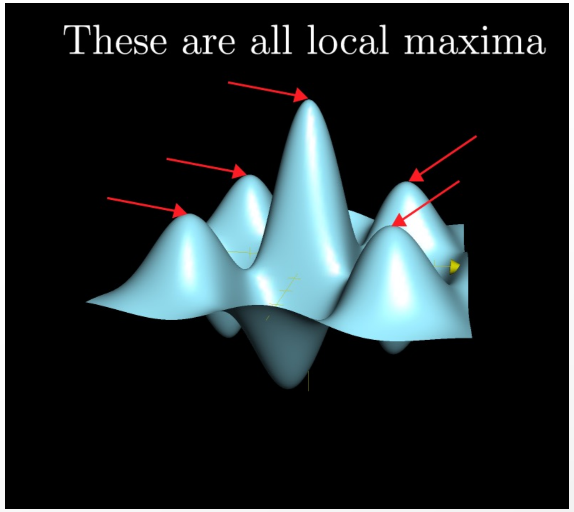
- Hill-climbing can solve large instances of n-queens ($n = 10^6$) in a few (ms)seconds
- 8 queens statistics:
  - State space of size ≈17 million
  - Starting from random state, steepest-ascent hill climbing solves 14% of problem instances
  - It takes 4 steps on average when it succeeds, 3 when it gets stuck
  - When "sideways" moves are allowed, performance improves ...
  - When multiple restarts are allowed, performance improves even more

