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REVIEW PAPER

A Review of Particle Swarm Optimization

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Abstract This paper presents an overview of the research progress in Particle Swarm Optimization (PSO) during 1995–2017. Fifty two papers have been reviewed. They have been categorized into nine categories based on various aspects. This technique has attracted many researchers because of its simplicity which led to many improvements and modifications of the basic PSO. Some researchers carried out the hybridization of PSO with other evolutionary techniques. This paper discusses the progress of PSO, its improvements, modifications and applications.

Keywords Iteration · Optimization · Particles · PSO

Introduction

Particle Swarm Optimization (PSO) was introduced in 1995 for solving non-linear functions [1]. It is simpler than other Evolutionary techniques as it requires only the specification of the problem and a few parameters to solve it. For this reason and to identify the untouched areas of research about PSO, this paper has been compiled. This paper presents an overview of research work concerning PSO. The research papers concerning PSO have been considered from 1995 to 2017.

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The papers have been categorized depending upon the aspects of PSO explored as follows:

1. Review Papers
2. Acceleration Coefficients (C_p, C_g) and IW
3. Stopping Criterion
4. Applications of PSO
5. Comparison of various types of PSO
6. Hybridization of PSO with other Optimization techniques
7. Multiobjective Optimization using PSO
8. Modified PSO
9. Velocity Upgradation

PSO Algorithm

PSO is a population based self-adaptive, stochastic optimization technique. The PSO begins by creating the initial particles, and assigning them initial velocities. It evaluates the objective function at each particle location, and determines the best function value and the best location. It chooses new velocities, based on the current velocity, the particles' individual best locations, and the best locations of their neighbors. It then iteratively updates the particle locations (the new location is the old one plus the velocity, modified to keep particles within bounds), velocities, and neighbors. Iterations proceed until the algorithm reaches a stopping criterion.

In an n-dimensional search space, position and velocity of particle j is represented by vectors $X_j = (X_{j1}, X_{j2},\ldots X_{jn})$ and $V_j = (V_{j1}, V_{j2},\ldots V_{jn})$ respectively. Let X_{pbest} and X_{gbest} be the personal best position of particle j and global best position of group. The modified velocity and position of each particle can be calculated using current velocity and distance from X_{pbest} and X_{gbest} as follows:

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$$V_{ij}^{k+1} = V_{ij}^{k} + C_p r_p \left(X_{pbestij}^{k} - X_{ij}^{k}\right) + C_g r_g \left(X_{gbesti}^{k} - X_{ij}^{k}\right)$$
$$i = 1, 2 \ldots N \quad j = 1, 2 \ldots p \qquad (1)$$

where,

k	Iteration Count.
V_{ij}^{k+1}	Velocity of j^{th} particle of i^{th} variable at $k+1^{th}$ iteration.
X_{ij}^{k}	Value of position of j^{th} particle of i^{th} variable at k^{th} iteration.
C_p, C_g	Cognitive and social acceleration coefficients.
N	Total number of variables.
$X_{pbestij}^{k}$	Personal best position of jth particle of i^{th} variable at k^{th} iteration.
X_{gbesti}^{k}	Global best value of i^{th} variable until k^{th} iteration.
r_p, r_g	Separately generated uniformly distributed random numbers.

Velocities are updated by Eq. (1) and position of each particle is updated by Eq. (2)

$$X_{ij}^{k+1} = X_{ij}^{k} + V_{ij}^{k+1} \quad i = 1, 2 \ldots N \quad j = 1, 2, \ldots p \qquad (2)$$

To increase the convergence rate of PSO, the inertia weight is proposed in the velocity Eq. (1). By using inertia weight, the suggested particle velocity will be changed to

$$V_{ij}^{k+1} = W * V_{ij}^{k} + C_p r_p \left(X_{pbestij}^{k} - X_{ij}^{k}\right) + C_g r_g \left(X_{gbesti}^{k} - X_{ij}^{k}\right) \qquad (3)$$
$$i = 1, 2 \ldots N \quad j = 1, 2 \ldots p$$

where, W is the Inertia Weight.

The algorithm is shown in flow chart as shown in Fig. 1.

The papers under various categories are discussed below:

1. Review Papers

The concept of PSO was implemented on non-linear benchmark function. The relationship of PSO was presented with A-life (Artifical life) as well as Genetic Algorithm.

