BOX 1 (5.5pts per item X 2 items=11pts)

1. If there are 3 students in a classroom, the probability of 2 or more people to have the same birthday is _____0.008204166_____

$$\binom{3}{2}\left(\frac{365}{365}\right)\left(\frac{1}{365}\right)\left(\frac{364}{365}\right)+\binom{3}{3}\left(\frac{365}{365}\right)\left(\frac{1}{365}\right)\left(\frac{1}{365}\right)=0.008204166$$

$$or\ 1-\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)=0.008204166$$

2. If there are 23 students in a classroom, the probability of 2 or more people to have the same birthday is _____0.50729723_____

$$1-\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)\left(\frac{362}{365}\right)\ldots\left(\frac{365-22}{365}\right)=0.5072972$$

Can use SAS output to confirm

Same Birthday Probability

Number in a Room	Probability
2	0.00273972602740
3	0.00820416588478
4	0.01635591246655
5	0.02713557369979
6	0.04046248364911
7	0.05623570309598
8	0.07433529235167
9	0.09462383388917
10	0.11694817771108
11	0.14114137832173
12	0.16702478883806
13	0.19441027523243
14	0.22310251200497
15	0.25290131976369
16	0.28360400525285
17	0.31500766529656
18	0.34691141787179
19	0.37911852603154
20	0.41143838358058
21	0.44368833516521
22	0.47569530766255
23	0.50729723432399

BOX 2 (5.5pts per item X 5 items=27.5 pts)

1. If we roll **a** fair die, the probability of having **5** on the top is __1/6_____
2. If we roll **two** fair dice, the probability of the sum to be **11** is ___0.05556___
   P(5)*P(6)+P(6)*P(5)=0.05556

   Can use SAS output to confirm

Rolling 2 dice 100000000 times

Value Rolled	Frequency	Probability
2	2,778,719	0.027787
3	5,556,249	0.055562
4	8,337,108	0.083371
5	11,113,274	0.111133
6	13,887,799	0.138878
7	16,668,423	0.166684
8	13,890,936	0.138909
9	11,104,051	0.111041
10	8,332,303	0.083323
11	5,554,599	0.055546
12	2,776,539	0.027765
.	100,000,000	1.000000

3. If we roll **five** fair dice, the probability of the sum to be **5** is ___0.000129___
   P(1)*P(1)*P(1)*P(1)*P(1)=(1/6)^5=0.000129

   Can use SAS output to confirm

Rolling 5 dice 100000000 times

Value Rolled	Frequency	Probability
5	12,741	0.000127
6	64,241	0.000642
7	193,054	0.001931
8	449,997	0.004500
9	900,712	0.009007

4. If we roll **five** fair dice, the probability of the sum to be **6 or less** is __0.00077____

$$P(sum \leq 6) = P(sum = 5) + P(sum = 6) = \left(\frac{1}{6}\right)^5 + \binom{5}{1}\left(\frac{1}{6}\right)^5$$
$$= 0.0001286 + 0.000643 = 0.00077$$

   Can use SAS output to confirm

Rolling 5 dice 100000000 times

Value Rolled	Frequency	Probability
5	12,741	0.000127
6	64,241	0.000642
7	193,054	0.001931
8	449,997	0.004500
9	900,712	0.009007

5. If we roll **two** fair dice, the probability of the sum to be **11 or more** is __0.0833__

$$P(sum \geq 11) = P(sum = 11) + P(sum = 12) = \binom{2}{1} P(5)P(6) + P(6)P(6)$$
$$= 0.05556 + 0.02778 = 0.0833$$

Can use SAS output to confirm

Rolling 2 dice 100000000 times

Value Rolled	Frequency	Probability
2	2,778,719	0.027787
3	5,556,249	0.055562
4	8,337,108	0.083371
5	11,113,274	0.111133
6	13,887,799	0.138878
7	16,668,423	0.166684
8	13,890,936	0.138909
9	11,104,051	0.111041
10	8,332,303	0.083323
11	5,554,599	0.055546
12	2,776,539	0.027765
.	100,000,000	1.000000

BOX 3 (5.5pts per item X 3 items=16.5pts)

1. If we flip a fair coin **four** times, the probability of getting **four** heads is_0.06252_____

$$X \sim Bin(4, 0.5), so\ P(four\ heads) = P(X = 4) = \binom{4}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} = 0.0625$$

Can use SAS output to confirm

Tossing 4 coins 100000000 times

Number of Heads	Frequency	Probability
0	6,253,719	0.062537
1	24,999,318	0.249993
2	37,495,820	0.374958
3	24,999,173	0.249992
4	6,251,970	0.062520
.	100,000,000	1.000000

2. If we flip a coin **10** times, the probability of getting all **tails** is __0.000977____

$$X \sim Bin(10, 0.5), so\ P(no\ heads) = P(X = 0) = \binom{10}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10-0} = 0.0009765625$$

Can use SAS output to confirm

Tossing 10 coins 100000000 times

Number of Heads	Frequency	Probability
0	97,734	0.000977
1	974,813	0.009748
2	4,396,051	0.043961
3	11,717,358	0.117174
4	20,510,226	0.205102
5	24,608,221	0.246082
6	20,509,436	0.205094
7	11,716,203	0.117162
8	4,396,416	0.043964
9	975,537	0.009755
10	98,005	0.000980
.	100,000,000	1.000000

3. If we flip a fair coin **8** times, the probability of getting **four** heads and **four** tails is___0.27347_____

$$X \sim Bin(8, 0.5), so\ P(four\ heads) = P(X = 4) = \binom{8}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{8-4} = 0.2734375$$

Can use SAS output to confirm

Tossing 8 coins 100000000 times

Number of Heads	Frequency	Probability
0	389,688	0.003897
1	3,125,314	0.031253
2	10,941,086	0.109411
3	21,870,994	0.218710
4	27,346,984	0.273470
5	21,879,459	0.218795
6	10,934,420	0.109344
7	3,121,370	0.031214
8	390,685	0.003907
.	100,000,000	1.000000

Survey question (no credit): I spent on average _____ hours to complete Lab 3.