P=14%

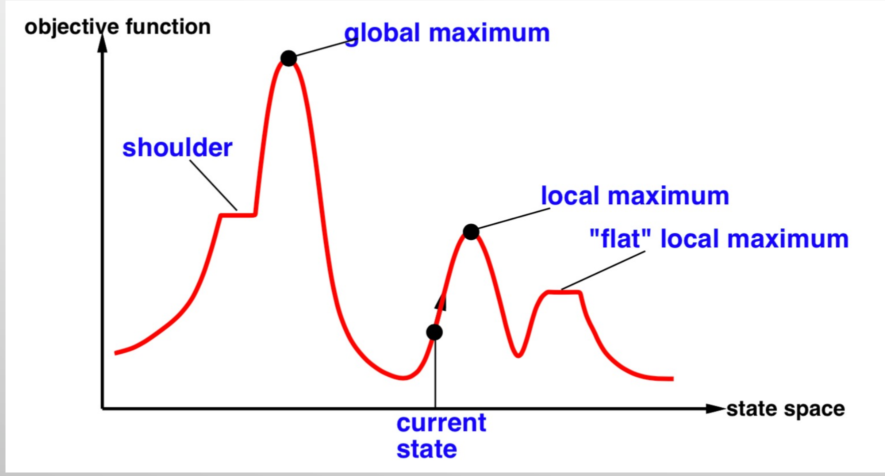
Hill Climbing Drawbacks

Local maxima



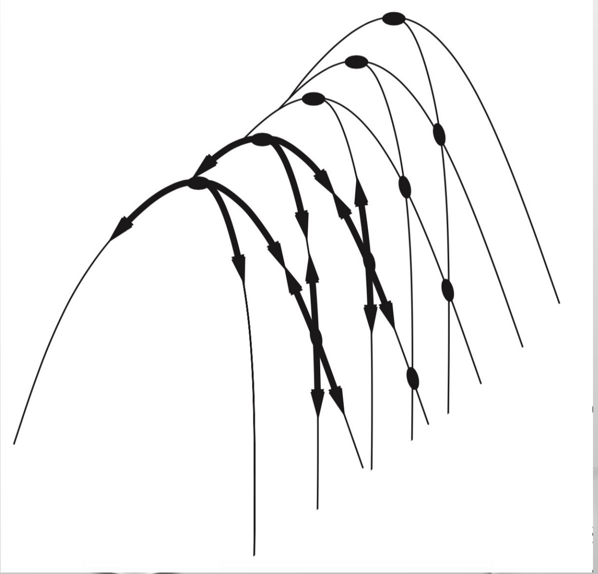


Plateaus

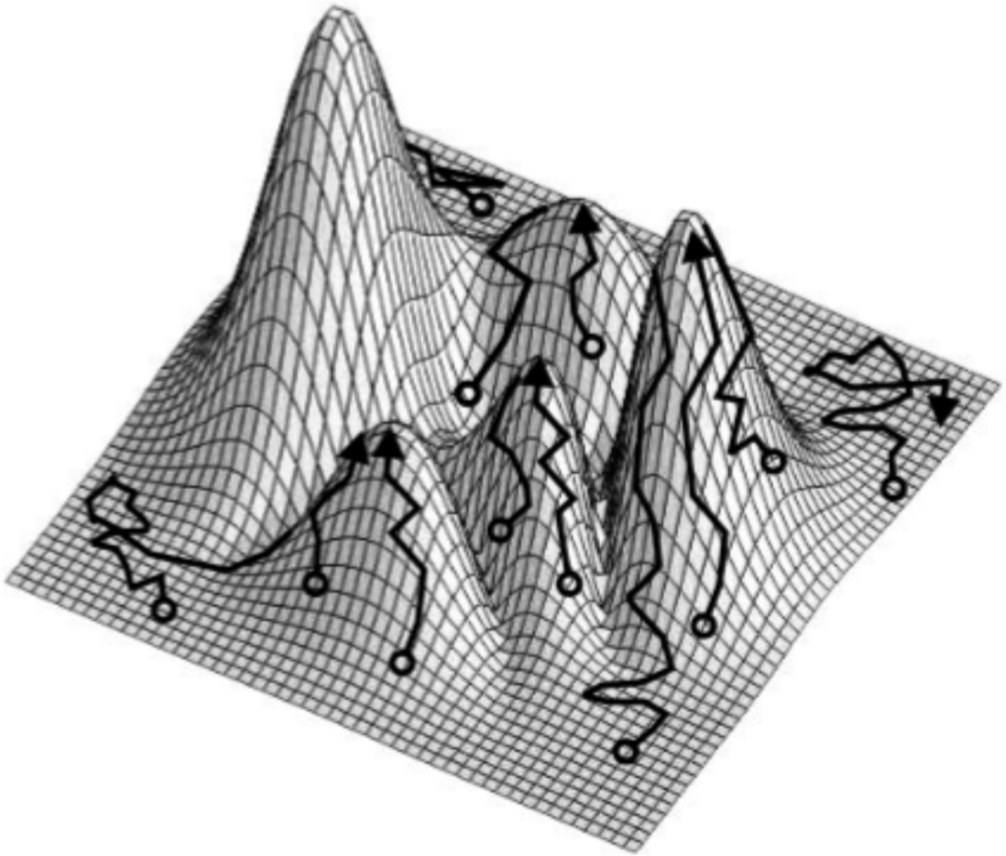


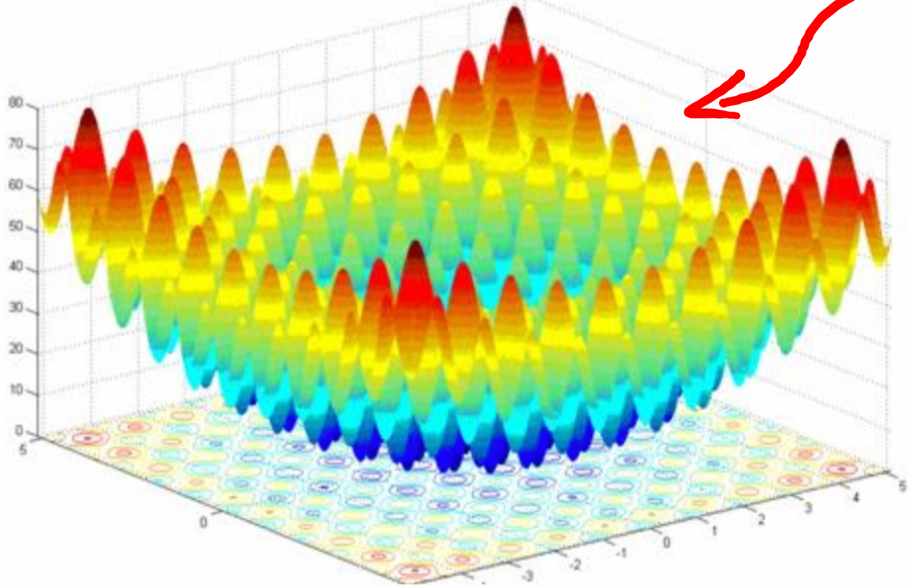
Diagonal

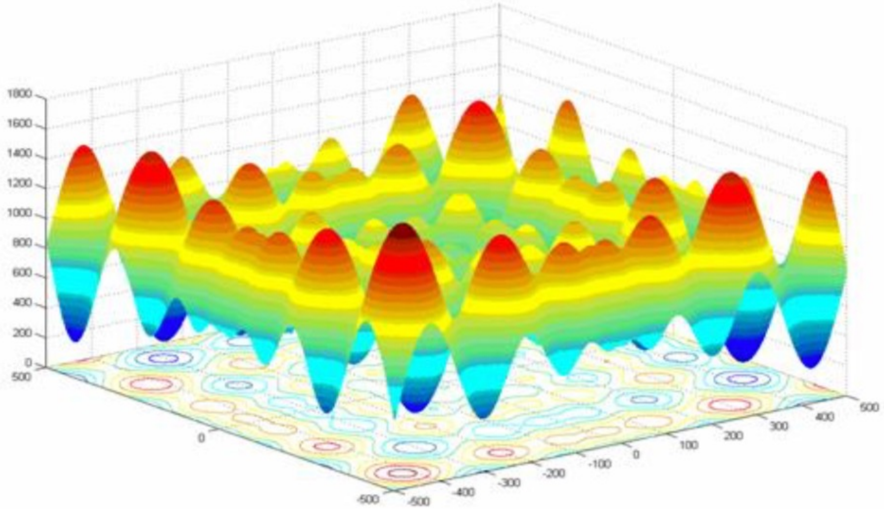
ridges



Trajectories, difficulties



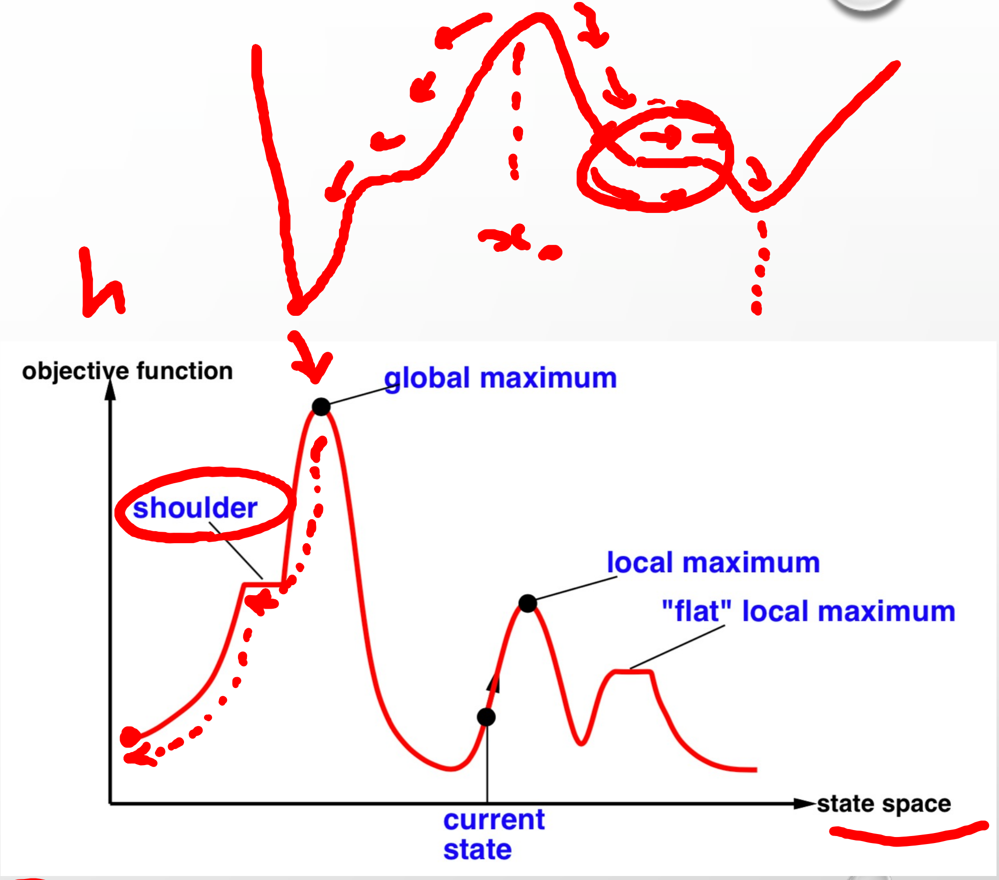


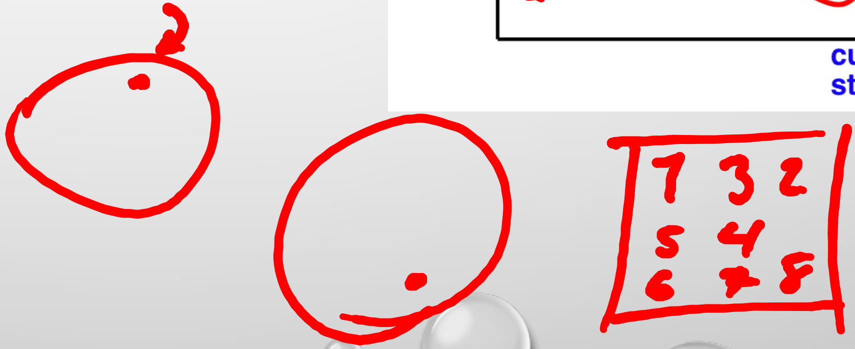


Escaping Shoulders: Sideways Move

- If no downhill (uphill) moves, allow sideways moves in hope that algorithm can escape
  - Must limit the number of possible sideways moves to avoid infinite loops
- For 8-queens
  - Allow sideways moves with limit of 100
  - Raises percentage of problems solved from 14 to 94%
  - However.... 4
    - 21 steps for every successful solution
    - 64 for each failure