Four review papers have been published so far. Parsopoulas and Vrahatis [2] published a review paper covering research papers up to 2002, discussing ability of PSO for multiobjective, mini-max, integer programming etc. Kameyama [3] have covered papers from 1995 to 2008, and discuss the progress of PSO, modifications in the basic PSO for improving exploitation and exploration.

Floudas and Gounaris [4] presented progress in global optimization during 1998–2008. The area of twice continually differentiable non-linear optimization, optimization with differentiable algebraic models, semi- infinite programming, optimization with grey box/nonfactorable models and bi-level nonlinear optimization were covered. Zhang et al. [5] provided advances with PSO, including its modifications, population topology, hybridization, extensions (to multiobjective, constrained, discrete, and binary optimization), theoretical analysis and parallel implementation mainly during 2010–2014.

2. Acceleration Coefficients (C_p, C_g) and IW

You et al. [6] proposed an adaptive weight PSO with constriction factor to overcome the problem of premature convergence. The value of inertia weight is set according to changes in the value of objective function.

Bansal et al. [7] studied 15 popular inertia weight strategies. They found Chaotic IW to be the best for better accuracy and random IW for efficiency.

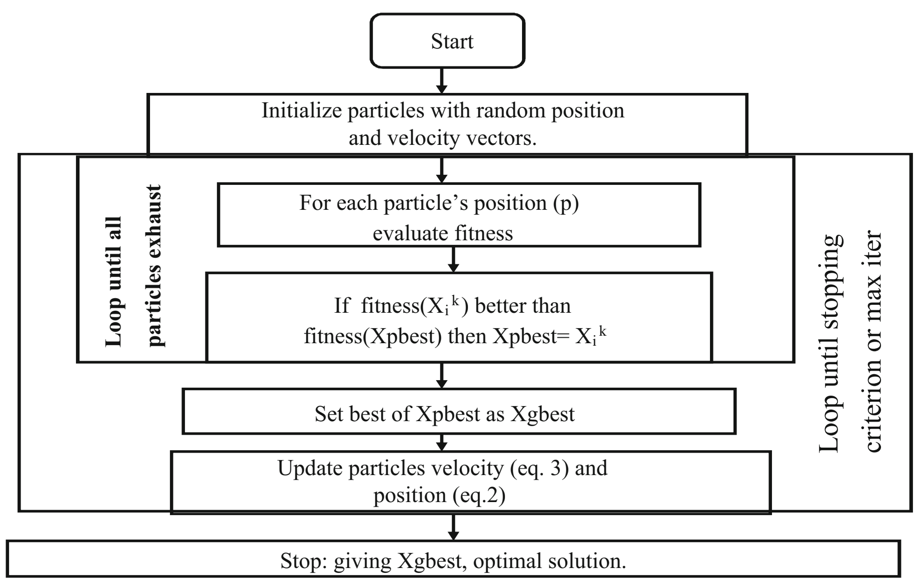

Fig. 1 Flow chart of PSO algorithm

Bonyadi and Michalewicz [8] investigated the movement patterns of a particle in the PSO algorithm. The Movement patterns were characterized on the basis of correlation between positions generated by particles and its range of movement.

3. Stopping Criterion:

Zielinski and Laum [9] evaluated various stopping criteria on the basis of movements made by the particles, improvements based criteria and distribution based criterion for a constrained single—objective PSO algorithm.

4. Applications of PSO:

A number of papers have been published on applications of PSO.

Engineering Applications

Wu et al. [10]. suggested various application of PSO in the field of Railway for scheduling, active controls, and network layout planning. In scheduling applications, PSO was used to find the optimal schedules. Al Rashidi and El-Hawary [11] suggested the application of PSO in solving optimization problems in the area of electric power systems. Jain et al. [12] applied PSO to multiobjective economic load dispatch problem considering various objectives. PSO has also been implemented for Optimal power flow [13, 14], load flow [15, 16], and optimum design of PID controller in AVR system [17], power loss minimization [18] etc. Nimtawat and Nanakom [19], suggested a PSO algorithm for beam-slab layout design of rectangular floor. Mac et al. [20] proposed a novel hierarchical global path planning method for mobile robots in cluttered environments using PSO.