3



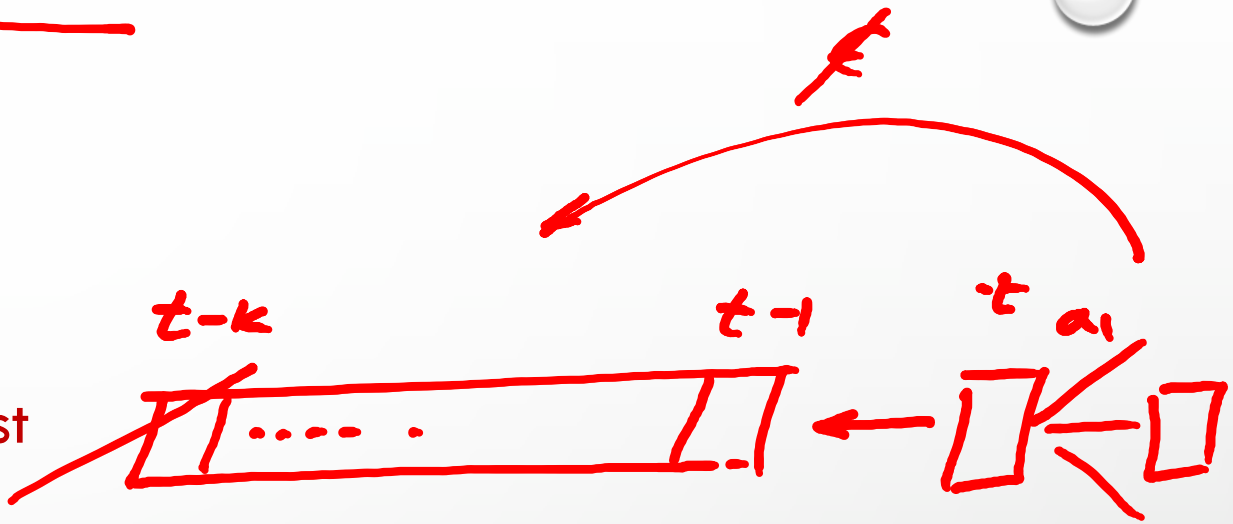


Hill Climbing Properties

- Not complete. Why?
- Terrible worst case running time.
- Simple, O(1) space, and often very fast.

Tabu Search

- Prevent returning quickly to the same state
- Keep fixed length queue ("tabu list")
- Add most recent state to queue; drop oldest
- Never move to a tabu state
- Properties:
  - As the size of the tabu list grows, hill-climbing will asymptotically become "non-redundant" (won't look at the same state twice)
  - In practice, a reasonable sized tabu list (say 100 or so) improves the performance of hill climbing in many problems

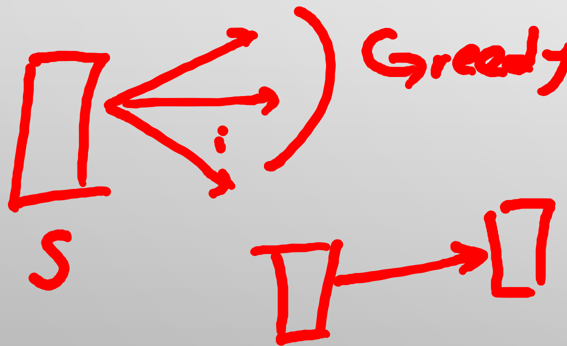


Hill Climbing: Stochastic Variations

- When the state-space landscape has local minima, any search that moves only in the greedy direction cannot be complete
- Random walk, on the other hand, *is* asymptotically complete

- ***Idea:*** Combine random walk & greedy hill-climbing
- At each step do one of the following:
  - Greedy: With prob. p move to the neighbor with largest value
  - Random: With prob. 1-p move to a random neighbor

$P_t \to 1$

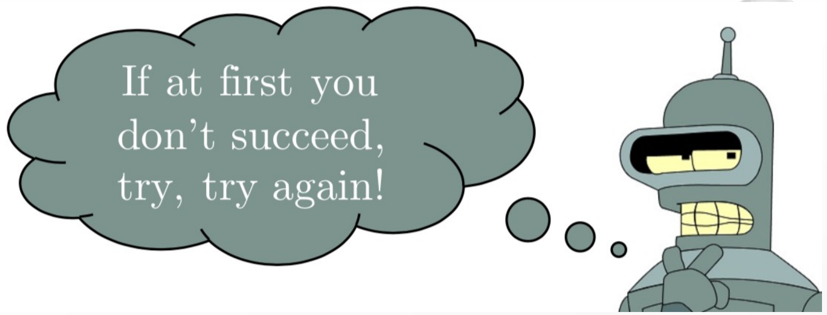


Greedy Choice with prob. p

0

$1-p \to$ Exploration

Hill-climbing with random restarts



- If at first you don't succeed, try, try again!
- Different variations $\mathbb{E}_Y(\mathbb{E}_X(X|Y)) = \mathbb{E}(X)$
  - For each restart: run until termination vs. run for a fixed time
  - Run a fixed number of restarts or run indefinitely
- Analysis $\mathbb{E}(3(T-1)+4) = 3\cdot 6.14 + 4$
  - Say each search has probability p of success
  - e.g., for 8-queens, p = 0.14 with no sideways moves
- Expected number of restarts? $1/p = 1/0.14 \simeq 7.14$
- Expected number of steps taken?

Failure → Expected 3 moves

Success → Expected 4 moves

$$\underbrace{\mathbb{E}(S_1|T)}_{3}+\dots+\underbrace{\mathbb{E}(S_T|T)}_{4}$$

$$3(T-1)+4$$

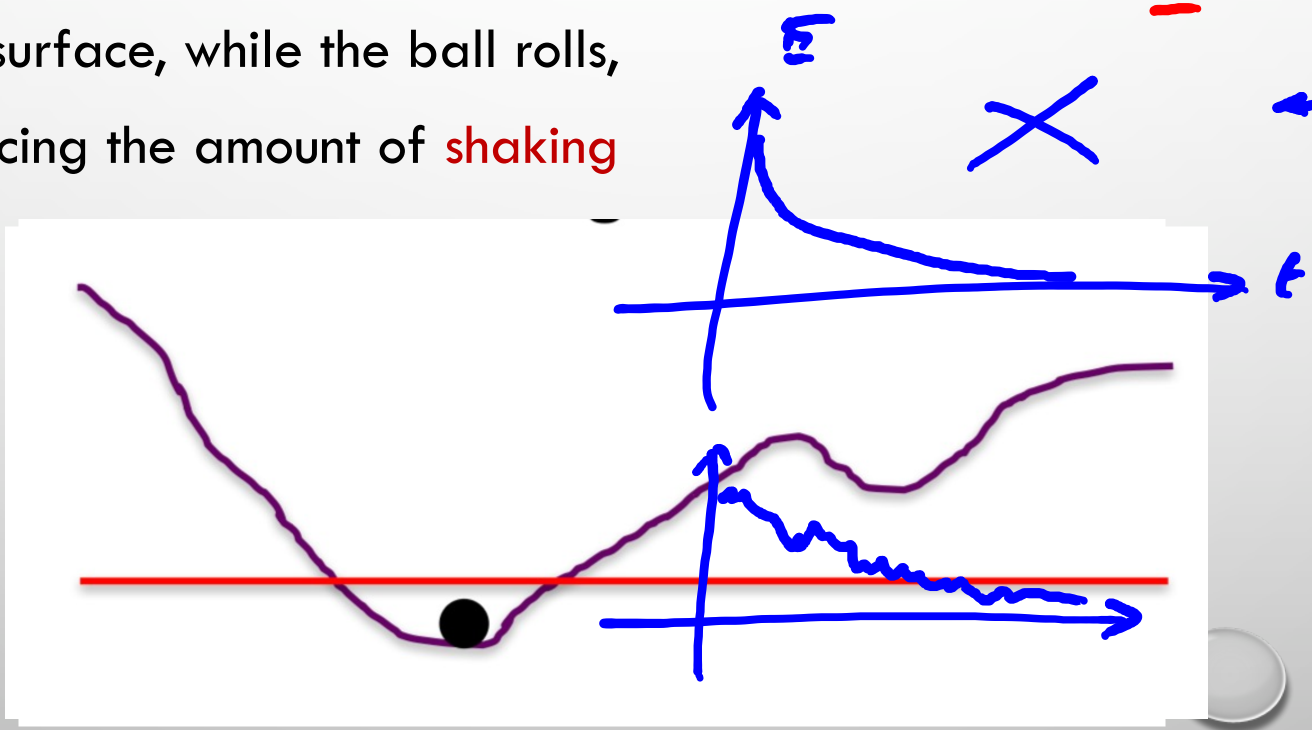
$$\mathbb{E}\{S\} = \mathbb{E}\{S_1 + S_2 + \dots + S_T\} = \mathbb{E}_T \underbrace{\mathbb{E}\{S_1+\dots+S_T|T\}}_{g(T)}$$

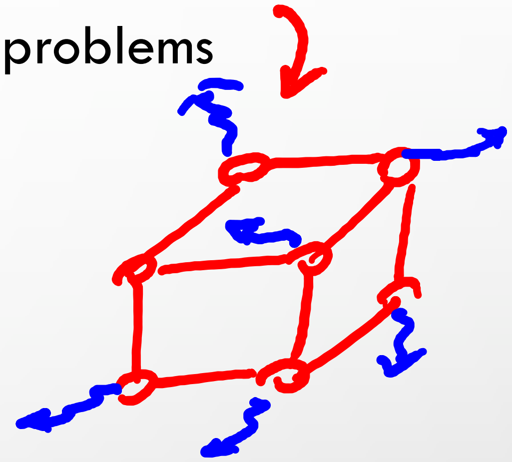
Hill-Climbing with Both Random Walk & Random Sampling

- At each step do one of the three
  - **Greedy**: move to the neighbor with largest value P_1
  - **Random Walk**: move to a random neighbor P_2
  - **Random Restart**: Start over from a new, random state $1-P_1-P_2$

Simulated Annealing

- Idea: escape local maxima by allowing some “bad” moves
  - **but gradually decrease their size and frequency**
  - method proposed in 1983 by IBM researchers for solving VLSI layout problems
- A Physical Analogy:
  - Imagine letting a ball roll downhill on the function surface
  - Now shake the surface, while the ball rolls,
  - Gradually reducing the amount of shaking





Simulated Annealing (cont.)

- Annealing = physical process of cooling a liquid → frozen
- simulated annealing:
  - free variables are like particles
  - seek “low energy” (high quality) configuration
  - slowly reducing temp. T with particles moving around randomly
- high T: probability of “locally bad” move is higher
- low T: probability of “locally bad” move is lower
- typically, T is decreased as the algorithm runs longer
  - i.e., there is a “temperature schedule”

Simulated Annealing (cont.)

function SIMULATED-ANNEALING(*problem, schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