Islam et al. [21] suggested a time-varying transfer function in the BPSO, namely TVT-BPSO, for systematic analysis of its exploration and exploitation capability. Suresha et al. [22] implemented PSO to predict the length of stay of patient in hospital and found PSO to be better than BP for prediction.

Other Applications

PSO has been implemented for various applications. Some of these are Source Seeking Problem [23] Elevator Door System [24], Quad Assignment Problem [25], Equipment Possession Quantity [26], Job Shop Scheduling Problem [27] etc.

5. Comparison of various type of PSO

Bhushan and Pillai [28] compared PSO and firefly algorithm (FFA). Ten standard nonlinear functions were chosen. Elapsed time and mean value of the function were evaluated for PSO and FFA. The mean value of function and elapsed time were found to be much smaller than FFA.

Angleline [29] attempted particle selection with PSO on four test functions and claimed that his method offered same improvement in performances in respect of some functions but not all functions.

6. Hybridization of PSO with other optimization techniques

Many researchers have attempt to Hybridize PSO with various other techniques and claimed improvements from performance point of view.

Chen and Peng [30] proposed a new hybrid methodology called PSO with recombination and dynamic linkage discovery (PSO-RDL) and implemented on four benchmark functions and a real world power system problem of Economic Dispatch. Authors found its performance to be comparable to that of classical EP, Fast EP(FEP), modified EP, improved FEP, as well as modified PSO(MPSO). Jiao et al. [31] have developed Elite particles swarm optimization with mutation to avoid local convergence. In this method bad particles were replaced by elite particles and to avoid local convergence mutation has been used.

Other hybrid algorithms reported are hybridized simplex method with PSO [32], combined mutative scale chaos [33] and mutation in PSO [34].

7. Multiobjective optimization using PSO

Multiobjective approach to optimization has been attempted through PSO using Discrete multiobjective PSO [35], a competitive co-operative and co-evolutionary approach [36], and vector evaluated PSO [37]. In all these papers the result have been discussed qualitatively.

8. Modified PSO

Many modifications have followed the original algorithm, like Memory Enhanced PSO [38], Predator–Prey PSO [39] There are newer variants like: CLPSO [40], SLPSO [41] and OLPSO [42] which do not require parameter tuning. These algorithms, unlike previously mentioned variants, have a structure entirely different from the PSO. They make use of the current information and usually do not include the previous iterations information.

Schutte and Groenwold [43] studied the variants of PSO algorithms and applied to Dixan-Szego test set. Liu and Wang [44] introduced Evolutionary game (EGPSO) in which the behavior of particles are modelled using Replicator dynamics and multi-start technique.

Hossen et al. [45] also tried an adaptive PSO based on behavior of spider.

Benmessabel and Touahria [46] illustrated the effect of excluding the redundant particle from current iteration. Jii and Wangi [47] combined PSO with gradient method, which avoids immature convergence.

Beheshti et al. [48] proposed binary accelerated PSO using two parameters that is no. of generations and population size. Rios and Sahinidis [49] presented a review of derivative free algorithms including PSO for constrained problems. Chen et al. [50] presented an organizational adjustment PSO based particle filter (OAPSO-PF) algorithm which allowed the particles to adopt to environment and reach the global optimum.

9. Velocity Upgradation:

Arasomwan and Adewumi [51] tried a PSO which upgrades the velocities based on Euclidean distance between particles. Baiquan et al. [52] have suggested a control system based strategy for speed and position formulas of PSO, which resulted in improvement in speed of convergence as well as premature convergence.

Conclusions

PSO has attracted many researchers from mathematical point of view. But, from application point of view, only a few researchers have developed their efforts. So there is a lot of scope of research work from application point of view.

The various aspects of PSO which need further exploration are the following:

1 Recombination and linkages discovery in PSO.
2 Elite particles, mutation and engineered crossover have potential to add to the efficiency and reliability of PSO.
3 Investigations on multistart techniques to avoid premature terminations.
4 For repetitive problems as is the case in practical field, PSO parameters need be tuned keeping in view the constraints on independent variables of the problems.
5 In multiobjective optimization we need various solutions (noninferior solutions) rather than one (optimum) solution. In other techniques dealing with multiobjective optimization, the problem is to be solved many times in order to achieve noninferior set. However, PSO starts with a number of points and, therefore, may be capable of identifying noninferior set in a single run. This aspect is under investigation of authors.

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