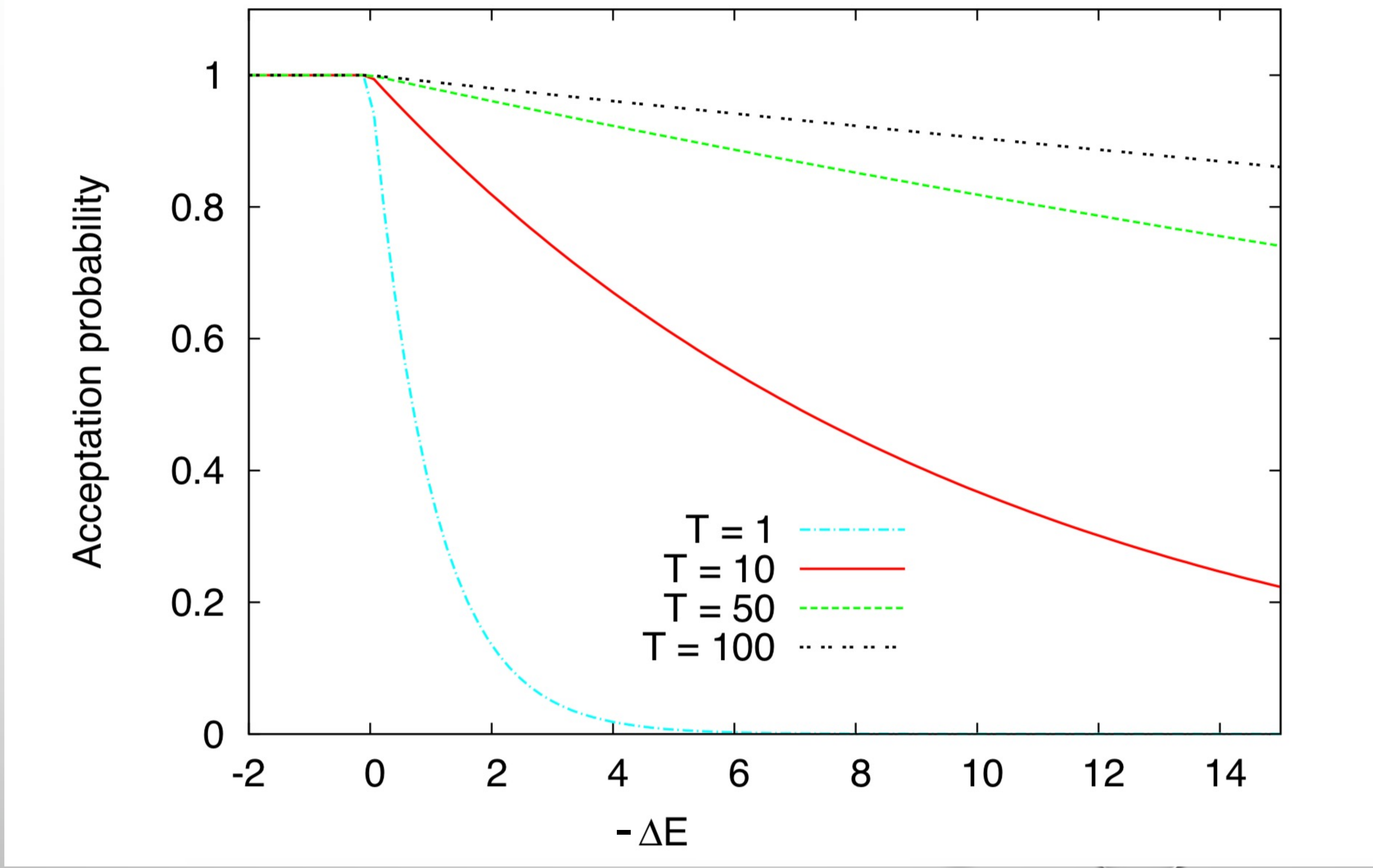
State

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    Temperature → $T \leftarrow$ *schedule*[*t*] e.g. $\frac{c}{\log t}$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*] → max Value
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$ $= \begin{cases} \approx 1 & T = \infty \\ \approx 0 & T \to 0 \end{cases}$

50%

a possible next move

Effect of temperature

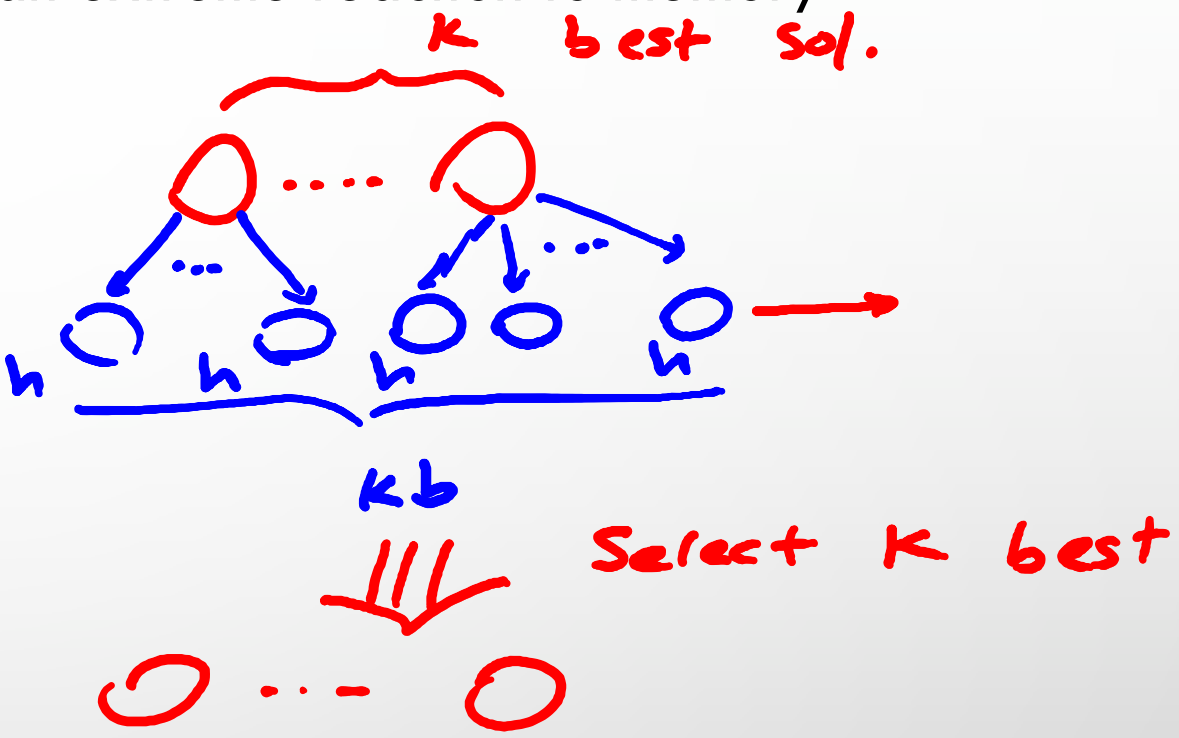


Simulated Annealing in practice

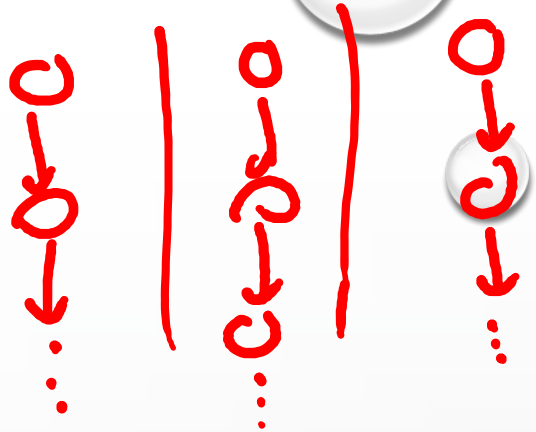
- Other applications:
  - Traveling salesman, Graph partitioning, Graph coloring, Scheduling, Facility Layout, Image Processing, …
- Optimal, given that T is decreased **sufficiently slow**.
  - Is this a useful guarantee?
- Convergence can be guaranteed if at each step, T drops no more quickly than C/log n, C=constant, n = # of steps so far.

Local beam search

- Idea: Keeping only one node in memory is an extreme reaction to memory problems.
- Keep track of *k* states instead of one
  - Initially: *k* randomly selected states
  - Next: determine all successors of *k* states
  - If any of successors is goal → finished
  - Else select *k* best from successors and repeat



Local Beam Search

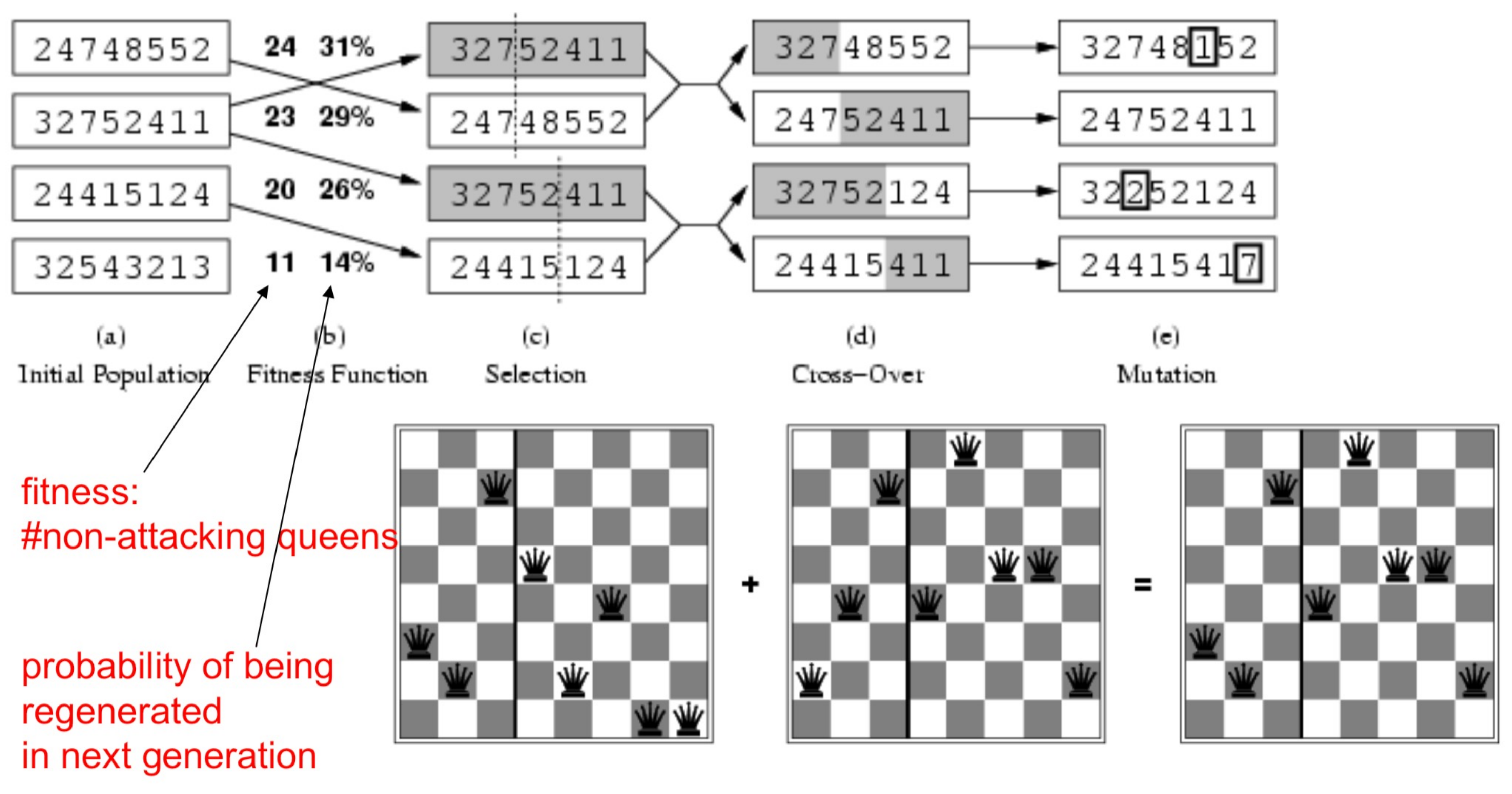


- Not the same as *k random-start searches run in parallel!*
  - Searches that find good states recruit other searches to join them
- Problem: quite often, all *k states end up on same local hill*
- Idea: Stochastic beam search
  - Choose *k successors randomly, biased towards good ones*
- Observe the close analogy to natural selection!

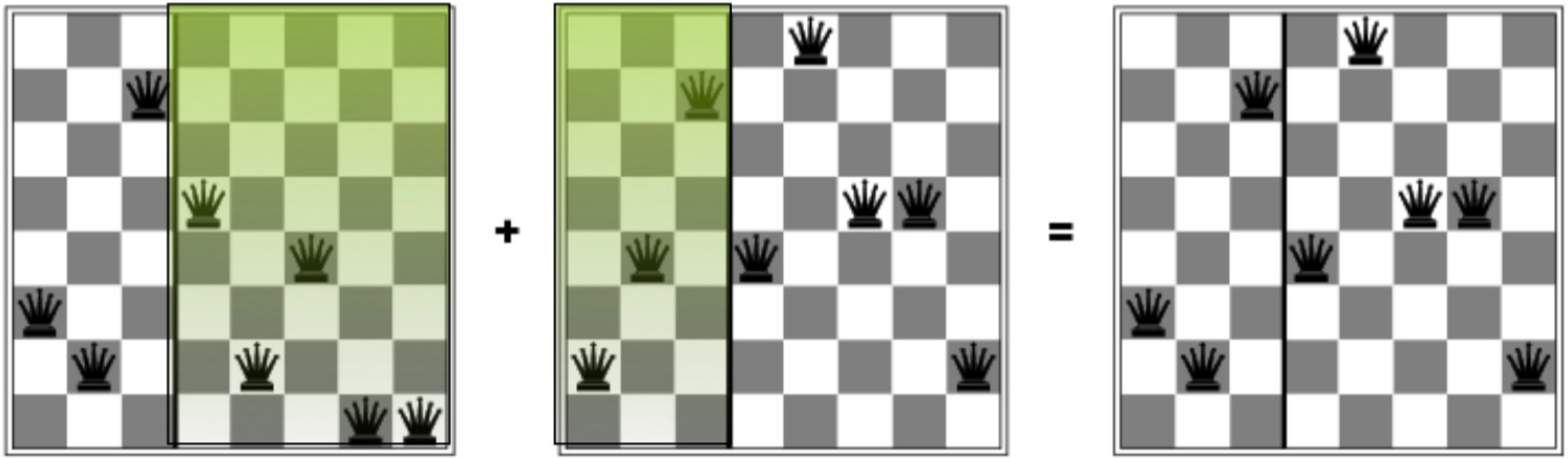
Genetic algorithms

- Local beam search, but…
  - A successor state is generated by ***combining two parent states***
- Start with *k* randomly generated states (population)
- A state is represented as a ***string*** over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (fitness function). Higher = better
- Produce the next generation of states by selection, crossover, and mutation

n-queens example



n-queens example (cont.)



Has the effect of “jumping” to a completely different new part of the search space (quite non-local)

Comments on Genetic Algorithms

- Genetic algorithm is a variant of “stochastic beam search”
- Positive points
  - Random exploration can find solutions that local search can’t
    - (via crossover primarily)
  - Appealing connection to human evolution
    - “neural” networks, and “genetic” algorithms are **metaphors**!
- Negative points
  - Large number of “tunable” parameters
    - Difficult to replicate performance from one problem to another
  - Lack of good empirical studies comparing to simpler methods
  - Useful on some (small?) set of problems but no convincing evidence that GAs are better than hill-climbing w/random restarts in general